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SOURCES OF SCATTER IN THE FUNDAMENTAL PLANE AND THE D_n-σ RELATION[1]

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ABSTRACT

On the basis of new photometric data for early-type galaxies in the Coma Cluster we have determined the relation $\log r_e = a \log \sigma + b\langle\mu\rangle_e + c$ known as the fundamental plane (FP). The scatter in the FP is equivalent to 11% uncertainty on distances. The residuals for the FP show no dependence on other available photometric and geometric parameters. Analysis of a nearby sample of ellipticals indicates that differences in dynamical structure for these galaxies may cause a scatter in the FP equivalent to 10% uncertainty on the distances. Thus, it may be possible to decrease the scatter by using the mean velocity dispersion within half the effective radius instead of the central velocity dispersion.

The scatter in the D_n-σ relation for the Coma Cluster galaxies is 17%, much higher than the 11% scatter for the FP. The residuals depend on the mean surface brightness. A similar result was obtained by Lucey, Bower, & Ellis, who used a linear fit in a restricted interval in $\langle\mu\rangle_e$ in order to incorporate the effect in the distance determination. The D_n-σ relation is an approximation to the FP, as first noted by Dressler et al. and Phillips. We find that the dependence on $\langle\mu\rangle_e$ can be understood (without restrictions in $\langle\mu\rangle_e$) as originating from this approximation. The FP projects into a slightly tilted and bended surface in the $\log D_n - \log \sigma - \langle\mu\rangle_e$ space. The available data in literature support this interpretation. Under the assumption that the FP is linear and the galaxies follow a de Vaucouleurs $r^{1/4}$ law, the D_n-σ relation can cause systematic errors of the order 10%–15% on distance determinations. It is possible that some existing discrepancies between distances determined from the D_n-σ relation and from the Tully-Fisher method or the surface brightness fluctuation method are caused by the systematic errors of the D_n-σ relation. There is no evidence that the large-scale motions in the universe derived by Lynden-Bell et al. on the basis of the D_n-σ relation are severely affected by the systematic errors. We conclude that the FP is to be preferred as a distance indicator.

Subject headings: galaxies: clustering — galaxies: distances and redshifts — galaxies: elliptical and lenticular, cD — galaxies: fundamental parameters — galaxies: photometry

1. INTRODUCTION

Understanding the methods for distance determination is critical when mapping large-scale motions for galaxies in the universe. In recent years two new methods for early-type galaxies have been invented, namely the fundamental plane (Djorgovski & Davis 1987) and the D_n-σ relation (Dressler et al. 1987b). The methods can be regarded as improvements of the Faber-Jackson method (Faber & Jackson 1976).

The fundamental plane (FP) relates three parameters for the galaxies in a relation linear in log-space: the effective radius r_e, the mean surface brightness $\langle SB\rangle_e$ within r_e, and the central velocity disperion $\sigma_{\rm cen}$. It follows that FP can be used for distance determinations. The physical origin of the FP is not clear. If we assume that ellipticals are well described by an $r^{1/4}$ law, and that their dynamical structure is similar, then the virial theorem implies that the mass-to-light ratio, M/L, can be expressed as a function of the parameters r_e, $\sigma_{\rm cen}$, and $\langle SB\rangle_e$, only. If some physical process during galaxy formation causes the M/L ratio to be a unique function of the same parameters, a tight relation is expected between r_e, $\sigma_{\rm cen}$, and $\langle SB\rangle_e$ (see also Djorgovski, de Carvalho, & Han 1988). It is not known with certainty what this physical process is. The FP has not yet been used for distance determinations for a large sample of galaxies. It requires reliable determinations of r_e and $\langle SB\rangle_e$ and these are difficult to achieve from aperture photometry. However, CCD surface photometry makes the determination possible (see, e.g. Jørgensen, Franx, & Kjærgaard 1992a).

The D_n-σ relation has been used by both its inventors, the group called the "seven Samurai" (Dressler et al. 1987a, b; Lynden-Bell et al. 1988; Faber et al. 1989), and by others (e.g., Lucey & Carter 1988; Lucey et al. 1991a, b). Dressler et al. (1987b) introduced the diameter parameter D_n, which combines the surface brightness and the effective radius. Dressler et al. found that $D_n \propto r_e\langle SB\rangle_e^{0.8}$ for normal elliptical galaxies, and that D_n therefore provides an edge-on view of the FP. Dressler et al. interpreted the D_n-σ relation as a reformulation of the FP (we will return to this topic in § 5). D_n has the advantage that it can be determined reliably from aperture photometry. The seven Samurai have used the D_n-σ relation to determine distances to more than 400 galaxies.

The D_n-σ relation gives distances to single galaxies accurate to about 20%. But severe discrepancies between distances derived from the D_n-σ relation and from the infrared Tully-Fisher relation (Lucey et al. 1991b; Mould et al. 1991) remain unexplained. Also comparisons with distances derived from the surface fluctuation method show discrepancies (Tonry 1991). The scatter in the D_n-σ relation does not seem to be constant from cluster to cluster (Lynden-Bell et al. 1988) and is not well understood. Recently Lucey et al. (1991a) found that the residuals of the D_n-σ relation correlate with the mean surface

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brightness, $\langle\mu\rangle_e$, of the galaxies. Lucey et al. used this to improve the relation by including a surface brightness term. In a later paper, however, Lucey et al. (1991c) found no significant dependence on $\langle\mu\rangle_e$ for a sample of 51 galaxies in the Coma Cluster.

It is very important to understand the differences between the FP and the D_n-σ relation to test the universality of these relations and understand the causes of the internal scatter. First then the reliability of distances derived from the different methods can be judged.

On bases of new photometric data for the Coma Cluster (Jørgensen et al. 1992a) we here determine the FP and the D_n-σ relation for the cluster and investigate the sources of scatter in the two relations.

The paper is organized as follows: The data are shortly described in § 2. The FP for the Coma Cluster is determined in § 3. In § 4 we investigate whether the scatter can be reduced by inclusion of extra parameters, and we estimate the scatter caused by differences in photometric and dynamical structure. The D_n-σ relation for Coma is determined in § 5 and the differences between the FP and the D_n-σ relation are analyzed. The dependence of the residuals for the D_n-σ relation on the mean surface brightness is investigated. In § 6 we discuss the systematic errors in peculiar velocities derived from the D_n-σ relation. The conclusions are given in § 7.

2. THE DATA

CCD photometry for a sample of 33 elliptical and S0 galaxies in the Coma Cluster has been obtained; see Jørgensen et al. (1992a) for a full description of reduction procedures and determination of global parameters. The galaxies have been observed in Johnson B and Gunn r. Here we use the global parameters derived by Jørgensen et al.: effective radius, r_e, mean surface brightness within this radius, $\langle\mu\rangle_e$, and the radius parameter r_n. In Johnson B r_n is the radius within which the mean surface brightness in Johnson B is 20.75 mag arcsec^{-2}. This is the same definition as used for D_n by the seven Samurai (Dressler et al. 1987b) and by Lucey & Carter (1988) and Lucey et al. (1991a, b). In Gunn r we use 19.60 mag arcsec^{-2} for the definition of r_n. The radii in Gunn r and Johnson B are very similar as the mean color of the galaxies is $(B - r) = 1.15$. All magnitudes were corrected for galactic extinction, K-corrected, and corrected for cosmological dimming. Photometric parameters like color, color gradient, and geometrical parameters for the galaxies have also been taken from Jørgensen et al. (1992a).

