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THEORY OF JETS FROM YOUNG STARS

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ABSTRACT

Simple equations are derived for the long-distance propagation of magnetohydrodynamic (MHD) jets. Solutions of these equations are fitted to two observed jets providing estimates of the fast magnetosonic speeds (v_f) and the distances of the fast magnetosonic points. The relation of the jet properties at large distances to a complete family of MHD jet solutions is discussed, and it is shown that there is one key dimensionless parameter, $\mathscr{B} \equiv (2B_0^2 a_i^2/\dot{M}_a v_K)^{1/2}$, where B_0 is the poloidal magnetic field at the inner disk radius a_i, v_K is the Keplerian velocity at a_i, and $\dot{M}_a$ is the disk accretion rate. The dependences of the fast magnetosonic speed and of the fluxes of mass, energy, momentum, and angular momentum of the jet on $\mathscr{B}$ are discussed. For $\mathscr{B}$ larger than a critical value (≈ 0.45), the central star spins down, while for smaller values it spins up. For $\mathscr{B}$ increasing from the critical value, v_f increases while the mass and momentum fluxes of the jet decrease.

Subject headings: ISM: jets and outflows — MHD — stars: pre–main-sequence

1. INTRODUCTION

High-velocity bipolar outflows and optical jets appear as characteristic features of star-forming systems (Lada 1985; Mundt 1985). Direct and indirect evidence indicates that most of these systems have Keplerian accretion disks (Sargent & Beckwith 1987; Kenyon & Hartmann 1987; Bertout, Basri, & Bouvier 1988; Basri & Bertout 1989). In turn, there is strong evidence that outflows and jets occur only in systems with accretion disks (see review by Königl & Ruden 1992). Regarding the theory of the outflows, it is well known that radiatively driven winds can be ruled out (because the radiation has insufficient momentum), and that thermally driven winds can also be ruled out (because of the excessively high temperatures required). Thus, most of the recent theoretical work on outflows and jets from star-forming systems have investigated magnetohydrodynamic (MHD) outflows which are a consequence of having a poloidal (r, z) magnetic field threading an accretion disk. The high specific angular momentum of MHD outflows can remove most of the angular momentum remaining in the inner part of an accretion disk, and this can have the important consequence that the formed stars rotate very slowly as observed (Hartmann et al. 1986).

Most of the theoretical work on MHD outflows and jets from star-forming systems has been based on the stationary self-similar model of Blandford & Payne (1982) (Königl & Ruden 1992). However, important physical quantities of this model, the fluxes of mass, energy, and angular momentum, are infinite; finite values are obtained by introducing cutoffs (at both small and large cylindrical radii). As a result the most important region of the flow dynamically (at small radii) is neglected. A different approach to the theory of MHD winds and jets, in which all of the physical quantities are finite, is based on envelope equations derived from the main MHD conservation equations by averaging over the jet cross section (Lovelace, Mobarry, & Contopoulos 1989, hereafter LMC; Koupelis & Van Horn 1989; Lovelace, Berk, & Contopoulos 1991, hereafter LBC). The outflow is assumed to originate from the inner part of the accretion disk where the power available from the accretion flow is largest (Shu et al. 1988; Pringle 1989) and where the magnetic field is predicted to be strongest (Wang, Lovelace, & Sulkanen 1990).

New impetus for the development and exploration of theoretical models of jets comes from recent spectral line observations of the radial velocity and other jet properties as a function of distance and from corresponding proper motion measurements in cases of nearby objects (see, for example, Mundt et al. 1990, and Reipurth 1989a). Although there is evidence of intrinsic temporal variability in the jet properties of some sources (for example, in HH 111 [Reipurth 1989b] and in HH 46/47 [Raga et al. 1990; Reipurth & Heathcote 1991]), it is reasonable to first fully explore the predictions of stationary jet models. Figure 1 shows an overview of the jet geometry. Section 2 of this paper simplifies the equations of LBC to apply to the propagation of MHD jets at large distances, and it applies the results to two observed jets. Section 3 of the paper summarizes the dependences of the main jet properties on the accretion disk parameters for a complete family of solutions obtained from the equations of LBC.

2. STEADY JET PROPAGATION AT LARGE DISTANCES

The equation for the conservation of axial momentum of a stationary ideal MHD jet can be written as

$$\frac{d}{dz}\mathscr{F}_P = \frac{F_z}{v_z} + 2\pi r\frac{dr}{dz}p_{ex}\,, \tag{1}$$

(LBC). Here, z is the axial distance from the origin, v_z is the mean axial speed of the jet flow, r is the mean jet radius, $p_{ex}(z)$ is the pressure (kinetic plus magnetic) of the medium outside the jet,

$$\mathscr{F}_P = \int_{z=\mathrm{const}} d^2x\left(\rho v_z^2 + p_{\mathrm{int}} + \frac{\boldsymbol{B}^2}{8\pi} - \frac{B_z^2}{4\pi}\right)$$

is the axial momentum flow of the jet, $F_z \approx -GM_* \dot{M}_j/z^2$ is the gravitational force of the central star (of mass M_*), $\dot{M}_j$ is the mass flow rate of the jet, and p_{int} is the internal kinetic pressure of the jet. Following the discussion of LBC, we assume that jet carries *no* net current. The jet collimation results from the focusing effect of the external medium.

