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COOPERATIVE GALAXY FORMATION AND LARGE-SCALE STRUCTURE

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ABSTRACT

We consider a model in which galaxy formation occurs at high peaks of the mass density field, as in the standard picture for biased galaxy formation, but is further enhanced by the presence of nearby galaxies. This modification is accomplished by assuming the threshold for galaxy formation to be modulated by large-scale density fluctuations rather than to be spatially invariant. We show that even a weak modulation can produce significant large-scale clustering. In a universe dominated by cold dark matter, a 2%–3% modulation on a scale exceeding 10 h^{-1} Mpc produces enough additional clustering to fit the angular correlation function of the APM galaxy survey. Such an effect might be detectable as a small (about 0.2 mag rms) spatial modulation of the characteristic scale in the galaxy luminosity function or as an apparent dependence of large-scale clustering on luminosity or distance in flux-limited samples. Although such effects are not necessarily present in all simple versions of the model, they could provide an observational indicator for cooperative galaxy formation. We discuss several astrophysical mechanisms for which there are observational indications that cooperative effects could occur on the scale required. Although the importance of such effects is highly uncertain, it would clearly be rash to exclude the possibility that large-scale power in the galaxy distribution is a consequence of the physics of galaxy formation. There is thus no compelling reason to reject the density fluctuation predictions of the simplest inflationary models.

Subject headings: dark matter — galaxies: formation — large-scale structure of universe

1. INTRODUCTION

Recent measurements of galaxy clustering on large scales have provoked a great deal of interest and controversy (Maddox et al., 1990; Efstathiou et al. 1990; Saunders et al. 1991). Using a variety of data, these studies all reveal rms fluctuations of order $\sim 50\%$, within spheres of radius 20 h^{-1} Mpc.[3] Relative to nonlinear clustering on scales $\sim 5\ h^{-1}$ Mpc, these amplitudes are 2–3 times stronger than predicted by the standard cold dark matter (SCDM) model. As a result, the new data have been widely interpreted as ruling out this model. The present paper focuses on the connection between the statistical properties of the galaxy and mass distributions, in order to explore whether, in the absence of a detailed theory for galaxy formation, the mass distribution predicted by the SCDM model (or any other model) can safely be excluded on the basis of observed large-scale clustering.

Our work is motivated by the realization that the galaxy-mass connection is a particularly uncertain ingredient of the standard model, and one that belongs in a completely different category from its other assumptions. We shall argue that we know so little about the details of star and galaxy formation that model predictions for the galaxy distribution must be treated with caution. In particular, we will show that plausible modifications of the standard high peak model can substantially change the predicted pattern of large-scale clustering. We do not aim to argue for any particular biasing model. Unfortunately, our understanding of the relevant processes is too incomplete for any such argument to be convincing. Rather, we show that the range of possibilities is much broader than has previously been recognized. The implication, of course, is that even such a well-specified theory as the cold dark matter model has less predictive power than one might have hoped. Nevertheless, although aspects of the *galaxy* distribution are uncertain, a number of tests of the theory depend almost solely on the *mass* distribution: microwave background fluctuations, large-scale streaming motions, abundance of massive clusters, structure of galaxy halos, and so on. These are not affected by the uncertainties we discuss in this paper.

Let us recall the four basic tenets of the SCDM theory: (1) the dark matter consists of weakly interacting elementary particles which had a low velocity dispersion at early times; (2) the universe has the critical density, $\Omega = 1$, and an expansion rate given by $h = 0.5$; (3) primordial density fluctuations are of the type predicted in standard inflationary models of the early universe: Gaussian, adiabatic, and scale-invariant; (4) the distribution of galaxies is related to the distribution of mass by

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[3] We write the Hubble constant as $H_0 = 100\, h$ km s^{-1} Mpc^{-1}.

the "high-peak" biasing model. The first three assumptions are motivated by fundamental, even if speculative, physics. Particles with the appropriate properties arise, for example, in supersymmetric theories; a critical density and the assumed type of primordial fluctuations are generic predictions of inflationary models of the early universe; a low Hubble constant is needed to get an acceptable age for any flat universe. The final assumption is motivated by considerations of a different kind. In a universe with critical density, galaxies must be more strongly clustered than the underlying mass up to scales at least as large as the Local Supercluster; otherwise dynamical analyses imply $\Omega < 1$. This requirement does not single out any specific form for the bias; the high-peak model is just a particularly plausible and convenient prescription.

It is fair to ask whether there is any observational support for the assumptions of the SCDM model. As yet, of course, there has been no direct experimental evidence for elementary particles of the required type. For the first time there is now some direct evidence in favour of a high mean cosmic density. New dynamical determinations have been made possible by all-sky redshift surveys of *IRAS* galaxies. Assuming these to be representative of the underlying mass density, one can predict the peculiar velocity field in regions of space where there are independently measured peculiar velocities for other galaxies (including our own). Unless *IRAS* galaxies are substantially *less* clustered than the mass, all recent determinations favor a value of Ω near unity (Strauss & Davis 1989; Rowan-Robinson et al. 1990; Bertschinger et al. 1990; Kaiser et al. 1991). On the other hand, recent measurements of the Hubble constant consistently prefer values larger than $h = 0.5$ (Pierce & Tully 1988; Jacob, Ciardullo, & Ford 1990; Tonry 1991). Testing assumption 3 is more difficult because it refers to properties of the *mass* distribution which are not readily observable;[4] inferences from the corresponding properties of the *galaxy* distribution require, in addition, assumption 4 (but see Moore et al. 1992; Weinberg 1992).

In principle, there is tremendous predictive power in the first three of the above assumptions. With only one free parameter—the initial fluctuation amplitude—one can specify the mass distribution accurately at all times on scales larger than individual galaxies. This is most easily done using N-body simulations (see Frenk 1991 for a review). Unfortunately, there are only a handful of observables which probe the mass distribution directly, the most readily accessible of which are the mass distributions in galaxy halos and in rich galaxy clusters, the peculiar motions of galaxies, the abundance of rich clusters, and the spatial structure of the microwave background radiation. Further predictions are possible only by resorting to assumption 4. With this assumption, the SCDM model has been singularly successful in accounting for many observed properties of galaxies and their distribution out to scales $\sim 10\ h^{-1}$ Mpc.[5] Among its most notable successes are agreement with the low order statistics of galaxy clustering, the inferred structure and abundance of galactic halos, the structural properties and abundances of galaxy groups and clusters, and the general topology of the large-scale galaxy distribution (see the review by Frenk 1991).