The parameters r_e, $\langle\mu\rangle_e$, and r_n from Jørgensen et al. agree very well with data from Faber et al. (1989); the mean offsets and uncertainty on the mean offsets are ("our"-"their") $\Delta \log r_e = -0.002 \pm 0.009$, $\Delta\langle\mu\rangle_e = -0.046 \pm 0.037$ (31 galaxies from the whole sample by Jørgensen et al. with "quality" 1 data from Faber et al.), and $\Delta \log r_n = 0.002 \pm 0.003$ (38 galaxies including four S0 galaxies from Dressler 1987). The photometry in Johnson B agrees within 0.02 mag with the photoelectric photometry from Burstein et al. (1987).

We compared our Gunn r parameters to the Johnson V parameters given by Lucey et al. (1991a). We assumed a constant color $(V - r) = 0.20$ [14 of the galaxies in common have $(B - V)$ from Faber et al., the mean is 0.95]. The mean offsets and uncertainty of the mean offsets are ("our"-"their") $\Delta \log r_e = 0.036 \pm 0.014$, $\Delta\langle\mu\rangle_e = 0.12 \pm 0.05$, and $\Delta \log r_n = 0.001 \pm 0.003$ (22 galaxies in common). The cause of the systematic difference in $\log r_e$ (which is reflected in $\langle\mu\rangle_e$, too) is not clear. For early-type galaxies, for which the color turns bluer at larger radii, it is expected that $\log r_e$ in Gunn r is smaller than in Johnson V. It is remarkable that there is no systematic difference in $\log r_e$ for the comparison with the data from Faber et al.

The central velocity dispersions for the galaxies are from Faber et al. (1989) (E galaxies) and Dressler (1987) (S0 galaxies). The median offset between the original velocity dispersions published by Dressler et al. (1987b) and those from Faber et al. is 0.013. This correction has been added to the velocity dispersions from Dressler (1987) in order to correct these to the system of Faber et al.

For the 27 galaxies in Coma observed in both colors and with known velocity dispersions Table 1 lists the velocity dispersions together with the photometric parameters as published by Jørgensen et al. (1992a). The table also includes the c_4-parameter and the ellipticity for the galaxies. The measurement errors of the parameters are typically: $\log r_e$: ± 0.037, $\langle\mu\rangle_e$: ± 0.15, $\log r_n$: ± 0.015, $\log \sigma$: ± 0.025.

3. THE FUNDAMENTAL PLANE FOR COMA

The FP has been determined by minimizing the residuals in each of the parameters independently and using the geometrical average of the coefficients as the final fit. Galaxies classified as S0 by Dressler (1980) have been excluded from the determination. We have applied the same lower limit on $\log \sigma$ as Lynden-Bell et al. (1988) and have therefore excluded the galaxy no. 107 (identification from Dressler 1980) as it has $\log \sigma < 2.0$. The sample then consists of 22 galaxies. For the two colors we find

Johnson B:

$$\log r_e = \underset{\pm 0.09}{1.203} \log \sigma + \underset{\pm 0.011}{0.352}\langle\mu\rangle_e - \underset{\pm 0.36}{9.310} \qquad \sigma_{\rm fit} = 0.046\,,$$

Gunn r: (1)

$$\log r_e = \underset{\pm 0.09}{1.256} \log \sigma + \underset{\pm 0.011}{0.354}\langle\mu\rangle_e - \underset{\pm 0.33}{9.041} \qquad \sigma_{\rm fit} = 0.043\,.$$

The scatter in the FP corresponds to errors on distances to single galaxies of only 11%. This is remarkably smaller than the 20% found by Dressler et al. (1987b) and by Djorgovski & Davis (1987). These estimates were based on more clusters. Lucey et al. (1991a) reported an even lower scatter for a sample of galaxies in the Coma Cluster. They excluded the galaxies with the lowest surface brightnesses.

Dressler et al. used a combined fit for galaxies in the Coma Cluster and galaxies in Virgo. We have determined the scatter of their data relative to their fit, $\log r_e = 1.33 \log \sigma + 0.332\langle\mu\rangle_e + \text{const}$, to be 13%–14% for Coma and Virgo. The photometric data for Coma galaxies from Faber et al. (1989) produce the same FP within the uncertainty as given in equation (1) and with the same scatter.

The sample of both field galaxies and galaxies from many different clusters analyzed by Djorgovski & Davis shows a larger scatter of 20%. This can be partly due to the uncertainties in the distances to their galaxies, their large measurement errors, and to truly larger scatter in the field (see de Carvalho & Djorgovski 1992).

An estimate of the measurement errors is needed to derive the intrinsic scatter in the FP. The errors in $\log r_e$ and $\langle\mu\rangle_e$ are

TABLE 1

DATA FOR THE COMA CLUSTER SAMPLE

			Johnson B			Gunn r				
Galaxy	Type	log σ	log r_e	$\langle\mu\rangle_e$	log r_n	log r_e	$\langle\mu\rangle_e$	log r_n	c_4	ϵ
31 NGC 4839	E/S0	2.413	1.49	22.49	0.94	1.45	21.14	0.97	0.009	0.417
49 NGC 4926	E	2.431	1.10	21.24	0.95	1.06	19.89	0.97	−0.009	0.132
69 IC 3959	E	2.301	0.78	20.81	0.77	0.77	19.60	0.77	0.007	0.135
70 IC 3957	E	2.176	0.71	20.98	0.64	0.65	19.61	0.64	0.000	0.073
105 NGC 4869	E	2.312	0.97	21.31	0.81	0.93	19.97	0.83	−0.003	0.103
106	S0	2.214	0.46	20.44	0.53	0.45	19.28	0.53	0.006	0.143
107	E	1.845	0.87	22.11	0.41	0.83	20.87	0.39	0.009	0.408
118 NGC 4906	E	2.225	0.96	21.60	0.71	0.93	20.33	0.72	−0.006	0.127
124 NGC 4876	E	2.260	0.79	20.95	0.73	0.80	19.82	0.74	0.001	0.306
125	E	2.223	0.28	19.63	0.57	0.27	18.47	0.57	0.007	0.057
128	S0	2.032	0.57	21.12	0.44	0.52	19.80	0.44	0.037	0.523
129 NGC 4874	D	2.389	1.75	23.12	0.98	1.73	21.85	1.00	0.004	0.112
130 NGC 4872	E/S0	2.326	0.55	20.01	0.74	0.54	18.82	0.75	0.017	0.048
131	S0	2.238	0.99	21.70	0.70	0.98	20.49	0.71	0.032	0.391
133 NGC 4867	E	2.346	0.58	20.01	0.77	0.54	18.78	0.76	0.003	0.283
136	E	2.262	0.27	19.65	0.55	0.23	18.30	0.56	0.016	0.250
143 IC 4051	E	2.348	1.36	22.58	0.78	1.30	21.18	0.81	−0.006	0.269
148 NGC 4889	D	2.581	1.54	21.94	1.18	1.51	20.59	1.20	−0.004	0.364
150 IC 4011	E	2.025	0.85	21.68	0.56	0.78	20.34	0.55	0.006	0.088
151 NGC 4886	E	2.215	1.06	21.83	0.74	1.02	20.57	0.73	0.004	0.018
152	SB0	2.205	1.06	22.30	0.59	1.05	21.09	0.59	0.034	0.334
153	E	2.130	0.68	21.22	0.54	0.66	19.98	0.55	0.003	0.008
159 NGC 4864	E	2.297	0.95	21.18	0.82	0.93	19.95	0.83	...	0.158
168 IC 4045	E	2.324	0.73	20.37	0.83	0.69	19.03	0.84	0.009	0.332
217 NGC 4881	E	2.340	1.07	21.54	0.84	1.07	20.38	0.84	0.002	0.048
239 NGC 4841B	E	2.360	1.07	21.54	0.84	0.99	20.10	0.84	0.004	0.037
240 NGC 4841A	E	2.422	1.30	21.73	1.01	1.24	20.40	1.01	−0.009	0.165