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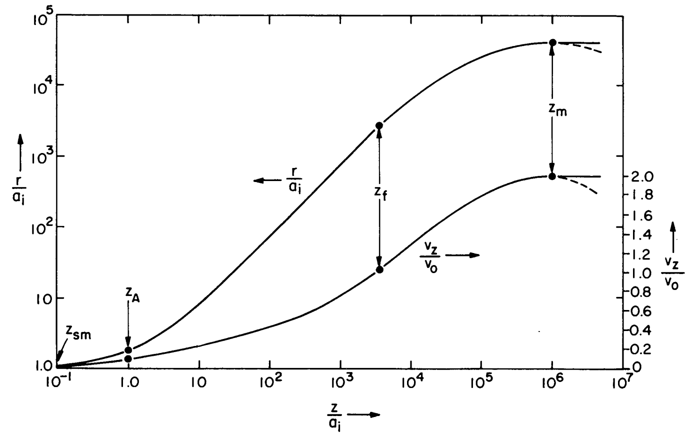

FIG. 1.—The figure shows a semiquantitative sketch of the jet radius (r) and axial velocity (v_z) a function of axial distance (z) from LBC. Here, a_i is the initial jet radius assumed equal to the inner radius of the disk, and v_0 is approximately the Keplerian velocity of the disk at a_i. The important axial distances are indicated: the slow magnetosonic critical point at z_{sm}; the Alfvén singular point at z_A; the fast magnetosonic critical point at z_f; and the distance of perfect collimation (where $dr/dz = 0$) at z_m. The third critical point, the cusp or slow wavefront point, is at a distance z_c very close to but less than z_{sm} (Lovelace et al. 1986). Section 2 gives expressions for z_f and z_m and the corresponding flow speeds, and it discusses the possible recollimation of the jet indicated in the figure by the dashed lines beyond z_m. Section 3 discusses z_{sm} and z_A and the corresponding flow speeds.

At large distances, $z \gg z_A$, where z_A is the distance to the Alfvén point discussed in § 3, the jet radius is much larger than its initial value (at the accretion disk, $z = 0$), the jet's rotation rate is much smaller than its initial rate, and the kinetic pressure of the jet is negligible due to adiabatic expansion (LBC). Consequently, to a good approximation $\mathscr{F}_P \approx \dot{M}_j v_z + \mathscr{F}_0/(2v_z^2)$, where the first term represents the momentum carried by the matter and the second term of the momentum carried by the predominantly toroidal magnetic field. The left-hand side of equation (1) is therefore $[1 - (v_f/v_z)^3](dv_z/dz)$, where $v_f \equiv (\mathscr{F}_0/\dot{M}_j)^{1/3}$ is the fast magnetosonic speed of the jet. From LBC, $\mathscr{F}_0 \approx a_i^2 B_0^2 v_0^2/4$, where a_i is the initial jet radius assumed equal to the inner radius of the disk, v_0 is the initial azimuthal velocity of the jet which is equal to the disk's rotational velocity at $r = a_i$, and B_0 is the value of poloidal magnetic field threading the disk at a_i. Thus, $v_f \approx (a_i^2 B_0^2 v_0^2/4\dot{M}_j)^{1/3}$ or, equivalently,

$$\dot{M}_j \approx 0.62 \times 10^{-9} \frac{M_\odot}{\rm yr} a^{5/2} m^{-1/2} \delta^{1/2} \left(\frac{B_0}{10G}\right)^2 \left(\frac{v_0}{v_f}\right)^3 , \quad (2)$$

where we have written $v_0 = (GM_*/a_i)^{1/2}\delta^{-1/2}$ with δ a numerical factor [$\gtrsim O(1)$; LBC]. Also, we have introduced $m \equiv M_*/M_\odot$ and $a \equiv a_i/(2 \times 10^{11}$ cm). We consider that a, m, and δ are all of order unity. Note, however, that a_i may be larger than the radius of the star, if the star has an appreciable intrinsic dipole magnetic field (Bertout et al. 1988). The quantities B_0, $\dot{M}_j$, and therefore v_f/v_0 are *intrinsic parameters* of the jet in that they are determined by the physical conditions of the disk close to the central object. In § 3, we derive the conditions which interrelate $\dot{M}_j$, B_0, v_f/v_0, and the disk accretion rate.

In order for the jet flow to pass through the fast magnetosonic critical point, the two terms on the right-hand side of equation (1) must cancel at this point. It is clearly appropriate to introduce the dimensionless variables: $V_z \equiv v_z/v_f$, $Z \equiv z/z_f$, $R = r/z_f$, and $P_{\rm ex} = p_{\rm ex}/(p_{\rm ex})_f$, where the f subscript indicates evaluation at $z = z_f$. The cancellation of the right-side equation (1) at z_f gives

$$z_f = \left\{\frac{GM_* \dot{M}_j}{2\pi[p_{\rm ex} R(dR/dZ)]_f v_f}\right\}^{1/3} ,$$

or

$$z_f \approx 0.75 \times 10^{15} \text{ cm } (am\delta)^{1/6} \left(\frac{\dot{M}_j}{10^{-9} M_\odot \text{ yr}^{-1}}\right)^{1/3}$$
$$\times \left[\frac{10^{-10} \text{ dyn cm}^{-2}}{(p_{\rm ex})_f}\right]^{1/3} \left(R_f \frac{dR}{dZ}\bigg|_f\right)^{-1/3} \left(\frac{v_0}{v_f}\right)^{1/3} . \quad (3)$$

Notice that $V_z = M_A^{2/3}$, where $M_A \equiv v_z/v_A$ is the Alfvén Mach number of the flow, and v_A is the Alfvén speed. Equation (1) can now be written as

$$\left(1 - \frac{1}{V_z^3}\right) \frac{dV_z}{dZ} = -\kappa \left\{\frac{1}{V_z Z^2} - \frac{P_{\rm ex} R(dR/dZ)}{[R(dR/dZ)]_f}\right\} , \quad (4)$$

where

$$\kappa \equiv \frac{GM_*}{z_f v_f^2} \approx 2 \times 10^{-4} a\delta \left(\frac{10^{15} \text{ cm}}{z_f}\right) \left(\frac{v_0}{v_f}\right)^2 .$$

Taylor expansion of equation (4) at $Z = 1$ gives $(dV_z/dZ)|_f = \{\kappa/3[2 + (dP_{\rm ex}/dZ) + (1/R)(dR/dZ)]_f\}^{1/2}$. Because z_f and κ depend on the external pressure, it is appropriate to refer to them as *extrinsic parameters* of the jet.