However, recent studies of clustering on scales $\gtrsim 10\ h^{-1}$ Mpc pose serious difficulties for the SCDM model. The most compelling data are angular correlation functions for the Automatic Plate Measuring Facility (APM) survey of $\sim 10^6$ galaxies. These decline much less rapidly on large scales than the SCDM predictions (Maddox et al. 1990). An independent indication of strong clustering on similar scales came from the Queen Mary and Westfield College Durham, Oxford, and Toronto (QDOT) redshift survey of over 2000 *IRAS* galaxies (Efstathiou et al. 1990; Saunders et al. 1991; Moore et al. 1992). This refers, necessarily, to a much smaller volume than the APM survey, and the signal is dominated by a few strong superclusters in the more distant part of the sample. Preliminary analysis of a significantly larger, but as yet unpublished, redshift survey of *IRAS* galaxies, suggests that much of the signal in the QDOT data is an upward statistical fluctuation (M. Davis & G. Efstathiou 1991, private communication; Fisher et al. 1992); confirmation of this must await final joint analysis of the two samples.

The suggestion of "excess power" first arose from estimates of the autocorrelation of Abell clusters (Hauser & Peebles 1973; Bahcall & Soneira 1983; Klypin & Kopylov 1983). The numerical simulations of White et al. (1987a) showed that cluster correlations in SCDM should have an amplitude 2–3 times smaller than these estimates. Unfortunately, observations of cluster correlations have had a checkered history, and the integrity of the Abell cluster has been repeatedly called into question (Sutherland 1988; Dekel et al. 1989; Olivier et al. 1990; Sutherland & Efstathiou 1991; but see Postman, Huchra, & Geller 1992). A new sample of rich clusters identified from the APM survey exhibits weaker correlations which are (marginally) consistent with the SCDM predictions (Dalton et al. 1992), but X-ray cluster samples have a large correlation length (Lahav et al. 1989). Finally, Peacock (1991) and Peacock & Nicholson (1991) show that bright radio galaxies are also strongly clustered on large scales. It is unclear how these objects should be related either to more normal galaxies or to rich clusters.

Crucial in the comparisons of these clustering data with SCDM are (often implicit) assumptions which relate the galaxy and mass distributions. Galaxy formation is a highly nonlinear, and perhaps nonlocal, transformation of the mass density field. As such it is unlikely to preserve the correlation statistics of the density field, even if the transformation is effectively local (P. Coles 1992, in preparation). The morphology-density relation (Dressler 1980; Postman & Geller 1984; Haynes 1988), in conjunction with growing evidence that galaxies of different luminosity (Loveday et al. 1992) or selected at different wavelength (Saunders, Rowan-Robinson, & Lawrence 1992) cluster differently, shows that the correlation statistics of all types of galaxy cannot separately parallel those of the mass distribution. Nevertheless, observational analyses almost always assume that this is the case, and where justification is required they cite the standard high-peak model.

The idea that the sites of galaxy formation might in some way be associated with the peaks of a suitably smoothed version of the linear density field seems eminently reasonable. In the simplest realization of this scheme, the statistics of real catalogs are identified with those of the peaks found when the smoothing scale, R_g, and threshold peak height, $\nu\sigma$, are both spatially invariant [here $\sigma^2 = \langle(\rho - \bar{\rho})^2\rangle/\bar{\rho}^2$ is the variance of

[4] The nature of the fluctuations in the microwave background radiation detected by *COBE*, and announced after this paper was submitted, provides substantial support for inflationary model assumption 3 (Smoot et al. 1992; Wright et al. 1992). This detection constrains the overall amplitude of the mass fluctuations, but does not affect the major issue addressed in this paper, namely, the shape of the galaxy autocorrelation function on large scales.

[5] However, a somewhat lower bias than originally envisaged is required to fall within the error bands of the *COBE* measurement.

the smoothed field]. This has become the standard model for "biased galaxy formation," and has been investigated extensively since Kaiser (1984) first suggested a similar model to explain the strong clustering of Abell clusters (Davis et al. 1985; Bardeen et al. 1986; White et al. 1987a; Coles 1989; Lumsden, Heavens, & Peacock 1989). In this particular model the "galaxy" correlation function on large scales is indeed proportional to that of the underlying mass fluctuations.

In fact, there is no convincing justification for assuming that peaks should make successful galaxies if and only if they rise above some *universal* threshold. Soon after the concept of biasing was introduced, there were suggestions that feedback effects from a first generation of galaxies, themselves formed by the collapse of the highest peaks, might suppress star formation in neighboring protogalaxies (Rees 1985; Silk 1985). Such "action-at-a-distance" mechanisms might also act to *stimulate* the formation of nearby galaxies (see Dekel & Rees 1987 for a review). Another possibility is that bias might arise "naturally" through nonlocal dynamical processes reflecting the effect of a galaxy's environment on its collapse and evolution. Such mechanisms have been explored by White et al. (1987b), Frenk et al. (1988), Carlberg & Couchman (1989), and Carlberg, Couchman, & Thomas (1990), but it is still unclear how they relate to the high-peak model or whether their predictions are consistent with observation (Cole & Kaiser 1989; White, Tully, & Davis 1988; Eder et al. 1989). In this paper we study a simple extension of the high-peak model which represents nonlocal effects of the first type.

If feedback effects are important, the visibility of a galaxy may depend on the large-scale statistical properties of the density field in a complex way. For example, protogalaxies collapsing within range of the radiative or hydrodynamic effects of other protogalaxies might produce slightly brighter or slightly fainter galaxies than those forming elsewhere. We investigate such cooperative effects using a simple model in which the threshold for producing a visible (i.e., cataloged) galaxy depends linearly on the mean mass density averaged over some large surrounding volume; this should itself be a (nonlinear) measure of the galaxy density in the same volume. Within this model a surprisingly small (but observable) spatial modulation of galaxy properties is sufficient to reconcile the SCDM model with the observed APM correlations. Although we cannot prove their importance, we mention several physical mechanisms that could plausibly produce such modulation. Our main point is that processes related to galaxy formation can substantially modify the spatial distribution of galaxies on large scales. Related ideas have been discussed recently by Babul & White (1991), but our mathematical formalism and our physical picture are fundamentally different from theirs. Our model is much closer in spirit to the original high-peak model, and is distinguished mainly by the fact that nonlocal effects cause the mass and galaxy correlation functions to have different shapes. In § 2 we develop the requisite mathematical tools. In § 3 we apply them to scale-free models, for which various analytic relations are easily derived, and to the CDM model. In § 4 we discuss physical processes which might give rise to cooperative galaxy formation, and observational tests of its consequences.