NOTES.—Galaxy identifications and types—from Dressler (1980). log σ—from Faber et al. (1989) and Dressler (1987). The values from Dressler have been corrected for the median offset relative to the Faber et al. system by adding 0.013. Photometric parameters—from Jørgensen et al. (1992a). Note that $\langle\mu\rangle_e$ listed in this table is corrected for the cosmological dimming. ϵ is the ellipticity at r_e (Gunn r). c_4 is the maximum or minimum of the coefficient for the cos 4 ϕ-term in a Fourier expansion of the deviation of the isophotes from ellipses.

correlated; Figure 1, adapted from Jørgensen et al. (1992a), shows $\Delta\langle\mu\rangle_e$ versus Δ log r_e when comparing our data with data for Faber et al. The correlation coefficient for the "quality 1" data from Faber et al. is 0.93. With typical uncertainties in log r_e and $\langle\mu\rangle_e$ of 0.037 and 0.15, respectively, this results in a typical uncertainty of 0.023 on log $r_e - 0.35\langle\mu\rangle_e$, which is the combination used in the FP. If the correlation is ignored, the error analyses would result in an incorrect value of 0.064. The dashed line in Figure 1 is the expected correlation resulting from fitting an $r^{1/4}$ growth curve. The solid line is the combination for which the point moves within the FP. A direct comparison of log $r_e - 0.35\langle\mu\rangle_e$ with data from Faber et al. (1989) indicates an uncertainty of 0.021. Because the correlation between the errors in the two parameters nearly matches the combination used in the FP the combined contribution is lower than the individual errors. Using 0.022 as the contribution from measurement errors on log $r_e - 0.35\langle\mu\rangle_e$ we find an intrinsic scatter for the FP of 0.027 in Johnson B and 0.019 in Gunn r. The intrinsic scatter is equivalent to a distance uncertainty of 6% and 4.5%, respectively.

It is surprising that the scatter in the FP for Coma is so small. The scatter can, however, not fully be explained by measurement errors. In the following we, therefore, investigate the cause of the intrinsic scatter and the possibility of extra parameters in the FP.

4. SOURCES OF SCATTER IN THE FUNDAMENTAL PLANE

4.1. *Extra Parameters*

In order to find eventual extra parameters in the FP we have studied the dependence of the residuals of other available parameters from our surface photometry (Jørgensen et al. 1992a).

The residuals show no dependence on color or color gradient. We have tested if the residuals for the FP and for a linear relation between the color $(B - r)$ and log σ are correlated. This is not the case. The absence of any correlation involving the color indicates that for this sample of galaxies effects of recent star formation are not present, or do not have any influence on the scatter in the FP.

The residuals in the FP are insensitive to the isophotal twist defined as the logarithmic radial gradient in the position angle, ΔPA/Δ log r.

Figure 2 shows the residuals plotted against the ellipticity at r_e. The dashed line shows the projection of an oblate galaxy with intrinsic ellipticity 0.4. The ratio of the intrinsic velocity dispersions for this galaxy was calculated with the tensor virial theorem (Binney & Tremaine 1987). The dependence of ellipticity, surface brightness, effective radius, and observed velocity dispersion on inclination angle can be calculated in a straightforward way. Figure 2 demonstrates that the galaxy is projected along a trajectory nearly parallel to the FP, as found earlier by Faber et al. (1987). The contribution to the scatter from projection effects is smaller than 6%.

Bender et al. (1989) found boxy galaxies to have a higher mass-to-light ratio (M/L) than galaxies containing a disk. If the M/L ratio depends on the shape of the galaxy it could be a cause of scatter in the FP. In Figure 3 we plot the residuals for the FP against c_4. c_4 is the maximum or minimum of the Fourier coefficient for the cos 4 ϕ-term in a Fourier expansion of the deviation of the isophotes from ellipses (Jørgensen et al.

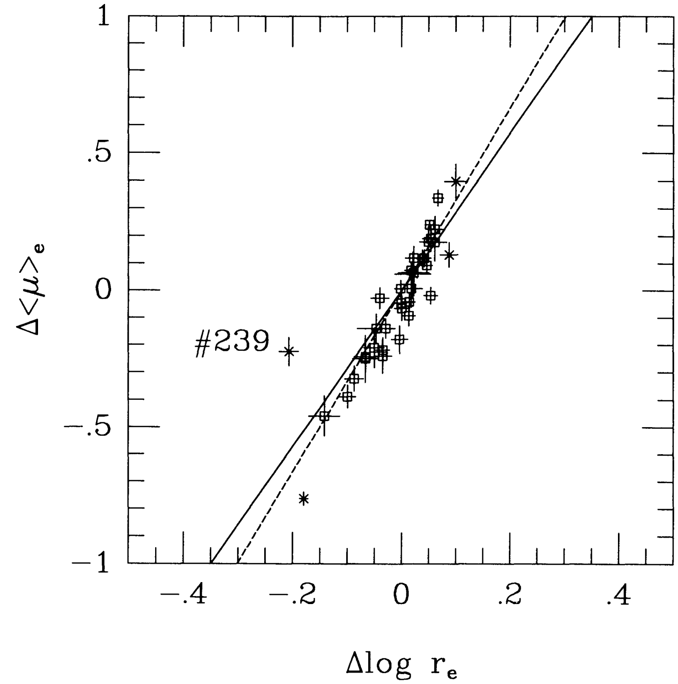

FIG. 1.—Correlation between errors in log r_e and $\langle\mu\rangle_e$. The figure shows the differences "our"-"their" for the comparison with data from Faber et al. (1989). Data of "quality 1" (Faber et al.) are plotted as boxes, "quality 2" as crosses. The error bars are our internal uncertainties. The dashed line shows the expected correlation resulting from fitting an $r^{1/4}$ growth curve, Δ log $r_e = 0.302\Delta\langle\mu\rangle_e$. The solid line shows for which correlation the errors would move a data point within the FP, but not away from the plane. The correlation coefficient for the differences for galaxies having "quality 1" data from Faber et al. is 0.93.