The energy flux, $\mathscr{F}_E$, of a stationary ideal MHD jet is a constant,

$$\mathscr{F}_E \approx \frac{1}{2} \dot{M}_j \left[1 + k_\rho \left(\frac{dR}{dZ}\right)^2\right] v_z^2 + \frac{\mathscr{F}_0}{v_z} = \text{const} , \quad (5a)$$

where k_ρ is a numerical constant of order unity (LBC). The terms $\propto \dot{M}_j$ represent the energy carried by the matter, while the term $\propto \mathscr{F}_0$ represents the Poynting flux. Evaluating equation (5a) at the fast magnetosonic point gives

$$\mathscr{F}_E \approx 0.033\ L_\odot \left(\frac{\mathscr{K}}{3}\right) \left(\frac{m}{a\delta}\right) \left(\frac{\dot{M}_j}{10^{-9} M_\odot \text{ yr}^{-1}}\right) \left(\frac{v_f}{v_0}\right)^2 , \quad (5b)$$

where $\mathscr{K} \equiv (3/2) + (k_\rho/2)(dR/dZ|_f)^2$, and $L_\odot \approx 3.8 \times 10^{33}$ ergs s^{-1}. In turn, equation (5a) implies that

$$\frac{dR}{dZ} = \pm \left\{\frac{2\mathscr{K} - [V_z^2 + (2/V_z)]}{k_\rho V_z^2}\right\}^{1/2} . \quad (6)$$

Equations (4) and (6) can be integrated away from the fast magnetosonic point to give the dimensionless axial velocity of the jet $V_z(Z)$. In general $V_z(Z)$ depends on κ, k_ρ, $dR/dZ|_f$, and the profile of the external pressure, $P_{\rm ex}(Z)$, If $P_{\rm ex}(Z) =$ const, the velocity increases and reaches a maximum at a large distance $z_m \gg z_f$ at which point $dR/dZ|_m = 0$. For example, for $\mathscr{K} = 2.75$, the velocity reaches a maximum value of $V_z \approx 2.14$ at a distance $z_m \approx 3.8 z_f \kappa^{-1/2} \propto z_f^{3/2}$. At this distance the fraction ϵ of the total energy flux carried by the electromagnetic field, the Poynting flux, is $\epsilon = 1/(1 + V_z^3/2)$; the fraction of the axial momentum flux carried by the B field is $1/(1 + 2V_z^3)$; and the fraction of the angular momentum carried by the B field is $\approx 1/(15.6 V_z)$. If $P_{\rm ex}(Z)$ is a decreasing function, the distance to the maximum of V_z is in general larger than its value for $P_{\rm ex} =$ const.

Beyond z_m, the jet velocity and radius predicted by equations (4) and (6) decrease; that is, the jet "recollimates." Earlier studies (Chan & Henrikson 1980; LBC; Contopoulos & Lovelace 1992) suggest that the decrease of the jet velocity and radius reverses—the jet "bounces"—at a large distance $\sim 2z_m$, and subsequently reaccelerates and reexpands. Note that the treatment of the passage of the flow through the bounce requires the full equations of LBC. Approximate fittings of solutions to equations (4) and (6) to observations of two jets are shown in Figure 2.