2. A MATHEMATICAL MODEL FOR COOPERATIVE GALAXY FORMATION

In the standard high-peak model, galaxy formation is assumed to occur from material initially located near high peaks of the *linear* density fluctuation field. The density contrast at early times, $\delta(\boldsymbol{x}) = [\rho(\boldsymbol{x}) - \bar{\rho}]/\bar{\rho}$, is taken to be a Gaussian random field, and to have been smoothed with a kernel of characteristic scale R_g, chosen so that the enclosed mass matches the halo mass of a bright galaxy. Peaks rising above some *global* threshold, $\delta(\boldsymbol{x}_{\rm pk}) > \nu\sigma$, are then identified as galaxy formation sites. Here σ is the rms value of δ, and ν is typically taken to be 2–3. We wish to extend this model to incorporate the possibility that peaks form galaxies more easily (or, equivalently, form brighter galaxies) if there are other galaxies nearby. One might imagine making the threshold level, ν, a decreasing function of the mean galaxy density in some surrounding domain of influence of characteristic size $R_s > R_g$. However, this is mathematically intractable because of the highly nonlinear relation between galaxy and mass densities in the high-peak model. We therefore adopt the simpler assumption that the threshold depends on the mean *mass* density in the domain of influence. This is strongly correlated with the galaxy density and is much easier to analyze. We take the simplest possible dependence and assume that galaxies form from material near peaks satisfying

$$\delta(\boldsymbol{x}_{\rm pk}) > \nu\sigma - \kappa\bar{\delta}(\boldsymbol{x}_{\rm pk}) , \tag{1}$$

where $\bar{\delta}$ is the density contrast smoothed on the scale, R_s, of the domain of influence, and κ is a constant which we will refer to as the *modulation coefficient*. We have thus introduced two parameters, R_s and κ, which characterize the scale and amplitude of cooperative effects. If κ is positive, peaks in "protosupercluster" regions have to cross a lower threshold in order to form a significant galaxy than peaks in "protovoids."

It is easy to see that the model of equation (1) is mathematically equivalent to the standard high-peak model, but for a different density field. Let us define a new field by

$$\delta'(\boldsymbol{x}) = \delta(\boldsymbol{x}) + \kappa\bar{\delta}(\boldsymbol{x}) . \tag{2}$$

Since $\bar{\delta}$ is just a convolution of δ with a fixed filter, it is clear that δ' retains the random phase property of δ, and so is also a Gaussian random field. Hence, rewriting equation (1) as $\delta' > \nu\sigma$, one finds that our cooperative galaxy formation model predicts a galaxy distribution identical to that found if the original high-peak model is applied to δ' rather than to δ.[6] We can therefore apply all the analytic machinery developed for the standard model to our extension of it. In order to keep the mathematics simple, we will follow Kaiser (1984) and approximate the correlation function of *peaks* above a given threshold by that of *points* above the same threshold. On scales larger than R_g this approximation is reasonably good. The result differs from the true peak-peak correlation function primarily in giving more weight to higher peaks (Bardeen et al. 1986). This approach is quite adequate for our purpose, which is to demonstrate how cooperative effects can modify the predictions of the standard biasing model.

We start from Kaiser's (1984) definition of the relevant correlation function as the fractional excess probability that two points at separation r are both above the global threshold

$$1 + \xi_{>\nu}(r) = \frac{\Pr(\delta_1 > \nu\sigma, \delta_2 > \nu\sigma)}{[\Pr(\delta > \nu\sigma)]^{-2}} = \frac{\mathscr{P}_2(r)}{\mathscr{P}_1^2} . \tag{3}$$

[6] This is not quite accurate, because the peaks of δ' do not coincide exactly with those of δ. However, the correspondence is good for the parameters which interest us. Furthermore, the analogy *is* exact in Kaiser's (1984) approximate treatment of the problem, which we follow in this paper.

In this definition, δ_1 and δ_2 are used to denote the density contrast at the positions, x_1 and x_2, of the two field points, and $|x_1 - x_2| = r$. Since δ is a Gaussian random field, these probabilities are given by

$$\mathscr{P}_1 = \int_v^\infty f_1(y)dy , \tag{4}$$

$$\mathscr{P}_2(r) = \int_v^\infty \int_v^\infty f_2(y_1, y_2; \xi(r)/\sigma^2) dy_1\, dy_2 , \tag{5}$$

where x, y are scaled variables with unit variance, $f_1(x)$ is the univariate Gaussian probability distribution function (pdf), $f_2(x, y; C)$ is the bivariate Gaussian pdf with covariance, $\langle xy \rangle = C$, $\xi(r) = \langle \delta_1 \delta_2 \rangle$ is the two-point correlation function of the matter distribution. Thus $\xi_{>v}(r)$ is a function only of $\xi(r)/\sigma^2$ and v. Kaiser (1984) showed that in the limit where $v \to \infty$ and $\xi \to 0$, this dependence is simply

$$\xi_{>v}(r) \simeq \frac{v^2}{\sigma^2}\, \xi(r) , \tag{6}$$

so that there is a multiplicative amplification of the correlations of peaks relative to those of the underlying matter distribution. For other approximations to $\xi_{>v}$ see Politzer & Wise (1984) and Jensen & Szalay (1986).

In our extension of the high-peak model, $\xi_{>v}$ will be given by these same formulae, except that ξ, σ, and v must be replaced by

$$\xi'(r) = \langle \delta_1' \delta_2' \rangle = \xi(r) + 2\kappa \langle \bar{\delta}_1 \delta_2 \rangle + \kappa^2 \langle \bar{\delta}_1 \bar{\delta}_2 \rangle , \tag{7}$$

$$\sigma' = \sqrt{\xi'(0)} , \tag{8}$$

$$v' = v\sigma/\sigma' . \tag{9}$$

If $R_s \gg R_g$, then $\sigma' \approx \sigma$, $v' \approx v$, and $\xi'(r) \approx \xi(r)$ for $r \ll R_s$. In this limit our modification of the standard model does not affect the "galaxy correlations", $\xi_{>v}$, on scales much smaller than that of the domain of influence. On scales much larger than R_s there are two possibilities. Either the smoothing involved in going from δ to $\bar{\delta}$ has little effect on the correlations, so that $\xi' \approx (1 + \kappa)^2 \xi$, or the tail of the smoothing kernel dominates the correlations so that $\xi' \approx \kappa^2 \langle \bar{\delta}_1 \bar{\delta}_2 \rangle$. In both cases $\xi'(r)$ is enhanced above $\xi(r)$ for large r, with the result that $\xi_{>v}$ in our cooperative formation model has extra large-scale power when compared with the standard high-peak model. Notice that if κ is too large or R_s is too small, then v' will differ significantly from v, and it is clear from equation (4) that our modification will change the *abundance* of galaxies predicted by the standard model. In practice, we will be interested in parameter sets for which this is not a problem.

Neither the Kaiser approximation for $\xi_{>v}$ (eq. [6]) nor the published alternatives to it (Politzer & Wise 1984; Jensen & Szalay 1986) are particularly accurate on the scales of interest to us (Coles 1986). We have therefore chosen to evaluate $\xi_{>v}(r)$ numerically. We use the techniques outlined in the Appendix to reduce the dimensionality of the integrals, in order to end up with

$$\xi_{>v}(r) = \left\{ \xi'(r) \int_0^1 [\xi'(0)^2 - \lambda^2 \xi'(r)^2]^{-1/2} \exp\left[-\frac{v'^2 \xi'(0)}{\xi'(0) + \lambda \xi'(r)} \right] d\lambda \right\} \times \left[\int_{v'}^\infty \exp\left(-\frac{y^2}{2} \right) dy \right]^{-2} . \tag{10}$$

The line integral in the first line of the equation may be evaluated by standard numerical techniques. In the next sections we calculate $\xi_{>v}(r)$ for cosmologically interesting perturbation spectra.

