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NONLINEAR PLASMA MECHANISMS OF PULSAR ECLIPSE IN BINARY SYSTEMS

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ABSTRACT

Several nonlinear mechanisms are considered as mechanisms for DC (continuum) pulsar eclipse. Induced photon decay (into plasmon plus scattered photon) is found to be a viable mechanism at 300 MHz, but not at 1400 MHz, unless the density profile is extremely sharp. Induced scattering off electrons and ion charge clouds is found to be less important. Raman scattering by beam-induced plasma turbulence is found to depend less strongly on frequency and could work at both frequencies. Refined observations of residual pulse-smeared emission during the pulse eclipse, especially near its edges, could provide further tests of these mechanisms.

Subject headings: binaries: eclipsing — MHD — plasmas — pulsars: general

1. INTRODUCTION

Since their discovery (Fruchter et al. 1988, 1990; Lyne et al. 1990; Nice et al. 1990) the eclipsing binary pulsars PSR 1957+20 and PSR 1744−24A remain a puzzle. They are observed to undergo eclipses, the durations of which greatly exceed what could be expected if produced by the bodies of the companion stars. It is widely believed that the eclipses are caused by the plasma surrounding the companion either in the wind from the companion (Fruchter et al. 1988, 1990; Cheng 1989) or in the companion magnetosphere (Michel 1989). The wind can be excited off the companion by the pulsar, but the mechanism remains unresolved. Wind excitation by high-energy radiation was discussed prior to the eclipsing pulsar discovery (Eichler & Ko 1988; Ruderman et al. 1989; Ruderman, Shaham, & Tavani 1989), with the last paper actually proposing a millisecond pulsar as the source of the radiation, and this possiblity was the first to be considered following the discovery (Phinney et al. 1988; Kluzniak et al 1988; Cheng 1989). Optical observations of the eclipsing pulsar 1957+20 companion, which show that the illuminated side of the companion is several times brighter than the "dark" side (Fruchter et al. 1988; van Paradijs et al. 1988), support the view that the companion surface is indeed heated. However, it is found that the illumination of the companion by X-rays and soft γ-radiation cannot provide the necessary mass loss for the companion (Eichler & Levinson 1988; Levinson & Eichler 1991), and other mechanisms of the energy transfer of the pulsar into the companion wind have been proposed, including diffusion of mildly relativistic pulsar pairs through a thermal wind of the companion with subsequent deposition of their energy (Krolik & Sincell 1990) and KHz radiation absorption combined with inward heat conduction (Levinson & Eichler 1991). The question of efficient excitation of a wind, which could provide the required mass loss of the companion, remains unresolved as yet. The mass loss is now suspected on the basis of an observed orbital period derivative (Ryba & Taylor 1991) to exceed 10^{16} g s^{-1}, much higher than suggested by the pulse dispersion just outside of the eclipse boundaries, and much higher than could be accounted for by high-energy radiation. It has been proposed (Eichler 1992) that a magnetosphere with a much larger cross section than the star itself could capture magnetic energy and conduct it to the star's surface, as does Earth's magnetosphere with solar wind energy. There is also the possibility that the companion fills its Roche lobe (Fruchter & Goss 1992) and that most of the mass loss is due to Roche lobe overflow that is excreted in the orbital plane. This could account for the discrepancy between the high mass-loss rate inferred from the orbital period derivative and the much lower rate suggested by pulse dispersion, for the latter would include only the outflow that is fairly sampled by our line of sight. In any case the companion is surrounded by a plasma that apparently produces eclipse whose nature is observationally constrained, as discussed below.

The very recent discovery of partially eclipsing pulsar PSR 1259−63 (Johnston et al. 1992) has provided more information, and perhaps more questions. Because the companion is a Be star, which presumably emits its wind without the assistance of the pulsar, the wind itself is better understood, but the eclipse mechanism is no less mysterious. Pulse dispersion measurements in this case provide an upper limit of 0.2 cm^{-3} pc over a distance of order $10^{-6}(\sin i)^{-1}$ pc. The partial eclipse of order 70%, which is observed at 660 and 1520 MHz, is therefore produced by a plasma whose density is only of order 10^5 cm^{-3}. A partial eclipse that is somewhat weaker is observed in the time-integrated component. A particularly puzzling feature is the absence of any frequency dependence on the strength of the pulse eclipse. This is in contrast to the other eclipsing pulsars, and no explanation is offered here.

Several eclipse mechanisms have been proposed. The plasma frequency cutoff mechanism (Fruchter et al. 1988; Phinney et al. 1988; Kluzniak et al. 1988; Michel 1989) assumes that the plasma density is high enough to reflect the radio waves. This mechanism requires a density of $n \approx 1.4 \times 10^{10}$ for the eclipse at $f = 1400$ MHz, and $n \approx 3 \times 10^9$ if the cutoff eclipse occurs only at $f = 318$ MHz. Unless there is a discontinuity in the density profile, these quantities are unlikely, since the observations of the pulse delays give the estimate of the column density just before and after the eclipse $\int n\,dl \approx 10^{17}$, while $\int dl \approx 10^{11}$ cm. Moreover, the observation of partial eclipse in the case of the eclipsing pulsar 1744−24A rules out the mechanism of plasma frequency cutoff. In the case of a contact discontinuity the eclipse duration should be independent of frequency, in contrast with the observed $f^{-\alpha}$ dependence with $\alpha = 0.41 + 0.09$ for PSR 1957+20 (Fruchter et al. 1990) and $\alpha = 0.63 + 0.18$ for PSR 1744−24A (Nice et al. 1990).

More recently a mechanism of plasma *refraction* was proposed (Emmering & London 1990) in which the plasma frequency is assumed to be below cutoff. Instead, the refraction bends the rays, thus producing an eclipse. Radiation beam refraction could in principle be responsible for partial eclipse. However, the partial eclipse of PSR 1744−24A, which exhibits no pulse smearing during the partial eclipse, appears to rule this out. The model also requires a density cutoff to fit the observed frequency dependence. It also fails to explain the observed low time delays. On the other hand, refraction *does* occur even in a low-density plasma although its magnitude is in question, and it remains a viable pulse smearing mechanism.

The free-free absorption mechanism (Wasserman & Cordes 1988; Rasio, Shapiro, & Teukolsky 1989) gives a frequency dependence of the eclipse duration which seems consistent with the observations ($\alpha = \frac{2}{3}$ for an isothermal wind and $\alpha = 2$ for an adiabatic wind). However, the mechanism requires very low wind temperatures $T \approx 300$ K for the density $n \approx 10^7$ cm^{-3}. For this temperature even the very existence of the wind is doubtful. Moreover, the pulsar radiation itself is intense enough to heat the plasma well beyond this value.