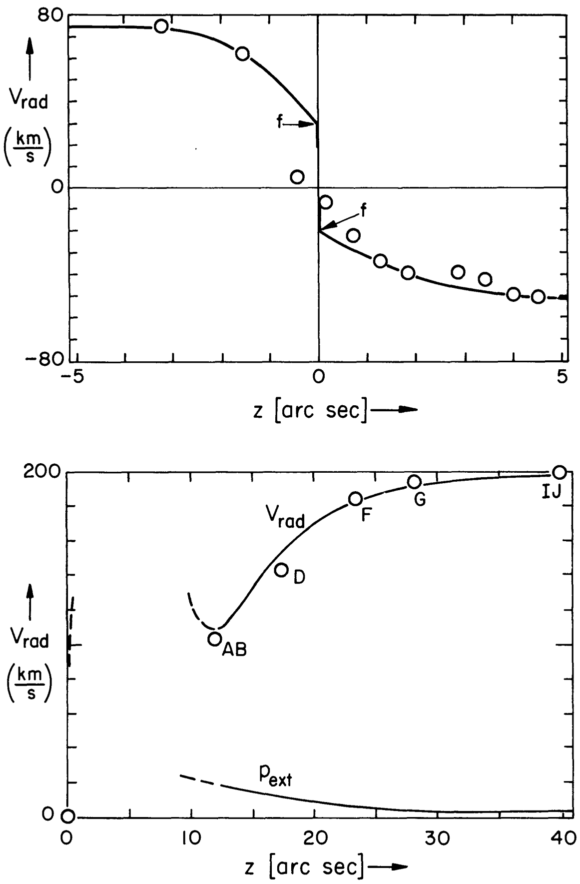

FIG. 2.—The top panel of the figure shows an approximate fitting of data on the jet of *XZ* Tau (*circles*) of Mundt et al. (1990) to $V_z(z)$ from the theory of § 2. For the solid curve for $z > 0$, we have taken $z_f \sin(\gamma) = 10^{14}$ cm, $\kappa_+ = 1.7 \times 10^{-3}$, $v_{f+} \cos(\gamma) = 23$ km s^{-1}, $\mathscr{K} = 2.75$ and $v_{f+}/v_0 = 1.1(\sin\gamma)^{1/2}$, where γ ($\approx 82°$; Mundt et al. 1990) is the angle between the jet axis and the line of sight. For the solid curve for $z < 0$, $z_f \sin(\gamma) = 0.6 \times 10^{14}$ cm, $\kappa_- = 1.3 \times 10^{-3}$, $v_{f-} \cos(\gamma) = 35$ km s^{-1}, $\mathscr{K} = 2.75$, and $v_{f-}/v_0 = 1.6(\sin\gamma)^{1/2}$. The approximate distance to the source is 160 pc so that $1'' \approx 2.3 \times 10^{15}$ cm. The *f*-arrows indicate the fast magnetosonic points. The pressure of the external medium is assumed to have the dependence $p_{ex} \propto (1 + z^2/H^2)^{-1}$ with $H = 10^{16}$ cm. For the value $\dot{M}_j \approx 5 \times 10^{-10}\ M_\odot$ yr^{-1} of Mundt et al., equation (3) implies $(p_{ex})_f \sim 2 \times 10^{-8}$ dyn cm^{-2}, and equation (5) gives $\mathscr{F}_E \sim 1.5 \times 10^{-2}\ L_\odot$, where we have omitted for simplicity the differences between the $\pm z$ jets. The discrepancy between the points and the model near $z = 0$ may be due to scattering in this region (see Mundt et al. 1990).

The lower panel shows an approximate fitting of data on the jet of HH 83 (*circles*) of Reipurth (1989a) to the theory of § 2. For the upper solid curve, we have taken $z_f \sin\gamma = 2.8 \times 10^{14}$ cm, $\kappa = 10^{-4}$, $v_f \cos(\gamma) = 93$ km s^{-1}, $\mathscr{K} = 2.75$, $v_f/v_0 = 2.67(\sin\gamma)^{1/2}$, and we have started the integration of eq. (4) at $z \sin(\gamma) = 8.7 \times 10^{16}$ cm and $v_z/v_f = 1.1$. The lower solid curve shows the assumed dependence of the external pressure. Note that we have assumed that the jet has recollimated and gone through one bounce near knot A. The approximate distance to the source is 460 pc so that $1'' \approx 6.9 \times 10^{15}$ cm. For example, for $\gamma = 45°$, $z_f \approx 3.9 \times 10^{14}$ cm and $v_f/v_0 \approx 2.2$. In turn, for $\dot{M}_j = 10^{-9}\ M_\odot$ yr^{-1}, eq. (3) implies $(P_{ex})_f \sim 3 \times 10^{-10}$ dyn cm^{-2}, and eq. (5) gives $\mathscr{F}_E \sim 0.15\ L_\odot$.

In addition to the mass and energy fluxes, the angular momentum flux, $\mathscr{F}_L$, of a stationary ideal MHD jet is a constant,

$$\mathscr{F}_L = \int_{z=\text{const}} d^2x\, r\left(\rho v_\phi v_z - \frac{B_\phi B_z}{4\pi}\right) = \text{const},$$

$$\approx 4.1 \times 10^{-5}\, \frac{J_\odot}{\text{yr}}\, (am)^{1/2} \left(\frac{\kappa_\rho}{2.5}\right) \left(\frac{\dot{M}_j}{10^{-9}\ M_\odot\ \text{yr}^{-1}}\right) \left(\frac{v_f}{v_0}\right)^2 , \quad (7)$$

where $J_\odot \approx 1.6 \times 10^{48}$ g cm^2 s^{-1} is the angular momentum of the Sun. The angular momentum removed from the disk by the jets may account for the slow rotation rates of T Tauri stars as discussed further in § 3.

If the fraction of the total energy flux of the jet carried by the Poynting flux is ϵ, then the jet's magnetic field is

$$(B_\phi)_{\text{rms}} = \left(\frac{2\epsilon \mathscr{F}_E}{r^2 v_z}\right)^{1/2} ,$$

$$\approx 1.4 \times 10^{-3} G \left(\frac{\epsilon}{0.2}\right)^{1/2} \left(\frac{\mathscr{F}_E}{0.1\ L_\odot}\right)^{1/2}$$

$$\times \left(\frac{10^{15}\ \text{cm}}{r}\right) \left(\frac{400\ \text{km s}^{-1}}{v_z}\right)^{1/2} . \quad (8)$$

The poloidal field is weaker by a factor $\sim a_i/r = 2 \times 10^{-4}$ $(10^{15}$ cm$/r)$; LBC. Because the toroidal field vanishes outside the jet (LBC), the radial field tension and pressure forces balance exactly. Consequently the B_ϕ field acts neither to pinch nor expand the jet channel. The jet density is $n_j \approx 300$ cm^{-3} $(\dot{M}_j/10^{-9}\ M_\odot$ yr$^{-1})(10^{15}$cm$/r)^2(400$ km s$^{-1}/v_z)$ for a mean particle mass that of hydrogen.

3. INTRINSIC PROPERTIES OF JETS

Earlier studies (LMC, LBC) developed a simplified theory for magnetically driven jets and winds. The basic equations of the theory follow from the conservation of mass, momentum, angular momentum (about the *z*-axis), and energy for the ideal MHD jet flow. The theory allows the determination of the jet's axial velocity, radius, rotation rate, and temperature from the surface of an accretion disk to very large distances. A method for solving the equations and a representative solution was presented in LBC. Here, we describe a different, more accurate method for solving the equations, and we present a complete family of solutions. The method we use is that of "inner (close to the disk) and outer (far from disk) expansions." This approach uses the fact that the slow magnetosonic critical point of the jet flow occurs close to the disk, at z_{sm}, far removed from the Alfvén point of the flow, at $z_{\text{A}} \gg z_{\text{sm}}$. We match the two solutions at a distance z_* between z_{sm} and z_{A}.