3. APPLICATIONS

In this section we apply our model for cooperative galaxy formation to specific Gaussian random fields. This shows how its predictions differ from those of the standard high-peak model, and allows us to assess its relevance to large-scale power in the observed galaxy distribution. We first examine scale-free random fields for which analytical calculations of the integrals are possible. We then consider the fluctuation field predicted in a universe dominated by cold dark matter. Let $P(k) = \langle |\delta_k|^2 \rangle$ be the power spectrum of density fluctuations as a function of spatial frequency k. This function is sufficient to specify all the properties of a Gaussian random field. We require versions of the initial fluctuation field smoothed on several scales. For a Gaussian filter of scale R, the smoothed field, $\delta_s(x, R)$, is obtained from the unsmoothed field, $\delta(x)$, through

$$\delta_s(x, R) = \frac{1}{(2\pi R^2)^{3/2}} \int d^3 s\, \delta(s) \exp\left(-\frac{|x - s|^2}{2R^2} \right) . \tag{11}$$

It is straightforward to show that

$$\langle \delta_s(x_1, R_1) \delta_s(x_2, R_2) \rangle = \frac{1}{2\pi^2} \int_0^\infty dk\, k^2 P(k) \frac{\sin kr}{kr} \exp\left[-\frac{1}{2} k^2 (R_1^2 + R_2^2) \right] \tag{12}$$

is the covariance between versions of the field with different smoothing and measured at different field points. Here $r = |x_1 - x_2|$. Notice that this notation differs slightly from that of the last section, where $\delta(x)$ was used to denote $\delta_s(x, R_g)$, and $\bar{\delta}(x)$ to denote $\delta_s(x, R_s)$.

3.1. *Scale-free Fluctuations*

A scale-free power spectrum has the form

$$P(k) = \langle |\delta_k|^2 \rangle \propto k^n , \tag{13}$$

with n constant. Power spectra of this sort not only are a mathematical covenience but also can provide a good approximation to physically motivated models over a limited range of scales. For such spectra, the variance of the density field varies with smoothing as $\langle \delta_s^2 \rangle \propto R^{-n-3}$. We are interested in the field smoothed on two different scales, R_g and R_s, for which, in our previous notation,

$$\frac{\langle \delta^2 \rangle}{\langle \bar{\delta}^2 \rangle} \equiv \frac{\sigma^2}{\bar{\sigma}^2} = \left(\frac{R_s}{R_g} \right)^{n+3} . \tag{14}$$

This simple relation enables us to be more precise about the requirement, $v' \approx v$, needed for our model to preserve the abundance and the small-scale clustering of galaxies in the standard high-peak model (i.e., for $\kappa = 0$). From equations (7), (8), (9), (12), and (14) we find that the condition $|v - v'| \ll 1$ requires

$$\kappa^2 + 2^{(n+5)/2} \kappa \ll \frac{2}{v} \left(\frac{R_s}{R_g} \right)^{n+3} . \tag{15}$$

Thus, as long as $R_s \gg R_g$, even a large value of κ can be invoked without affecting abundances or small-scale clustering.

For a power-law spectrum, equation (12) can actually be evaluated analytically. We find

$$\langle \delta_s(x_1, R_1)\delta_s(x_2, R_2)\rangle = \sigma^2\left(\frac{2R_g^2}{R_1^2 + R_2^2}\right)^{(n+3)/2} F_n\left(\frac{r^2}{2(R_1^2 + R_2^2)}\right), \quad (16)$$

where $F_n(x) = M((n + 3)/2, 3/2; -x)$ is Kummer's function (Abramowitz & Stegun 1972). Hence, in our previous notation,

$$\langle \delta_1 \delta_2 \rangle = \xi(r) = \sigma^2 F_n(r^2/4R_g^2), \quad (17)$$

$$\langle \bar{\delta}_1 \bar{\delta}_2 \rangle = \bar{\sigma}^2 F_n\left(\frac{r^2}{4R_s^2}\right) = \sigma^2\left(\frac{R_g}{R_s}\right)^{n+3} F_n\left(\frac{r^2}{4R_s^2}\right), \quad (18)$$

$$\langle \delta_1 \bar{\delta}_2 \rangle = \sigma^2\left(\frac{2R_g^2}{R_s^2 + R_g^2}\right)^{(n+3)/2} F_n\left(\frac{r^2}{2(R_s^2 + R_g^2)}\right). \quad (19)$$

The Kummer function, $F_n(x)$, has asymptotic behavior, $F_n \to 1$ as $x \to 0$, and $F_n \propto x^{-(n+3)/2}$ as $x \to \infty$. Thus, for $r \gg R_s$, all three of these correlations are approximately equal, demonstrating the result, noted earlier, that $\xi' \approx (1 + \kappa)^2 \xi$ on large scales. Provided that equation (15) is satisfied, our modification of the standard high-peak model increases the galaxy correlations, $\xi_{>\nu}$, predicted on these scales by the same factor. The transition between these enhanced correlations and the unchanged small-scale correlations clearly occurs at $r \sim R_s$. Its detailed shape may be expected to depend on the shape of the filter function, representing the distance dependence of a galaxy's influence on its neighbors (here assumed Gaussian).

Figures 1*a* and 1*b* show $\xi_{>\nu}(r)$ for fluctuation spectrum indices $n = -1$ and $n = 0$, respectively. In both cases, we chose $R_s/R_g = 10$ and $\nu = 2.5$, and used the four values of κ indicated in the plot. Our Gaussian representation of the domain of influence clearly leads to a relatively abrupt transition between the small- and large-scale regimes. Only for the largest values of κ does condition (15) begin to be violated in the $n = -1$ case. When this happens, the bias on small scales begins to drop below that predicted by the standard model. This is easily understood as a consequence of σ' beginning to increase significantly above σ, thereby forcing ν' to drop below ν—the peaks accepted as galaxies start to become less "rare." However, these effects are negligible for parameters of interest in our current problem. Figure 1*c* illustrates the effect of altering the averaging scale. As expected, the modification of the standard model retains its shape as R_s is adjusted, provided that it does not approach R_g too closely. Clearly, wide variations in the shape of galaxy correlations could be produced by superposing a number of feedback effects with varying strength, sign, and scale.