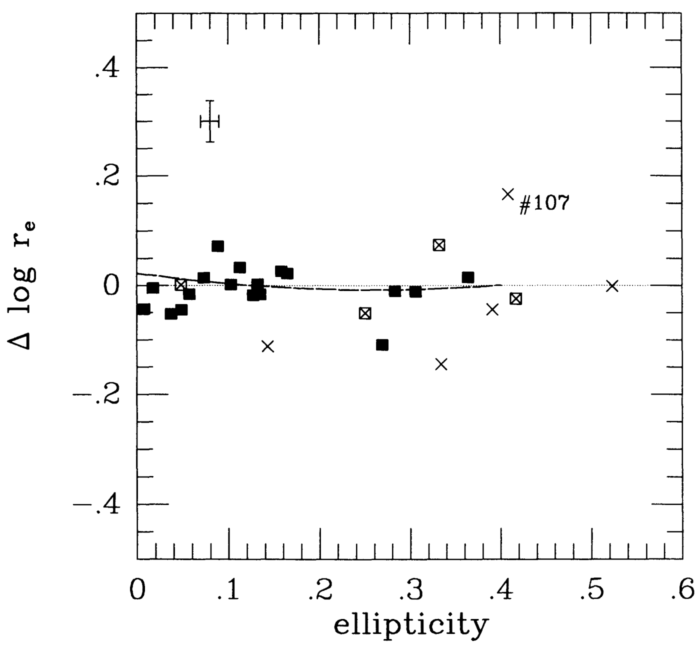

FIG. 2.—The residuals, log r_e − fit, for the determined FP (Gunn r) plotted against the apparent ellipticity at the effective radius. S0 galaxies and no. 107: *crosses*; E galaxies with $c_4 > 0.008$; *squares with crosses*; and E galaxies with $c_4 \leq 0.008$: *solid squares*. Typical errors are shown in the upper left corner. The residuals show no dependence on the apparent ellipticity. The dashed line shows the projection of an oblate galaxy with intrinsic ellipticity of 0.4. The galaxy projects along the FP within 6%.

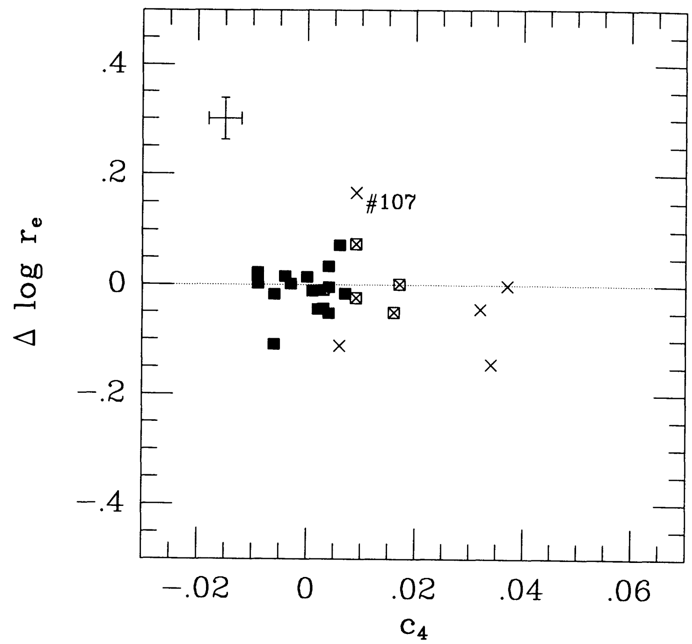

FIG. 3.—The residuals for the determined FP (Gunn r) plotted against c_4. Symbols as in Fig. 2. Typical errors are shown in the upper left corner. The residuals do not depend on c_4. Thus the FP can be used without knowledge of the presence of a weak disk or "boxiness" of the galaxies.

1992a; Franx, Illingworth, & Heckman 1989). As so c_4 is a tracer of the presence of a disk (c_4 positive) and of the "boxiness" of the galaxy (c_4 negative). There is no dependence of the residuals on c_4. A similar result was obtained by van der Marel (1991) for a sample of mainly field galaxies. Thus it is not critical for the use of the FP if the galaxies are disky or boxy. It requires a larger sample to investigate if the FP for S0 galaxies is the same as for elliptical galaxies. Inclusion of the four S0 galaxies with known log σ increases the scatter to 0.053 (in Johnson B).

It has been suggested that cluster environment could give differences in the FP (Silk 1989; Djorgovski et al. 1988; de Carvalho & Djorgovski 1992; see also Burstein, Faber, & Dressler 1990). Our sample in Coma does not cover a large enough range in distance from the cluster center to test for such effects. Lucey et al. (1991c) have analzed a large sample of galaxies in Coma covering a factor of 150 in projected galaxy density. They found that the FP zero point does not depend on the distance from the cluster center. Note that this result was not correctly attributed to these authors in Jørgensen et al. (1992b).

4.2. *How Similar is the Structure of Galaxies?*

It is well documented that galaxies show deviations from the $r^{1/4}$ law, and that they show differences in dynamical structure. Such differences between galaxies may cause scatter in the FP, as the current definition employs only the central velocity dispersion and the parameters from the $r^{1/4}$ law fit. A rigorous application of the tensor virial theorem may reduce such scatter, but that requires two-dimensional photometric and kinematic data. Not many galaxies have two-demensional kinematic data, but there is a reasonable sample of galaxies with kinematic data along the apparent major and minor axis. van der Marel (1991) modeled the available data, under the assumption that galaxies are axisymmetric and have a simple distribution function. We used his sample to estimate how much scatter is produced by the structural differences between galaxies.

First, we tested whether the use of the $r^{1/4}$ law profile with constant ellipticity instead of the true photometric profile introduces significant scatter. We used van der Marel's software to deproject the observed and synthetic photometric profiles and calculated the intrinsic velocity dispersions. The projected second velocity moment $\mu_2 = (\sigma^2 + v_{\rm rot}^2)^{1/2}$ was evaluated, and the luminosity weighted moment within an effective radius $\mu_2(r_e)$. This second moment is less sensitive to the details of dynamical structure of the galaxies than the central velocity dispersion. The second moments based on the photometry $\mu_2(r_e)$ and the moments based on the $r^{1/4}$ law fit $\mu_{2,\,1/4}(r_e)$ compare very well. The scatter in the ratio is 0.9%, so the contribution to the scatter in the FP is equivalent to uncertainties in the derived distances of the order of 1%. This quantifies the photometric similarity of our sample of elliptical galaxies. This estimate ignores the contribution to the scatter due to disks which are probably present in a large fraction of ellipticals (e.g., Rix & White 1990). Thus, the result may not hold for samples taken at larger redshifts, where accurate galaxy classification is much more difficult.

Next we estimated the possible scatter due to differences in dynamical structure. It is well known that anisotropies in the velocity dispersions can cause large differences in velocity dispersion profiles. The central velocity dispersions can vary widely for spherical models with a fixed density profile (Richstone & Tremaine 1984). These authors found that the maximum allowed variation of the velocity dispersion within circular apertures to be $\Delta \log \sigma = 0.41$ for apertures with a radius of $0.1r_e$. The variation decreases to 0.09 for an aperture with $r = r_e$.

