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ANOMALOUS ENERGY AND MOMENTUM TRANSFER OF INTENSE NEUTRINOS IN SUPERNOVA EXPLOSIONS

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ABSTRACT

A novel mechanism, by which the intense flux of neutrinos in a supernova explosion can effectively transfer energy and momentum into the stellar matter outside the neutrinosphere, is proposed. The energy and momentum rate can be greater by orders of magnitude than that conventionally accepted. Roughly, the new rate scales with the first power of the Fermi constant G_F, instead of the second power resulting from collision between individual neutrinos and fermions. Through scattering of neutrinos and sound waves, it increases the scattering cross section by a factor G_F^{-1}, which may shed new light on the problem of the stalled supernova explosion.

Subject headings: hydrodynamics — instabilities — supernovae: general

Late-type stars, of masses greater than 8 $M_\odot$, can undergo core collapse leading to a Type II supernova explosion. The explosion drives a strong outgoing shock wave and pushes the overlying stellar matter into the interstellar space. After the collapsed matter reaches a hard core, the shock is pushed out at an initially increasing speed. Because of the strong stellar gravity and loss of energy in dissociating nuclei, the shock has to strive to plow through the overlying matter to travel outward; the shock loses energy and slows down. Whether it can successfully escape to the interstellar space depends on the amount of the gravitational binding energy of the initially infalling matter and the details of nuclear reactions (Bethe 1990). In the past, theoretical calculations have shown that for progenitor stars of masses less than 25 $M_\odot$, either the explosion energy is too low and the time scale is too long to account for the typical Type II supernova light curves (several times 10^{51} ergs) (Wilson et al. 1986), or the explosion is inhibited and left with a stalled shock (Hillebrandt 1987). These calculations depend sensitively on the controversial equation of state in the stellar core and lead to different results. This problem is further exacerbated by SN 1987A. The explosion promptly gave out an energy of about 2×10^{51} ergs, and yet its progenitor star is believed to be a blue star of 15–20 $M_\odot$ (Arnett 1987; Woosley, Pinto, & Ensman 1988).

Although the ultimate energy source of the explosion is the gravitational binding energy of the infalling matter bounced back from the core, it is the neutrinos, released by converting protons into neutrons in the neutron stars, that carry most of the binding energy (several times 10^{53} ergs s^{-1}) (Burrows, Mazurek, & Lattimer 1981). In the extremely dense stellar core, neutrinos scatter matter rapidly and take about 1 s to diffuse out of the neutrino "photosphere." Because the shock density is relatively low, neutrinos leaving the neutrinosphere will pass through the shocks located about several hundred kilometers from the core. Neutrinos interact little with the shock, according to the conventional calculations of neutrino-fermion collision, and, not surprisingly, only a small fraction of neutrino energy and momentum is deposited into the shock. The bounced shock may become stalled due to an inadequate initial kinetic energy and a lack of subsequent energy input. In some calculations using a favorable equation of state, the small amount of additional neutrino heating can help the shock succeed in moving away (Bethe & Wilson 1985); however, the explosion energy is often found to be too low for typical Type II supernovae (Wilson et al. 1986). Generally speaking, the stalled shock problem of Type II supernovae is a serious one, and thus far no consistent theory has been developed to remedy this difficulty.

In our view, the difficulty may stem from the usual assumption of particle collision between the neutrinos and the electrons and nucleons, which results in too small a cross section for rapid transfer of neutrino energy and momentum. The cross section σ_ν of the neutrino-Fermion collision scales as G_F^2, where G_F is the Fermi constant (Bahcall 1989). The mean free path l_c and the time scale for the neutrinos to dump their streaming energy into the matter are respectively $(\sigma_\nu n)^{-1}$ and $(\sigma_\nu nc)^{-1}$, which also scale as G_F^{-2}, where c is the speed of light and n is the matter density. Approximately at the time when l_c exceeds the density scale length, neutrinos and matter will be decoupled, and the location where the decoupling occurs is called the neutrinosphere. Beyond the neutrinosphere, l_c becomes too large for neutrinos to exchange energy or momentum efficiently with matter.