3.2. *Cold Dark Matter Models*

We now apply our cooperative galaxy formation model to a CDM universe. We take the power spectrum in the form given by Davis et al. (1985):

$$|\delta_k|^2 = 2.43 \times 10^3 b^{-2} k(1 + \alpha k + \beta k^{3/2} + \gamma k^2)^{-2}\ \mathrm{h}^{-3}\ \mathrm{Mpc}^3, \quad (20)$$

where b is the usual "biasing parameter," defined as the square root of the ratio of the variances of the galaxy and mass fluctuations within randomly placed spheres of radius 8 h^{-1} Mpc (see, e.g., Bardeen et al. 1986). If k is expressed as 2π divided by

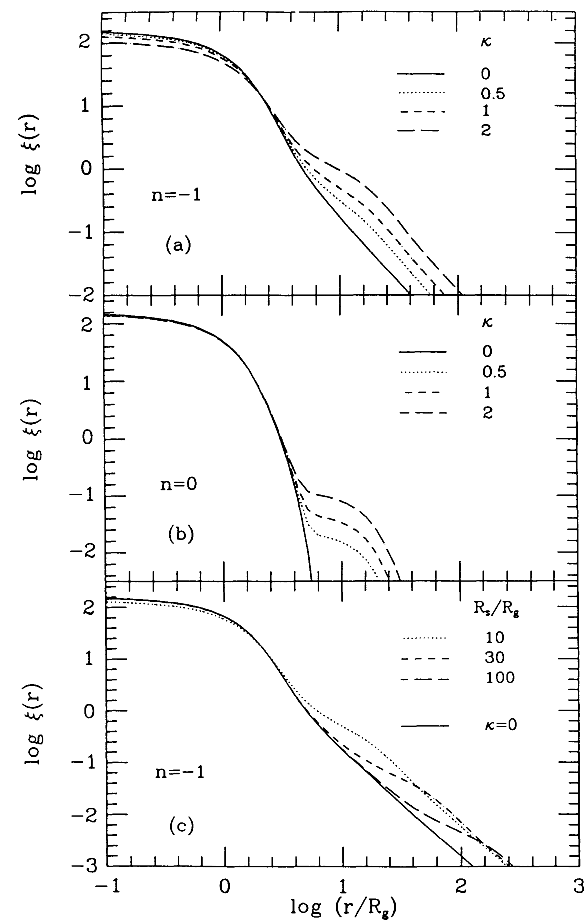

FIG. 1.—Autocorrelation functions for models with scale-free fluctuation spectra, $P(k) \propto k^n$. The unperturbed threshold overdensity for galaxy formation is taken to be $\nu = 2.5$. (*a*) Correlations for $n = -1$, with fixed $R_s/R_g = 10$ and κ equal to 0 (*solid line*), 0.5 (*dotted line*), 1 (*short-dashed line*), and 2 (*long-dashed line*). (*b*) As in (*a*), but for $n = 0$. (*c*) Correlations for $n = -1$, with fixed $\kappa = 1$ and R_s/R_g equal to 10 (*dotted line*), 30 (*short-dashed line*), and 100 (*long-dashed line*); the solid line corresponds to $\kappa = 0$ as in (*a*). Note that since the threshold is specified relative to the amplitude of the linear density fluctuations, the size of these correlations is independent of the normalization of the underlying linear power spectrum.

the present wavelength in Mpc, then $\alpha = 1.7l$, $\beta = 9.0l^{3/2}$ and $\gamma = 1.0l^2$, where $l = (\Omega h^2)^{-1}$; in the standard CDM model, $\Omega = 1$ and $h = 0.5$, so $l = 4$. Since our model, like the original high-peak model, calculates galaxy correlations from ξ/σ^2, the amplitude of the power spectrum, and so the parameter b drops out of our analysis. All the numerical results given and plotted below assume $h = 0.5$.

We now proceed as in the previous section, substituting the power spectrum, equation (20), in equation (12) in order to calculate the three terms in the equation for $\xi'(r)$ (eq. [7]). There are four parameters that must be fixed. The first two specify the standard high-peak model: these are the galactic smoothing scale, R_g, and the global threshold, ν. The remaining two describe the modulation of the threshold due to cooperative effects: these are the scale of the domain of influence, R_s, and the modulation coefficient, κ. For the first two we take values typical of those used in N-body work: $R_g = 0.5\ h^{-1}$ Mpc and $\nu = 2.8$. Figure 2 shows spatial correlation functions for three different choices of the modulation parameters. For each smoothing length, κ was adjusted so as to produce the

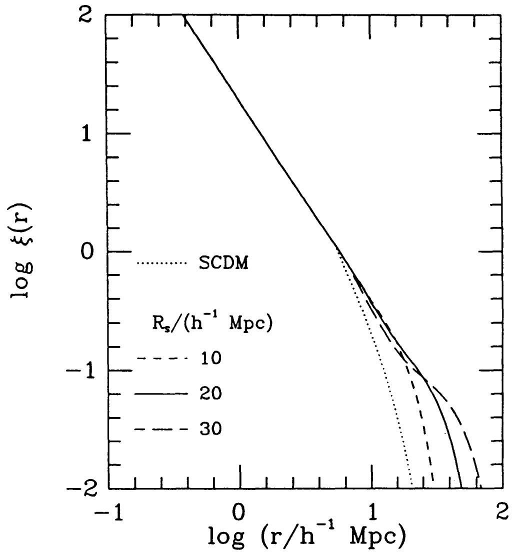

FIG. 2.—Autocorrelation functions for the cold dark matter model. The results of our calculations are plotted for $\xi \lesssim 1$; on smaller scales we plot the power law $\xi(r) = (r/r_0)^{-1.7}$, with $r_0 = 5.7\ h^{-1}$ Mpc. (This power law is plotted from the point at which it first intercepts the model prediction.) The dotted line gives correlations in the standard "high-peak" model with $R_g = 0.5\ h^{-1}$ Mpc and $\nu = 2.8$. The remaining lines show results for models of cooperative galaxy formation with these same values for R_g and ν, but with different sizes of the domain of influence: $R_s = 10\ h^{-1}$ Mpc (*short-dashed line*), $R_s = 20\ h^{-1}$ Mpc (*solid line*), and $R_s = 30\ h^{-1}$ Mpc (*long-dashed line*). In each case, the modulation coefficient was adjusted so as to give a constant, 2.5% rms modulation of the threshold: $\kappa = 0.84$, 2.29, and 4.48, respectively. All the calculations depicted here assume $h = 0.5$.

same rms modulation of the threshold, $\kappa\bar{\sigma}/\nu\sigma \equiv \Delta\nu/\nu = 0.025$ (cf. eq. [1]). This required taking $\kappa = 0.84$, 2.29, and 4.49 for $R_s = 10$, 20, and 30 h^{-1} Mpc, respectively.