Proposed models based on pulse smearing (Fruchter et al 1990; Rasio, Shapiro, & Teukolsky 1990) require excess dispersive delays greater than the pulsar width near the eclipse, while no delays of this magnitude have been ever observed. Moreover, at 318 MHz, Fruchter & Goss (1992) report a disappearance of the time-integrated signal (a "DC" or "continuum" eclipse, as termed by previous authors) of PSR 1957+20, and it is very hard to explain this with mere pulse smearing

Mechanisms based on the interaction of the pulsar radio emission with the companion wind plasma have been proposed in Eichler (1991). These include Raman scattering, parametric instabilities, and absorption by energetic electrons whose energy is below the brightness temperature of the incident radiation at the eclipse site. Most of the mechanisms appeared to fail, but the Raman scattering by Langmuir waves (plasmons) appeared plausible, if there is enough energy in the waves. In particular, it can account for the observation (Fruchter & Goss 1992) of a true eclipse of a time-integrated signal at low frequency (318 MHz), while the eclipse at higher frequency (1400 MHz; Ryba & Taylor 1991) appears to be largely pulse smearing, as it is not observed as a DC eclipse.

We expect plasma mechanisms to play an important role. In the present paper we consider two plasma mechanisms, which might be responsible for the eclipses, depending on the eclipsing plasma parameters. These mechanisms can, in principle, account for both DC eclipse and partial eclipse.

The mechanisms we discuss here are closely related to the nonlinear interaction of the electromagnetic waves with other electromagnetic waves and/or with plasmons. They can lead to the loss of the photons in the initially beamed radiation either due to the effective isotropization or by a simple disappearance of the photons due to some sort of decay. The most important types of such an interaction are electromagnetic wave decay and electromagnetic wave scattering. An important feature of the processes is that they are induced processes, that is, the process probabilities (or the reaction rates) are proportional to the number of waves taking part in the interaction, thus the typical reaction rates can be greatly enhanced with respect to the spontaneous processes rates.

The first of the two classes of processes we consider is three-wave interactions. A transverse electromagnetic wave (hereafter denoted by t) interacts with another electromagnetic wave plus plasmon (hereafter denoted by l), two plasmons, or two other transverse waves. This can be described as a resonant decay of the electromagnetic wave into two others, that is, (1) $t \to t + l$, (2) $t \to l + l$, or (3) $t \to t + t$. In principle, all three processes can result in the effective removal of radiation from our line of sight. We show that only the first of the three possibilities is important in our case, and we consider in detail case 1 only. This process occurs with and without an ambient plasmon field. The rate of scattering of photons out of a beam in the presence of longitudinal waves (denoted by subscript l) is, by detailed balance, given by an equation of the following form:

$$\frac{\partial N_t}{\partial t} = \sum_{t_1, l} K_{t_1, l}(-N_t N_{t_1} - N_t N_l + N_{t_1} N_l) , \tag{1}$$

where the sum is taken over the possible final states of the photon t_1 and over the possible plasmon states l.

When there are no plasmons, the rate of the process is determined by the first term under the summation sign of equation (1). The second and third terms represent induced Raman scattering out of and into the beam, respectively. The three-wave interaction (case 1) is called Raman instability or coherent decay when the three waves are monochromtic coherent waves. In the case of the Raman instability, N_l grows exponentially, but too slowly to be of interest for PSR 1957+20 parameters (Eichler 1991). However, high-energy particles from the pulsar can excite plasma turbulence, which may be just enough to account for the eclipse (Eichler 1991). The first term on the right-hand side, however, was not considered previously. It represents *induced photon decay* into plasmon plus scattered photon (or, alternatively, *induced plasmon emission* by the photon—we shall use the terms interchangeably) and is directly connected to Raman scattering by equation (1). It is also accompanied by the induced scattering off electrons and ions, as shown below. Thus, it is possible to compare and contrast all of these mechanisms in any given context with a minimum of free parameters. It is clear, for example, that photon-induced plasmon emission is more important than Raman scattering if the number of scattered photons per mode exceeds the number of plasmons per mode. This could be the case for the parameters of the PSR 1957+20 system. Photon-induced plasmon emission is also found to be more important than induced scattering off individual electrons and ions (Appendix C).

In § 2 we review briefly the probably eclipsing plasma and electromagnetic beam parameters. In § 3 we consider induced processes including electromagnetic waves only (with no ambient plasmon field). We propose induced plasmon emission as a possible mechanism for the pulsar eclipse. We find that it seems sufficient to account for a DC eclipse of PSR 1957+20 at 318 MHz even in the case of low densities, but it predicts a very steep frequency dependence for the DC eclipse duration unless the density gradient is sharper inside the eclipsing region than outside it. A similar difficulty exists for the observations of PSR 1744+24A, where the only detections are at higher frequencies.

In § 4 we consider Raman scattering by plasma turbulence and find the conditions under which this process can account for the eclipses. The mechanisms require high but not unreasonable levels of the Langmuir turbulence, while the corresponding frequency dependence of the eclipse duration seems to be consistent with the observations even with a moderate density gradient in the flow. We also analyze frequency dependence of the optical depth obtained from the proposed mechanisms and compare it with the observational data. Given the uncertainties of the experimental data, we find them to be consistent.

The minimum of free parameters is the evident advantage of both the proposed mechanisms, and these models can be therefore easily tested by future observations. Similarly, the plasma parameters are tightly constrained within any given model.

2. BASIC PARAMETERS

Since the eclipse depends on basic plasma parameters, it is useful to make some rough estimates of them. For PSR 1957+20 the distance from the pulsar to the eclipse site, presumably close to the orbital radius, is $d \sim 1.7 \times 10^{11}$ cm, while the eclipse radius, R_E, is $\sim 6 \times 10^{10}$ cm, that is, several times greater than the companion radius, which can be at most the Roche lobe radius 2×10^{10} cm. The lower limit on the plasma density is determined by the time delay observations (see Introduction) and apparently cannot be less than $\sim 10^6$ cm^{-3} just outside the eclipse. Upper limits have been estimated using the conditions of (1) the pulsar wind standoff at a distance of the same order as R_E, and (2) the requirement for the observed companion mass loss. The first condition gives (see, e.g., Harding & Gaisser 1990)

$$\frac{L}{4\pi d^2 c} \approx n m_p v^2 , \tag{2}$$

and with $L \approx 2 \times 10^{35}$ ergs s^{-1} one obtains

$$n \sim 10^9 \text{ cm}^{-3} \left(\frac{v}{0.003c}\right)^{-2} . \tag{3}$$

The assumption of pressure standoff requires problematic energetics for the wind in many models. The second condition gives

$$\dot{M} \sim 4\pi R_E^2 n m_p v \tag{4}$$

and with the same assumption

$$n \sim 10^8 \text{ cm}^{-3} \left(\frac{\dot{M}}{10^{15} \text{ g cm}^{-3} \text{ s}^{-1}}\right) . \tag{5}$$

