

SEARCHING GAMMA-RAY BURSTS FOR GRAVITATIONAL LENSING ECHOES:  
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## ABSTRACT

The first available 44 gamma-ray bursts (GRBs) detected by the Burst and Transient Source Experiment on board the *Compton Gamma-Ray Observatory* have been inspected for echo signals following shortly after the main signal. No significant echoes have been found. Echoes would have been expected were the GRBs distant enough and the universe populated with a sufficient density of compact objects composing the dark matter. Constraints on dark matter abundance and GRB redshifts from the present data are presented and discussed. Based on these preliminary results, a universe filled to critical density of compact objects between  $10^{6.5} M_{\odot}$  and  $10^{8.1} M_{\odot}$  are now marginally excluded, or the most likely cosmological distance paradigm for GRBs is not correct. We expect future constraints to be able either to test currently popular cosmological dark matter paradigms or to indicate that GRBs do not lie at cosmological distances.

*Subject headings:* dark matter — gamma rays: bursts — gravitational lensing

## 1. INTRODUCTION

There is much evidence that dark matter composes a large fraction of the mass of the universe (see, for example, Kormendy & Knapp 1987). There is no present consensus as to what form this dark matter takes. Assuming the dark matter is composed of individual components, there are only bounds on the masses of these components. This dark matter may take the form of massive compact objects (COs) such as black holes which formed in the early universe and dominate the universe's mass (Carr 1990). One particularly popular mass scale for hypothesized COs is near  $10^6 M_{\odot}$ , which is the Jeans mass at recombination for many universe scenarios (Carr & Rees 1984; Gnedin & Ostriker 1992).

There is also no present consensus as to the cause or location of gamma-ray bursts (GRBs). Their isotropic angular distribution combined with an apparently "confined" peak brightness distribution (Meegan et al. 1992) generates an interesting puzzle. A paradigm presently gaining in popularity is that these bursts occur at cosmological distances (Paczynski 1986). Lensed bursts at cosmological distances can be used to trace the matter between the observer and the burst (Press & Gunn 1973; Webster & Fitchett 1986; Nemiroff 1991a; Blaes & Webster 1992), including potential diffraction effects (Bliokh & Minakov 1975; Gould 1992). It has even been suggested that lensing causes GRBs (McBreen & Metcalfe 1988), but this idea has been criticized (Kovner 1990).

The primary effect of a gravitational lens on a GRB would be to create more images of the burst than one. These images could not be angularly resolved with present technology, but they could be temporally resolved. A typical result would be that some bursts would have "echoes" of themselves arriving at later times. These echoes would have identical spectral and temporal profiles, although they would appear either dimmer or brighter than the previously recorded burst. Detecting

lensing by temporal analysis was first suggested by Paczyński (1987) and has been discussed by Krauss & Small (1991), who showed that the mass of the lens can be determined uniquely; by Mao (1992), who estimated the frequency of this effect due to the known galaxy distribution; and by Blaes & Webster (1992), who estimated the dark matter mass scales that could be explored with GRBs. Narayan & Wallington (1992) have determined what lens characteristics can be determined from such a temporal detection.

In this work we report on an active search for gravitational lens-induced echoes in the Burst and Transient Source Experiment (BATSE) data on board the *Compton Gamma-Ray Observatory* (CGRO), and the implications these results suggest.

## 2. THE ECHO SEARCH PROCEDURE

We have searched the first 44 available BATSE burst time profiles looking for signals in the data that could be gravitational lens-induced echoes of previous, stronger signals. By "available" BATSE bursts we mean those GRBs which had complete uncompromised data sets delivered into the public domain by the time of the submission of this paper. The search procedure was as follows. The GRB light curve (more specifically, a plot of the number of counts measured for the GRB integrated over all energies greater than 30 keV vs. time) was scanned. We searched only for echoes that came after the original GRB signal in time and were not overlapping the original burst time profile: there must be at least one time bin of data at the background level between the main GRB signal and the candidate echo. Echoes superposed on the original burst signal would be harder to identify, since data from the original signal would disrupt any statistical comparison.