In this work we propose a new mechanism of the neutrino-matter interactions through scattering of neutrinos with sound waves. We will show that in the presence of streaming neutrinos, the scattered sound waves grow at a rate scaled as $G_F n_\nu$, a much larger rate, where n_ν is the neutrino density. This result is not unlike the interactions of photons and electrons. For individual electrons to scatter photons, the cross section is the Thomson-scattering cross section, which scales as e^4. However, in the presence of streaming electrons, collective electromagnetic waves can be excited, whose growth rate is a fraction of the squared plasma frequency, scaled as $e^2 n_e$, where n_e is the electron density. Streaming neutrinos here are analogous to streaming electrons, and sound waves are analogous to electromagnetic waves. In fact, the analogy is not quite exact, since the way in which neutrinos interact with matter is different. As we will show later, entropy irregularities in the matter are required to make the sound waves grow at the aforementioned rate; otherwise, the growth rate will still scale as G_F^2.

In the downstream region of the strong supernova shock, there are fluctuations of all kinds. Besides sound waves and vortices, there should also be an abundance of entropy irregularities, which will be shown to trap the proposed unstable sound waves. We suggest that it is in this downstream region of the supernova shock that the physical conditions are appropriate for the matter to tap the neutrino energy efficiently beyond the conventional neutrinosphere, thereby making the explosion successful.

It has been pointed out that plasma waves can be excited by the streaming neutrinos (Bingham, Dawson, & Su 1993). This is in fact a second-order effect, because the growth rate scales as $G_F^2 n n_\nu$. Moreover, in the case of a supernova explosion, the stellar matter is too collisional for plasma waves to excite. We therefore consider the interaction of neutrinos and hydrodynamical waves. To justify this point, we shall make an estimate of the parameter regime of interest. For a collapsed star of 20 $M_\odot$, the electron density at the rebounced shock 1 s after the collapse is $\sim 10^{31}$ g cm^{-3}, and the electron temperature is of the order of mega–electron volts. The electron-electron collisional frequency is $\omega_p/n_e \lambda_D^3$, where ω_p and λ_D are the plasma frequency and the Debye length, respectively, and its value is about $10^{-3}\omega_p$, much larger than the growth rate of the plasma wave or even the frequency of the proposed sound wave. Consequently, plasma waves have no chance to grow in the presence of collisional damping of electrons.

This makes us turn to the fluid picture, where the collision between the fermions is so frequent that all species of fermions are tightly coupled. The only dissipation is through viscosity, which is dominated by the ion viscosity: $\eta_i \sim 10^{-10} T_6^{5/2} n_{31}^{-1}$ cm^{-2} s^{-1}, where the subscripts of temperature T and matter density n denote their values in units of 10^6 eV and 10^{31} particles cm^{-3}, respectively. For sufficiently long-wavelength sound waves, the fluid is effectively dissipationless for the waves to have a sufficient time to grow; we will come back to this point and make a comparison later.

The Lagrangian density of neutrinos in the presence of electrons can be written as

$$L_\nu + L_{\rm int} = \bar{\psi}_\nu (i\hbar \partial\!\!\!/ - m_\nu c^2)\psi_\nu - \frac{G_F}{\sqrt{2}} [\bar{\psi}_\nu \gamma^\lambda (1+\gamma_5)\psi_\nu][\bar{\psi}\gamma_\lambda(1+\gamma_5)\psi] , \quad (1)$$

where the first term represents the Lagrangian of free neutrinos and the second the interaction Lagrangian (Bethe 1986). The interaction Lagrangian is actually the coupling between neutrino current j_ν^λ and fermion current j_λ by the weak interaction manifested by the presence of the Fermi constant G_F. The projection operator $1+\gamma_5$ for neutrinos equals 2.

In the presence of fluid perturbation, the total Lagrangian density must include the fluid component. The linear sound wave obeys the perturbed fluid Lagrangian density, and, for the convenience of later application, we shall write the perturbed fluid and interaction Lagrangians alone:

$$\delta L_{\rm fluid} + \delta L_{\rm int} = \frac{1}{2} M n_0 \left[\left(\frac{\partial \xi}{\partial t}\right)^2 - C_s^2 (\nabla \cdot \xi)^2 \right] - \sqrt{2} G_F \left(\delta n \, \delta n_\nu - \frac{n_0 \, \delta \boldsymbol{v} \cdot \delta \boldsymbol{j}_\nu + \delta n \, \delta \boldsymbol{v} \cdot \boldsymbol{j}_{0\nu}}{c^2} \right) , \quad (2)$$

where ξ is the fluid displacement, C_s the sound speed, M the average nucleon mass, and the subscript zero denotes the equilibrium quantities. The perturbed quantities with subscript ν are for neutrinos, and those with no subscript are for matter. Furthermore, we have also assumed that the fluid perturbation is nonrelativistic.