These estimates, however, yield a density that is much higher than that suggested by pulse delay at the eclipse boundary. On the other hand, the estimate (4) can be misleading if the mass outflow from the companion occurs mostly in the orbital plane, while the eclipsing plasma lies well above the plane. In the context of radio eclipses, we shall assume that the density is in the range 10^6–10^7 electrons cm^{-3}, which yields a plasma frequency of order 10^8 s^{-1}. If the pulsar wind is balanced by a companion magnetosphere the magnetic field in the eclipsing region should be

$$B \sim \left(\frac{2L}{d^2 c}\right) \sim 40 \text{ G} \tag{6}$$

at the nose and, as is the case for Earth's bow shock, within an order of magnitude of this downstream, that is, at least several gauss. The corresponding cyclotron frequency ω_B is of the order of *several* times 10^8 s^{-1} and is of the same order or less than the plasma frequency. In the case of PSR 1957+20 the eclipses are observed at frequencies from 2×10^9 and up to 2.8×10^{10}. Therefore, we adopt the relation $\omega_B < \omega_p < \omega$ or even $\omega_B \ll \omega_p \ll \omega$. The observational situation for PSR 1744−24 appears to yield similar parameterization.

Thus we assume that in a typical situation radiation frequencies considerably exceed both characteristic plasma frequencies, so that radio waves can penetrate the companion plasma without being reflected. As shown above the radio frequencies fall in the range where plasma effects are expected to play a decisive role. The linear absorption at cyclotron frequency harmonics seens to be negligible or even ruled out by the observations of small Faraday rotations in the eclipse vicinity (Fruchter et al. 1990). Although it cannot be guaranteed that there is no sufficiently large *perpendicular* magnetic field, such a geometry would have no obvious justification in a magnetotail configuration. We delay a discussion of a possible role of the magnetic field for elsewhere. In the present paper the effect of the magnetic field effect is neglected.

3. INDUCED PHOTON DECAY

In this section we consider only the nonlinear processes that are induced by the electromagnetic waves (as opposed to by plasmons), that is, we consider the first term in equation (1) and the induced photon scattering on the (dressed) plasma particles $t + e(i) \rightarrow t_1 + e(i)$. To do this we assume that an initially sufficiently narrow radiation beam penetrates a plasma with an ambient radiation field. This ambient radiation field is assumed to exist primarily due to the scattering of the initial radiation beam, that is, it can be either the previously scattered radiation or the same finite width beam radiation (see below). It is convenient to describe the t-wave distribution in terms of "photon number"

$$N_{\boldsymbol{k}} = \frac{W_{\boldsymbol{k}}}{2\hbar\omega_{\boldsymbol{k}}} , \tag{7}$$

where $W_{\boldsymbol{k}}$ is the spectral energy density, which is related to the total energy density by the relation

$$W = \int W_{\boldsymbol{k}} d\boldsymbol{k} . \tag{8}$$

The photon number dynamics is governed by the so-called kinetic equation for t-waves (see, e.g., Tsytovich 1970, 1977; Pustovalov & Silin 1975), which is given in Appendix A. One can easily see that the rate of the process is proportional to the number of the waves in the final state, the fact that represents stimulation due to the Bose statistics of photons. Equation (A1) describes the induced scattering of the t-waves on the particles, while equation (A7) describes the t-wave decay onto the t-wave and l-wave. It should be understood that the induced scattering part is resonant with electrons or ions only and is far from the three-wave resonance, while the decay term does not contain any resonances with particles.

The resonance condition for the induced scattering is picked up by the δ-function with the argument $\omega - \omega_1 = (\boldsymbol{k} - \boldsymbol{k}_1)\boldsymbol{v}$. For the conditions listed in § 2 the typical resonant velocities are

$$v \approx \frac{\omega - \omega_1}{2k \sin^{2(\theta/2)}} = c \frac{\Delta\omega}{\omega} \frac{1}{2 \sin^{2(\theta/2)}} . \tag{9}$$

The resonance velocity is much larger than the electron thermal velocity, unless

$$\frac{\Delta\omega}{\omega} < \frac{v_T}{c} \sin^2 \frac{\theta}{2} . \tag{10}$$

If the inequality (10) holds, the corresponding phase space in the induced scattering integral is small, and the contribution is small also. In the opposite case of large resonance velocities the integrand contains exponentially small factor $\exp(-v^2/2v_T^2)$, and the integral is again small. Thus, we can conclude that the largest contribution should come from the induced plasmon emission (decay) term. (A more detailed analysis can be found in Appendix C.)

The induced decay kinetic equation for the t-waves can be written in the more transparent and useful form of an optical depth Σ as follows:

$$\Sigma = \int dl \int \sin\theta \, d\theta \, \frac{2\omega_p \omega^2}{(2\pi)^2 c^4} P(k, \theta) \frac{dN_t(\omega, \theta)}{d\omega}, \quad (11)$$

where θ is the angle between $\boldsymbol{k}$ and $\boldsymbol{k}_1$, and the relations

$$\omega_p \ll \omega, \omega_1, \; N(\omega_1) - N(\omega) \approx (\omega_1 - \omega) \frac{dN(\omega)}{d\omega} \quad (12)$$

are taken into account along with the resonance conditions

$$\Delta\omega = \omega_l = \omega - \omega_1 \approx \omega_p, \quad k_l \approx 2k \sin^2 \frac{\theta}{2}. \quad (13)$$

The integral is easily done in the approximation $N_t = \text{const}$ in a cone with opening angle θ_0, and the result takes the following form

$$\sigma \approx \sigma_T \frac{\hbar\omega}{m_e c^2} \frac{dN_t}{d\ln\omega} \sin^4 \frac{\theta_0}{2}, \quad (14)$$

where σ_T is the ordinary Thomson scattering cross section $\sigma_T = 8\pi e^4/3m_e^2 c^4$ and the angle θ_0 is treated as small. The simplifying approximation $N_t = \text{const}$ can be justified by the observation that there is a substantial fraction of backscattered electromagnetic waves in the considered process so that the radiation is effectively isotropized. Moreover, one can easily see that the frequency shift in the process is negligible, so it is natural to conclude that the penetrating radiation beam interacts with the same but isotropized radiation. In this case the total photon number in the ambient electromagnetic field should be determined by the energy conservation, which requires that the total number of photons in the isotropic field should be equal to the total number of photons in the pulsed radiation (beam). With these arguments one can estimate $\sin(\theta_0/2) \approx 1$ and $N_\gamma \times 4\pi \approx N_b \times \Omega_0$ where Ω_0 is the solid angle filled with the pulsed radiation. N_b is related to the intensity through the relation

$$I_\nu = \frac{\nu^2}{c^2} (h\nu N_b \Omega_0), \quad (15)$$

and since $I_\nu \propto \nu^{-3}$ (Fruchter et al. 1990), one has $N_b \propto \nu^{-6}$, and the optical depth can be written in the form

$$\Sigma \approx 5 \times \int n \, dl \sigma_T \frac{I_\nu}{\nu^2 m_e}. \quad (16)$$