For the *outer solution*, we solve the radial virial equation of LMC from $z_* \gg z_{\text{sm}}$ and $v_z = v_* \gg v_{\text{sm}}$ to $z \gg z_{\text{A}}$. This equation has the form $\Lambda(d^2r/dz^2) = F_r$, with $\Lambda = \Lambda(r, z, r', v_z \mid R_{\text{A}}, V_{\text{A}})$ and $F_r = F_r(r, z, r', v_z \mid R_{\text{A}}, V_{\text{A}})$, where $r' = dr/dz$, $R_{\text{A}} \equiv r_{\text{A}}/a_i$ is the dimensionless Alfvén radius, $V_{\text{A}} \equiv v_{\text{A}}/v_0$ is the dimensionless Alfvén velocity, a_i is the initial radius of the jet, and v_0 is the disk's rotation rate at a_i. For the outer solutions there are just two important parameters, R_{A} and V_{A}. [There is a weak dependence on the parameter $\delta = v_{\text{K}}^2/v_0^2$, where $v_{\text{K}} = (GM_*/a_i)^{1/2}$ is the Keplerian velocity of the disk, and on the thermal pressure of the gas.] We note that the radial virial equation has a singularity at the Alfvén point at $z = z_{\text{A}}$, $r = r_{\text{A}}$, $v_z = v_A$, where $\Lambda = 0$. In order for the flow solution to pass smoothly through this point, the initial slope $r'(z_*)$ must be chosen uniquely. Smooth flows can readily be obtained by numerical shooting method solution of the virial equation. The required values of r'_* are of order unity. However, there is a further constraint of the solution space which arises from the fact that the fast magnetosonic point is necessarily at a very large distance, $z_{fm} \gg z_A$, if the pressure of the medium external to the jet is small compared with the jet's internal magnetic pressure at z_A. Note that in the absence of an external medium the outflow would have a conical shape with half-opening angle of order 45°. The family of outer jet solutions obtained under these conditions is shown in Figure 3 as $v_f/v_0 = (k_\rho V_A R_A^2)^{1/3}$ as a function of R_A. Notice that *no* solutions are found to exist for $R_A \lesssim 1.75$ or $v_f/v_0 \lesssim 0.73$ for the conditions of the figure.

The inner solution applies from the surface of the disk at $z \approx 0$ to $z_* \gg z_{sm}$. For this region, we use the Bernoulli constant for the flow which can be written as

$$\mathscr{E} = \tfrac{1}{2}\boldsymbol{v}_p^2 + w + \Phi_g + \tfrac{1}{2}(r\omega)^2 + \omega_i(r_A^2\omega_i - r^2\omega)\,, \quad (9)$$

where $\boldsymbol{v}_p$ is the poloidal (r, z) velocity, $w = \int dp/\rho$ the enthalpy, Φ_g the gravitational potential, $\frac{1}{2}(r\omega)^2$ the toroidal kinetic energy, the ω_i term is the Poynting flux per particle, and $\omega_i = v_0/a_i$ is the initial angular rotation rate of the jet (which is equal to the disk's rotation rate). Conservation of angular momentum gives $\omega = \omega_i r_A^2(v_z - v_A)/(r^2 v_z - r_A^2 v_A)$, (LBC). To a good approximation, we have $(\omega^2 - 2\omega\omega_i)/\omega_i^2 = -1 + (v_z/v_A)^2[1 - (r/r_A)^2]^2$ and $r = a_i + r'_* z$, where r'_* is known from the outer solution. For the inner region, we assume the gas to be isothermal so that $w = c_{si}^2 \ln(\rho)$, where $c_{si} \equiv (kT_i/\bar{m})^{1/2}$ is the Newtonian sound speed of the disk corona, with $c_{si} \ll v_0$, and $\bar{m}$ is the mean particle mass. Equation (9) has the form $\mathscr{E} = \mathscr{E}(z, v_z) = \text{const}$, or, equivalently, $(dv_z/dz)(\partial\mathscr{E}/\partial v_z) + (\partial\mathscr{E}/\partial z) = 0$. At the slow magnetosonic critical point, $(\partial\mathscr{E}/\partial v_z)_{sm} = 0$, at $v_z = v_{sm} = c_{si} A^{-1/2}$, where A is a numerical factor (between 1 and 2). At this point, we must have $\partial\mathscr{E}/\partial z = 0$, which gives $z_{sm} \approx (\delta - 1)r'_* a_i/[(2\delta + 1)(r'_*)^2 - \delta]$. The density ratio between $z = 0$ and the slow magnetosonic point is roughly $\exp[-k(v_K/c_{si})^2]$. Here, $k = \frac{1}{2}(\delta - 1)^2(r'_*)^2/[(2\delta + 1)(r'_*)^2 - \delta]$ is much less than unity if $(\delta - 1)^2 \ll 1$. In contrast, for a nonmagnetic, isothermal wind, this same ratio is $\exp[-(v_K/c_{si})^2]$ (Parker 1963), which is vanishingly small (10^{-300}) for plausible values of c_{si}.

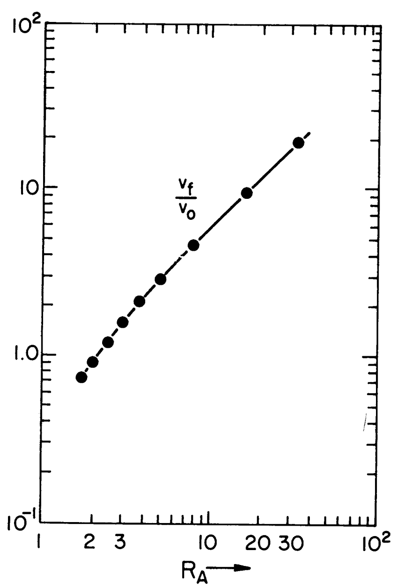

FIG. 3.—The figure shows the dependence of the fast magnetosonic speed of the jet, v_f, on the dimensionless Alfvén radius $R_A = r_A/a_i$. Here, a_i is the inner radius of the disk, $v_0 = v_K/\delta^{1/2}$, and $v_K = (GM_*/a_i)^{1/2}$. For the figure we have taken $\delta = 1.1$ and $k_\rho = 2.5$. The dots represent the numerically determined values.

