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STRUCTURE OF MERGER REMNANTS. III. PHASE-SPACE CONSTRAINTS

LARS HERNQUIST[1]
Board of Studies in Astronomy and Astrophysics, University of California at Santa Cruz, Santa Cruz, CA 95064

AND

DAVID N. SPERGEL[2] AND JEREMY S. HEYL[3]
Princeton University Observatory, Princeton, NJ 08544
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ABSTRACT

This paper explores the evolution of the coarsely grained phase-space density in mergers and in galaxy formation. In particular, numerical simulations are used to determine the properties of remnants produced by "major" mergers between equal-mass galaxies. Contrary to some existing claims, remnants of mergers between stellar disks are found to lack sufficient material at high phase-space densities to be identified as elliptical galaxies. We quantify this effect by computing the cumulative coarsely grained phase-space distribution, $s(\bar{f})$, for the remnants and compare it to that derived from simple models of the mass profiles of ellipticals. In so doing, we estimate that the discrepancy is confined to the inner $\sim 15\%$ of the stellar mass. In principle, this problem can be circumvented by dissipation in gas and star formation, but this process by itself probably requires that the progenitors comprise a gas fraction $\sim 25\%$–30% of their luminous mass. More directly, as shown by additional simulation, the phase-space discrepancy can be reconciled by including compact bulges in the progenitors having $\sim 20\%$–25% the mass of the disks.

We further speculate on the relevance of our analyses to more general situations where the progenitor galaxies have very different masses or to remnants produced from repeated mergers in a dense galactic environment. A number of observational signatures are noted which may help to establish the importance of merging to the structure and origin of early-type galaxies. In addition, we apply the methods developed here to the sample of hot stellar systems cataloged recently by Bender et al. A strong correlation is found between the luminosity of these objects and the "effective" coarsely grained phase density ($\bar{f}_{\rm eff} \propto 1/\sigma r_{\rm eff}^2$). Implications of these findings for the interpretation of the fundamental plane of elliptical galaxies are discussed.

Subject headings: cosmology: theory — galaxies: interactions — methods: numerical

1. INTRODUCTION

In many theoretical scenarios of galaxy formation, large stellar systems are built up from mergers of less massive components. Motivated by observations indicating that remnants of "major" mergers of comparable-mass spirals relax to objects with structural properties similar to elliptical galaxies, Toomre & Toomre (1972) and later Toomre (1977) put forward the "merger hypothesis": that mostly stellar disks are the building blocks from which more massive early-types are produced. At roughly the same time, Ostriker & Tremaine (1975) suggested that dynamical friction and repeated merging near the centers of clusters might be responsible for the massive cD galaxies often seen there. Later, through increasingly sophisticated numerical simulation, Barnes (1985, 1989) demonstrated that compact groups may be subject to a "merging instability," effectively destroying the group on a time scale short compared to the age of the universe and yielding remnants which resemble elliptical galaxies. As argued by White (1990) and Efstathiou (1990), such an evolutionary history is a natural consequence of the turnaround of larger and larger mass scales with time in a hierarchical universe. These notions have been extended through to subgalactic scales by, e.g., Dubinski & Carlberg (1991), Katz (1991), and Katz & Gunn (1991) who show that the collapse of individual protogalactic density perturbations is not smooth, as in the Eggen, Lynden-Bell, & Sandage (1962) picture of galaxy formation, but is often fragmented and clumpy, as envisaged by Searle & Zinn (1978), implying that "minor" mergers may play a role in structuring even late-type galaxies (Toth & Ostriker 1992; Quinn, Hernquist, & Fullager 1993).

Observations and modeling suggest that telltale signs of galaxy interactions, including various types of "fine structure" and kinematic anomalies, can persist for extended periods of time, perhaps making it possible to identify merger remnants long after they are born (e.g., Schweizer et al. 1990). If hot stellar systems, in particular, are generally formed from the agglomeration of less massive objects, it may be possible to infer the nature of the progenitors by a systematic "archeology" of early-type galaxies. For example, the "starpiles" of major mergers like those imagined by Toomre (1977) may differ subtly from the remains of former compact groups or from spheroids produced by fragmented collapse in an expanding universe. Identifying traits unique to remnants produced by a given scenario may, therefore, help to establish the relative import of ancient and present-day mergers to galaxy formation.

In this paper, we examine one aspect of this problem by studying to what extent the phase-space structure of remnants can be used to constrain the nature of their progenitors. The results here extend and expand on ones already presented by Hernquist (1992, 1993a), who considered some aspects of the structural and kinematic properties of merger remnants but did not analyze their phase-space distribution. For simplicity,

[1] Alfred P. Sloan Foundation Fellow, Presidential Faculty Fellow.
[2] Alfred P. Sloan Foundation Fellow, Presidential Young Investigator.
[3] Current address: University of Durham, Durham, DH1 3LE, England.

we restrict the first phase of our investigation to mergers between identical galaxies consisting of dark halos, luminous disks, and, optionally, compact luminous bulges. In addition, we speculate on the relevance of our results for more general situations involving more than two progenitors having a spectrum of masses, but defer a detailed numerical study of these considerations for future works.

Many of the concepts presented and discussed here are not new and have been put forward by many authors at various times in the past. In particular, Carlberg (1986) was among the first to note that the high maximum phase-space densities in elliptical galaxies could not be attained by mergers of stellar disks, owing to Liouville's theorem. This sentiment was echoed by Gunn (1987), who argued that ". . . [it does not seem plausible] you can make rocks by merging clouds." Nevertheless, the significance of Carlberg's findings was disputed by Lake (1989), who suggested that the central phase density in a remnant might be boosted by infall of outlying material during a merger; he also proposed that Carlberg's assertion would be restricted to a sufficiently small fraction of the remnant mass as to be insignificant observationally. To date, none of these various ideas has been tested with detailed simulations comprising realistic models for the progenitor galaxies.

Here we consider these issues by quantifying the evolution of the coarsely grained distribution function in major mergers and by computing the phase-space structure of the remnants and comparing them to simple models of elliptical galaxies. In § 2, we summarize the nature of phase-space constraints in stellar-dynamical systems and develop related tools which can be applied to numerical simulations and observations of the global structure of hot stellar systems. In addition, the simulation techniques and galaxy models are discussed briefly. Results of the analysis applied to mergers of identical disk-halo and disk-bulge-halo galaxies are presented in § 3. Implications and extensions of the findings are described in § 4, along with possible applications of our methods to observed properties of elliptical galaxies. Finally, we speculate on the overall significance of the results in § 5 and suggest other tests which might be used to infer the relevance of mergers to the origin and observed properties of present-day galaxies.