The integration here is taken along the line of sight, and l denotes the distance from the pulsar. According to a naive model of the eclipsing region, the integrand is nonzero only for $l > l_0$ where l_0 denotes the location of the eclipsing plasma outer edge. At distances significantly larger than l_0 the integrand should decrease. Notice that n is a decreasing function of l no slower than $l^{-\alpha}$, $\alpha > 1$, and $I_\nu \propto l^{-2}$ or even decreases faster when the scattering is significant. One can easily see that the optical depth then can be estimated as

$$\Sigma \approx 5 \times n(R_E) R_E I_\nu(R_E) \sigma_T/\nu^2 m_e, \quad (17)$$

where R_E is the shortest distance between the companion center and the line of sight. Thus the condition for eclipse to occur at a frequency ν and phase ψ (see Fig. 1) is

$$\Sigma = 0.8 \times n_7 R_{E11} \nu_8^{-2} I_\nu > 1, \quad (18)$$

where $n_7 = n/10^7 \text{ cm}^{-3}$, $R_{E11} = R_E/10^{11}$ cm, $\nu_8 = \nu/100$ MHz and I_ν is in ergs cm^{-2} s^{-1} Hz^{-1}. In terms of measurable parameters

$$\Sigma = 0.8 \times 10^2 \times n_7 R_{E11} \nu_8^{-2} (D_{10}/\alpha_{11})^2 I_\nu(\text{Earth}), \quad (19)$$

where D_{10} is the distance to the system in units of 10 kpc, a is the separation in 10^{11} cm, and I_ν(Earth) is in mJy.

The observed parameters imply $\Sigma \sim 1$ near $\nu \sim 1000$ MHz and therefore that the eclipses can be produced by the mechanism of induced plasmon emission.

Let us now turn to the discussion of the frequency dependence of the eclipse duration and to the time delay asymmetry. The assumed geometry (e.g., Rasio et al. 1989) and notation are clear from Figure 1. The time delay is determined by the integral $\int n \, dl$ along the line of sight. One can easily see that if the dependence $n(l)$ is l^{-2} or weaker, the time delays at the egress side (radiation coming through the tail) are several times greater than the delays at ingress side, and this produces the observed asymmetry in time delays (Fruchter et al. 1990).

On the other hand, the condition for the eclipse is determined by $\Sigma \sim 1$ with Σ from equation (16). Since $I_\nu \propto l^{-2}$ along the line of sight at the eclipse edge (I_ν decreases even faster inside the eclipse region), the whole integral is proportional to $\int n \, dl/l^2$ along the line of sight. One can easily see that the contribution of the tail (large l) is in this case small, while the main contribution comes from the region where $l \sim a$. Therefore, the tail does not produce any substantial asymmetry, and nearly symmetric eclipses should be observed.

An analysis of time-delay dependence on phase ψ can in principle provide an information on the $n(\psi)$ dependence. Given the lack of exact time delay distribution published in

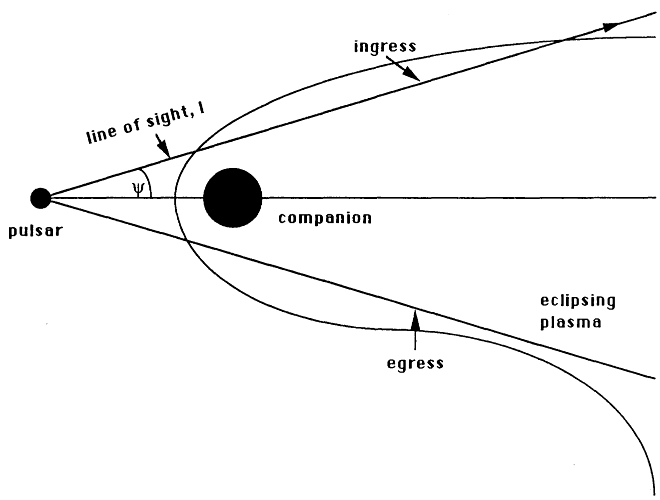

FIG. 1.—Pulsar eclipse geometry. The companion moves upward in the diagram. The asymmetry in the tail is due to the orbital motion of the companion. The angle ψ is that seen from the pulsar between the companion and observer.

digital form, we restrict ourselves with a rough analysis of the slopes of the curves in Figure 7 of Fruchter et al (1990) for the pulsar PSR 1957+20. If we assume power law $n \propto \psi^{-\beta}$, then we can estimate $\int n\,dl \propto \psi^{-(\beta-1)}$, and the curves show average $\beta - 1 \approx 3.7$ in the range $\psi < 0.06$, but a substantially greater average $\beta - 1 \approx 7.2$ when the range is taken more narrow, $\psi < 0.05$. These results show that the density logarithmic derivative increases fast when coming deeper inside the companion plasma, so that it is not unreasonable to expect that $\beta - 1 \sim 10$ at the eclipse edge. As one can see from equation (16), with $I_\nu \propto \nu^{-3}$ taken into account, the eclipse duration frequency dependence is determined by the relation

$$\int n\,dl/\nu^5 \approx \text{const} , \tag{20}$$

and therefore

$$\psi \propto \nu^{-\mu}, \quad \mu = \frac{5}{\beta - 1} . \tag{21}$$

Using the value $\beta - 1 \approx 7.2$ one has $\mu = 0.7$. This is not in good agreement with the observed value of 0.4 that is measure for the pulse eclipse of PSR 1957+20 unless large astrophysical uncertainties are invoked.

However, we emphasize the crucial point that the DC eclipse is observed only at 300 MHz and *not*, with any statistical significance, at higher frequencies. No quantitative measurement of its frequency dependence is yet available. A tentative prediction, then, that emerges from this particular model is that the DC eclipse should be more strongly dependent on frequency than the pulse eclipse. The absence of a prolonged eclipse at 1400 MHz is consistent with this prediction.