We take an aperture of $0.1r_e$ as the typical aperture size for determination of central velocity dispersions for nearby galaxies like those in van der Marel's sample. We calculated the observed mean central velocity dispersions within $0.1r_e$, $\sigma_{0.1r_e}$, for these galaxies from the long-slit kinematic data. We found a scatter in $\log [\mu_2(r_e)/\sigma_{0.1r_e}]$ of 0.04, equivalent to uncertainties of 10% in the distances derived from the FP. This scatter is much smaller than allowed by the dynamical models, and ellipticals must, therefore, have rather similar dynamical structure, presumably caused by the formation process. However, the scatter is large enough to cause significant uncertainties in distances.

If we instead compare $\mu_2(r_e)$ with $\bar{\sigma}$, which is the unweighted average of the velocity dispersion inside $r_e/2$, calculated from the long-slit kinematics, the scatter is equivalent to a distance uncertainty of 8%. The scatter is lowest, equivalent to 6% uncertainty on distances, if we compare $\mu_2(r_e)$ to $(\bar{\sigma}^2 + v_{\rm rot}^2)^{1/2}$, $v_{\rm rot}$ is the rotational velocity.

It is difficult to verify whether the use of the $\mu_2(r_e)$ will indeed lower the scatter in the FP, as most galaxies in van der Marel's sample are field galaxies. His six galaxies in Virgo show a scatter of 0.064 with respect to the FP when the mean velocity dispersion within 2″ is used. Using $\mu_2(r_e)$ gives a scatter of 0.054. This indicates an improvement. The same can be achieved by using $\sigma_{0.1r_e}$ or $\bar{\sigma}$, which give a scatter of 0.052 and 0.048, respectively.

We conclude that long-slit observations of a well-defined cluster sample are needed to demonstrate convincingly that the differences in dynamical structure cause as much as 10% scatter in the FP. The modeling results imply that this contribution will decrease for galaxy samples taken at larger redshift, as the central velocity dispersions will be taken through apertures with larger physical size. Most likely, the differences in dynamical structure cause scatter lower than 10% for the Coma Cluster sample.

As a by-product of this analysis it is interesting to study the dependence of the M/L ratio on the luminosity. If the ratio $\mu_2(r_e)/\sigma_{0.1r_e}$ depends on luminosity or velocity dispersion, then the dependence of the M/L ratio on these parameters is wrongly estimated if $\sigma_{0.1r_e}$ is used to estimate the M/L ratio.

We find a weak dependence of $\mu_2(r_e)/\sigma_{0.1r_e}$ on $\sigma_{0.1r_e}$ and luminosity:

$$\mu_2(r_e)/\sigma_{0.1r_e} \propto \sigma_{0.1r_e}^{-0.11\pm0.05} \propto L^{-0.05\pm0.02} .$$

Thus the dependence of M/L on the luminosity will be overestimated if we use $\sigma_{0.1r_e}$ instead of $\mu_2(r_e)$. Using central velocity dispersions, Faber et al. (1987) derived that the M/L ratio depends on luminosity as $M/L \propto L^{0.25}$. Our results indicate that the coefficient may be too large by 0.10. We note, however, that the sample of galaxies is incomplete, and is biased toward round galaxies.

Van der Marel concluded the opposite from the same data—he found $M/L \propto L^{0.35}$ His conclusion was based on a direct correlation between the luminosity and the M/L ratios he derived from modeling the data. This analysis has the disadvantage that the derived M/L ratios depend on the assumed (uncertain) distances of the galaxies. If we use the same distances as van der Marel, and the central velocity dispersions of the galaxies to estimate the M/L ratio, we find a similar strong dependence of M/L on luminosity. We conclude, therefore, that van der Marel's result is due to the distance uncertainties and the incompleteness of the sample. The results from our differential analysis should be more robust.

5. THE D_n-σ RELATION AND SOURCES OF SCATTER

With the same selection of galaxies and the same fitting method as for the FP we find the following D_n-σ relations

Johnson B:

$$\log r_n = \underset{\pm 0.15}{1.395} \log \sigma - \underset{\pm 0.34}{2.428} \quad \sigma_{\rm fit} = 0.073$$

Gunn r: (2)

$$\log r_n = \underset{\pm 0.15}{1.451} \log \sigma - \underset{\pm 0.34}{2.549} \quad \sigma_{\rm fit} = 0.073 .$$

This is in agreement with the result from Dressler et al. (1987b). The scatter in the D_n-σ relation is remarkably larger than in the FP. It corresponds to 17% error on distances. Lynden-Bell et al. (1988) found a scatter of 16% for Coma and 19% for Virgo. We derive an intrinsic scatter of 0.062. If the D_n-σ relation was just an alternative expression of the FP the intrinsic scatter should be the same for the two relations. The difference indicates that they are actually not identical. Since the same values for the velocity dispersions are used for both the FP and for the D_n-σ relation the increased scatter must be due to the use of r_n instead of r_e and $\langle \mu \rangle_e$.

When Dressler et al. (1987b) introduced D_n, they showed that

$$r_n \propto r_e \langle SB \rangle_e^{0.8} . \quad (3)$$

This relation is expected for galaxies with $r^{1/4}$ profiles and surface brightnesses around 21.8 mag arcsec^{-2} (Johnson B) (Phillips 1988). If equation (3) was exact, the D_n-σ relation in Johnson B given in equation (2) should have a scatter equal to the scatter of the FP with coefficients 1.395 and 0.32. The scatter for the FP with these coefficients is 0.055, lower than the scatter for the D_n-σ relation.

The cause for the discrepancy is the approximate nature of equation (3). Dressler et al. (1987b) showed that r_n/r_e is a function of $\langle SB \rangle_e$, only. Phillips (1988) generalized this by introducing a general radius parameter R_n, within which the mean surface brightness has some particular value $\langle SB \rangle_n$. He showed that the relation between R_n and r_e is of the form

$$R_n \propto r_e \langle SB \rangle_e^{\kappa} F(\langle SB \rangle_e) , \quad (4)$$

where $F(\langle SB \rangle_e)$ is constant to first order in $\langle SB \rangle_e$. The parameter κ is 0.8 for the conventional definition of D_n: $\langle SB \rangle_n = 20.75$ mag arcsec^{-2}. The FP given by $\log r_e = a \log \sigma + b \langle \mu \rangle_e + c$ transforms into

$$\log R_n = a \log \sigma + \left(b - \frac{\kappa}{2.5}\right) \langle \mu \rangle_e + d + f(\langle \mu \rangle_e) . \quad (5)$$

A D_n-σ relation of the form $\log R_n = \hat{a} \log \sigma + e$ is therefore expected to show residuals of the form

$$\Delta_{\rm PRED}(D_n - \sigma) = (a - \hat{a}) \log \sigma + \left(b - \frac{\kappa}{2.5}\right) \langle \mu \rangle_e + f(\langle \mu \rangle_e) . \quad (6)$$

Since the parameters a and $\hat{a}$ are almost identical in our fits, the residuals of the D_n-σ relation depend mainly on $\langle \mu \rangle_e$. The linear term disappears for an appropriate choice of $\langle SB \rangle_n$. The conventional definition of D_n reduces the linear term to zero at $\langle \mu \rangle_e = 21.8$ mag arcsec^{-2} (Phillips 1988). The term $f(\langle \mu \rangle_e)$ will remain for any choice of $\langle SB \rangle_n$. The conventional definition of D_n produces $f(\langle \mu \rangle_e) \approx -0.0268(\langle \mu \rangle_e - 21.53)^2 + 6.672$ within 0.03 mag for $19 \leq \langle \mu \rangle_e \leq 25$ mag arcsec^{-2} (Johnson B). This confirms that f is a second-order correction (see also Phillips 1988). On basis of equation (6) the D_n-σ relation is expected to give a bended [the term $f(\langle \mu \rangle_e)$] and slightly tilted (when $a \neq \hat{a}$) view of the FP.