2. METHODS

2.1. *Phase-Space Diagnostics*

In a stellar system, the finely grained phase-space density, $f(\boldsymbol{x}, \boldsymbol{v})$, measures the probability of finding a star in the differential phase-space volume $d\boldsymbol{x}\, d\boldsymbol{v}$ centered on the point $(\boldsymbol{x}, \boldsymbol{v})$, when multiplied by $d\boldsymbol{x}\, d\boldsymbol{v}$. In the limit that individual stellar encounters are negligible, $f(\boldsymbol{x}, \boldsymbol{v})$ evolves according to the collisionless Boltzmann or Vlasov equation, which can be written

$$\frac{Df}{Dt} = 0 . \qquad (2.1)$$

Physically, this is simply a continuity equation and implies that the phase-space density around the phase-space point of any star is always preserved (e.g., Binney & Tremaine 1987). In particular, if a stellar-dynamical system initially has a maximum phase-space density, $f_{\max}$, and undergoes collisionless evolution to a new state, then the maximum phase-space density of the final system cannot exceed $f_{\max}$. This consideration motivated Carlberg (1986) to suggest that ellipticals cannot be made from mergers of *stellar* disks since $f_{\max}$ is larger in ellipticals than exponential disks. Indeed, numerical simulations have verified this argument but systematic attempts have not been made to quantify the amount of "dense" material that would be required to eliminate this problem. This is an important issue since, as noted by Lake (1989), if the mass in question is negligible, then the implied discrepancy between $f_{\max}$ in remnants of mergers between stellar disks and actual ellipticals might not be observable, in practice.

When dealing with numerical simulations and interpretations of observational data, $f(\boldsymbol{x}, \boldsymbol{v})$ usually cannot be computed reliably since six-dimensional phase space is not sampled smoothly owing to limited resolution. In these situations, the nature of phase space can, however, be explored quantitatively with the *coarsely* grained distribution function, $\bar{f}(\boldsymbol{x}, \boldsymbol{v})$, defined so that $\bar{f}(\boldsymbol{x}, \boldsymbol{v})\Delta \boldsymbol{x}\, \Delta \boldsymbol{v}$ is the probability of finding a star in the *macroscopic* volume $\Delta \boldsymbol{x}\, \Delta \boldsymbol{v}$ centered on the point $(\boldsymbol{x}, \boldsymbol{v})$. Even when numerical simulations are coarse-grained into relatively few δx and δv bins, it is still difficult to have enough particles to precisely estimate the phase-space density.

We have taken the approach of coarsely graining the distribution along energy hypersurfaces. This approach is motivated by noting that galaxy mergers effectively isotropize the velocity distribution. It has the technical advantage of being simple to calculate from numerical simulations. This approach differs from that of Lake (1989) who computed the coarse-grained density on a finer six-dimensional grid. Our approach leads to a much lower coarse-grained phase space density for spiral disks.

For example, in a spherical, isotropic system, the phase-space density is a function solely of the energy, E, and $f(E)dE$ is the probability of finding a particle in a phase-space volume around a point on the hypersurface given by the energy E. Correspondingly, the differential energy distribution, dN/dE, measures the number of stars in the energy range, E to $E + dE$ according to

$$\frac{dN}{dE} = f(E)g(E) , \qquad (2.2)$$

where $g(E)$ is the phase-space volume of a given energy hypersurface

$$g(E) = \int d^3r\, d^3v \delta[E - H(\boldsymbol{r}, \boldsymbol{v})]$$

$$= 16\pi^2 \int_0^{r_m(E)} r^2\, dr \sqrt{2[\Psi(r) - E]} . \qquad (2.3)$$

Here $\Psi(r)$ is the potential and $\Psi(r_m) \equiv E$. In the analysis below, we coarsely grain the phase-space density on energy hypersurfaces by computing dN/dE and then use equations (2.2)–(2.3) to measure $\bar{f}(E)$, the coarsely grained phase-space density averaged over these hypersurfaces.

For a *smooth* isotropic system, the coarsely grained distribution function calculated in this manner is identical to the finely grained distribution function, $f(E)$. As an illustration, consider the density-potential pair defined by

$$\rho(r) = \frac{M}{2\pi} \frac{a}{r} \frac{1}{(r + a)^3} , \qquad (2.4)$$

$$\varphi(r) = -\frac{GM}{r + a} , \qquad (2.5)$$

where M is the mass and a is a scale length. As shown by Hernquist (1990a), this model provides an excellent fit to the light distribution in elliptical galaxies and, moreover, is completely analytic. In particular, the corresponding distribution function is

$$f(E) = \frac{M}{8\sqrt{2}\pi^3 a v_g^3} \frac{1}{(1-q^2)^{5/2}} \times [3 \sin^{-1} q + q(1-q^2)^{1/2}(1-2q^2)(8q^4-8q^2-3)] , \quad (2.6)$$

where

$$q = \sqrt{-\frac{a}{GM} E} , \quad (2.7)$$

and

$$v_g = \left(\frac{GM}{a}\right)^{1/2} . \quad (2.8)$$

Here $f(E)$ is normalized so that $f\,d\boldsymbol{x}\,d\boldsymbol{v}$ is the *mass* in the six-dimensional volume element $d\boldsymbol{x}\,d\boldsymbol{v}$ centered on the phase-space point $(\boldsymbol{x}, \boldsymbol{v})$. The function $f(E)$ is shown as a solid line in Figure 1 for a model with unit mass and scale length in units where $G \equiv 1$, where it is compared with a coarsely grained estimate of $f(E)$, computed from an N-body realization with $N = 40{,}000$ particles, using the procedure described above. In the limit of large N, we expect $\bar{f}(E) \rightarrow f(E)$ exactly. Clearly, the agreement between $f(E)$ and $\bar{f}(E)$ is quite good in this, case and the scatter in the measured values of $\bar{f}(E)$ is consistent with Poisson fluctuations arising from sampling errors.

In an anisotropic system or one that is not completely phase-mixed, coarse-graining erases the small-scale irregularities that are represented in the finely grained distribution function. Consequently, $\bar{f}(E)$ does not satisfy the Vlasov equation. Nevertheless, as discussed in § 3 and 4 it is possible in some cases to predict roughly how $\bar{f}(E)$ will evolve, given some assumptions about the nature of the initial conditions (e.g., Hausman & Ostriker 1978).

As implied by Figure 1, stellar systems contain material at both very high and very low phase-space densities. To quantify

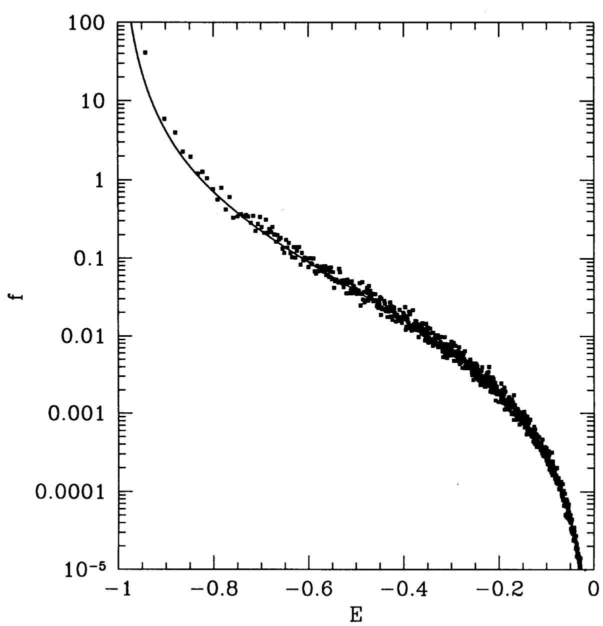

FIG. 1.—Finely grained distribution function, $f(E)$, for a Hernquist (1990) model with unit mass and unit scale length. Squares show the coarsely grained distribution function, $\bar{f}(E)$, computed from a numerical realization of the Hernquist model with 40,000 particles.