It is also emphasized that there are no free parameters in the calculation of optical depth except for the geometric uncertainties in the wind profile, the turbulence, and the radiation fields. This makes the mechanism hard to dismiss in the case of PSR 1957+20. On the other hand, it cannot work for the 1400 MHz eclipse of PSR 1744−24A; the brightness temperature of the photons is not sufficient. In the following section, a second mechanism is considered that could apply to both eclipsing pulsars. Because of its intimate connection with the first via equation (1), we do not consider the possibility that both may be important in nature a violation of Occam's razor.

4. RAMAN SCATTERING

Plasmons can stimulate a three-wave process just as photons do. The effective photon scattering is described in this case by the terms in equation (A7), containing N_{k_l}, where N_{k_l} denotes the number of plasmons taking part in the interaction.

While the induced plasmon emission mechanism depends only on the plasma density and the beamed radiation energy density, the stimulated Raman scattering requires much plasma turbulence. Such turbulence could be attributed to the penetration of the relativistic electron-positron pulsar wind into the eclipsing plasma, which produces a two-stream instability.

When a relativistic electron-positron beam with the density $\mathcal{N}_b$ and flow Lorentz-factor γ_b penetrates a plasma with the density n, Langmuir waves (plasmons) are excited with the typical growth rate

$$\Gamma \approx \left(\frac{\mathcal{N}_b}{n}\right)^{1/2} \omega_p \gamma_b^{-1/2}. \tag{22}$$

Let us assume that the electron-positron beam density near the pulsar surface is $\mathcal{N}_0$. The electron-positron plasma density varies approximately as $\sim r^{-3}$ inside the light cylinder $R_L = cP/2\pi$ and as r^2 outside it in the wind region. Then in the vicinity of the eclipsing region

$$n_b \approx \mathcal{N}_0 \left(\frac{R_p}{R_L}\right)^3 \left(\frac{R_L}{D}\right)^2 \sim 10\, \frac{\mathcal{N}_{12} R_6^3}{D_{11}^2 P_{\text{ms}}} , \tag{23}$$

where $R_p = R_6 \times 10^6$ cm is the pulsar radius, $P = P_{\text{ms}} \times 1$ ms in the pulsar period, and $D = D_{11} \times 10^{11}$ cm is the distance from the pulsar to the eclipsing plasma. The density $\mathcal{N}_{12}$ is the surface density of pairs at the pulsar measured in 10^{12} cm^{-3}, a typical value in most existing pulsar models. Typically, $\gamma_b \sim 10^2$ in standard pulsar models.

Thus, one obtains a typical growth rate of

$$\frac{\Gamma}{\omega_p} \approx 3 \times 10^{-3} \frac{\mathcal{N}_{12}^{1/2} R_6^{3/2}}{n_6^{1/2} D_{11} P_{\text{ms}}^{1/2}} . \tag{24}$$

This growth rate is fairly high, and one can expect that Langmuir turbulence is effectively excited. The excitation mechanism implies resonance Cerenkov excitation near the wavenumber $k_0 \approx \omega_p/v_{\text{res}}$, where v_{res} is the resonance particle velocity. In our case the resonant particles come from the relativisitic electron-positron beam. Since the temperature is also relativistic and the velocity distribution is essentially isotropic, the typical resonant velocities are in the range $v_T \ll v_{\text{res}} < c$, where v_T is the ambient plasma electron thermal velocity. Thus, one can estimate that the typical wavevector of the excited waves will be $k_0 \approx \omega_p/c\eta$ where $\eta < 1$ describes the resonant velocities' spread and can be estimated as $\eta \approx \frac{1}{3}$.

If the growth rate Γ is not sufficiently high, the most important nonlinear process which affects the plasmon spectral distribution is the induced scattering of the plasmons on the ions (see, e.g., Tsytovich 1977; Zakharov 1984). This process results in the plasmon transfer into the long wavelength range $k < k_0$, and these plasmons also cannot take part in the decay. However, if the growth rate satisfies the inequality

$$\frac{\Gamma}{\omega_p} > (k_0 r_{\text{D}})^2 \approx 5 \times 10^{-3} T_6^{3/2}, \tag{25}$$

where $r_{\text{D}} = v_T/\omega_p$ is the Debye radius and $T = T_6 \times 10^6$ K is the plasma temperature, then the plasmon spectrum becomes modulationally unstable, and the plasmons are converted eventually into the short wavelength region $k_0 < k < r_{\text{D}}^{-1}$ due to the Langmuir collapse (see, e.g., Goldman 1984; Shapiro & Shevchenko 1984; Zakharov 1984). In this case the total energy density in plasmons is determined by the Langmuir collapse at the level

$$\frac{W}{nT} \approx (k_0 r_{\text{D}})^2 \left(\frac{m_i}{m_e}\right) \sim 1 . \tag{26}$$

Present theories of the strong Langmuir turbulence do not properly describe the source region $k_l \approx k_0$ where the waves are excited, and the sink region $k_l \approx r_{\text{D}}^{-1}$ where they are Landau damped. The spectrum is widely believed to be Kolomogorov-like in the inertial range $k_0 \ll k_1 \ll r_{\text{D}}^{-1}$,

$$W_k = 4\pi k^2 N_k \hbar\omega_p = W_0 \frac{1}{k_0} \left(\frac{k_0}{k}\right)^{5/2} , \tag{27}$$

and is flatter near the source $k_l \sim k_0$ and steeper near the sink $k_l \sim r_D^{-1}$ (see, e.g., Goldman 1984).

The photon scattering angle in the interaction with the Langmuir turbulence is determined by the wavevectors of the Langmuir waves taking part in the scattering through the resonance condition, which reads in our case $\omega, \omega_1 \gg \omega_p$:

$$\sin\frac{\theta}{2} \approx \frac{k_l c}{2\omega} . \tag{28}$$

One can easily see that the smallest scattering angle is determined by k_0: for $\omega_p/\omega = 0.1$,

$$\left(\sin\frac{\theta}{2}\right)_{\min} \approx \frac{k_0 c}{2\omega} \approx \frac{\omega_p}{2\omega\eta} , \tag{29}$$

or $\theta \approx 20°$. Scattering by only 20° or so could not produce a nearly total eclipse in the case of PSR 1957+20, for which the eclipse region subtends only about 30° as seen by the pulsar. However, for the spectrum (27), the optical depth as a function of scattering angle varies as $\theta^{-1/2}$, so the DC eclipse trails the pulse eclipse by only two or three optical depths, and the duration of the former could be almost that of the latter.