The Euler-Lagrange equation of this perturbed Lagrangian can be obtained by using two additional constraints which relate δn and $\delta \boldsymbol{v}$ to ξ. The perturbed velocity is defined as $\delta \boldsymbol{v} = \partial \xi / \partial t$; furthermore, the pressure equation demands that $\delta P = -\Gamma P_0 \nabla \cdot \xi$, which is related to the density perturbation by $\delta n = (n_0/\Gamma P_0)\delta P \equiv \delta P / C_s^2$, where Γ is the adiabatic index and C_s is the sound speed. The Euler-Lagrange equation for the fluid perturbation is obtained by taking a variation with respect to the vector field ξ. The equation of motion follows

$$\left(\frac{\partial^2}{\partial t^2} - C_s^2 \nabla^2 - \frac{\sqrt{2} G_F \boldsymbol{j}_{\nu 0}}{Mc^2} \cdot \nabla \frac{\partial}{\partial t} \right) \delta P + \frac{\sqrt{2} G_F C_s^2}{Mc^2} \nabla^2 (n_0 \boldsymbol{j}_{0\nu} \cdot \delta \boldsymbol{v}) = \frac{\sqrt{2} G_F C_s^2}{Mc^2} \left(\frac{\partial}{\partial t} \nabla \cdot \delta \boldsymbol{j}_\nu + C_s^2 \nabla^2 \delta n_\nu \right) , \quad (3)$$

where we have multiplied $C_s^2 \nabla \cdot$ by the Euler-Lagrange equation. Upon taking the Fourier transformation in t and $\boldsymbol{x}$, we thus obtain

$$\left(\omega^2 - k^2 C_s^2 + \frac{2\sqrt{2} G_F \omega \boldsymbol{k} \cdot \boldsymbol{j}_{0\nu}}{Mc^2} \right) \delta P = \frac{\sqrt{2} G_F C_s^2}{Mc^2} (C_s^2 k^2 \delta n_\nu - \omega \boldsymbol{k} \cdot \delta \boldsymbol{j}_\nu) . \quad (4)$$

To solve equation (4), we need to relate δn_ν and $\delta \boldsymbol{j}_\nu$ to δn. This is achieved by a kinetic theory approach. We assume that the neutrino is ultrarelativistic and its wavelength is so short that the neutrino can be regarded as a particle. In a collisionless system, the distribution of the neutrinos, $f(\boldsymbol{p}, \boldsymbol{x}, t)$, must satisfy the Vlasov equation. The neutrinos experience a mean force given by the matter, as described by the interaction Lagrangian. To the lowest order in the limit of small C_s/c, the matter can be regarded as static, and the individual neutrino energy is modified by the presence of matter in the following manner:

$$E = \sqrt{2} G_F n + \sqrt{p^2 c^2 + m_\nu^2 c^4} , \quad (5)$$

where E is the neutrino energy and $\boldsymbol{p}$ the neutrino momentum. The Hamilton equations of motion for a particle read

$$\frac{d\boldsymbol{x}}{dt} = \frac{\partial E}{\partial \boldsymbol{p}} \approx c\hat{\boldsymbol{p}} , \quad \frac{d\boldsymbol{p}}{dt} = -\nabla E = -\sqrt{2} G_F \nabla \delta n , \quad (6)$$

where $\hat{\boldsymbol{p}}$ is the unit vector along the direction of particle motion. It follows that the linearized Vlasov equation becomes

$$\left(\frac{\partial}{\partial t} + c\hat{\boldsymbol{p}} \cdot \nabla \right) \delta f_\nu = \sqrt{2} G_F \nabla \delta n \cdot \frac{\partial f_{\nu 0}(\boldsymbol{p})}{\partial \boldsymbol{p}} , \quad (7)$$

where δf_ν and $f_{\nu 0}$ are the perturbed and equilibrium distribution functions, respectively.