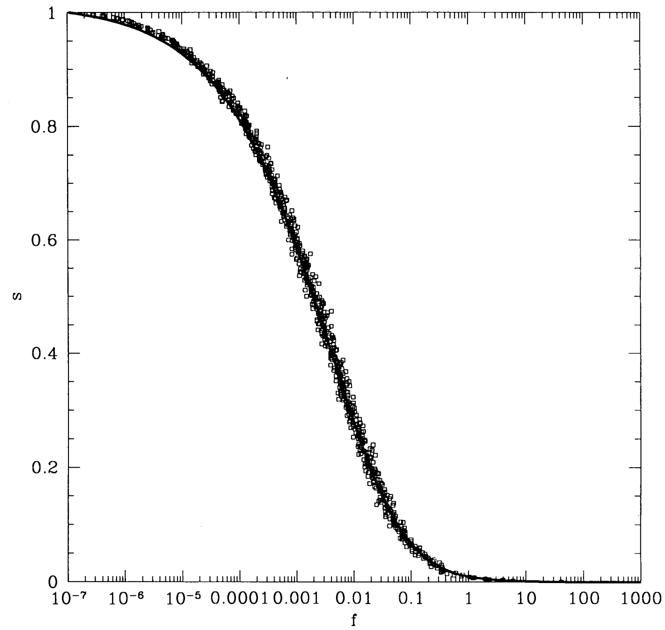

FIG. 2.—Cumulative phase-space distribution function, $s(f)$ for the profiles shown in Fig. 1. Solid line shows the result for the finely grained distribution function, while squares show the quantity $s(\bar{f})$ for the coarsely grained distribution function, computed from the numerical realization.

the amount of material at a given phase-space density, it is expedient to work with integral rather than differential quantities. We will coarse-grain the distribution function into energy bins and compute the cumulative distribution from the differential energy distribution:

$$s(\bar{f}) = \int_{E_0}^{E(\bar{f})} \frac{dN}{dE} dE , \quad (2.9)$$

where $E_0 = \psi(0)$ is the central binding energy and $E(f)$ is given by inverting the distribution function, $f(E)$. As always, when computed from, e.g., a simulation with limited resolution, the cumulative distribution refers to the coarsely grained phase-space density, $\bar{f}(E)$, and will be denoted by $s(\bar{f})$. For an isotropic distribution function, $s(\bar{f})$ should approach $s(f)$, the fine-grained distribution function, with increasing particle number. Figure 2 shows a comparison between $s(f)$ and $s(\bar{f})$ for the analytic model shown in Figure 1 and its numerical realization with $N = 40{,}000$ particles. As expected for the large N limit, $s(f)$ and $s(\bar{f})$ are quite close and the scatter is greatly reduced relative to that in Figure 1, owing to the fact that $s(f)$ is an integral quantity.

The notion of coarse-graining can be extended to produce global averages which provide insight into how the phase-density will evolve in certain cases and which can be employed to interpret observed properties of hot stellar systems. For simplicity, it is useful to introduce an "effective phase-space density," which is a measure of the mean phase-space density within the effective radius of a galaxy. Begin by defining $\bar{f}_{1/2}$ to be the average phase-space density within the spherical half-mass radius of the galaxy according to

$$\bar{f}_{1/2} \equiv \frac{M/4}{(4\pi r_{1/2}^3/3)(4\pi\sigma_{3d}^3/3)} , \quad (2.10)$$

where, by definition, half of the total mass, M, lies within the radius $r_{1/2}$ and it is assumed that half of the mass in this region has speeds less than σ_{3d}, the maximum three-dimensional velocity dispersion. This expression can be simplified by eliminating the mass according to the virial theorem $\sigma_{3d}^2 \approx 2GM/5r_{1/2}$ (e.g., Binney & Tremaine 1987), and by using the fact that $\sigma_{1d} = \sigma_{3d}/3^{1/2}$. Equation (2.11) then becomes

$$\bar{f}_{1/2} \simeq \frac{15\sqrt{3}}{128\pi^2} \frac{1}{G} \frac{1}{\sigma_{1d} r_{1/2}^2} . \quad (2.11)$$

For the model shown in Figure 2, $r_{1/2} = 1 + 2^{1/2}$ and $\sigma_{1d} \approx 0.3$, depending weakly on the velocity distribution (Hernquist 1990a), implying $\bar{f}_{1/2} = 0.012$. Figure 2 suggests that this is indeed a good measure of the median phase-space density within the half-mass radius.

In what follows, we will estimate the effective phase-space density from observational samples. In so doing, we will employ the central line-of-sight dispersion, σ_0, and the projected half-light radius, R_e, to define an effective phase-space density for the luminous matter:

$$f_{\rm eff} \equiv \frac{1}{\sigma_0 R_e^2} . \quad (2.12)$$

For an assumed mass model, $f_{\rm eff}$ can be related to $f_{1/2}$.

2.2. *Simulation Techniques*

As a detailed application of the various concepts introduced in § 2.1, we consider numerical simulations of mergers between identical galaxies constructed to resemble present-day spirals. The models ignore the effects of dissipation in gas, but include the various collisionless components thought to dominate the mass distribution of real galaxies. In the calculations, two identical galaxies with self-gravitating disks, halos, and optional compact bulges merge following close collisions from parabolic orbits.

The galaxy models are constructed according to the procedure described by Hernquist (1993b). Briefly, particles are distributed in space according to analytic density profiles chosen to mimic those which are thought appropriate for the various components of galaxies like the Milky Way (e.g., Bahcall & Soneira 1980; Caldwell & Ostriker 1981). The disks are assumed to follow an exponential distribution in cylindrical radius and are modeled by isothermal sheets perpendicular to the disk plane; that is, the vertical density is proportional to $\mathrm{sech}^2\, z/z_0$. Throughout, we adopt a system of units in which the radial scale length of the disks, h, the masses of the disks, $M_{\rm disk}$, and the gravitational constant are all unity. Scaled to the Milky Way, this implies that unit length, mass, time, and velocity are roughly 3.5 kpc, $5.6 \times 10^{10}\ M_\odot$, 1.3×10^7 yr, and 260 km s^{-1}, respectively. For simplicity, we use $z_0 = 0.2$ in all simulations reported here. Since the vertical scale height is constant across each disk, the Toomre Q-parameter (Toomre 1963) varies weakly with radius and is normalized so that its mean value $\langle Q \rangle \approx 1.5$.

