

STELLAR EVOLUTION AS A PROBE OF NEUTRINO PROPERTIES

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ABSTRACT

We present the results of evolutionary computations devoted to a study of the effect of a nonvanishing neutrino magnetic moment (n.m.m.) on the evolution of stars. In agreement with previous investigations, we found that n.m.m. affects the evolution of low-mass stars, increasing the cooling mechanisms in the core of red giants and thus increasing the mass M_c of the He core at the onset of central He burning. One finds that in the range $0 < \mu_\nu < 10^{-11} \mu_B$ (Bohr magnetons), M_c appears linearly dependent on the value of μ_ν . However, pushing this investigation toward larger stellar masses, giants around the so called transition phase shows a much larger nonlinear dependence of M_c on μ_ν , because n.m.m. is governing whether or not the star experiences electron degeneracy in the core.

The evolution of selected models through the whole phase of He burning has been followed. Contrary to previous suggestions, we found that the direct influence of n.m.m. on these structures appears negligible. As a whole, we calibrate the luminosity of He-burning stars in terms of n.m.m. Comparison with observational constraints concerning the pulsational properties of RR Lyrae variables in the well studied globular cluster M3 gives an upper limit to n.m.m. of $\mu_\nu < 10^{-12} \mu_B$. The same upper limit is derived by considerations of the luminosity of He-burning giants in the galactic clusters Hyades and Praesepe.

Subject headings: elementary particles — globular clusters: general — stars: evolution — stars: variables: other (RR Lyrae)

1. INTRODUCTION

Using stellar structures as natural laboratories for testing fundamental physics is the exciting possibility brought to light by the continuous improvement of present knowledge of stellar evolution. As a matter of fact, theoretical evolutionary computations concerning the two major phases of H and He burning are already available in literature for a rather complete set of assumptions about the star masses and their chemical compositions. On the other hand, the continuous improvement of observational techniques is providing us with valuable observational data which can be used to produce stringent constraints on such theoretical prescriptions. As a result, one expects to be able to constrain even minor variations in the adopted physical input which is at the basis of the theoretical evolutionary scenario. In recent times, such an approach has been used to investigate open questions about particle physics, like the occurrence of weakly interacting massive particles (the so-called WIMPs: see, e.g., Rood & Renzini 1989; Faulkner & Swenson 1991; Faulkner 1991). In particular, a series of papers has been devoted to the search for an upper limit to the neutrino magnetic moment (n.m.m.). A nonvanishing n.m.m. of the order of $10^{-10} \mu_B$ has been suggested as a possible solution to the solar neutrino puzzle (Voloskin, Vysotskii, & Okun' 1986a, b, 1987). With similar values of the magnetic moment, a significant fraction of solar neutrinos should be rotated by the solar magnetic field into right-handed neutrinos, escaping detection.

The evolutionary approach to the problem of an upper limit for n.m.m. appears rather straightforward: the larger the neutrino magnetic moment, the larger should be the production of

plasma neutrinos in stellar interiors and, in turn, the more efficient the neutrino cooling. Thus one may hope to put observational constraints to the n.m.m. looking at the evolutionary behavior of stellar structures where neutrino cooling plays a critical role. Following such an approach, Blinnikov (1988) compared the theoretical lifetimes of cooling white dwarfs with observational data, deriving an upper limit of $\mu_\nu = 10^{-11} \mu_B$ to the value of n.m.m., a value to be compared with the upper limit of $\mu_\nu = 10^{-10} \mu_B$ (Kyudjiev 1984) derived from laboratory physics. More recently, Raffelt (1990) followed an alternative approach based on the evolutionary properties of Population II stars in galactic globular clusters (GGC). After the depletion of central H, low-mass stars populating GGC start burning H in a shell surrounding an inert He core. During this phase, the star appears as a red giant (RG), whose He core becomes electron degenerate, growing with time as a consequence of the conversion of H into He in the layers surrounding the He core. It will remain as a RG until the He core reaches in its interior the 10^8 K needed to ignite the burning of He into carbon. After He ignition, the star will become a horizontal-branch (HB) star, burning both He at the center and H in shell. Figure 1 shows the typical color-magnitude diagram for stars in a GGC, illustrating the evolutionary phases we are dealing with. The connection with n.m.m. is given by the theoretical evidence for which the temperature of the He core is governed by the balance between the gravitational heating and the cooling by neutrinos. As a consequence, the larger the neutrino cooling, the more delayed is the ignition of He and the larger is the He core at this ignition. Since the size of the He core governs the luminosity of the HB stars, one expects that an increase in the cooling should be followed by a detectable increase in the expected luminosity of HB stars. Thus the luminosity level of HB stars can be used as an indicator of n.m.m.