The key parameter for the family of jet solutions of Figure 3 is the Alfvén radius R_A. We determine R_A by using the conservation of mass and angular momentum for the disk/jet system. We simplify the subsequent formulae by assuming $\delta \approx 1$. Conservation of mass gives $\dot{M}_a = \dot{M}_* + 2\dot{M}_j$, where $\dot{M}_a$ is the mass accretion rate of the disk of large r, $\dot{M}_*$ is the rate of increase of the star's mass, and the mass outflow rate $2\dot{M}_j$ is assumed for simplicity to be equally divided between the $\pm z$ jets. Conservation of angular momentum gives $\dot{M}_a a_i v_K = dJ_*/dt + 2\mathscr{F}_L$, where J_* is the angular momentum of the star, and $2\mathscr{F}_L$ is the angular momentum efflux assumed equally divided between the $\pm z$ jets. Conservation of energy gives an equation of the form $\dot{M}_a v_K^2/2 = d/dt[J_*^2/(2I_*)] + 2\mathscr{F}_E + \cdots$, where I_* is the moment of inertia of the star, the ellipsis denotes other terms such as the boundary layer radiation (Pringle 1989; LBC), and $2\mathscr{F}_E$ is the energy efflux of the $\pm z$ jets. For fixed $\dot{M}_a$, the maximum of $\mathscr{F}_E$ as a function of $\dot{J}_*$ occurs for $J_* \dot{J}_* < 0$, which corresponds to J_*^2 decreasing with time. Small values of $\dot{J}_*$ during most of the star's disk accretion are suggested by observations of T Tauri stars by Hartmann et al. (1986) which show that most of the stars have small rotational velocities ($\lesssim 10\%$ of the breakup velocity).

Therefore, we first consider the limit where $\beta^2 \equiv [\dot{J}_*/(\dot{M}_* a_i v_K)]^2 \ll 1$ so that $\dot{M}_a a_i v_K \approx 2\mathscr{F}_L$. In this limit, the jets remove most of the angular momentum still in the incoming matter at the inner disk radius. In this limit, the azimuthal motion of the disk matter in the very inner part of the disk is significantly less than Keplerian. Because $J_* \approx \text{const}$, the star's rotation rate decreases with time, $\Omega_* \propto I_*^{-1}$. Using the fact that the specific angular momentum of the jet is $\mathscr{F}_L/\dot{M}_j = a_i v_K R_A^2 k_\rho$ (LBC), gives $R_A^2 = 1/(2k_\rho \xi)$, where $\xi = \dot{M}_j/\dot{M}_a$. In turn, equation (9) gives $\mathscr{F}_E = \dot{M}_j v_K^2[R_A^2 - (3/2)]$, with our earlier weak assumption that $c_{si} \ll v_K$. We can also express the energy flux of the jet at the fast magnetosonic point as $\mathscr{F}_E = \mathscr{K}\dot{M}_j v_f^2$, where $\mathscr{K}$ is defined in equation (5b). Combining this expression with equation (2) gives

$$\mathscr{B}^{4/3} = g_1 \xi^{-1/3} - g_2 \xi^{2/3}\,, \quad (10a)$$

where $g_1 = 2/(k_\rho \mathscr{K})$, $g_2 = 6/\mathscr{K}$, and $\mathscr{B} = [B_0^2 a_i^2 v_K/(\dot{M}_a v_K^2/2)]^{1/2}$ is a useful dimensionless measure of the strength of the disk magnetic field,

$$\mathscr{B} \approx 0.22\left(\frac{a^{3/2}}{m^{1/2}}\right)\left(\frac{B_0}{10G}\right)\left(\frac{10^{-7}\ M_\odot\ \text{yr}^{-1}}{\dot{M}_a}\right)^{1/2}. \quad (10b)$$

Because there is a minimum value of the Alfvén radius R_A (Fig. 2), there is a maximum value of ξ which we denote $\xi_c = 1/[2k_\rho(R_A^2)_{min}]$. In turn, there is a corresponding minimum value of the magnetic field denoted $\mathscr{B}_c$ and given by $\mathscr{B}_c^{4/3} = g_1 \xi_c^{-1/3} - g_2 \xi_c^{2/3}$. The dependences of ξ, R_A, and v_f/v_0 on $\mathscr{B}$ are shown as the solid curves in Figure 4. For the conditions of the figure, $\xi_c = (\dot{M}_j/\dot{M}_a)_{max} \approx 0.068$ and $\mathscr{B}_c \approx 0.45$. For $\mathscr{B} \gg \mathscr{B}_c$ and thus $\xi \ll \xi_c$, we find the asymptotic results $\xi \approx g_1^3 \mathscr{B}^{-4}$ or $\dot{M}_j \propto (\dot{M}_a)^3/B_0^4$, $R_A \approx \mathscr{B}^2/(2k_\rho g_1^3)^{1/2}$, and $v_f/v_0 \approx \mathscr{B}^2/(4g_1^2)$. The rate of energy release from the accretion flow near the inner part of the disk is ηL_a, where $L_a \equiv \dot{M}_a v_K^2/2 = 0.56\ L_\odot$