Halos are represented by a truncated mass profile resembling an isothermal sphere with density structure $\rho \propto \exp(-r^2/r_t^2)/(1 + r^2/\gamma^2)$, with $\gamma = 1$ and $r_t = 10$. With these choices, the rotation curve of a disk plus halo galaxy is relatively flat for radii $r \lesssim 10$ (Hernquist 1992).

When added, bulges are represented by nonspherical versions of Hernquist (1990a) models (Dubinski & Carlberg 1991; Hernquist 1993a, b) with scale lengths $a = 0.2$ and $c = 0.1$. Rotation is included in some calculations by requiring all particles to circulate in the same sense around the symmetry axis, yielding ratios of circular to random velocity $v/\sigma \sim 1$ within the half-mass ellipsoids of the bulge (Hernquist 1993a).

The analysis described in § 3 was based on approximately 25 simulations of mergers between identical galaxies: the four bulgeless calculations presented by Hernquist (1992), the eight models reported by Hernquist (1993a) which include bulges with and without rotation, a subset of seven of these 12 with 4 times the particle number used in Hernquist (1992, 1993a), and about a half-dozen other simulations employing different orbital geometries from the other sets. The ratio of halo-to-disk-to-bulge mass in each galaxy is always $H{:}D{:}B = 5.8{:}1{:}M_b$, where M_b is either 0 or $\frac{1}{3}$, depending on whether or not the galaxies are bulgeless.

The simulations described and presented here were performed using a hierarchical tree algorithm (Barnes & Hut 1986), optimized for vectorizing supercomputers (Hernquist 1987, 1990b). Forces are computed with a tolerance parameter $\theta = 0.7$, including terms up to quadrupole order in the multipole expansions. Interparticle forces are softened with a cubic spline (e.g., Hernquist & Katz 1989; Goodman & Hernquist 1991) and different species of particles have their own softening lengths to mitigate discreteness effects. Particle coordinates are integrated with a leap-frog algorithm (e.g., Press et al. 1986), employing a sufficiently small time step to guarantee energy conservation to better than 1%.

Low-resolution simulations were performed with 16,384, 4096, and 16,384 particles in each disk, bulge, and halo, respectively. High-resolution models employed 4 times as many particles in each component. All calculations were done on the Cray Y-MP and Cray C-90 at the Pittsburgh Supercomputing Center.

3. MAJOR MERGERS

As is by now well established, close encounters of galaxies from parabolic orbits lead to complete merging in just a few dynamical times (e.g., van Albada & van Gorkom 1977; White 1978, 1979; Gerhard 1981; Farouki & Shapiro 1982; Barnes 1988, 1992), and the ensuing remnants have properties reminiscent of elliptical galaxies. Little is known in detail, however, about how the distribution function behaves in such events. When dissipation can be neglected, Liouville's theorem applies and the fine-grained phase-space density is exactly conserved. Owing to sampling errors we cannot determine reliably how the finely grained distribution function evolves in simulations like those reported here, but the structure of phase space can still be quantified by appealing to coarse-graining.

From simple energetic considerations, it is possible to predict a priori how quantities like the effective phase density introduced in § 2.1 will evolve in parabolic mergers between identical *spherical* galaxies. Prior to the encounter, the rms three-dimensional velocity dispersion, σ, the mass, M, the gravitational radius, R, and the total energy of each galaxy, U, are related by the virial theorem according to

$$U = -\frac{M\sigma^2}{2} = -\frac{GM^2}{2R} . \quad (3.1)$$

After a merger from a parabolic orbit, assuming that negligibly few particles are lost, the mass of the remnant is $2M$ and the total energy is $2U$. Thus, the gravitational radius of the remnant is double that of each progenitor, and the velocity dispersion is unchanged (Hausman & Ostriker 1978). Hence, in parabolic mergers, the effective phase density should drop by a factor ~ 4 as the luminosity doubles.

Real galaxies are not spherical, of course, and consist of several distinct components; it is not clear to what extent this energy argument applies. During a merger, energy and angular momentum are transferred from the luminous material to the dark halo, with a resulting tendency for the velocity dispersion of the luminous material to increase (Okumura, Ebisuzaki, & Makino 1991).

We can estimate these effects by studying simulations of mergers of multicomponent galaxies. We begin by computing the cumulative phase-space distribution, $s_{\rm initial}(\bar{f})$, for the luminous component in the progenitors, using the approach outlined in § 2. First, the differential energy distribution, dN/dE, and the radial mass profile, $M(r)$, are determined by averaging on spheres. The mass profile is then used to compute the potential and the density of states, $g(E)$. Finally, equations (2.3) and (2.4) are used to obtain $s(\bar{f})$. This approach isotropizes the disks and treats the material in the outer portions of the disks as being at relatively low coarsely grained phase-space density.

Once the merger is complete, in the sense that most of the system has relaxed into an equilibrium state, we employ the same approach to compute the final cumulative phase-space density function for the remnant, $s_{\rm final}(\bar{f})$. Most of the simulations show distinct tidal tails and faint "shells" (Hernquist & Spergel 1992) which are smoothed out by the radial averaging process.

The result of this procedure applied to four mergers involving progenitors having compact bulges is shown in Figure 3. The simulations began with the same pair of identical galaxies, but the models differed in the orientation of the initial spin planes of the merging galaxies, as in models 1–4 of Hernquist (1992). The solid line in Figure 3 shows the cumulative phase-space density for the luminous progenitors, while the light curves show the $s(\bar{f})$ profiles for the luminous remnants. Despite the fact that the various disk orientations produce remnants having rather different shapes and outer isophotes, the final cumulative phase-space profiles are remarkably similar. This suggests that the evolution of the coarsely grained distribution function is rather insensitive to the precise details of the merging process and that the various averages performed in the analysis have not biased the results significantly.

Figure 4 shows the ratio of the initial to final coarsely grained phase-space density for the remnants shown in Figure 3. Throughout most of the remnant, the effect of violent relaxation and phase-mixing is to decrease the coarsely grained distribution function by roughly a factor of 3–4, with a somewhat steeper decline in the outer regions. This behavior is likely due to the transfer of energy and angular momentum from tightly bound to loosely bound material in the remnants as the merger progresses; an effect which acts to concentrate the inner regions and puff up the outer ones. These findings are in good agreement with the energy argument presented above, which predicts that the effective phase density should drop by a factor of 4 in a parabolic merger of identical, spherical galaxies.