Following such an approach, Raffelt derived an upper limit to n.m.m. of $10^{-12} \mu_B$, lowering by a further order of magni-

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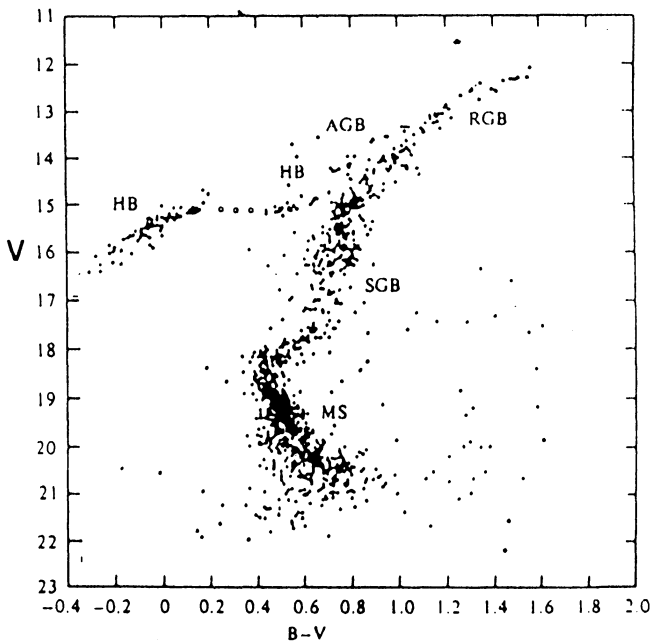


FIG. 1.—The color-magnitude diagram of stars in the Galactic globular cluster M5. The main evolutionary phases of a low-mass star are indicated.

tude Blinnikov's estimate. However, as it will be discussed in the next section, we are not completely comfortable with some of the assumptions made by Raffelt. Thus we decided to reconsider the problem on the basis of new evolutionary computations. In this paper we will report the results of such an investigation. We will find that there are at least two observational evidences, in addition to those considered by Raffelt, that confirm the important result given by him. As a result, we feel rather confident of presenting as a firm astrophysical upper limit the value of $10^{-12} \mu_B$ for n.m.m. This bound seems to rule out n.m.m. as an explanation of the low solar neutrino flux.

2. He CORES AS A FUNCTION OF NEUTRINO MAGNETIC MOMENT

In the advanced evolutionary phases of low-mass stars, the production of neutrinos from plasmon decay is the only mechanism which significantly contributes to the neutrino energy losses. A nonvanishing n.m.m. allows the decay of plasmons into two different types of neutrinos, increasing the production of plasma neutrinos and, thus, the cooling by such particles. As discussed in the previous section this would affect the evolution of electron degenerate stellar cores of red giants, delaying the ignition of helium; the anomalous neutrino losses would, therefore, lead to a core-mass excess beyond the standard values. According to the well-established evolutionary scenario, one easily derives how and where such an increase in the core mass affects the evolutionary behavior. Namely, one expects (1) larger luminosity of the RG at the He flash, and (2) larger luminosity, lower lifetimes, and larger effective temperatures for the He-burning giants. By handling all these features, Raffelt (1990) estimated an upper limit of $10^{-12} \mu_B$ for n.m.m.

However, as discussed by Degl'Innocenti (1991), such a result is not completely free of problems. Raffelt assumes that the mass of the He core at the He flash is influenced by the

amount of mass loss during the RG phase, a quite reasonable assumption which, however, is not supported by detailed evaluations (Castellani & Castellani 1992): RG evolutions with mass loss show that the star near the helium flash does retain a strong memory of the initial mass because the evolution of the inner degenerate core is barely affected by the H-rich envelope. As a result, one may say that stellar cores do not recognize that the envelope is losing mass. Moreover, the evolutionary scenario adopted by Raffelt has been improved in recent times. New computations of low-mass stellar models with improved input physics (Sweigart 1987; Chieffi & Straniero 1989; Castellani, Chieffi, & Pulone 1991) slightly change the evolutionary scenario adopted by Raffelt. However, in our feelings, the most important point is that we are not completely confident of the use of the top luminosity of the RG branch, which should be estimated no better than $\approx \pm 0.1$ mag (Frogel, Cohen, & Persson 1983; see also Vandenberg & Durrell 1990).