The data, originally in 64 ms time bins, were rebinned in time optimally so that all the signal from all the channels was combined into two main adjoining time bins of nearly equal amplitude. This was done to ensure that any observed echo signal would not be diminished in amplitude relative to the main signal by being distributed over two adjoining time bins. A search interval (from  $\Delta t_{\min}$  to  $\Delta t_{\max}$ ) was defined starting from one time bin past the end of the main signal ( $\Delta t_{\min}$ ) and ending at the conclusion of the data stream recorded by

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BATSE with 64 ms resolution ( $\Delta t_{\max}$ ). This search interval was then checked for candidate echoes. Any fluctuation in the background greater than  $5\sigma$  over the mean that was separated in time from the main signal was considered a candidate echo.

Candidate echoes were then checked in two ways. First the time series of the echo was expanded to the finest time resolution available (usually 64 ms) and compared with the main signal to see if the time profiles were similar. During intervals of light curve similarity, a  $\chi^2$  test between the reduced main signal and the candidate echo was carried out to see if a series of time bins were consistent with being drawn from the same distribution, for any amplification or time offset. A similar comparison test has been proposed by Wambsgans (1993). Usually this  $\chi^2$  test was decisive, except in the cases where the candidate echo signal was either too dim and noisy, or not well time-resolved. We note that more distant bursts are more likely to be lensed, but also more likely to be dim and noisy, which generally creates a bias against lensing. In the remaining dim or time-unresolved cases, the candidate echo signal was compared to the main signal in the four separate energy channels. Gravitational lens effects would demand the echo strength not be significantly different in every energy channel.

No candidate echo survived the above stated criteria. For each GRB we then recorded the minimum and maximum time delay of the search interval,  $\Delta t_{\min}$  and  $\Delta t_{\max}$ , respectively, as well as the minimum acceptable burst echo strength,  $e_{\min}$ , that could have been found. To be conservative,  $e_{\min}$  was typically defined as the ratio of  $5\sigma$  of the background level divided by the height of the second tallest of the two peaks that defined the original signal. The values of  $\Delta t_{\min}$ ,  $\Delta t_{\max}$ , and  $e_{\min}$  are listed in Table 1. The first two entries in Table 1 correspond to the BATSE trigger number of the GRB and the date of the GRB. A "B" after the date usually signifies that the particular GRB was only the second brightest trigger occurring that day. Some GRBs have more than one entry in the table as different values of  $e_{\min}$  were recorded for different time intervals following the main signal.

### 3. GRAVITATIONAL LENS THEORY

If the universe were filled with a significant density of CO dark matter in a certain mass range, relative to the closure density, and if GRBs lie at cosmological distances, one might expect there to have been a gravitational lens-induced echo. The nonexistence of such an echo can be used either to rule out the existence of dark matter in this form and abundance or to indicate that GRBs do not lie at cosmological distances.

Similar "null lensing result" arguments have been used with quasars and QSOs as the candidate source objects to eliminate compact dark matter in other mass ranges. Hewitt (1986) eliminated closure density of COs between  $10^{11}$  and  $10^{13} M_{\odot}$  with radio observations using the Very Large Array. Using published optical data on QSOs (Crampton et al. 1989), Nemiroff (1991b) was able to rule out a closure density of COs greater than  $10^{9.9} M_{\odot}$ . Using VLBI observations, Kassiola, Kovner, & Blandford (1991) were able to rule out a closure density of COs between  $10^7$  and  $10^9 M_{\odot}$ .