For the estimates of the optical depth we use the Kolmogorov-like spectrum (27). In this case W_0 is related to the total turbulent energy density $W_{\rm tot}$ approximately in the following way:

$$W_{\rm tot} \approx \int_{k_0}^{r_D^{-1}} W_k \, dk \approx \int_{k_0}^{r_D^{-1}} W_0 \frac{1}{k_0}\left(\frac{k_0}{k}\right)^{5/2} dk \approx \frac{2W_0}{3} . \tag{30}$$

The further details of the calculations of the optical depth are given in Appendix B, and the result can be expressed in the following form:

$$\Sigma \approx \int \frac{dl\omega_p}{4\pi^2 c}\left(\frac{\omega_p}{\omega}\right)^2\left(\frac{T}{m_e c^2}\right)^2\left(\frac{m_i}{m_e}\right)\eta^{-1/2} , \tag{31}$$

where the plasmon frequency and wavevector obey the decay resonance conditions and in the case of high-frequency electromagnetic waves are given by equation (13).

Thus, the only parameters that determine the optical depth are the same as in the previous case with the addition of the electron temperature. With the same numerical data and estimates $T \approx 10^6$ K, $l \approx 10^{11}$ as in Eichler (1991), one has $\Sigma \geq 1$ for $\omega_p/\omega \approx 10^{-1}$. The estimate of T and its radial dependence are uncertain, particularly since heating by the pulsar emission itself may be significant. Thus an eclipse due to Raman scattering seems to be possible in the case of each of the two eclipsing pulsars.

One can see that the frequency dependence of the optical depth is less steep than in the case of the induced plasmon emission. The frequency dependence of the eclipse duration is determined by the ω_p and T spatial dependence. We assume a polytropic law for the temperature: $T \sim n^{\Gamma-1}$, $1 < \Gamma < 2$, and power dependence for the density, as above (see § 2). In this case the eclipse duration frequency dependence takes the following form

$$\mu = \frac{4}{[\beta(4\Gamma - 1) - 2]} \tag{32}$$

with the same notations as in equation (21). Since β and Γ are rather uncertain, we cannot determine eclipse duration frequency dependence with confidence. For a wind-type distribution of the matter $\beta = 2$ in the isothermal case $\Gamma = 1$ one obtains $\mu = 1$, while in the adiabatic case $\Gamma = 5/3$ one obtains $\mu = 0.43$. Given the uncertainties of the parameters in PSR 1744−24A, the results are consistent with the observational data.

5. CONCLUSIONS

Both induced photon decay ($t \to t + l$) and Raman scattering ($t + l \to t' + l$) can in principle produce pulsar eclipse. As required by observations of Fruchter & Goss (1992), these eclipse mechanisms can scatter the pulsed radiation out of the line of sight, and not merely smear the pulses. The frequency dependence of the DC eclipse is mechanism dependent and is steeper for the case of eclipse by induced photon decay, so the latter mechanism, already problematic for PSR 1744−24A, could be tested for the case of 1957+20 by future observations. Raman scattering also makes quantitative predictions, but they depend on the theory of plasma turbulence responsible for the scattering. A given spectrum of plasma turbulence, however, predicts a specific lag between the pulse eclipse and the DC (continuum) eclipse. The most prevalent theory, in which cascading by caviton collapse is the dominant mechanism for nonlinear spectral transfer, would predict a lag of two to three optical depths for 1957+20. If the optical depth scales as the column density, one might estimate on the basis of the time profile of the pulse residuals that this occurs within 5 minutes or so at eclipse egress and perhaps even faster at eclipse ingress. Future observations might be able to detect this lag.

Observations of the eclipse edges are admittedly difficult in the case of 1957+20, since the time scale for the eclipse transition is so short. However, the recently discovered pulsar 1259−63 (Johnston et al. 1992) that is eclipsed by the wind of a Be star may provide a much better opportunity to study the ragged edge of pulsar eclipse, for here the time scale for change in the local density is of order 1 yr or more. Time-integrated observations, in particular, could be done at greater sensitivities. This, and the likelihood that the pulsar is completely enveloped by the eclipsing material, may sharpen the distinction between absorption and scattering models. It is conceded, however, that the apparent lack of observed frequency dependence in the partial eclipse near periastron is puzzling.

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APPENDIX A

The interaction between photons and plasmons can be described in terms of the so-called kinetic equation for waves (see, e.g., Tsytovich 1970, 1977; Pustovalov & Silin 1975). As a rule the corresponding kinetic equations are divided onto two parts, the first of which being associated with the resonance interaction of waves with particles (with dynamical Debye screening taken into account), while the second one is due to the resonant interaction of waves with waves of the same or any other type, this interaction being hydrodynamical by nature. Thus, the first part includes interaction with a group of resonant particles only, while, in the second part, the particles interact through their wave motion, that is, in a nonresonant way. Usually the second type of interaction is more effective if it is allowed by the resonance conditions, that is, wave energy-momentum conservation. In the case of the two processes, considered in the present paper, the corresponding kinetic equations for the photons can be written as follows:

1. For the induced scattering $t + e(i) \leftrightarrow t + e(i)$,

$$\frac{dN_{\boldsymbol{k}}}{dt} = \frac{\partial N_{\boldsymbol{k}}}{\partial t} + \boldsymbol{v}_g \frac{\partial N_{\boldsymbol{k}}}{\partial \boldsymbol{r}} = \Sigma_s \int \frac{dbfk_1\, d\boldsymbol{p}}{(2\pi)^6} \hbar(\boldsymbol{k} - \boldsymbol{k}_1) \frac{\partial f_s}{\partial \boldsymbol{p}} \delta[\omega - \omega_1 - (\boldsymbol{k} - \boldsymbol{k}_1)\boldsymbol{v}] N_{\boldsymbol{k}} N_{\boldsymbol{k}_1} P_s(\boldsymbol{k}, \boldsymbol{k}_1) , \tag{A1}$$

where the group velocity $\boldsymbol{v}_g \approx c\boldsymbol{k}/k$, s denotes electrons and ions, and f_s is the species s distribution function which is assumed Maxwellian:

$$f_s = (2\pi m_s T_s)^{-3/2} \exp\left(-\frac{p^2}{2m_s T_s}\right) . \tag{A2}$$

The resonant particle velocities are determined by the δ function, which gives

$$v_{\rm res} = (\omega - \omega_1)/|\boldsymbol{k} - \boldsymbol{k}_1| . \tag{A3}$$