The results embodied by Figures 3 and 4 are quite encouraging and suggest that analyses of the type proposed in § 2.1 can serve as useful guides in interpreting the phase-space evolution of numerical simulations. Perhaps most important in this regard is the fact that the coarsely grained distribution function evolves in a manner similar to that predicted from mass and energy conservation. The energy argument is independent of the detailed processes which bring the system into equilibrium. In addition, the agreement between this estimate and the simulation results presented in Figure 4 strongly imply that the latter are not corrupted by numerical artifacts, like enhanced two-body relaxation. Together, these considerations suggest that Liouville's does not play a significant role in determining the overall properties of the remnants: the coarsely grained phase-density is reduced relative to the progenitors throughout the remnants, except perhaps near their very centers. These expectations are consistent with the statistical

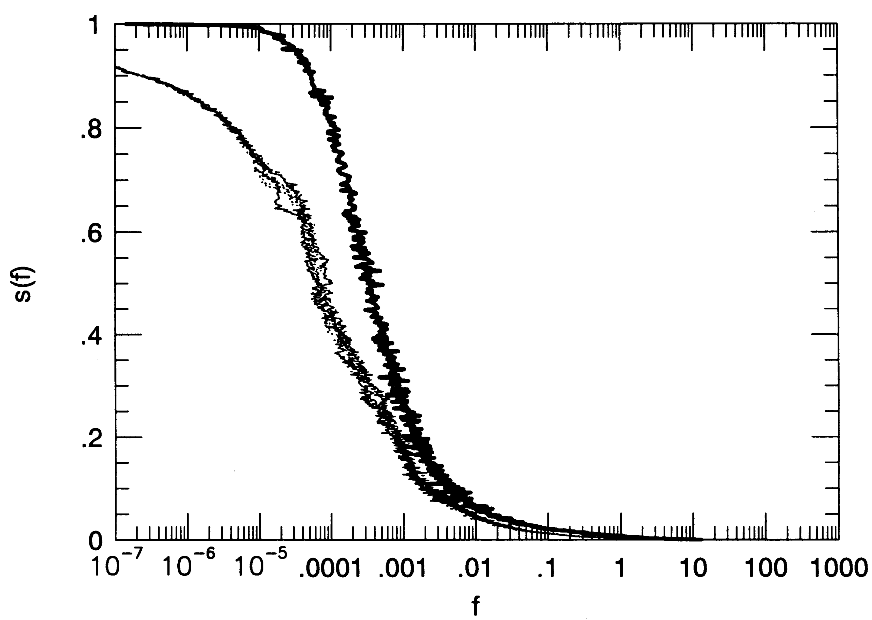

FIG. 3.—Initial and final cumulative distributions of coarsely grained phase-space density, $s(\bar{f})$, for different mergers. All simulations began with the same pair of identical galaxies. Heavy solid line shows the profile for the progenitor galaxies, and light curves show $s(\bar{f})$ for remnants produced by four mergers of galaxies comprising disks, bulges, and halos from different orbital geometries (cf. models 1–4 of Hernquist 1992).

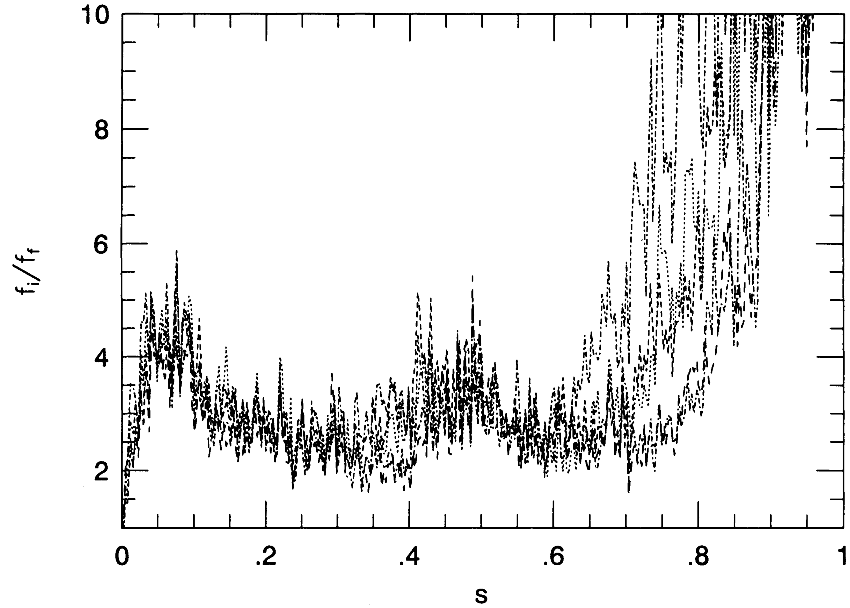

FIG. 4.—Ratio of initial to final coarsely grained phase-space densities, as a function of the cumulative profile, $s(\bar{f})$, for the simulations shown in Fig. 3.

mechanical treatment of violent relaxation outlined in Spergel & Hernquist (1992).

Of greatest interest to the present discussion are comparisons between the phase structure of the remnants in the simulations and that expected for observed elliptical galaxies. An illuminating example is provided by Figure 5, which shows the cumulative distribution of the coarsely grained distribution function is the remnant of a simulated merger between bulgeless disk-halo progenitors, where it is compared with that for a Hernquist model with the same effective radius. In view of the fact that the structure of real ellipticals is well-fitted by the latter, the departure between the two curves in Figure 5 at high phase-space density implies that elliptical galaxies cannot be constructed from mergers of pure stellar disks. Note that the deficit of high phase-space material is not restricted to a negligible fraction of the remnant mass. At small values of $\bar{f}$, the difference between the curves in Figure 5 is not statistically significant. Nevertheless, based on comparisons between models with varying particle number, we estimate conservatively that there is a true deficit of high phase-space density material in the inner $\approx 10\%$–20% of the remnant by mass.

4. DISCUSSION

4.1. *The Merger Hypothesis*

The findings summarized in § 3 have a number of implications for the suggestion that ellipticals are produced by mergers of disk progenitors. Our results appear to be quite

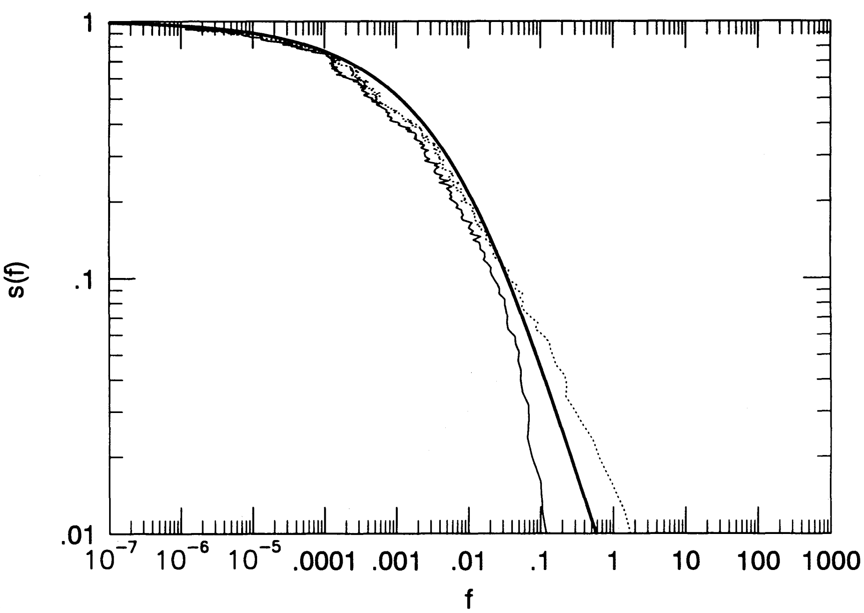

FIG. 5.—Cumulative phase-space density, $s(\bar{f})$, found by coarsely graining the remnant of a merger between two galaxies consisting of only disks and halos. Dashed line shows the form of $s(\bar{f})$ predicted for a Hernquist model with the same effective radius as the simulation remnant.