Lucey et al. (1991a) first demonstrated that the residuals for the D_n-σ relation depend on $\langle \mu \rangle_e$. In Figure 4*a* this effect is shown for our data. The residuals are taken with respect to the D_n-σ relation defined in equation (2). The predicted residuals given by equation (6) are overplotted. Furthermore, the predicted residuals $\Delta_{\rm PRED}(D_n - \sigma)$ are plotted against the measured residuals from the D_n-σ relation in Figure 5. The correlation coefficient for the 22 galaxies in the sample is 0.77. The galaxies clearly lie along the line *residual* $(D_n - \sigma) = \Delta_{\rm PRED}(D_n - \sigma)$. The scatter around the line is the same as the scatter for the FP (0.046), as expected. This figure demonstrates that the FP is a true improvement of the D_n-σ relation.

Lucey et al. (1991a) corrected the D_n-σ relation by including a term linear in $\langle \mu \rangle_e$. As we have seen above, such a linear correction can also be achieved by redefining r_n to enclose another mean surface brightness $\langle SB \rangle_n$. The correction is not quite complete, since it does not include the quadratic term $f(\langle \mu \rangle_e)$. It reduces the scatter, because most of the outlying galaxies in the sample of Lucey et al. have high surface brightness. In a later paper Lucey et al. (1991*c*) analyzed a much larger sample of galaxies in Coma and found the linear dependence on $\langle \mu \rangle_e$ to be much weaker. This is due to the fact that many of the newly observed galaxies have $\langle \mu \rangle_e$ in a rather small interval. The quadratic nature of the deviation reduces the effect when galaxies in a narrow surface brightness range are considered.

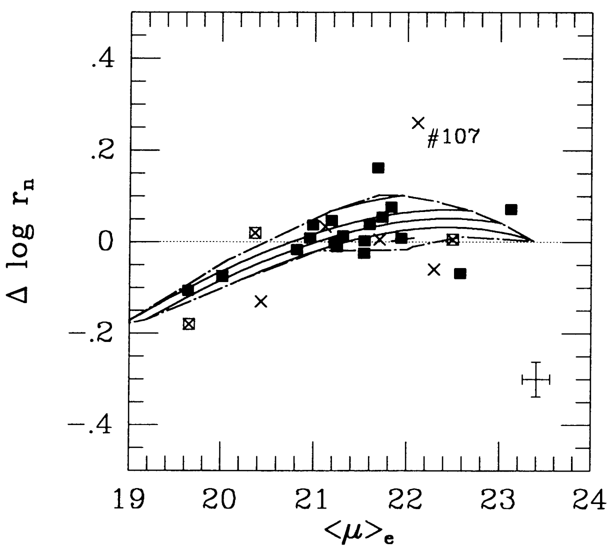

FIG. 4*a*

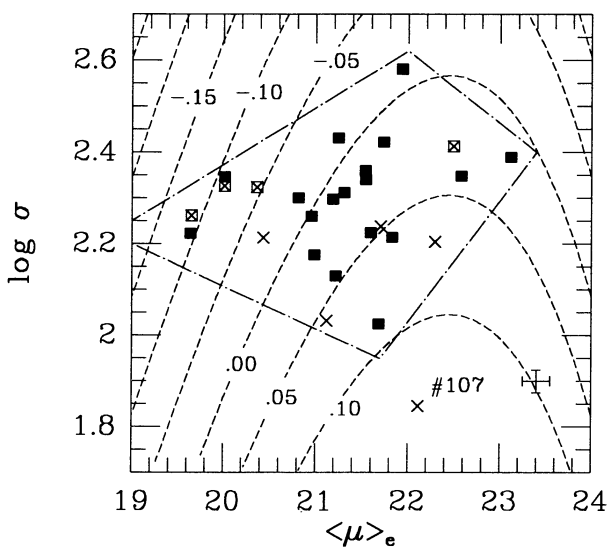

FIG. 4*b*

FIG. 4.—(*a*) The residuals, $\log r_n$ − fit, for the determination of the D_n-σ relation (Johnson B) plotted against the mean surface brightness, $\langle \mu \rangle_e$. Symbols as in Fig. 2. Typical errors are shown in the lower right corner. Overplotted is the prediction of the residuals calculated from the FP, the D_n-σ relation, and the theoretical relation between $\log r_n/r_e$ and $\langle \mu \rangle_e$ assuming an $r^{1/4}$-law for the galaxies. The solid lines show locations of constant $\log \sigma$, 2.0, 2.2, 2.3, 2.4, and 2.6, values increasing downward in the plot. The dashed-dotted line surrounds the distribution of the galaxies in $\log \sigma - \langle \mu \rangle_e$.

FIG. 4.—(*b*) $\langle \mu \rangle_e$ plotted against $\log \sigma$; symbols as in Fig. 2. Typical errors are shown in the lower right corner. The dashed-dotted line encloses the distribution which is shown projected in the residual plot, Fig. 4*a*. The dashed contours have constant predicted residual from the D_n-σ relation $\Delta_{\rm PRED}(D_n - \sigma)$. The lines are labeled with the expected residual.

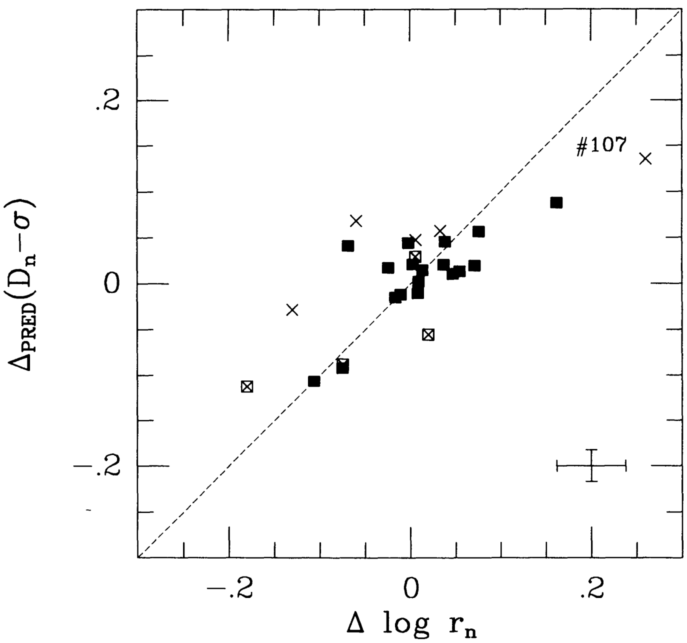

FIG. 5.—The predicted residuals for the D_n-σ relation as described by eq. (6) plotted against the observed residuals. Typical errors are shown in the lower right corner. The correlation eoefficient for the 22 galaxies included in the sample is 0.77. The dashed line is the one-to-one line.