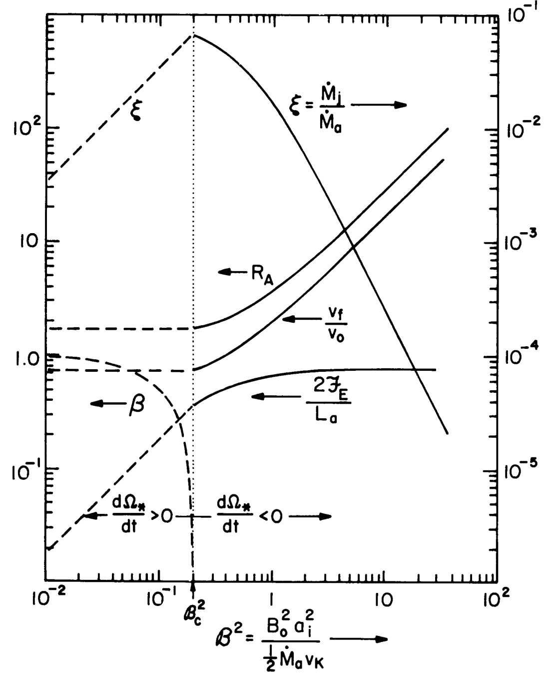

FIG. 4.—The figure shows the dependence of the important jet parameters on the dimensionless disk magnetic field $\mathscr{B}$. For the figure we have taken $\delta = 1.1$, $k_\rho = 2.5$, and $\mathscr{K} = 2.75$. The rotation rate of the star decreases (toward zero) for $\mathscr{B} > \mathscr{B}_c$, whereas it increases for $\mathscr{B} < \mathscr{B}_c$.

$(m/a)(\dot{M}_a/10^{-7}\ M_\odot\ \mathrm{yr}^{-1})$, and $\eta = 1 - (\Omega_*/\Omega_K)^2 \leq 1$ (Pringle 1977). For the conditions of Figure 4, we have $2\mathscr{F}_E/L_a \approx 0.76$–$5.8\xi$. The excess power, $L_{bl} = \eta L_a - 2\mathscr{F}_E$, appears as electromagnetic (UV) "boundary layer" radiation (Bertout 1987; Kenyon & Hartmann 1987; Bertout et al. 1988).

For weak magnetic fields, $\mathscr{B} < \mathscr{B}_c$, it is not possible for the jets to carry away all of the incoming angular momentum ($\dot{M}_a a_i v_K$). Consequently, a fraction of this angular momentum goes into the star, $\beta = \dot{J}_*/(\dot{M}_* a_i v_K) > 0$. This results in spinning up the star, $d\Omega_*/dt > 0$. In this limit angular momentum conservation gives $R_A^2 = [(1-\beta)/\xi + 2\beta]/(2k_\rho)$. We again obtain equation (10), but now $g_1 = 2(1-\beta)/(k_\rho \mathscr{K})$ and $g_2 = (4/\mathscr{K})[(3/2) - \beta/k_\rho]$. For a given value of $\mathscr{B} < \mathscr{B}_c$, the smallest possible value of β occurs for the minimum value of R_A and this corresponds to the largest possible value of $\xi = \dot{M}_j/\dot{M}_a$. The dashed curves in Figure 4 show these dependences. In this limit, we find $\xi/\xi_c \approx (\mathscr{B}/\mathscr{B}_c)^2 < 1$, or $\dot{M}_j \propto B_0^2$, $R_A = 1.75$, $v_f/v_0 = 0.73$, $\mathscr{F}_E \propto \dot{M}_j$, and $2\mathscr{F}_E/L_a \approx 5.3\xi < 0.36$.

In order that the formed star be slowly rotating as observed (Hartmann et al. 1986), it is necessary that $\beta^2 \ll 1$, or equivalently $\mathscr{B} > \mathscr{B}_c$, during most of the disk accretion of a significant fraction of the star's mass. The rate of increase of the star's angular momentum is $\dot{J}_* = \dot{M}_* a_i v_K \beta$ so that its increase in angular momentum is $\Delta J_* = \Delta M_* a_i v_K \langle\beta\rangle$, where ΔM_* is the mass change. In order to have, say, $\Delta J_* < 50\ J_\odot$, we need $\langle\beta\rangle < 0.1(\Delta M_*/0.1\ M_\odot)$. Formation of a slowly rotating star with significant mass addition by accretion from an approximately Keplerian disk appears to require outflows or jets with high specific angular momentum. A contrary view is given by Königl (1992).