consistent with Carlberg's (1986) claim that stellar disks lack sufficient material at high phase-space densities to serve as building blocks for elliptical galaxies. This deficit of high phase-space density material is also apparent in surface density profiles of the remnants. A convincing demonstration of this is shown in Figure 6, where the projected mass density of the remnant from the disk-halo simulation in Figure 5 is compared with a remnant formed from a merger with identical orbital properties, but where the progenitors also include 25% of their luminous mass in compact bulges. In this figure an $R^{1/4}$ profile would be a straight line. Obviously, the remnant in the simulation where the progenitors lack bulges is too diffuse in the center to be identified with an elliptical galaxy, as expected on the basis of the analysis presented in § 3 above. Figure 6 also demonstrates that when the progenitors include sufficient mass in dense spheroids, the projected surface density of the remnant is remarkably like that of an elliptical galaxy. (The apparent turnover of the dotted curve in Figure 6 for $Q^{1/4} \lesssim 0.5$ is an artifact of limited resolution in the simulation.)

The magnitude of the deficit of high phase-space density material in remnants produced in mergers of bulgeless progenitors can be estimated from results like that in Figure 6, by subtracting the surface density from that appropriate a pure $R^{1/4}$ law of the same mass and effective radius. In the particular example shown in Figure 6, this difference amounts to $\approx 15\%$ the luminous mass and is confined to radii $\lesssim \frac{1}{3}$ the effective radius of the remnant, again in good agreement with Figure 5. These results are contrary to the suggestion made by Lake (1989), who speculated that stellar material from the outer disk might settle into the center of the remnant and boost the phase-space density there. In the simulations, there is a very good correlation between initial and final binding energy (e.g., Barnes & Efstathiou 1987; Quinn & Zurek 1988; Spergel & Hernquist 1992). Furthermore, while our results show that the half-mass properties of galaxies do not constrain merging scenarios, the discrepancy between the phase-space structure of remnants of bulgeless stellar disk mergers and real ellipticals is sufficient to exclude the formation of ellipticals by such mergers.

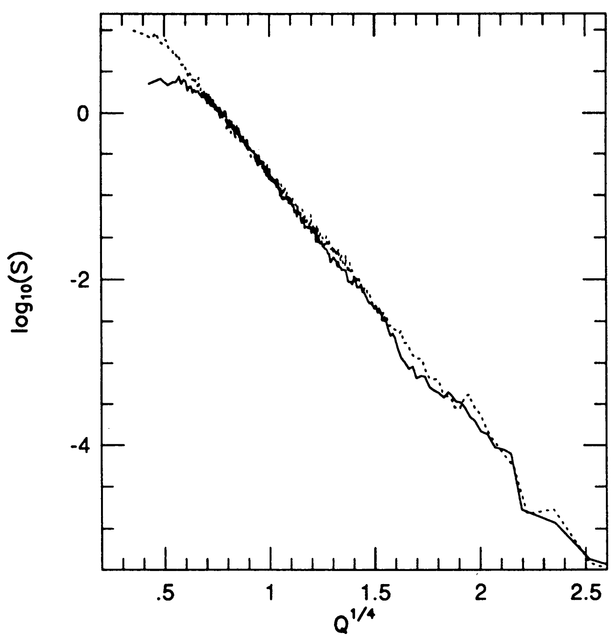

FIG. 6.—Logarithm of surface density of luminous components of two merger remnants. Solid line is for a remnant produced from the merger of two equal-mass galaxies consisting of only disks and dark halos. Dotted line is from a simulation with identical orbital properties, but where the progenitors also contain compact bulges. In the latter simulation, the bulge-to-disk mass ratio is 1:3. Surface density is shown as a function of fourth root of the "elliptical radius," Q (see Hernquist 1992 for details).

The overall significance of these results for the merger hypothesis is unclear. The deficit of matter at high phase-space density can be obviated by adding bulges to the progenitors. We have not determined precisely the fraction of luminous mass which must reside in compact bulges to eliminate the problem, but the results above indicate that $\approx 25\%$ is sufficient. Much smaller bulge masses will probably not be viable since not all bulge material falls into the center of the remnants, and the estimate above suggests that pure stellar disks fail to produce good $R^{1/4}$ laws by $\approx 15\%$ of their mass.

In principle, the deficit can also be removed by dissipation in gas and star formation. Hydrodynamical simulations (Negroponte & White 1983; Hernquist 1989, 1991; Barnes & Hernquist 1991; Hernquist & Barnes 1991) show that tidal forces in a merger tend to drive a significant fraction ($\sim 50\%$) of the gas in progenitors into the centers of the remnants. Since the gas is not subject to Liouville's theorem, it can become much more centrally concentrated than the stars. This sudden infall of gas will likely fuel a starburst, perhaps forming enough stars at high phase-space density to account for the central structure of ellipticals. By itself, this route does appear to require a progenitor gas fraction $\sim 30\%$, which exceeds that in present-day early spirals, but is probably consistent with the gas content of galaxies in the early universe when the merger rate was higher.

4.2. *Galaxy Catalogs*

Somewhat more speculative than our analysis of remnants of major mergers is the application of the various tools developed in § 2.1 to observed properties of elliptical galaxies. One of the remarkable properties of elliptical galaxies is that they comprise a two-parameter system lying in a "fundamental plane" (Djorgovski & Davis 1987; Dressler et al. 1987). This correlation is apparent in the recent sample compiled by Bender, Burstein, & Faber (1992), who tabulated velocity dispersions, effective radii, and luminosities of nearby hot stellar systems. We will use these quantities to compute the effective phase-space density of these systems and to see whether or not it correlates with the luminosity.

Figure 7 shows the correlation between absolute magnitude and $\bar{f}_{\rm eff}$ for the giant and intermediate ellipticals in Bender et al.'s sample. The correlation is extremely good: a least-squares fit finds that $M = -1.35(\pm 0.06) \log \bar{f}_{\rm eff} +$ constant, with an rms scatter of only 0.15 mag, and so is nearly as tight as the correlation found by Bender et al. between absolute magnitude and mass-to-light ratio (κ_1 and κ_3 in their notation). A least-squares fit to the fundamental plane viewed edge-on for the same data set yields an rms scatter of only 0.18 mag. While the effective phase density is computed from similar quantities as κ_3, it has a different dependence on velocity dispersion and effective radius: $\kappa_3 \propto \sigma^2 R_{\rm eff}$, while $\bar{f}_{\rm eff}^{-1} \propto \sigma R_{\rm eff}^2$.