Once again we see that our modulation of the threshold for galaxy formation has little effect on small-scale correlations. However, at large r, deviations from the standard biasing model can be substantial, even for relatively small values of the modulating scale, $R_s \simeq 10\ h^{-1}$ Mpc. Large changes in $\xi_{>\nu}$ occur even though the fractional change in the threshold is small. (Since the number density of galaxies depends exponentially on ν, it is reassuring that only a small perturbation is required.) Effects are more complex than for the scale-free spectra of the last section because the linear mass correlation function for CDM has a zero crossing on the scales of interest [at $r = 18(\Omega h^2)^{-1}$ Mpc] and thus is not well approximated by any power law. The smoothing involved in obtaining $\bar{\delta}$ from δ can push the corresponding zero crossing of ξ' out to considerably larger scales, resulting in a prediction of similar behavior for the galaxy correlations. This is the reason why the ξ' curves in Figure 2 remain positive at separations for which ξ_ρ is negative.

In order to find out whether our model can match observed estimates of large-scale power in the galaxy distribution, we now calculate the predicted angular autocorrelation function, $w(\theta)$, for comparison with the estimate of Maddox et al. (1990) from the APM survey. Their data (kindly provided by the APM group) are reproduced in Figure 3, which plots $w(\theta)$ for six disjoint bins in apparent magnitude, all scaled to the magnitude limit of the Lick catalog (Groth & Peebles 1977), which is deemed to be complete to $b_j = 18.4$. The dotted line shows the angular correlation function expected in the SCDM model, which, of course, assumes a uniform biasing threshold. The "power crisis" of the standard model on large scales is clearly illustrated in this plot. As Maddox et al. discuss, the fit cannot be improved by changing the bias strength in the CDM predictions or by changing the assumed Hubble constant within acceptable limits; the theoretical $w(\theta)$ has the wrong shape.

The angular correlation function, $w(\theta)$, is related to its spatial counterpart, $\xi(r)$, through Limber's integral equation (which involves the luminosity function or, alternatively, the redshift distribution of the sample under consideration). Thus, to calculate our predictions for $w(\theta)$, we need to know $\xi(r)$ for all r. Our analytic models, however, are only valid on large scales, since they do not include dynamical effects which alter small-scale clustering. This limitation is of little relevance here, since we are primarily interested in large-scale effects, and, in any case, our model predictions do not differ from those of the standard model on small scales. We have therefore used a similar approach to that of Maddox et al. (1990). We extrapolate our model correlation functions to small scales by $\xi(r) = (r/r_0)^{-1.7}$, where $r_0 = 5.7\ h^{-1}$ Mpc, as recommended by Maddox et al. The transition from our model prediction to this power law is made where the two functions cross: $r = 5.7\ h^{-1}$ Mpc for the standard model ($\kappa = 0$) and slightly larger radii for our cooperative formation models (see Fig. 2). We also adopt the luminosity function recommended by Maddox et al. (their eq. [3]) and their evolutionary prescription. For the standard model, the resulting $\xi(r)$ is a reasonable match to the N-body simulations of Frenk et al. (1990), and the corresponding $w(\theta)$ is very similar to the one plotted by Maddox et al.

Predictions of $w(\theta)$ made in this way are compared with the APM data in Figure 3 for the same three values of R_s shown in Figure 2. Recall that in each case κ was chosen so that the rms modulation of the effective threshold is only 2.5%. The model

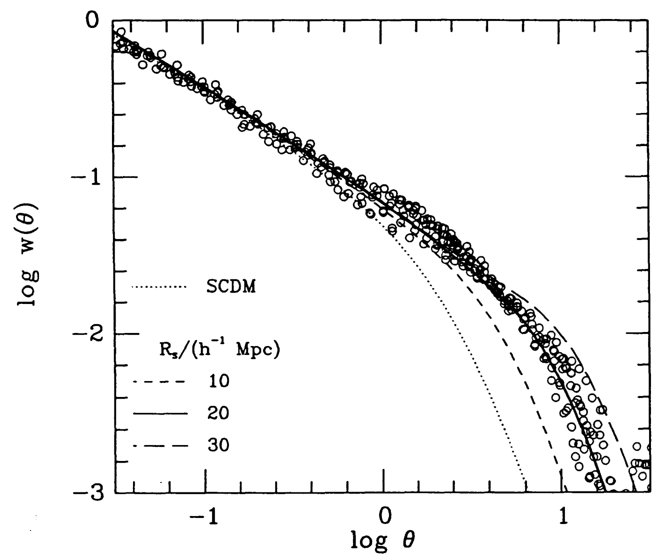

FIG. 3.—Angular autocorrelation function for galaxies in the APM survey and in cold dark matter models. The circles give the APM result for galaxies in the magnitude range $17 \leq b_j \leq 20$, split into six disjoint lines in apparent magnitude, all scaled to the magnitude limit of the Lick catalogue, $b_j = 18.4$. The dotted line shows correlations in the standard CDM model. The remaining three lines show correlations in models of cooperative galaxy formation assuming the same sizes of the domain of influence used in Fig. 2: $R_s = 10\ h^{-1}$ Mpc (*short-dashed line*), $R_s = 20\ h^{-1}$ Mpc (*solid line*), and $R_s = 30\ h^{-1}$ Mpc (*long-dashed line*). The angular correlations were obtained from the spatial correlations depicted in Fig. 2 using the procedure described by Maddox et al. (1990).

with $R_s = 20\ h^{-1}$ Mpc gives an excellent fit to the observations, while the other two models bracket the range of acceptable possibilities. A modulating scale of $\sim 10\ h^{-1}$ Mpc seems to be the smallest acceptable value, but arbitrarily large amounts of large-scale power can be produced in the galaxy distribution by choosing a sufficiently large R_s. In the next section we explore physical mechanisms that may produce a modulation on the scales required to account for the APM data.

4. DISCUSSION

We envisage cooperative effects as arising through radiative and hydrodynamical processes during protogalactic evolution. In a model like CDM, the dominant dark matter component will be almost unaffected by these processes. Thus, the structure and abundance of massive halos should differ little between cooperative formation and standard models. As a result, the most plausible implementation of our ideas would seem to be through a modulation of the luminosity of the galaxy formed by a peak of given height. We require peaks to form somewhat brighter galaxies when there are other peaks nearby. Lower peaks would then make it into a magnitude-limited catalog more easily if they occurred in "protosuperclusters" rather than in "protovoids." This brightening could occur in a variety of ways and could even conspire to produce no detectable dependence of the luminosity function on clustering. However, it is more plausible that such a dependence would be induced, and it is easy to estimate how large a modulation of the galaxy luminosity function might be required.