Approximate expressions for the probabilities P_s can be found in (Tsytovich 1970)

$$P_e = \frac{(2\pi)^3 e^4}{2\omega\omega_1 m_e^2}\left(1 + \frac{(\boldsymbol{k}\boldsymbol{k}_1)^2}{k^2 k_1^2}\right)\left|\frac{\varepsilon_i(\omega_-, \boldsymbol{k}_-)}{\varepsilon(\omega_-, \boldsymbol{k}_-)}\right|^2 , \tag{A4}$$

$$P_i = \frac{(2\pi)^3 e^4}{2\omega\omega_1 m_e^2}\left(1 + \frac{(\boldsymbol{k}\boldsymbol{k}_1)^2}{k^2 k_1^2}\right)\left|\frac{\varepsilon_e(\omega_-, \boldsymbol{k}_-) - 1}{\varepsilon(\omega_-, \boldsymbol{k}_-)}\right|^2 , \tag{A5}$$

where $\omega_- = \omega - \omega_1, \boldsymbol{k}_- = \boldsymbol{k} - \boldsymbol{k}_1$ and the dielectric permittivities are, as usual, defined as follows:

$$\varepsilon_s(\omega, \boldsymbol{k}) = 1 - \omega_{\rm ps}^2 \int d\boldsymbol{p} f_s \frac{1}{(\omega + i\epsilon - \boldsymbol{k}\boldsymbol{v})^2} , \tag{A6}$$

$$\varepsilon = \varepsilon_e + \varepsilon_i - 1 , \quad \epsilon \to +0 , \quad \omega_{\rm ps}^2 = 4\pi q_s^2 n/m_s = \omega_p^2(m_e/m_s) . \tag{A7}$$

In these formulae it is implicitly assumed that they include only $\boldsymbol{k}_-$, ω_- which are far from the resonance $Re\varepsilon = 0$, so that the singularity in the denominator is excluded and does not contribute to equations (A4) and (A5).

2. For the decay $t \leftrightarrow t + l$ the corresponding expressions can in fact be obtained from the same equation as above, when the contribution of the poles in the probabilities P_e and P_i are considered. Since the poles imply $Re\varepsilon = 0$, it means that the beat wave is an exact eigenmode, which in this case corresponds directly to the plasmon (Langmuir wave) with the approximate dispersion relation $\omega = \omega_p$ in our case. Thus the corresponding wave resonance conditions would read

$$\omega = \omega_1 + \omega_p, \qquad \boldsymbol{k} = \boldsymbol{k}_1 + \boldsymbol{k}_l , \tag{A8}$$

and it can be shown that the small exponential due to the δ function in equation (A1) is balanced by the large factor coming from the pole. Eventually one can obtain the well-known kinetic equation for the decay in the form (Tsytovich 1970)

$$\frac{dN_{\boldsymbol{k}}}{dt} = \frac{\partial N_{\boldsymbol{k}}}{\partial t} + \boldsymbol{v}_g \frac{\partial N_{\boldsymbol{k}}}{\partial \boldsymbol{r}} - \int \frac{d\boldsymbol{k}_l\, d\boldsymbol{k}_1}{(2\pi)^6} Q(\boldsymbol{k}, \boldsymbol{k}_1, \boldsymbol{k}_l)(N_{\boldsymbol{k}} N_{\boldsymbol{k}_1} + N_{\boldsymbol{k}} N_{\boldsymbol{k}_l} - N_{\boldsymbol{k}_1} N\boldsymbol{k}_l) + \int \frac{d\boldsymbol{k}_l\, d\boldsymbol{k}_1}{(2\pi)^6} Q(\boldsymbol{k}_1, \boldsymbol{k}, \boldsymbol{k}_l)(N_{\boldsymbol{k}} N_{\boldsymbol{k}_1} + N_{\boldsymbol{k}_1} N_{\boldsymbol{k}_l} - N_{\boldsymbol{k}} N_{\boldsymbol{k}_l}) , \tag{A9}$$

where

$$Q(\boldsymbol{k}, \boldsymbol{k}_1, \boldsymbol{k}_l) = \delta(\omega - \omega_1 - \omega_l)\delta(\boldsymbol{k} - \boldsymbol{k}_1 - \boldsymbol{k}_l) \frac{\hbar e^2 (2\pi)^6 \omega_p k_l^2}{16\pi m_e^2 \omega\omega_1}\left(1 + \frac{(\boldsymbol{k}\boldsymbol{k}_1)^2}{k^2 k_1^2}\right) . \tag{A10}$$

The first term in equation (A9) describes the direct process—the decay of the electromagnetic wave onto the electromagnetic one and plasmon, while the second term describes the inverse process—the coalescence of the electromagnetic wave and plasmon into the electromagnetic wave.

APPENDIX B

In this Appendix we analyze in detail the decay of an electromagnetic wave (t-wave or photon) onto electromagnetic and Langmuir waves (l-wave or plasmon). The kinetic equation for this process is given by equation (A9) in Appendix A. We shall analyze separately the part containing photon numbers only (which we call induced plasmon emission), and the part containing plasmon numbers as well (Raman scattering).

1. With the observation that $v_g = c$ and $(1/c)\partial/\partial t = \partial/\partial l$, where l is the distance along the propagation line of the radiation beam (i.e., line of sight) one can reduce the equation for the induced plasmon emission (A9) to the following form:

$$\frac{1}{N_k}\frac{dN_k}{dl} = \frac{\hbar e^2 \omega_p \omega^2}{2m_e^2 c^4}\int_0^\pi d\theta \sin^2\left(\frac{\theta}{2}\right)\sin(\theta)(1+\cos^2\theta)[N(\omega+\omega_p, \theta) - N(\omega-\omega_p, \theta)] , \tag{B1}$$

where $\omega_l \approx \omega_p \ll \omega, \omega_1$ and $k_l \approx 2\omega \sin(\theta/2)/c$ is taken into account. We also have used the expressions for the probabilities, given in Appendix A, and have carried out the integrations using δ functions. In what follows we approximate for simplicity the angle dependence of the stimulating photon spectrum in the integrand by $N(\theta) = \text{const}$ for the angles $\theta < \theta_0$. With this assumption we use the inequality $\omega_p \ll \omega$ for the Taylor expansion

$$N(\omega \pm \omega_p) \approx N(\omega) \pm \frac{dN}{d\omega}\omega_p , \tag{B2}$$

so that we have eventually

$$\frac{1}{N_k}\frac{dN_k}{dl} = \frac{\hbar e^2 \omega_p^2 \omega^2}{m_e^2 c^4} N(\omega)\int_0^{\theta_0} d\theta \sin^2\left(\frac{\theta}{2}\right)\sin(\theta)(1+\cos^2\theta) , \tag{B3}$$

which gives the result (14) in the main text, when defining the optical depth as

$$\Sigma = \int dl \frac{1}{N_k}\frac{dN_k}{dl} , \tag{B4}$$

where the integration is carried out along the line of sight.