To compute the probability of finding a CO by gravitational lensing in a given data set, one can use the detection volume formalism (Nemiroff 1989). The detection procedure of § 2 defines a detection volume between the observer and each GRB, at redshift  $z_{\text{GRB}}$ , within which a compact object lens of mass  $M_{\text{CO}}$  must fall to create a detectable gravitational lens echo. The procedure used here closely follows Nemiroff

TABLE 1  
BATSE ECHO LIMITS

Trigger Number	Date	$\frac{C_{\text{peak}}}{C_{\text{complete}}}$	$z_{\text{GRB}}$ (Theoretical)	$e_{\min}$	$\Delta t_{\min}$ (s)	$\Delta t_{\max}$ (s)
105.....	910421	11.165	0.324	0.0170	6.00	33.70
107.....	910423	0.225	1.930	0.4500	8.00	230.00
108.....	910424	0.126	2.498	0.5800	0.60	240.60
109.....	910425	3.764	0.538	0.0300	128.00	191.00
111.....	910426	0.610	1.234	0.1900	95.60	207.10
114.....	910427	0.580	1.262	0.1500	29.80	230.10
121.....	910429	1.267	0.886	0.0580	47.80	111.60
130.....	910430	3.164	0.583	0.0150	127.80	159.60
138.....	910502B	0.409	1.477	0.3700	0.80	237.00
142.....	910502	9.019	0.358	0.0390	6.00	234.10
143.....	910503	42.285	0.172	0.0130	27.80	73.40
143.....	910503	42.285	0.172	0.0024	73.40	234.10
148.....	910505	1.424	0.841	0.0830	30.00	209.60
160.....	910507	2.903	0.607	0.0560	21.60	214.30
171.....	910509	0.676	1.178	0.1700	30.00	209.70
179.....	910511	2.809	0.616	0.0580	9.30	235.50
185.....	910512	0.185	2.102	0.3000	0.39	240.70
204.....	910517B	0.576	1.266	0.1590	11.20	234.10
207.....	910518	0.222	1.938	0.2300	0.11	1.39
207.....	910518	0.222	1.938	0.4000	1.39	240.70
211.....	910518B	0.524	1.322	0.3100	18.90	94.21
211.....	910518B	0.524	1.322	0.2400	94.21	232.50
214.....	910521B	0.729	1.138	0.3000	50.40	215.40
218.....	910521	0.747	1.126	0.2600	3.20	237.90
219.....	910522	14.684	0.284	0.0200	59.90	119.60
222.....	910523	3.376	0.566	0.0430	59.90	149.70
223.....	910523B	0.420	1.459	0.4310	2.56	237.40
228.....	910526B	0.918	1.026	0.1840	9.80	226.30
229.....	910526	0.324	1.640	0.3810	1.03	237.50
235.....	910528	0.455	1.408	0.2320	47.70	222.20
237.....	910529	0.718	1.146	0.1830	50.40	236.10
249.....	910601	35.573	0.187	0.0028	71.50	208.30
253.....	910602B	0.321	1.645	0.3260	3.07	239.40
257.....	910602	1.678	0.780	0.0036	127.00	214.20
289.....	910607	0.183	2.114	0.6140	0.13	238.00
298.....	910609	6.938	0.405	0.0146	0.90	240.00
332.....	910612	1.266	0.887	0.0472	127.00	222.20
394.....	910619	4.557	0.492	0.0110	152.40	191.20
401.....	910620B	0.270	1.779	0.6700	6.14	238.00
404.....	910621B	0.929	1.020	0.0930	143.40	191.20
408.....	910621	1.357	0.859	0.5120	13.30	133.90
414.....	910622	0.720	1.145	0.2470	6.40	238.70
432.....	910625	0.420	1.459	0.1040	0.06	0.18
432.....	910625	0.420	1.459	0.1710	0.18	241.20
444.....	910626	4.908	0.476	0.0210	0.64	240.60
451.....	910627	17.288	0.263	0.0150	28.80	233.20
465.....	910629B	0.257	1.817	0.5920	40.40	238.00
467.....	910629	4.917	0.475	0.0390	23.10	237.50

(1991b). These volumes are then added (for each source) and compared to the volume per lens expected for a universe with a given lens density. If the former volume is significantly larger than the latter volume, then that universe (with the given lens density) is ruled out.