Other available good-quality CCD photometry like the photometry for Abell 2199 and Abell 2634 from Lucey et al. (1991b), for which Lucey et al. also noted a dependence on $\langle\mu\rangle_e$, and the photometry for Virgo, DC 2345−28, and Fornax from Faber et al. (1989), is in agreement with the predicted residuals as described by equation (6).

The reduced scatter of the FP with respect to the D_n-σ relation is a strong argument to use the FP. We, therefore, reviewed the original motivation for the use of the D_n-σ relation. Dressler et al. introduced D_n because the parameter is easier to measure than r_e and $\langle\mu\rangle_e$ separately, especially when aperture photometry is used. Since these authors derived D_n from an $r^{1/4}$ growth curve fitted to the data, this is equivalent to the statement that the parameter D_n depends on r_e and $\langle\mu\rangle_e$ in a stable way. Similarly, the combination $\log r_e - 0.35\langle\mu\rangle_e$ which enters the FP is well determined. This is caused by the same reason that D_n is well determined: the errors in $\langle\mu\rangle_e$ and r_e are highly correlated and the correlation nearly resembles the combination in the FP, and D_n (see Fig. 1). As an example, the measurement errors in our data on $\log r_e - 0.35\langle\mu\rangle_e$ are as small as 0.022 (§ 2). Thus, the uncertainty on $\log r_e - 0.35\langle\mu\rangle_e$ determined from CCD photometry is comparable with the uncertainty on $\log r_n$, and the original motivation for the use of $\log r_n$ has greatly diminished.

If the FP is linear and universally valid the use of the D_n-σ relation will introduce *systematic* errors dependent on $\langle\mu\rangle_e$ and $\log \sigma$. This was also noted by Phillips (1988). In Figure 4*b* locations in ($\log \sigma$, $\langle\mu\rangle_e$) giving the same predicted residuals $\Delta_{PRED}(D_n - \sigma)$ are connected with dashed lines. The error on the distance varies from +17% to −10% for galaxies with surface brightnesses between $\langle\mu\rangle_e = 20$ mag arcsec^{-2} and 21.5–23 mag arcsec^{-2}, respectively. These effects are uncomfortably large compared to the observed streaming motions.

6. CONSEQUENCES FOR DETERMINATION OF PECULIAR VELOCITIES

We have investigated the expected systematic errors on the distances derived by the seven Samurai (Faber et al. 1989) due to their use of the D_n-σ relation. Since this question concerns distance determinations and, therefore, determination of expected effective radii, we used the FP for Coma determined by minimizing the residuals in $\log r_e$. For our data in Johnson B we find $\log r_e = 1.167 \log \sigma + 0.351\langle\mu\rangle_e - 9.198$. The constant is the median for our sample. We have then calculated the predicted systematic errors for the D_n-σ relation with slope 1.2 ($\log r_n = 1.2 \log \sigma - 1.967$). In Figure 6 we show the ratio of the flow velocity to the group velocity determined by the seven Samurai plotted against the predicted systematic error given as $\Delta_{PRED} \ln r_n$. Clusters with six or more members observed are indicated with filled squares and clusters with three to five members observed with open squares. The dashed-dotted curve on the figure marks the expected location of the data points if the derived flow velocities are due to the systematic errors, $\Delta_{PRED} \ln r_n = -\ln(1 - v_{flow}/v_{group})$. In Table 2 we list the distances derived by Faber et al. using the D_n-σ relation and derived by us using the FP.

Figure 6 does not show systematic effects. The predicted systematic errors are on the order of 5%. For clusters with only two galaxies this increases to 10%. Using the FP given in equation (1) gives the same result. We verified that the differential Malmquist bias corrections do not change this conclusion. Hence there is no evidence that the use of the D_n-σ relation caused the large-scale motions seen by the seven Samurai, but

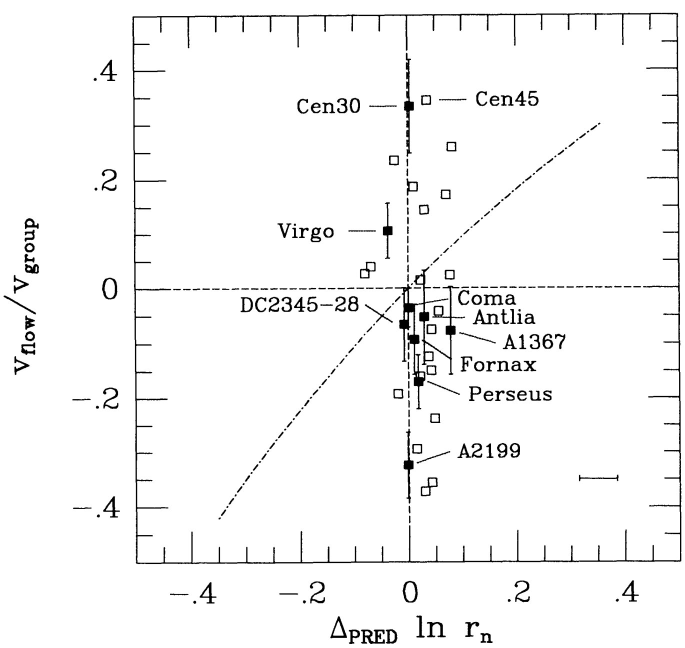

FIG. 6.—Relative flow velocities derived from the D_n-σ relation plotted against the predicted systematic errors, $\Delta_{PRED} \ln r_n$. The systematic errors are calculated from published values of $\log \sigma$ and $\langle\mu\rangle_e$ assuming that the FP of Coma is universal. The relative flow velocities are taken from the literature. *Solid squares*: data from Faber et al. (1989), clusters with six and more members observed. *Open squares*: data from Faber et al., clusters with three to five members observed. The uncertainty on $\Delta_{PRED} \ln r_n$ for a single galaxy is shown in the lower right corner. The dashed-dotted curve indicates how the systematic effects can affect the derived flow velocity of a cluster with a true flow velocity of zero, $\Delta_{PRED} \ln r_n = -\ln(1 - v_{flow}/v_{group})$.