Thus we decided to revisit the whole problem, relying on updated evolutionary computations and looking for further and more direct indications about n.m.m. This has been done by computing suitable evolutionary models covering both the H- and He-burning phases under different assumptions about the value of n.m.m. To this purpose, we implemented our evolutionary code (FRANEC, i.e., Frascati Raphson Newton Evolutionary Code) with the contribution of μ_ν to the neutrino energy emissivity. The numerical processes and the basic features of the physical input adopted by the FRANEC code has been discussed in Chieffi & Straniero (1989). For the anomalous neutrino energy losses (as produced by a nonvanishing μ_ν) we adopted the formulations already given in the literature (Beaudet, Petrosian, & Salpeter 1967; Itoh et al. 1989; Raffelt 1990).

To explore the effect of n.m.m. on the mass of the He core at the He ignition we first evolved a model of $0.8 M_\odot$, $Z = 2 \times 10^{-4}$, $Y = 0.23$ which should be representative of stars evolving in metal-poor GGCs, like the well-known M15. The computations have been performed under the alternative assumptions $\mu_\nu = 0$, 5×10^{-12} , $10^{-11} \mu_B$. Details of the computation can be found in Degl'Innocenti (1991). As shown in Figure 2, we found that the mass of the He core at the flash (M_c) can be well approximated by

$$M_c = M_{c0} + 9.3 \times 10^{-3} \mu_\nu,$$

where the core mass is expressed in units of solar masses and the value of n.m.m. in units of $10^{-12} \mu_B$. Such a linear dependence on the value of n.m.m. appears in reasonable agreement with similar evaluations by Raffelt.

From a quick look at the evolutionary results one finds that, for $10^{-11} \mu_B$, the core mass excess is so large that the luminosity of stars at the RGB tip is about 1 mag higher than the standard previsions, a value which is certainly ruled out by observations. Thus, as already stated by previous authors (Fukugita & Yazaki 1987; Raffelt, Dearborn, & Silk 1989), $\mu_\nu = 10^{-11} \mu_B$ appears a clear upper limit for the value of the neutrino magnetic moment. Nevertheless observational uncertainties in the luminosity of the RG tip makes it difficult to reach a much more stringent limit on n.m.m. As a matter of fact, one can write equation (38) in the Raffelt paper as

$$\delta M_c \cong 1.4(M^{\text{tip}} - M^{\text{RR}}) + 0.364,$$

where M^{tip} represents the magnitude of the RG tip. Without entering into detailed discussion of Raffelt's paper, here let us

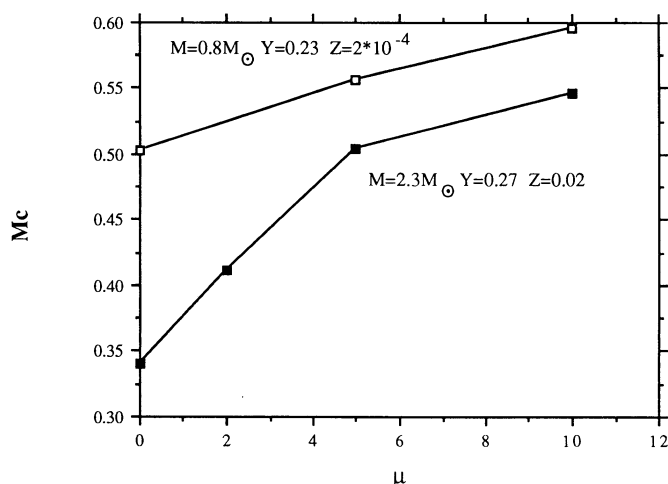


FIG. 2.—Theoretical expectations about the mass of the He core at the He-burning ignition (M_c), as a function of n.m.m. (μ in $10^{-12} \mu_B$). The figure shows the values of M_c for a low-mass model ($0.8 M_\odot$, $Z = 2 \times 10^{-4}$, $Y = 0.23$), representative for globular cluster stars, and for a $2.3 M_\odot$ model, with $Y = 0.27$, $Z = 2 \times 10^{-2}$, representative of the “transition giants” in Galactic clusters.

only notice that if one relies on the already quoted estimate about the error on M_c^{up} (± 0.1), this implies an error of $\delta M_c \approx 0.14 M_\odot$ and, in turn, an error of about $8 \times 10^{-12} \mu_B$ on the value of μ_ν . The next section will be devoted to a discussion of the evolutionary parameters best suited to place a limit on such increases in the M_c in GGC stars.