Specifically, for the two bright images expected with a CO lens (hereafter referred to as the "main signal" and the "candidate echo") to be within the minimum dynamic range  $e_{\min}$  of each other that is necessary for detection, the lens must be closer to the observer-source axis than

$$b_e = \sqrt{\frac{4R_{\text{Sch}} D_{\text{OL}}^4 D_{\text{LS}}^4 \Phi}{D_{\text{OS}}^4}}, \quad (1)$$

where  $\Phi$  is related to the ratio of the amplitude between images  $e$  by  $\Phi = (e^{1/4} - e^{-1/4})^2/2$ , where O, L, and S refer to the observer (CGRO), lens (CO), and source (GRB), respectively. Here

$R_{\text{Sch}}$  is the Schwarzschild radius of the lens,  $D^A$  refers to cosmological angular diameter distance (Dyer & Roeder 1973). Note that a relatively dim GRB would have smaller  $e_{\text{min}}$ , and hence smaller  $b_e$  and a lower probability of being detectably lensed.

For the two images expected with a CO lens to arrive within the maximum detectable search time  $\Delta t_{\text{max}}$  of each other, the distance of the lens from the observer-source axis must be less than a certain  $b_{\Delta t(\text{max})}$ . This impact parameter  $b_{\Delta t(\text{max})}$  can be determined from the cosmological time delay equation (see, for example, Mao 1992)

$$\frac{c \Delta t}{(1 + z_{\text{CO}})R_{\text{Sch}}} = \frac{|x_-^2 - x_+^2|}{2DR_{\text{Sch}}} - 2 \ln \left[ \frac{b_{\Delta t} - x_-}{x_+ - b_{\Delta t}} \right], \quad (2)$$

where

$$x_{\mp} = (b_{\Delta t}/2) \mp \sqrt{(b_{\Delta t}/2)^2 + 2DR_{\text{Sch}}}, \quad (3)$$

such that  $D = D_{\text{OL}}^A D_{\text{LS}}^A / D_{\text{OS}}^A$ ,  $c$  is the speed of light,  $H_0$  is the present value of the Hubble parameter.

Similarly, if the two images created by the CO gravitational lens arrive within the minimum detectable search time  $\Delta t_{\text{min}}$ , the echo image would not be noticed. To include this possibility in the analysis, we specify that the distance of the lens from the observer-source axis must be greater than a similarly determined  $b_{\Delta t(\text{min})}$  for the candidate echo to be noticed.

To generate the most probable CO limits, we have chosen a “best fit” relation between the peak flux and  $z_{\text{GRB}}$ , similar to the method used in Wickramasinghe et al. (1993), by fitting the shape of the integrated number versus peak flux (binned to 1.024 s bins). We note that estimates by Mao & Paczyński (1992), and Paczyński (1992), give considerably higher redshifts per burst, although the value they used for the shape of the spectral slope may be slightly low compared with present GRB data. These higher redshifts would give considerably higher probabilities for lensing and hence considerably less conservative limits on COs. We use the relation

$$\frac{C_{\text{peak}}/C_{\text{complete}}}{(1 + z_{\text{GRB}})^2 - 2(1 + z_{\text{GRB}})^{1.5} + (1 + z_{\text{GRB}})} = \frac{0.333}{}, \quad (4)$$

which assumes a cosmology with  $\Omega = 1$ . Here  $C_{\text{peak}}$  is the peak flux in 1.024 s bins submitted to the public domain in 1992 December which had been corrected for systematic orientation effects.  $C_{\text{complete}}$  is the peak flux (again with 1.024 s bins) at which the used sample was 99.6% complete, which was 0.89 photons  $\text{cm}^{-2} \text{s}^{-1}$ , where the photons are constrained to have energy between 50 keV and 300 keV.