TABLE 2

DISTANCES TO CLUSTERS

N_{grp}	N_{gal}	$R_{D_n-\sigma}$ (km s^{-1})	R_{FP} (km s^{-1})	Cluster Name
61........	36	7458	7458	Coma
30........	18	6050	6162	Abell 426, Perseus
56........	17	1333	1285	Virgo
73........	12	11918	11905	Abell 2199
31........	11	1422	1438	Fornax
72........	10	9134	9063	DC 2345−28
51........	7	7171	7753	Abell 1367
58........	6	2221	2228	Cen 30
46........	6	3488	3592	Antlia
25........	5	5049	5453	Abell 194
36........	5	6954	6820	Abell 569
32........	5	1990	2051	Eridanus
24........	5	5150	5347	Pisces
27........	5	4397	4498	Abell 262
87........	4	3581	3736	Pegasus
70........	4	2336	2451	NGC 5846 group
63........	4	2982	2752	NGC 5044 group
66........	4	3895	3933	IC 4329 group
80........	4	2328	2399	Telescopium
59........	4	3177	3291	Cen 45
82........	3	5648	5772	
69........	3	2819	2750	
75........	3	6611	7176	Abell 2197
47........	3	4201	4443	Abell 1060, Hydra
98........	3	3030	2777	GH 107
29........	3	2297	2398	NGC 1199 group
103........	3	5779	5866	HMS 0122+3305
34........	3	4019	4313	NGC 1600 group
26........	3	1500	1400	Cetus
48........	3	1991	2078	NGC 3607 group

NOTES.—$R_{D_n-\sigma}$—from Faber et al. (1989). N_{grp}—the cluster identification from Faber et al., and N_{gal}—the number of galaxies used for the determination. R_{FP}—determined as $\ln R_{FP} = \ln R_{D_n-\sigma} + \Delta_{PRED} \ln r_n$, where $\Delta_{PRED} \ln r_n$ is the predicted systematic error.

it could certainly be a source of scatter. It will be very valuable to redetermine the distances using new CCD photometry and the FP.

Lucey et al. (1991b) found very large negative peculiar velocities for the clusters Abell 2199 and Abell 2634. The authors addressed the problem of possible systematic errors due to the dependence on $\langle\mu\rangle_e$ and concluded that the derived distances were not significantly affected by this kind of bias. We here reanalyze the data for the elliptical galaxies as published by Lucey et al., and further use the data to illustrate the stability of distances derived from the FP. To limit the influence of possible zero differences we for this analysis use the FP derived from the data on the Coma Cluster from Lucey et al. (1991a); minimizing the residuals in $\log r_e$ these data give $\log r_e = 1.154 \log \sigma + 0.346\langle\mu\rangle_e - 8.706$ (the zero point is the median for the 26 galaxies). We use a D_n-σ relation with coefficient 1.2 ($\log r_n = 1.2 \log \sigma - 1.962$, zero point from Lucey et al. 1991b). The derived distances are listed in Table 3. Galaxies with $\log \sigma < 2.0$ were omitted. Following Davies et al. (1987) we have corrected $\log \sigma$ for the aperture effect by adding 0.007 to the values from Lucey et al. We did not correct for possible systematic offsets in $\log \sigma$ between data from Lucey et al. (1991b) and Davies et al. (1987). Such differences enters distances derived from the FP and from the D_n-σ relation with about the same amount, thus the relative differences are preserved but the absolute values of both estimates are likely to be affected by zero point differences.

TABLE 3

DISTANCES TO ABELL 2199 AND ABELL 2634

Cluster	$\langle\mu\rangle_e$(Johnson V)	$R_{D_n-\sigma}$ (km s^{-1})	R_{FP} (km s^{-1})	N_{gal}
Abell 2199.........	18.6–21.0	9771	9517	8
Abell 2199.........	No restrictions	(11265)	9623	13
Abell 2634.........	18.6–21.0	12071	11468	9

We have derived distances for Abell 2199 with the sample limited to $18.6 < \langle\mu\rangle_e < 21.0$ mag arcsec^{-2} as done by Lucey et al., and for the full sample without restrictions in $\langle\mu\rangle_e$. We emphasize that the determination using the D_n-σ relation without restrictions in $\langle\mu\rangle_e$ is included for illustrative purposes, only. All elliptical galaxies in Abell 2634 observed by Lucey et al. have $\langle\mu\rangle_e$ in the interval 18.6–21.0 mag arcsec^{-2}. The relative systematic error for the two clusters when using the D_n-σ relation is 3%–5% ($18.6 < \langle\mu\rangle_e < 21.0$ mag arcsec^{-2}). This is the same size as the quoted uncertainties on the cluster distances derived by Lucey et al. The result is somewhat sensitive to the adopted coefficients for the FP. With coefficients derived from our Gunn r photometry, but zero point from Lucey et al. (1991a), the relative systematic errors are 6%–8%. The peculiar velocities found by Lucey et al. are of the order 12%–25% of the radial velocity. The second determination for Abell 2199 shows that the FP results in a distance stable within 1% when including low and high surface brightness galaxies. The result from the D_n-σ relation deviates with 15% relative to the result for the limited sample, emphasizing the limited validity of the method (e.g., Lynden-Bell et al. 1988; Lucey et al. 1991b).

On basis of the analysis of the data from Faber et al. (1989) and from Lucey et al. (1991a, b) we conclude that for cluster samples the use of the D_n-σ relation occasionally introduces systematic errors of the order 5%; thus some of the discrepancies between distances derived from the D_n-σ relation and other methods might be explained as due to systematic errors.

7. CONCLUSIONS

We have found a remarkably low scatter in the FP for the Coma cluster, corresponding to 11% uncertainty on derived distances. The result is in agreement with other studies using good-quality CCD photometry (Dressler et al. 1987; Lucey et al. 1991a). The residuals for the FP do not show any dependence on other photometric or geometric parameters.

We have shown that galaxies have significant differences in dynamical structure, which may contribute to the scatter in the FP giving uncertainties in the derived distances of as much as 10%. The use of either a luminosity-weighted second velocity moment, or the mean velocity dispersion within a fraction of the effective radius, may decrease the scatter. This is expected to have the largest effect on nearby galaxies. The differences in dynamical structure appear to be much smaller than models suggest as allowed for spherical systems; thus, either the formation histories of ellipticals are similar, or the possible formation processes all lead to rather similar end-products. The current data suggest that the M/L ratio calculated from the second velocity moment depends somewhat weaker on luminosity than found for the M/L ratio calculated from the central velocity dispersion: $M/L \propto L^{0.15}$. Final conclusions on these issues require data for a complete sample of ellipticals with well-determined distances.

The D_n-σ relation has a scatter equivalent to distance uncertainties of 17%, much larger than the FP. The residuals for the D_n-σ relation depend on the mean surface brightness, $\langle\mu\rangle_e$. These residuals follow naturally from the linearity of the FP, and the imperfect approximation of the D_n-σ relation to the FP. The best improvement of the D_n-σ relation is the FP, and not the inclusion of the surface brightness correction as suggested by Lucey et al. (1991a).

The D_n-σ relation gives a bended slightly tilted view of the FP. The systematic errors arising from this, assuming the FP is linear, are of the order 10%–15% for single galaxies, while the errors for clusters depend on the sample selection.

The systematic errors could be a possible explanation for some of the discrepancies seen between distances derived from the D_n-σ relation and from other methods. It is, however, not expected that the presence of such errors would change the overall conclusion concerning the large-scale motions in the universe and the presence of very large peculiar velocities in the vicinity of the Centaurus Cluster.

If the very low scatter in the FP for the Coma Cluster is typical for clusters, the FP will be a major improvement compared to other methods. It is, therefore, very important to fully understand the FP. It should be studied if the FP is universally valid, and if the scatter is the same in other clusters. Preliminary results from our investigation of the poorer clusters Abell 3574 and Abell S753 show that these clusters follow the same FP as the Coma Cluster. The dependence on the dynamical structure of the galaxies should also be investigated, and the velocity dispersion to use in the relation probably redefined.

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