Again, the Fourier transformation of this equation yields

$$\delta f = -\left\{ \left[G_F \boldsymbol{k} \cdot \frac{\partial}{\partial \boldsymbol{p}} f_{\nu 0}(\boldsymbol{p}) \right] \Big/ (\omega - \boldsymbol{k} \cdot \hat{\boldsymbol{p}} c) \right\} \delta n . \quad (8)$$

The desired δn_ν and $\delta \boldsymbol{j}_\nu$ are simply $\int \delta f d^3\boldsymbol{p}$ and $\int \boldsymbol{p} \, \delta f d^3 \boldsymbol{p}/m_\nu$, both of which are proportional to $G_F \delta n$. In doing the angular integral, one encounters a singularity associated with the wave-neutrino resonance $\omega = \boldsymbol{k} \cdot \hat{\boldsymbol{p}} c$, which gives rise to the Landau damping or inverse Landau damping depending on whether the local slope of $f_{\nu 0}$ is negative or positive. As the bulk neutrinos stream outward relative to the matter beyond the neutrinosphere, it is likely that $f_{\nu 0}$ has some range of positive momentum gradient for the excitation of the unstable sound waves.

When substituting δn_ν and $\delta \boldsymbol{j}_\nu$ back in equation (4), we find that terms on the right-hand side are of order G_F^2, and therefore the growth rate is of order G_F^2, comparable to the collisional frequency between fermions and neutrinos and too small for any improvement beyond the conventional calculation of a supernova explosion.

However, we can drastically improve the growth rate by a factor of G_F^{-1} by considering, instead of free-propagating sound waves, the sound waves trapped in temperature cavities. Because the temperature cavity must persist for a sufficiently long time for the excited sound wave to grow, it must be in pressure equilibrium with the ambient matter, which means the temperature cavity is also a density bump. Since both the Reynolds number and the thermal insulation in the stellar matter are very high, there can be plenty of density irregularities created behind the supernova shock, persisting for a long time. It is these irregularities that can tap the neutrino streaming energy effectively to excite unstable sound waves, thereby dissipating the neutrino energy into the stellar matter via hydrodynamical processes.

To illustrate this mechanism, we shall consider a simple one-dimensional model, where the temperature cavity is modeled as

$$C_s^2 = \begin{cases} C_2^2 & \text{for } -L < x < L \,, \\ C_1^2 & \text{otherwise} \,, \end{cases} \tag{9}$$

where $C_1 > C_2$. For a given ω and some range of k_y, there exist trapped sound waves.

The sound wave must die away outside the two surfaces at $x = \pm L$, as $e^{-\kappa_+ x}$ for $x > L$ and $e^{\kappa_- x}$ for $x < -L$. The real parts of both κ_+ and κ_- must be positive. Ignoring the G_F^2 terms, we have two dispersion relations outside the two surfaces:

$$\omega^2 - (k_y^2 - \kappa_\pm^2)C_1^2 + \frac{2\sqrt{2}G_F}{Mc^2}\,\omega k_y j_{\nu 0y} = \mp \frac{i2\sqrt{2}G_F}{Mc^2}\,\omega \kappa_\pm j_{\nu 0x} \,. \tag{10}$$

To show the instability, let us choose $\kappa_- = a + ib$, where $a > 0$ and $a \gg |b|$. Since the imaginary part of the frequency ω_i is much smaller than the real part ω_r, we can expand ω^2 in equation (10). It then results that

$$\omega_i = \frac{\sqrt{2}G_F j_{\nu 0x} a}{Mc^2} \,. \tag{11}$$

Since, to the leading order, the problem has symmetry about the $x = 0$ plane, the solution must be either symmetric or antisymmetric to the first power of G_F. Solving for κ_+ from equation (10), we find that $\kappa_+ = a + ib - i4\sqrt{2}G_F j_{\nu 0x}\omega_r/Mc^2C_1^2$. It naturally follows that $b = 2\sqrt{2}G_F j_{\nu 0x}\omega_r/Mc^2C_1^2$.

The interior region has standing waves, and they obey the same dispersion relation (eq. [11]), except that C_1 and $\kappa_\pm$ are replaced by C_2 and $\pm ik_x$, respectively. Again, for a given ω, we can solve for k_x. That is,

$$k_x = \pm[k_0 + (i\omega_i\omega_r/C_2^2 k_0)] + \omega_i\omega_r/C_2^2 a \,,$$

where

$$k_0 = \sqrt{k_y^2(C_1^2 - C_2^2) - a^2C_1^2}/C_2 \,.$$

The forward-traveling wave has slightly shorter wavelength than the backward traveling wave, resulting in an envelope of length scale $\omega_i\omega_r/C_2^2 a$; moreover, the amplitudes of both waves decrease slightly as they propagate along.