The values of  $C_{\text{peak}}/C_{\text{complete}}$  and the theoretical  $z_{\text{GRB}}$  used in our analysis are also listed in Table 1. Note that many of the values of  $C_{\text{peak}}/C_{\text{complete}}$  are less than unity. This is allowed because some of the GRBs we use are too dim to appear in the complete sample. The results are not a function of the value of the Hubble constant used.

The probability  $P$  that a GRB at redshift  $z_{\text{GRB}}$  is detectably lensed is

$$P = 1 - e^{-\lambda}, \quad (5)$$

where

$$\lambda = \int_0^{D_{\text{OS}}^P} n_{\text{CO}} \pi \max [b_{\Delta t(\text{max})}^2 - b_{\Delta t(\text{min})}^2, 0] dD_{\text{OL}}^P, \quad (6)$$

where  $b_{\text{max}} = \min [b_e, b_{\Delta t(\text{max})}]$ ,  $D^P$  refers to cosmological proper distance (Dyer & Roeder 1973), and the proper number density of lenses is given by

$$n_{\text{CO}} = \frac{3H_0^2(1 + z_{\text{CO}})^3 \Omega_{\text{CO}}}{4\pi R_{\text{Sch}} c^2}, \quad (7)$$

where  $\Omega_{\text{CO}}$  is the fraction of the  $\Omega = 1$  universe composed of COs. The parameter  $\lambda$  corresponds to the number of echoes expected to be observed.

#### 4. LIMITS ON COSMOLOGICAL PARADIGMS

Figure 1 shows what proportion of the  $(M_{\text{CO}}, z_{\text{GRB}})$ -plane is excluded for the two GRBs of the first available 44 with the highest peak flux in a 64 ms bin. These bursts are identified here by their BATSE trigger numbers 143 and 298 and will be hereafter referred to by only their trigger numbers. Burst 143 has the highest total fluence of all the GRBs tested so far, as well as the second highest peak flux. Burst 143 extends for some tens of seconds before subsiding. Burst 298, on the other hand, has the highest peak flux but a relatively small fluence. This is because of the extremely short duration of the GRB. The duration of this GRB is shorter than 0.5 s.

The results shown in Figure 1 are not unexpected given these GRB attributes. The short duration of burst 298 allows one to search for echoes at shorter time delays—which correspond roughly to a search for a lower mass CO lens that would cause such time delays. Therefore, since no echoes were found, the excluded region extends to a lower excluded mass. Since 143 used wider bins for the echo comparison and since the echo took longer to reach its peak flux than 298, the excluded mass range for 298 also extends to a slightly higher excluded mass. Burst 143, however, is excluded from lying at a smaller distance than 298. This is because 143’s higher fluence, when summed into two bins, allows a greater dynamic range between the peak bin and the background (through a lower  $e_{\text{min}}$ ), which translates into a higher probability of finding an echo between  $\Delta t_{\text{min}}$  and  $\Delta t_{\text{max}}$ . Since closer GRBs have a lower chance of being lensed, the higher probability of lensing corresponds to a lower GRB redshift where lensing can be excluded. From inspection of Figure 1, we see that either 143 occurred at a

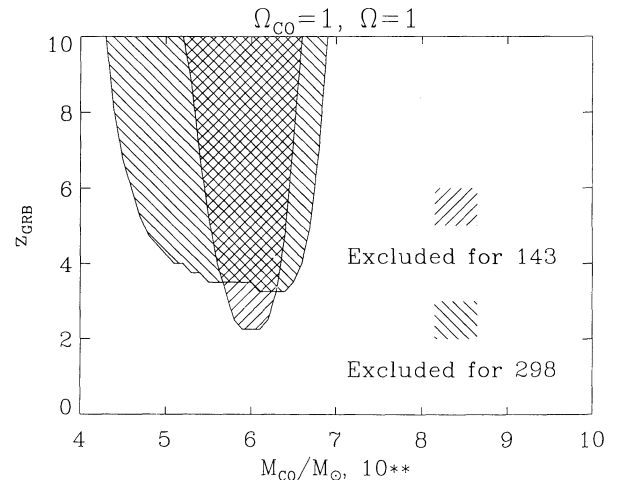


FIG. 1.—A plot of GRB redshift vs. CO lens mass (at closure density) showing the region excluded by BATSE observations for triggers 143 and 298. The shaded region is excluded at the 90% confidence level.