As discussed below, the significance of the result in Figure 7 is problematic. It is interesting that in terms of luminosity the fit translates into a correlation $L \propto \bar{f}_{\rm eff}^{-0.54 \pm 0.02}$, which, as argued in § 3, is consistent with the interpretation that higher luminosity objects are built from major mergers of roughly comparable mass progenitors. Furthermore, as we show in

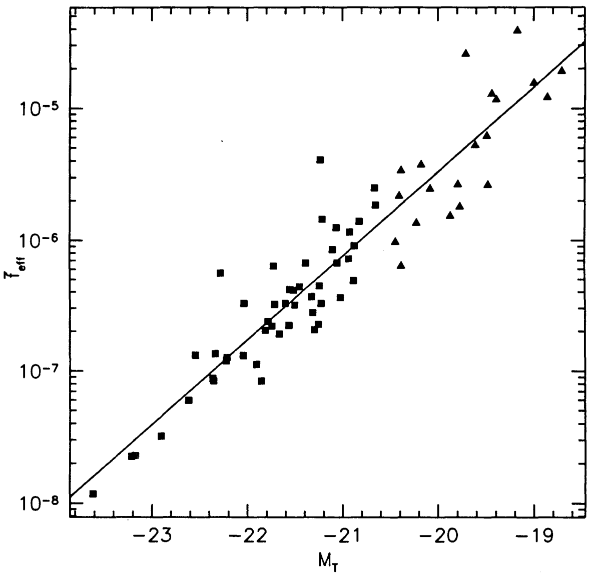

FIG. 7.—Correlation between effective phase-space density and absolute magnitude in a sample of nearby giant and intermediate mass ellipticals. Filled squares denote giant ellipticals and filled triangles denote intermediate ellipticals. Solid line is a least-squares fit to the data. The effective phase-space density was computed from the velocity dispersion and effective radius (see text).

§ 4.3, it is also consistent with some forms of hierarchical clustering.

A correlation of the type displayed in Figure 7 was, in fact, anticipated in earlier work by Carlberg (1986) who found a strong correlation between galaxy luminosity and central phase-space density. If all ellipticals are well described by de Vaucouleur's profiles (e.g., de Vaucouleurs 1948, 1987), there should be a dimensionless relation between core and effective radii and between central and effective velocity dispersion. Thus, there should be a dimensionless ratio relating the effective to central phase-space density. We have chosen to focus on the effective phase-space density since it is a much more robust quantity and is not dominated by the details of the density profile in the inner regions.

Interestingly, the strong correlation between effective phase-space density and galaxy luminosity can be extended to include less massive systems. Figure 8 shows the relationship between $\bar{f}_{\rm eff}$ and magnitude for giant ellipticals, intermediate ellipticals, dwarf ellipticals, compact ellipticals, bulges, and dwarf spheroidals. The fit shown in Figure 8 is derived from all the galaxies except for the dwarf spheroidals. Figure 8 suggests that dwarf spheroidals should be viewed as part of a formation sequence distinct from that of other hot stellar systems. A least-squares fit to the galaxy magnitudes and $\bar{f}_{\rm eff}$ yields $M = -2.04 \log \bar{f}_{\rm eff}$ + constant, with an rms scatter of 1.4 mag. If we restrict our attention to the compact, intermediate, and giant ellipticals, then the scatter in the diagram is reduced. The dotted line shows a fit to these three populations. It has a scatter of only 0.43 mag and a slope of -1.66. This implies that $L \propto \bar{f}_{\rm eff}^{-0.66}$. Bulges lie along this line but with a larger spread. This may be partly due to difficulties in separating bulges and disks or perhaps is a result of dynamical coupling between disks and bulges. Dwarf ellipticals appear to be a parallel but distinct branch of the family of hot stellar systems.

It is possible that the correlations in Figures 7 and 8 might be improved by removing peculiar motions from the Bender et al. sample; they assume a smooth Hubble flow when computing galaxy luminosity and effective radius. Deviations from the Hubble flow will lead to distance errors which may account for some of the scatter in Figures 7 and 8. The estimated galaxy luminosity is proportional to D^2 and the effective phase-space density is proportional to D^{-2}. Thus distance errors shift the galaxies along a line of slope -1 in the plane of Figures 7 and 8.

4.3. *Hierarchical Merging*

As noted in § 4.2, the observed correlation between luminosity and effective phase density in the Bender et al. sample can be interpreted as arising from the growth of hot stellar systems via major mergers of comparable mass progenitors. In this case, as suggested by the energy argument and simulations results in § 3, we expect a correlation of the form $\bar{f}_{\rm eff} \propto L^{-2}$, quite similar to the fit to the data in Figure 7. Clearly, however, it is not likely that all massive structures developed in this manner. Even if it is the case that mergers are indeed responsible for producing hot stellar systems, it is not plausible that the progenitors always have nearly the same mass, or that only one merger event necessarily gave rise to the structure seen in all ellipticals today.

A definitive interpretation of the correlation found in § 4.2 will await detailed modeling under a much wider set of initial conditions than considered here. We can, however, speculate on what trends might be expected in other situations. For example, the energy argument in § 3 can be generalized straightforwardly to successive mergers of identical subsystems from parabolic orbits. A remnant produced from mergers of p identical spherical progenitors will have binding energy, mass, and gravitational radius equal to p times that of each progenitor, and the velocity dispersion will be unchanged. The effective phase density of the remnant will, therefore, be a factor p^2 *smaller* than that of each progenitor. This suggests that in a

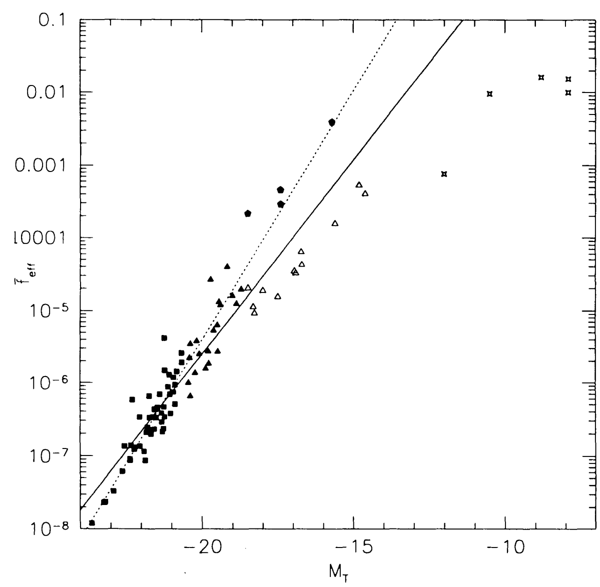

FIG. 8.—Same as Fig. 7, but including lower mass systems. Filled squares denote giant ellipticals, filled triangles denote intermediate ellipticals, filled pentagons denote compact ellipticals, open squares denote bulges, open triangles denote dwarf ellipticals, and crosses denote dwarf spheroidals. Solid line is a least-squares fit to all the data except that for dwarf spheroidals. Dotted line is a least-squares fit to the data for giant, intermediate, and compact ellipticals.

scenario in which galaxies are built up through mergers of identical weakly bound subsystems, we expect that $\bar{f}_{\rm eff} \propto L^{-2}$ from this naive argument, as in major mergers of comparable mass galaxies.