2. The second part of the process is related to stimulation by the ambient plasmons. To reduce the corresponding equation for the stimulated Raman scattering to a more simplified form, assume that the N_1 radiation is essentially the isotropized pulsar radiation, and therefore $N_1 \ll N$ in the complete kinetic equation (see Appendix A). In this case only the term proportional to the number of beamed photons and number of plasmons is important in equation (A9). Therefore, with the same observations as above, we can write the equation for the stimulated Raman scattering in the following form:

$$\frac{1}{Nk}\frac{dNk}{dl} = -\frac{\hbar e^2 \omega_P}{4m_e^2 c^4}\int_{0_{\min}}^{\pi} d\theta \sin\theta(1+\cos^2\theta)k_l^2 N_l , \tag{B5}$$

where the same high-frequency approximations are made and

$$k_l \approx 2\omega \sin(\theta/2)/c . \tag{B6}$$

The limits for integration over θ are determined by the plasmon spectrum. The lowest wavenumber for plasmons excited by a relativistic electron-positron beam is $k_{\min} = \omega_p/c\eta$, where η describes the resonant velocities spread in the beam and is estimated as $\eta \approx 0.3$ in the case of the isotropic relativistic temperature of the beam. The largest k_l is determined by the Landau damping and is of the order of $k_{\max} = r_D^{-1}$. Therefore, the integration limits are determined by the inequality.

$$\frac{\omega_p}{2\omega\eta} < \sin\left(\frac{\theta}{2}\right) < \frac{\omega_p}{2\omega}\frac{c}{v_T} . \tag{B7}$$

The spectrum $N_l(k_l)$ is rather uncertain. However, most theories of the Langmuir turbulence predict the Kolmogorov-like spectrum

$$W_k = 4\pi k^2 N_k \hbar\omega_p = W_0 \frac{1}{k_0}\left(\frac{k_0}{k}\right)^{5/2} \tag{B8}$$

in the inertial range $k_{\min} \ll k_l \ll k_{\max}$. For the calculation we use equation (B8) for the entire range of integration. The expression for the optical depth will take the form

$$\Sigma = \frac{e^2}{16\pi m_e^2 c^4}\int dl \int_{\theta_{\min}}^{\pi} \frac{\sin\theta\, d\theta(1+\cos^2\theta)}{(2\omega/c)^{5/2}(\theta/2)} W_0 k_0^{3/2} , \tag{B9}$$

$$\approx \frac{W_0 k_0^{3/2} e^2}{4\pi m_e^2 c^{3/2}\omega^{5/2}}\frac{1}{\theta_{\min}^{1/2}} , \tag{B10}$$

where $k_0 = \omega_p/c\eta$, $\theta_{\min} \approx k_0 c/\omega \approx \theta_p/\omega\eta$, and $W_0 \approx 3nT(m_i/m_e)(k_0 r_d)^2/2$ (see eqs. [27]–[30]). Now equation (31) is easily obtained.

APPENDIX C

In this Appendix we analyze the kinetic equation for the induced scattering. The resonance condition is picked out by the δ-function in equation (A1). For the present case of high photon frequencies the condition can be expressed as

$$v_r \approx \frac{\omega_-}{2k \sin(\theta/2)} \approx c\frac{\omega_-}{2\omega \sin(\theta/2)} , \tag{C1}$$

where θ is the angle between $\boldsymbol{k}$ and $\boldsymbol{k}_1$, and $\omega, \omega_1 \gg \omega_p$ is taken into account.

For the Maxwellian distributions the kinetic equation (A1) can be recast in the following form:

$$\frac{1}{N_{\boldsymbol{k}}}\frac{dN_{\boldsymbol{k}}}{dt} = -\Sigma_s \int \frac{d\boldsymbol{k}_1}{(2\pi)^6}\frac{\hbar\omega_-}{T_s}\frac{1}{(2\pi m_s T_s)^{1/2}}\exp\left(-\frac{\omega_-^2}{2k_-^2 v_{Ts}^2}\right)P_s(\boldsymbol{k}, \boldsymbol{k}_1)N_{\boldsymbol{k}_1} . \tag{C2}$$

Since $\varepsilon(\omega, \boldsymbol{k})$ depends greatly on the ratio ω/kv_t we divide the integration range into three ranges: (A) $\omega_-/k_- > v_{\mathrm{Te}}$, (B) $v_{\mathrm{Te}} > \omega_-/k_- > v_{\mathrm{Ti}}$, and (C) $v_{\mathrm{Ti}} > \omega_-/k_-$

$$\int(\ldots) = \int_A(\ldots) + \int_B(\ldots) + \int_C(\ldots) \tag{C3}$$

and estimate the integrals, substituting the inequalities (A), (B), and (C) by stronger ones: (A) $\omega_-/k_- \gg v_{\mathrm{Te}}$, (B) $v_{\mathrm{Te}} \gg \omega_-/k_- \gg v_{\mathrm{Ti}}$, and (C) $v_{\mathrm{Ti}} \gg \omega_-/k_-$.

In range (A) $\varepsilon_i \approx 1$, $\varepsilon \approx \varepsilon_e \approx 1 - \omega_p^2/\omega_-^2 \approx \max(1, \omega_p^2/\omega_-^2)$ far from the resonance. The probabilities take the form

$$P_e \approx \frac{(2\pi)^3 e^4}{2\omega\omega_1 m_e^2}(1 + \cos^2\theta) , \tag{C4}$$

$$P_i \approx P_e \frac{\omega_p^2}{\omega_-^2} . \tag{C5}$$

The exponential terms are very small, and since all other terms in the integral are not large, the whole integral can be eventually estimated to give an optical depth

$$\Sigma \sim \int ndl\sigma_{\mathrm{T}} \frac{\hbar\omega}{m_e c^2}\frac{dN}{d\ln\omega}\exp\left(-\frac{\omega_p^2 c^2}{\omega^2 v_{\mathrm{Te}}^2}\right) , \tag{C6}$$

where $\sigma_{\mathrm{T}} = 8\pi e^4/3m_e^2 c^4$ is the Thomson scattering cross section. One can see that because in our case $(\omega/\omega_p)^2 \sim 10^2$, $v_T^2/c^2 \sim 10^{-4}$ the exponent is very small and the resulting optical depth is negligible.

An analogous analysis of terms (B) and (C) shows that the maximal optical depth resulting from the induced scattering is at most

$$\Sigma \sim \int ndl\sigma_{\mathrm{T}} \frac{\hbar\omega}{m_e c^2}\frac{dN}{d\ln\omega}\left(\frac{m_e}{m_i}\right)^{1/2} . \tag{C7}$$

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