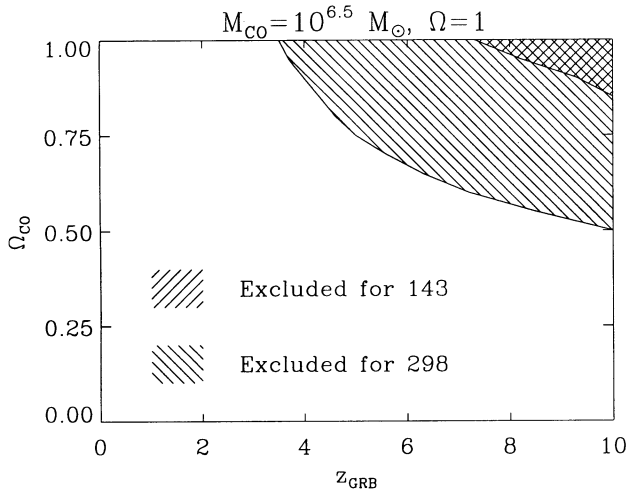


FIG. 2.—A plot of CO density vs. GRB redshift showing the region excluded by BATSE observations for a CO mass of  $10^{6.5} M_{\odot}$  for trigger numbers 143 and 298. If an  $\Omega = 1$  universe contains  $\Omega_{CO}$  of  $10^{6.5} M_{\odot}$  lenses, then a GRB with a  $z_{GRB}$  in the hatched region would have a greater than 90% chance that an echo would have been detected with the procedure described in the text. Since no echo was observed, the shaded region is most probably excluded.

redshift less than about 2, or the universe cannot be composed of COs of  $10^6 M_{\odot}$ .

Figure 2 shows the excluded region of the  $(z_{GRB}, \Omega_{CO})$ -plane for a specific cosmological dark matter candidate:  $10^{6.5} M_{\odot}$  COs, for the two GRBs 143 and 298. This is a popular hypothesized mass for COs, specifically discussed recently by Gnedin & Ostriker (1992), although for a universe where  $\Omega = \Omega_{CO} \approx 0.15$ . Here we see the most stringent constraint comes from the short duration GRB 298. From inspection of Figure 2 we can see that either these COs have  $\Omega_{CO} < 0.75$ , or this GRB occurred closer than a redshift of 5.

Given the cosmological redshifts of the GRBs stated in Table 1, as well as the maximal lensing assumption of  $\Omega_{CO} = 1$ ,

we found that no individual GRB is expected to show a lens echo.

These results are summarized more generally in Figure 3. This figure shows precisely the number of echoes expected to be found in all the GRB light curves, for each  $(\Omega_{CO}, M_{CO})$  pair. One can now see that although no individual GRB is expected to show a lens echo, for all the GRBs taken together one echo is in fact quite likely. That no echo has been found does not significantly rule out any  $(\Omega_{CO}, M_{CO})$  pair at this time; however, some mass scales are marginally excluded. For this plot one can see that the most likely mass range CO to create an echo detectable with the described procedure, which is now marginally excluded for an  $\Omega_{CO} = 1$  universe, is between  $10^{6.5}$  and  $10^{8.1} M_{\odot}$ .

There are several assumptions that are embedded in the cosmological model that may confound the derived results. First of all, for Figure 3 and the results that depend on redshift for the GRB, the model used in equation (4) may not be accurate. Specifically, GRBs may be closer than this relation implies, translating into fewer expected gravitational lens-induced images by any given population of COs. For example, if the “low-z fit” of Wickramasinghe et al. (1993) is used, the expected number of lens-induced extra images decreases by about a factor of 4. However, were the “high-z fit” used the expected number of lens-induced extra images would increase by about a factor of 4. The redshift fit does not affect the results summarized in Figures 1 and 2.