However, before leaving this argument, evolutionary considerations suggest an extension of the investigation to more massive stars, representative of evolved stars in clusters experiencing the so called red giant transition phase, i.e., when the evolutionary giants are near the limiting mass for experiencing electron degeneracy during H-shell burning. The philosophy motivating such an approach is rather simple: one expects that in similar cases the increased neutrino cooling can play the role of deciding whether or not the stellar core becomes degenerate, with—possibly—an increased effect on the mass of the He core at He-burning ignition. Thus we performed a series of computations for a $2.3 M_\odot$ model, with $Y = 0.27$, $Z = 2 \times 10^{-2}$, which appears adequately representative for the “transition giants” observed in the well-studied galactic clusters Hyades and Praesepe (Castellani, Chieffi, & Straniero 1992).

The results, as shown in Figure 2, nicely confirm such a belief. One finds that for $\mu_\nu < 5 \times 10^{-12} \mu_B$ the dependence of M_c on μ_ν is much larger than in the previous case. This is because n.m.m. is pushing the stellar core into the degenerate state. The dependence of M_c beyond $\mu_\nu = 5 \times 10^{-12} \mu_B$ shows that for these values the stellar core is degenerate, and the dependence of M_c on μ_ν becomes that already found for low-mass red giants. The smaller value of M_c in the $2.3 M_\odot$ model is easily understood in terms of well-known relations connecting the mass of the He core at the flash with star mass and chemical composition.

3. GLOBULAR CLUSTER HORIZONTAL-BRANCH STARS

In principle, a nonvanishing n.m.m. affects the evolution of HB stars in two different ways, following both the evolutionary influence of the larger helium core formed during the RGB phase and the direct influence of neutrino losses in the HB

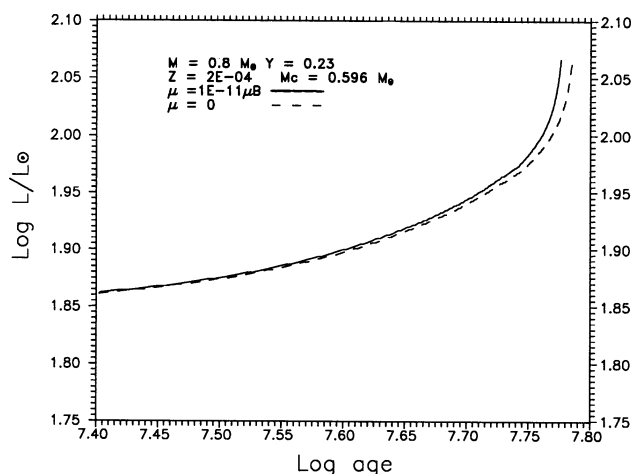


FIG. 3.—Time behavior of the luminosity of our HB model ($0.8 M_\odot$, $Z = 2 \times 10^{-4}$, $Y = 0.23$) as computed for $\mu_\nu = 0$ and $\mu_\nu = 10^{-11} \mu_B$. The age of the star (in years) is measured from the onset of central He-burning. The figure shows only details of the final phases of central He-burning up to the central helium exhaustion.

stage. However, during the helium flash the central density of stars drops rapidly (a HB star has a central density of $\approx 10^4 \text{ g cm}^{-3}$ against $\approx 10^6 \text{ g cm}^{-3}$ for a red giant) and the plasma neutrino losses decrease by an order of magnitude. As a consequence, our evolutionary calculations indicate that the direct influence of n.m.m. on HB stars is negligible. This is shown in Figure 3, where we report the time behavior of the luminosity of our HB model as computed for $\mu_\nu = 0$ and $\mu_\nu = 10^{-11} \mu_B$. This result disagrees with previous indications by other authors (Fukugita & Yazaki 1987; Raffelt & Dearborn 1988; Raffelt 1990) who assumed a large influence of the n.m.m. on the evolution of helium-burning stars. The reason for such a discrepancy may be found in the fact that these authors assumed the helium core as homogeneous and isothermal while, in reality, both temperature and density drop rapidly moving out from the center. This overestimate of the values of the temperature and density in the core lead to an overestimate of the neutrino energy losses too.