So, far, a has been arbitrary. It is determined by matching the solutions at the boundaries $x = \pm L$, which will yield discrete eigenvalues for a. The matching condition is to require that the pressure perturbation be continuous across the boundary. As the unstable modes of large growth rates require large a, therefore, from the expression for k_x, the instability favors large values of k_y. The self-excited modes will surely choose the smallest possible wavelengths. The most unstable waves oscillate many times within a distance of $2L$, and the discrete values of a are densely distributed and can be regarded as a continuum.

This slab model of a temperature cavity is just to illustrate the principle behind the instability. Since the most unstable waves are of short wavelength, any three-dimensional temperature cavity, which does not necessarily have a sharp temperature boundary as used in this toy model, can trap sound waves and make this instability operate. However, the growth rate of the sound wave can be reduced substantially when the wave is trapped by temperature irregularities of gentle gradients. This is because, first, the growth rate is proportional to the inverse length scale of the wave in the evanescent region, and, second, this length scale is comparable to the wavelength, which becomes larger as the wave approaches the boundaries. A short-wavelength sound wave propagating in a medium of gentle gradient can be described by

$$\exp\left[-i\omega t + ik_y y + i\int k_x(x)dx\right] ,$$

where

$$\int k_x\,dx = \pm \int \sqrt{\frac{\omega^2}{C_s^2(x)} - k_y^2}\,dx$$

$$\approx \pm \frac{\omega}{C_s}\int \sqrt{\frac{\Delta x}{L}}\,dx = \pm \frac{2\omega}{3C_s}\sqrt{\frac{\Delta x^3}{L}} \tag{12}$$

near the boundary. The above approximate equality arises from an expansion of the sound speed near the evanescent boundary, and L is the length scale of the temperature cavity. Apparently, the inverse length scale of the wave is of order $(\omega^2/C_s^2 L)^{1/3}$, smaller than that in a sharp boundary by a factor $(C_s/\omega L)^{1/3}$. Therefore, for temperature irregularities of general shape, we expect that the sound wave should grow at a rate

$$\omega_i \sim \frac{G_F j_{\nu 0}}{Mc^2}\left(\frac{1}{\lambda^2 L}\right)^{1/3} , \tag{13}$$

where λ is the average wavelength in the temperature cavity.

The growth rate given by equation (13) scales as the first power of G_F, and it is a fraction of a factor $(G_F j_{\nu 0}/Mc^2 C_s)(\lambda/L)^{1/3}$ of the wave frequency. In the case of a supernova explosion, this is about $10^{-12}(\lambda/L)^{1/3}R_{500}^{-2}$, given that the neutrino flux is about 3×10^{53} ergs s^{-1}, $Mc^2 \sim 1$ GeV, the matter and neutrino temperatures both about 1 MeV, and the location of the shock at about 500 km. As an example, since the proton-proton collisional mean free path is roughly

10^{-7} cm, let us choose a sound wave of wavelength, say, 10^{-5} cm and a cavity of length $L = 1$ cm. Such a wave will grow in 1 s, which is just offset by the viscous damping, at a time also about 1 s. Since the viscous damping scales as λ^{-2}, for sound waves with a slightly longer wavelength or sound waves trapped by a smaller cavity the neutrino-driven growth can dominate the viscous damping, resulting in a net growth. After the sound wave grows in several *e*-folding times, the wave energy can begin to be dissipated by nonlinear processes, whereby the neutrino streaming energy and momentum are indirectly transferred to the stellar matter. Note that the growth rate depends rather weakly on the $\frac{1}{3}$ power of L but sensitively on the second power of the location of the shock. The neutrino energy must be dumped to the matter at radii not much larger than 500 km; otherwise, the growth rate of the wave will be too weak to overcome the viscous damping. On the other hand, the intense neutrinos can only come out of the neutrinosphere in a time on the order of 1 s, which means the shock must have traveled for a distance on the order of a few hundred kilometers before the intense neutrinos arrive. Therefore, one expects that the stalled shock will only receive an additional kick after few seconds of the core collapse. Whether such a kick can produce a few times 10^{51} ergs s^{-1} of luminosity for a 15–20 $M_{\odot}$ progenitor star remains a question, which can only be answered by a detailed study.

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