As noted above, an rms modulation, $\Delta\nu/\nu = 2.5\%$, of the effective threshold can boost the large-scale power of a CDM model to the levels seen in the APM survey. In the neighborhood of our chosen parameters, $\nu = 2.8$, $R_g = 0.5\ h^{-1}$ Mpc, the formulae of Bardeen et al. (1986, Figs. 1 and 3) show that the abundance of peaks varies as $\Delta n/n \approx -10\Delta\nu/\nu$. Thus, we require cooperative effects to cause a 25% modulation of the probability that a peak produces a cataloged galaxy. Let us take a galaxy luminosity function of the form $n_g(L)dL \propto \exp(-L/L_*)dL/L$ and, for a simple model for brightening, assume that the modulation affects the characteristic luminosity, L_*, but preserves the shape. This is equivalent to assuming that all galaxies are brightened by the same factor. We can characterize the depth of a flux-limited survey by the expected median distance. For our chosen luminosity function and in the absence of cooperative effects, the flux limit corresponds to $0.62L_*$ at this distance. A 0.21 mag modulation of L_* at this limiting luminosity alters the abundance by 25%. This is a remarkably small amount. The luminosities of galaxies need to be modulated by only about 0.2 mag to explain the entire "excess" power in the APM data. Notice that in principle such an effect should be directly observable. Current studies of the luminosity dependence of clustering, however, come to contradictory conclusions (Hamilton 1988; White et al. 1988; Eder et al. 1989; Valls-Gabaud, Alimi, & Blanchard 1989; Loveday et al. 1992).

With a luminosity function of our assumed shape, this simple model predicts the abundance variation induced in flux-limited catalogs to be quite a strong function of distance. At the expected upper and lower quartile distances of such a catalog, the apparent magnitude limit corresponds to luminosities of $1.20L_*$ and $0.28L_*$. Above these limits a 0.21 mag. variation of L_* induces abundance variations of 40% and 17%, respectively, corresponding to threshold variations of $\Delta\nu/\nu \approx 4.0\%$ and 1.7%. This particular model for the influence of cooperative effects on the luminosity function thus predicts that the large-scale clustering seen in flux-limited surveys should increase strongly with distance.[7] (However, other equally simple models can be found in which no such dependence is present—for example, one in which the amplitude but not the shape of the luminosity function depends on environment.) Note that these distance-dependent effects occur within any one flux-limited sample but do not affect the scaling of $w(\theta)$ between samples to different flux limits. There is some weak but suggestive evidence for a distance dependence of clustering in current data. Most of the excess power detected by the QDOT survey is contributed by superclusters in the more distant part of the sample, as can be clearly seen in the counts-in-cells analysis of Efstathiou et al. (1990; see their Fig. 1). Notice that this same effect requires that the large-scale clustering—but *not* the small-scale clustering—galaxies should increase strongly at high luminosities. Weak evidence in favor of this comes from the fact that the most luminous galaxies known, cD galaxies and strong radio sources, do indeed have strong large-scale correlations (West & van den Bergh 1991; Peacock & Nicholson 1991; Peacock 1991).[8] Finally, a large-scale modulation of galaxy luminosities would give rise to a similarly modulated variation in the zero points of the Tully-Fisher or "D_n-σ" relations, commonly used as distance indicators. This effect might be misinterpreted as coherent large-scale galaxy flows (e.g., Lynden-Bell et al. 1988; Mathewson, Ford, & Buchhorn 1992; Silk 1989).

The amplitude of cooperative effects may not need to be large, but their coherence scale, in excess of $10\ h^{-1}$ Mpc, remains impressive. For our arguments to be convincing, we must clearly identify processes which could lead to the kind of modulation we are suggesting. Our poor understanding of the physics of galaxy formation is a severe handicap at this point. We are able to do little more than point to some observational indications that physical effects of large intrinsic scale are present at the epoch of galaxy formation, and can plausibly influence galaxy formation itself.

Along the line of sight to high-redshift quasars, the density of intervening hydrogen clouds is observed to drop as the redshift approaches that of the quasar—the so-called proximity effect. This is clear evidence that the UV radiation field of a typical quasar is affecting the ionization state, and perhaps the structure, of the intergalactic medium out to comoving distances of about $20\ h^{-1}$ Mpc (Bajtlik, Duncan, & Ostriker 1988). At higher redshifts there is also evidence that the observed quasar population emits insufficient UV radiation to explain the ionization of the diffuse intergalactic medium, and that a substantial additional radiation source, most plausibly massive stars in forming galaxies, is needed (Donahue & Shull 1987; Shapiro & Giroux 1987). If star formation is rapid, the UV luminosity of a massive protogalaxy can rival that of a QSO (e.g., Terlevich 1989), and so should influence the structure and ionization of

[7] As a result of such a dependence, our calculation of $w(\theta)$ is incorrect, since the standard form of Limber's equation assumes that the galaxy autocorrelation function is independent of redshift. We have repeated our calculations taking this into account by regarding κ in eq. (7) as a function of redshift. We find that this introduces only a small change in the inferred $w(\theta)$, and, by slightly adjusting the values of our parameters, we can recover almost exactly the results shown in Fig. 3.

[8] Some local biasing models can give a dependence of the amplitude of the correlation function on luminosity; the distinguishing feature of cooperative models is a dependence of the *shape* of this function on luminosity.

surrounding material over scales comparable to those of the observed proximity effect. Such changes in the thermodynamic state of pregalactic matter will affect later galaxy formation through changes in the effectiveness of cooling processes. The most likely consequences of UV radiation are the dissociation of preexisting molecular gas (Kang & Shapiro 1992) and the ionization and dispersal of dense cloudlets. It would therefore seem that although the scale of these radiative effects is similar to those of our cooperative model, their *sign* might be wrong.[9] However, additional effects, such as a change in the stellar initial mass function, could conceivably reverse the sign of the effect. Babul & White (1991) suggest that such radiative processes may give rise to large-scale structure in the galaxy distribution by creating large barren regions surrounding the burned-out-remnants of old quasars. This idea requires a much more drastic decoupling of the mass and galaxy distributions than the model we investigate in this paper.

Starbursting galaxies may be the best observed analogs of "typical" galaxies during their major formation phase. Recent observations by Heckman, Armus, & Miley (1990) suggest that such large-scale star formation is accompanied by massive superwinds, scaled-up versions of the strong, collimated wind seen in the nearby starbursting dwarf, M82. From their data these authors estimate wind velocities and mass fluxes of about 2000 km s^{-1} and 50 $M_\odot$ yr^{-1} in their larger systems. If such a wind blows for about 10^8 yr, approximately the dynamical time of the regions seen to be active, it gives off a total mechanical energy which is larger than the binding energy of the observed stars in a bright galaxy. Emitted by a protogalaxy at redshift 4 and unimpeded by surrounding intergalactic matter, such a wind would travel a comoving distance of 15 h^{-1} Mpc by a redshift of 2 (for $\Omega = 1$) and would still have a kinetic energy comparable to that of the stars in a bright galaxy. Whether propagation would actually occur over such distances depends on whether intergalactic space is nearly empty or contains sufficient diffuse matter to slow the wind. Continued failure to detect any diffuse absorption through the Gunn-Peterson test suggests that the mean density of any such component may be low. In this case the wind may propagate until it impacts a neighboring protogalactic cloud, thereby stimulating further galaxy formation over the scales envisaged by our model. This idea is a weaker and more inhomogeneous version of the kind of explosive galaxy formation suggested by Ostriker & Cowie (1981).