This is certainly not a "universal" relation, however, and it is not difficult to imagine merging scenarios where it fails to some degree. Consider a galaxy of mass M_1 which successively merges with p identical less massive objects from parabolic orbits, each having mass $M_2 = M_1/p$. Suppose, also, that the primary has gravitational radius, R_1, and dispersion, σ_1, and that the corresponding quantities for each less massive object are R_2 and σ_2. Since the energy argument involves simple linear combinations of conserved quantities, we can predict the structure of the ensuing remnant by first adding the p less massive objects together and then combining this system with the primary. Applying this procedure, it is trivial to show that the remnant mass, $M_{\rm rem}$, energy, $U_{\rm rem}$, dispersion, $\sigma_{\rm rem}$, and gravitational radius, $R_{\rm rem}$ satisfy

$$M_{\rm rem} = 2M_1 , \tag{4.1}$$

$$U_{\rm rem} = -\frac{M_1}{2}(\sigma_1^2 + \sigma_2^2) , \tag{4.2}$$

$$\sigma_{\rm rem}^2 = \tfrac{1}{2}(\sigma_1^2 + \sigma_2^2) , \tag{4.3}$$

and

$$R_{\rm rem} = R_1 \frac{4\sigma_1^2}{\sigma_1^2 + \sigma_2^2} . \tag{4.4}$$

The effective phase density of the remnant, $\bar{f}_{\rm rem}$, will be related to that of the original primary, $\bar{f}_1$ by

$$\frac{\bar{f}_{\rm rem}}{\bar{f}_1} = \frac{\sqrt{2}}{16}\left[1 + \left(\frac{\sigma_2}{\sigma_1}\right)^2\right]^{3/2} . \tag{4.5}$$

If $\sigma_2 = \sigma_1$, then we recover the result found earlier, *independent* of the mass ratio of the progenitors. If instead, we postulate a parameterized relation between dispersion and luminosity of the form $L \propto \sigma^n$, then

$$\frac{\bar{f}_{\rm rem}}{\bar{f}_1} = \frac{\sqrt{2}}{16}\left[1 + \left(\frac{L_2}{L_1}\right)^{2/n}\right]^{3/2} . \tag{4.6}$$

Hence, for a value $n \sim 4$, as in the Faber-Jackson relation, this result predicts a correlation of the form $\bar{f} \propto L^{-2.25}$ for $L_2 = L_1/2$ and $\bar{f} \propto L^{-2.7}$ for $L_2 = L_1/5$. Thus, at least in the context of these simple-minded considerations, we still anticipate a definite correlation between effective phase density and luminosity, but one that is somewhat steeper than in mergers between identical galaxies. Clearly, arguments like this are subject to considerable uncertainty, however, since, among other things, it is found experimentally that the dispersion in remnants produced by mergers of identical spherical galaxies is slightly greater than that in the progenitors (e.g., Okumura et al. 1991). The transfer of energy from the luminous material to the halo will increase $\bar{f}_{\rm rem}$ and flatten the $\bar{f} - L$ correlation.

4.4. *Implications for Galaxy Formation*

There appear to be many routes to forming giant ellipticals in cosmological models which involve hierarchical clustering. Giant ellipticals may have been built up through mergers of smaller spherical stellar systems, from mergers of spirals containing large bulges, or from mergers of gas-rich spirals in the early universe. Establishing definite trends that might be used to test these various suggestions observationally is clearly of highest priority. While violent relaxation tends to erase much information about initial conditions, there may be subtle, yet observable differences between the various routes just noted. For example, since phase-mixing is least effective in the outer regions, observations of the structural and kinematic properties of ellipticals at low surface density may provide some information on their origin (e.g., Heyl, Hernquist, & Spergel 1993a, b).

It would appear much more difficult, however, to form dwarf galaxies by merging, owing to the absence of suitable progenitors. The effective phase-space density of dwarf ellipticals is much larger than that of any known spiral. If dwarfs form dissipationlessly, their high phase-space densities require that these galaxies originated at very high redshifts. If dissipation played an important role in the formation of dwarfs, then either they started out with very little angular momentum or quite efficiently transported it outward.

Dwarf spheroidals do appear to have a very different formation history from other hot stellar systems. As Figure 8 shows, these objects deviate significantly from the mean relation between luminosity and effective phase-space density and are much less concentrated than other hot stellar systems. It is possible that some dwarf ellipticals comprise debris from mergers of massive disk galaxies (Barnes & Hernquist 1992) or that tidal effects have significantly influenced their formation and evolution (e.g., Kuhn & Miller 1989).

5. SUMMARY

In this article, we have explored the use of the coarsely grained phase-space density as a tool for understanding the evolution and formation of hot stellar systems. Unlike the finely grained phase-space density, the coarsely grained phase-space density can be easily measured in both computer simulations and from observations of elliptical galaxies.

In numerical simulations, the coarsely grained phase-space density shows a relative uniform decrease by factors $\sim$3–4 in mergers of equal mass systems. This phase space decrease is consistent with simple analytical estimates. Furthermore, the phase-space structure of remnants of major mergers proves conclusively that pure stellar disks do not represent viable progenitors from which ellipticals can be formed. This finding can be reconciled with the merger hypothesis either by adding compact bulges to the progenitors or, at least in principle, through the effects of dissipation in gas. This is not unreasonable, since while Sc galaxies contain a significant amount of gas, Sa galaxies contain large bulges.

The addition of bulges to the premerger galaxies may not, however, alleviate problems with metallicity. Giant ellipticals are more metal-rich than bulges and compact ellipticals (for a discussion, see Ostriker 1980). There is a very strong correlation between the strength of the magnesium line and luminosity, apparently necessitating that star formation must play some role in the formation of ellipticals. If, in fact, many or all ellipticals are formed via major mergers, it seems likely that both bulges and gas dynamics are required.

To simplify the interpretation of observations, we also have defined an effective coarsely grained phase-space density, $\bar{f}_{\rm eff} = 1/\sigma_0 r_e^2$ and found that it correlates extremely well with luminosity for elliptical galaxies and bulges of spiral galaxies. This correlation extends from dwarf up to giant ellipticals, hinting that it might provide important information about the origin of hot stellar systems. An intriguing result is that the slope of the luminosity–$\bar{f}_{\rm eff}$ correlation is consistent with a fully dissipationless scenario for forming hot stellar systems.

At present, we do not fully understand the physical significance of the correlation between effective phase-space density and luminosity found for the Bender et al. sample of elliptical galaxies, although, as we suggested earlier, it may be a direct consequence of the formation history of these objects. A correlation of this form would arise in dissipationless collapse models provided that the perturbation spectrum incorporates significant power on small scales. Alternatively, our findings may reflect the nature of merging and growth structure in a hierarchical universe. However, in this interpretation, significant amounts of dissipation would be required to form the smallest objects, which would necessarily have the highest phase-space densities. If such objects could be built at sufficiently early times though, then the correlation between coarsely grained phase-space density and luminosity may simply reflect the nature of the dynamics of merging of mostly stellar systems.

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