An upper limit on the value of $\mathscr{B}$ arises from the fact that a sufficiently strong field, $\mathscr{B}_{max} \sim (v_K/v_r)^{1/2} \gg 1$, where v_r is the radial accretion speed, leads to a nonaxisymmetric interchange instability (Kaisig, Tajima, & Lovelace 1992). Therefore, we assume $\mathscr{B}$ is in the range from $\mathscr{B}_c$ to $\mathscr{B}_{max}$. For an unbiased sample of objects, or one object at different epochs, we denote the probability of a value $\mathscr{B}$ in the interval $d\mathscr{B}$ as $W_\mathscr{B} d\mathscr{B}$ and consider as an example $W_\mathscr{B} \propto \mathscr{B}^{-q}$ with $q = $ const. Using the above-mentioned asymptotic dependence $v_f \propto \mathscr{B}^2$, we find $W_{v_f} \propto v_f^{-(q+1)/2}$. Note that speed of the collimated jet ($dr/dz = 0$), v_j, is larger than v_f by a constant numerical factor, $v_j = Cv_f$. For the conditions of Figure 4, $C = 2.14$. Notice that for a given value of $\dot{M}_a$ or L_a, a large value of v_f corresponds to a small jet momentum flux, $\mathscr{F}_P \propto \dot{M}_j v_f \propto L_a/v_f$, and an even smaller mass flux, $\dot{M}_j \propto L_a/v_f^2$, where we have used the fact that $\dot{M}_j v_f^2/L_a$ is approximately constant for $\mathscr{B}$ significantly larger than $\mathscr{B}_c$. A high v_f jet will tend to have a smaller bow shock velocity. Neglecting for simplicity, the momentum carried by the B field, the bow shock velocity is $v_{bs} \approx v_j/[1 + (\rho_a/\rho_j)^{1/2}]$, where ρ_a is the density of the ambient medium, and ρ_j is the density of the jet (Norman, Winkler, & Smarr 1983; Hartigan 1989). Because $\rho_j \propto \dot{M}_j/v_f \propto \dot{M}_a/v_f^3$, we have $v_{bs} = Cv_f/[1 + N(v_f/v_0)^{3/2}]$, where $N \approx 1.1(n_{ex}/10^4\ \mathrm{cm}^{-3})^{1/2}(10^{-7}\ M_\odot\ \mathrm{yr}^{-1}/\dot{M}_a)^{1/2}(r/10^{15}\ \mathrm{cm})(a/m)^{1/2}$ for a hydrogen gas. Thus, v_{bs} increases with v_f for $v_f/v_0 < (2/N)^{2/3}$. For larger v_f, it decreases with v_f and approaches $v_{bs} \propto v_f^{-1/2}$. The time scale t_s required for stationary intrinsic jet parameters ($\dot{M}_j$, $\mathscr{F}_L$, $\mathscr{F}_E$, etc.) to be established after altered conditions at the disk (ΔB_0 and/or $\Delta \dot{M}_a$) is roughly the flow time between the disk and the Alfvén point. We estimate $t_s \lesssim 10^2 a_i/v_K \approx 10$ days $(a^{3/2}/m^{1/2})$.

There are solutions to our equations which correspond to intrinsically lopsided jets where there are different values of $\xi_\pm = \dot{M}_{j\pm}/\dot{M}_a$, $R_{A\pm}$, $v_{f\pm}$, $\mathscr{F}_{E\pm}$, etc., for the $+z$ and $-z$ jets for a given value of $\mathscr{B}$. The power outflow $\mathscr{F}_{E+} + \mathscr{F}_{E-}$ is less than $2\mathscr{F}_E$ for the symmetric case considered. However, a non-reflection symmetric $|\boldsymbol{B}|$ field may favor intrinsically lopsided jets as discussed previously by Wang, Sulkanen, & Lovelace (1992) for extragalactic jets. The source XZ Tau shown in Figure 2 is best fitted by intrinsically lopsided jets.

In other systems with magnetically driven jets but large accretion rates, we expect to have $\mathscr{B} < \mathscr{B}_c$. This limit is of interest because there is a definite minimum value of the fast magnetosonic speed, v_f, and a corresponding minimum value of the speed of the collimated jet, $v_j = Cv_f$. For the conditions of Figure 4, these values are $v_f \approx 0.7v_K$ and $v_j \approx 1.49v_K$. This aspect of magnetically driven jets distinguishes them from other types of winds (Holzer & MacGregor 1985). The minimum jet speed for $\mathscr{B} < \mathscr{B}_c$ may underly the constant speed $0.26c$ observed for the jets of SS 433 (Margon 1984). Of course, outflow speeds lower than this mimimum may result if the jet entrains external material or if the jet flow originates from the disk at distances larger than a_i.

4. CONCLUSIONS

Application of the theory of LBC is made to observed radial velocity profiles of two jets (§ 2). Analysis of such velocity profiles allows the determination of the fast magnetosonic speed, v_f, and the axial distance of the fast magnetosonic point, z_f. In turn, the theory relates the mass flux of the jet, $\dot{M}_j$, to v_f/v_0 and B_0, where v_0 is approximately the Keplerian velocity at the inner radius of the disk, a_i, and B_0 is the poloidal magnetic field threading the disk at a_i. In addition, the theory gives three relationships between the energy, momentum, and angular momentum fluxes of the jet and $\dot{M}_j$ and v_f/v_0.

Section 3 discusses a complete family of magnetically driven jet solutions obtained from the equations of LBC. We show that the key dimensionless parameter which determines the essential nature of a jet is $\mathscr{B} = (2B_0^2 a_i^2/\dot{M}_a v_K)^{1/2}$, where $\dot{M}_a$ is the disk accretion rate. The dependence of v_f/v_0 and the jet fluxes on $\mathscr{B}$ is derived. For $\mathscr{B}$ larger than a critical value, $\mathscr{B}_c \approx 0.45$, the star spins down, whereas for $\mathscr{B} < \mathscr{B}_c$ it spins up. For $\mathscr{B}$ increasing from $\mathscr{B}_c$, the jet velocity increases, while the mass and momentum fluxes of the jet decrease. For a sample of objects, or one object at different epochs, a distribution of $\mathscr{B}$ values gives rise to a distribution of jet velocities and fluxes.

For some T Tauri sources, modeling of the IR-UV spectrum of the disk, the boundary layer, and the star allows approximate determination of $\dot{M}_a$, a_i, v_K, and the angle between the disk normal (or jet axis) and the line of sight (Kenyon & Hartmann 1987; Bertout et al. 1988; Basri & Bertout 1989). Detection and spectral measurements of optical jets in these sources would be of particular interest because v_f/v_0 could be determined assuming $v_0 \approx v_K$. From a given value of $v_f/v_0 > 1$, Figure 4 implies a definite value of $\xi = \dot{M}_j/\dot{M}_a$. Because $\dot{M}_a$ is known, $\dot{M}_j$ and the other jet fluxes could be predicted and compared with the observed jet. Furthermore, one could derive the disk magnetic field B_0.

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