As a third possibility, we note that disk galaxies are the dominant population in the flux-limited surveys which have so far been used to analyze large-scale clustering. There is considerable observational and theoretical reason for believing that disks were the last parts of galaxies to form, and that disk material was accreted at fairly low redshift, $z \lesssim 1$, through infall from large radii (e.g., White 1990). The total amount of material accreted, and so the luminosity of the final disk, could therefore be affected by the much larger cluster and supercluster structures which were beginning to collapse at the same time. For example, the formation of a filament or pancake might result in a significant increase in the pressure and density of any intergalactic medium, and so might enhance the final stages of protodisk accretion. If such processes were correlated with the size of the supercluster, this nonlinear coupling could give additional positive biasing beyond that predicted by the high-peak model. The observation that the well-known morphology-density relation extends to supercluster scales (e.g., Haynes 1988) would seem to give at least some support to this idea that disk formation is somehow affected by the very largest structures.

We are unable to demonstrate that any of the above three mechanisms will produce cooperative effects corresponding to the model of this paper. On the other hand, in each case there is some direct observational evidence which suggests that coherent effects can act on the scale required. Furthermore, the modulation amplitude needed to reproduce the observed large-scale structure is, as we have seen, quite small. Thus modifications of the SCDM theory which involve the physics of galaxy formation seem much more promising than ones involving the physics of the very early universe. Almost all of the latter involve relinquishing some of the most physically attractive assumptions of the SCDM theory, and often invoke speculative ideas which have no better prospect of empirical test than those of the simpler original model. In contrast, if cooperative effects were important during galaxy formation, they could be manifest as large-scale dependences of galaxy luminosity (and perhaps also surface brightness or morphology) on environment, although it is possible to have cooperative galaxy formation models in which these dependences are weak. The idea may thus be open to test, and although the present observational situation is confused, with published claims both for and against such effects (e.g., Haynes 1988; White et al. 1988; Eder et al. 1989; Hamilton 1988; Valls-Gabaud et al. 1989), improving data sets and a clear understanding of what is required should make it possible to detect the effects of cooperative galaxy formation directly, if it is indeed responsible for the apparent large-scale structure. For the time being, however, the best tests of the cold dark matter model remain those that are sensitive to the distribution of mass rather than to the distribution of galaxies.

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[9] In a recent paper, Efstathiou (1992) argues that the UV ionization would promote the formation of luminous galaxies by reducing the fraction of gas consumed in forming dwarf galaxies at early times.

APPENDIX

Consider the general problem of evaluating an integral of the form:

$$\Phi_n(a_1, a_2, \ldots, a_n; \theta) = \Pr(x_1 > a_1, x_2 > a_2, \ldots, x_n > a_n) ,$$

where the x_i possess a multivariate Gaussian probability distribution with $\langle x_i \rangle = 0$ and $\langle x_i x_j \rangle = \theta_{ij}$ which we denote by $f_n(x_1, \ldots, x_n; \theta)$. The direct evaluation of the necessary n-dimensional integral can be very expensive in terms of computer time, so instead we convert the integrals to line integrals as follows.

For each set of values of $(a_1, a_2, \ldots, a_n)$, let us regard Φ_n as a function of the θ_{ij} in ${}^nC_2$-dimensional space. Let P be the point with coordinates θ_{ij} in this space, and let $\Phi_n(P)$ be the function Φ_n evaluated at P. Suppose we know the value of $\Phi_n(Q)$, at another point Q in this space, with coordinates ϕ_{ij}. The point Q can be chosen in such a way that the evaluation of $\Phi(Q)$ is simpler than that of $\Phi(P)$. In the case we shall consider we take the diagonal elements ϕ_{ii} to be equal to the θ_{ii}, and in order to make the evaluation of $\Phi(Q)$ as simple as possible, we take $\phi_{ij} = 0$ for $i \neq j$. We can then express $\Phi_n(P)$ as

$$\Phi_n(P) = \Phi_n(Q) + \sum_{i<j} \int_{\phi_{ij}}^{\theta_{ij}} \frac{\partial \Phi_n(G)}{\partial \gamma_{ij}} \, d\gamma_{ij} ,$$

where the summation extends only over $i < j$ because the matrix Φ_n is symmetric in i and j. In the integral, the point G has coordinates given parametrically by

$$\gamma_{ij}(\lambda) = \lambda\theta_{ij} + (1 - \lambda)\phi_{ij} ,$$

where $0 \leq \lambda \leq 1$. Hence

$$\frac{d\Phi_n}{d\lambda} = \sum_{i<j} \frac{\partial \Phi_n}{\partial \gamma_{ij}} \frac{d\gamma_{ij}}{d\lambda} = \sum_{i<j} \frac{\partial \Phi_n}{\partial \gamma_{ij}} (\theta_{ij} - \phi_{ij}) .$$

Now for Gaussian distributions,

$$\frac{\partial f_n(x; \gamma)}{\partial \gamma_{ij}} = \frac{\partial^2 f_n(x; \gamma)}{\partial x_i \partial x_j}$$

(for $i \neq j$), which can be easily be proved by writing the multivariate Gaussian probability density function as the inverse of its characteristic function and then differentiating under the integral sign. Thus, we have

$$\frac{\partial \Phi_n}{\partial \gamma_{12}} = \int_{a_1}^{\infty} dx_1 \int_{a_2}^{\infty} dx_2 \cdots \int_{a_n}^{\infty} dx_n \frac{\partial^2 f_n}{\partial x_1 \partial x_2} = \int_{a_3}^{\infty} dx_3 \int_{a_4}^{\infty} dx_4 \cdots \int_{a_n}^{\infty} dx_n f_n(a_1, a_2, x_3, \ldots, x_n; \gamma) .$$

We can therefore reduce the dimensionality of the original integral.

In the case discussed in § 2 we have $n = 2$, so this method produces a particularly simple result. We have

$$\frac{\partial \Phi}{\partial \gamma_{12}} = f_2(\nu\sigma, \nu\sigma; \gamma) .$$

We can therefore reduce the original integral to the line integral:

$$\Phi(\theta_{ij}) - \Phi(\phi_{ij}) = (\theta_{12} - \phi_{12}) \int_0^1 f_2(\nu\sigma, \nu\sigma; \lambda(\theta_{12} - \phi_{12})) \, d\lambda .$$

Recall that we are free to pick $\phi_{ij} = 0$ for $i \neq j$. $\Phi(\phi_{ij})$ is then just $\mathscr{P}_1^2$. To recover the notation of § 2, we pick $\theta_{11} = \theta_{22} = \xi'(0)$ and $\theta_{12} = \theta_{21} = \xi'(r)$. $\Phi(\theta_{ij})$ is then just $\mathscr{P}_2$, so that from equation (3) we have

$$\xi_{>\nu} = \frac{\Phi(\theta_{ij}) - \Phi(\phi_{ij})}{\Phi(\phi_{ij})} ,$$

which leads directly to equation (10).

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