The cosmological relation we used between  $C_{peak}$  and  $z_{GRB}$  also assumes a standard candle model for GRBs. A more realistic model involving a luminosity function for GRBs would, however, necessarily increase the probability of lensing, assuming the comoving number density of GRBs were unchanged.

Were GRBs to undergo significant luminosity or number density evolution, this could also confound the probability of lensing. We note that there is no known luminous object in the universe which is *not* thought to undergo significant evolution out to a redshift of order unity. The only way that evolution

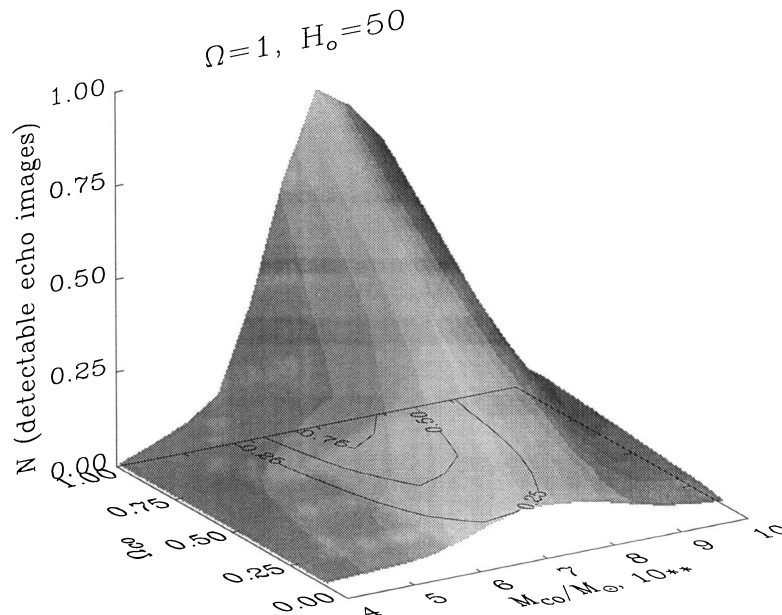


FIG. 3.—A plot incorporating the first available 44 BATSE detected GRBs showing how many detectable echo images would be expected for the given  $(\Omega_{CO}, M_{CO})$  values. We see that a lens-induced image is just about expected for a universe composed of between  $10^{6.5} M_{\odot}$  and  $10^{8.1} M_{\odot}$  COs. The null detection only marginally excludes this paradigm.

could reduce the probability of lensing, though, is if GRBs were either less numerous or less luminous in the past. The general trend of evolution, though, out to redshifts on order unity, is to create objects that are more numerous or luminous in the past, as hypothesized for QSOs (see, for example, Weedman 1986), and galaxies (see, for example, Koo and Kron 1992).

Future analysis of GRBs will be able to explore a larger range of the ( $\Omega_{\text{CO}}$ ,  $M_{\text{CO}}$ ) plane by increasing the number of bursts searched for echoes. Already, as of the publication of this paper, 10 times the number of bursts have been detected by BATSE than have been used in this analysis. Were these images also to show no sign of a gravitational lens-induced

echo, there would be a significant discrepancy between the number of images expected and the number observed for large regions of the ( $M_{\text{CO}}$ ,  $\Omega_{\text{CO}}$ )-plane. This result could either rule out two other popular universe models (Carr & Rees 1984; Gnedin & Ostriker 1992), one with  $\Omega = 1$  and the maximum amount of baryons in COs ( $\Omega_{\text{CO}} \simeq 0.15$ ) allowed by the baryons implied by nucleosynthesis elemental abundance analysis, and another with a  $\Omega = \Omega_{\text{CO}} \simeq 0.15$ , or indicate that GRBs do not lie at cosmological distances.

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