We found that for $10^{-11} \mu_B$ the HB model with no mass loss, which is the reddest possible model, becomes as hot as $\log T_e = 4.05$ with a He-burning lifetime reduced by about a factor of 2. Again, one easily finds that both these predictions are completely outside the range of values allowed by observations. This can be directly derived by the temperature of HB stars observed in the color-magnitude diagram of M15 or by the analysis given by Buzzoni et al. (1983) of the number ratio “R” of HB and RGB stars. However, when one looks for a safe upper limit to μ_ν , one finds that the quoted observational parameters are not the most useful ones. This is because the temperature of the HB is also governed by the amount of α -elements in the stellar matter (see, e.g., Bencivenni et al. 1991) on which we lack firm evaluations. Moreover, the parameter R, which is an indirect measure of the HB lifetime is known for globular clusters only with a rather large statistical error.

For all these reasons, we found that the most promising way to put a firm upper limit on μ_ν is to look at the expected properties of RR Lyrae pulsators in globular clusters. As a matter of fact, we know that pulsational period is known with extremely good precision and that this period depends on the mass, luminosity, and temperature of the pulsators.

Following the usual formulation, let us recall that for a variable:

$$\log P = 11.497 + 0.84A - 3.48 \log T_e,$$

where both P and T_e are given by observation, whereas

$$A = \log L/L_\odot - 0.81 \log M/M_\odot$$

is a function of M_c and, thus, of μ_v . The parameter A can be connected to n.m.m. following the relation given by Caputo, Martinez Roger, & Paez (1987):

$$A = 1.51 + 1.27Y + 2.15 \delta M_c,$$

where δM_c is the core-mass excess beyond the standard values. Then, comparing theoretical expectations about the value of A for different assumptions on n.m.m. with observational values of the "mass-to-luminosity parameter," one may constrain n.m.m.

To this end we show in Figure 4 observational values of A as derived from the values of period and effective temperature reported by Sandage (1990) for the well-studied globular cluster M3. As is well known, the lower boundary of the observed A distribution has to be connected to the so called zero-age horizontal branch (ZAHB), i.e., to stars just starting the phase of quiescent He burning at their center. As reported in the figure, the observational value of A corresponding to such ZAHB pulsators can be safely fixed at $A = 1.778 \pm 0.023$, as shown by the two dashed lines. Note that such a range of values takes into account uncertainty in the cluster reddening and in the relation between star color and temperature, both affecting the value of T_e and, in turn, the parameter A . The same Figure 4 shows theoretical expectations about the lower limit of the A distribution as a function of n.m.m. The uncertainty on the theoretical value of A is mainly due to the assumed uncertainty in the helium content of HB stars. Globular clusters are among the oldest objects in the Galaxy; therefore, their initial helium abundance will not be much larger than the primordial helium abundance. We assumed for this value $Y = 0.23$ as estimated by Buzzoni et al. (1983) for GGC stars and recently supported by observations of helium line strengths in the emission of metal-poor H II regions (Steigman 1989; Steigman, Gallagher, & Schramm 1989), adopting a con-

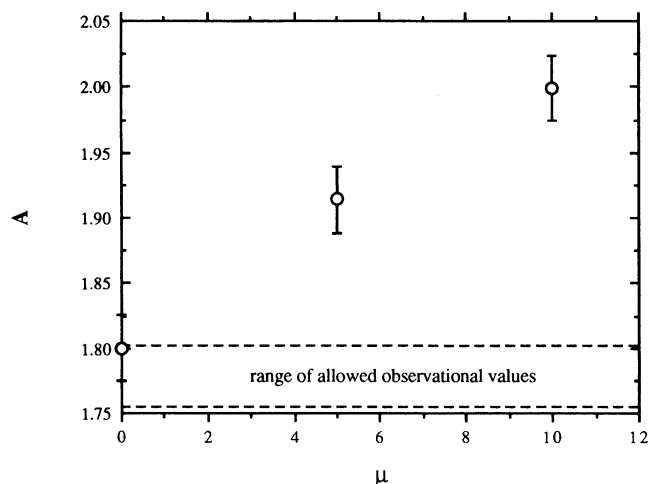


FIG. 4.—Theoretical expectations about the lower limit of the A distribution as a function of n.m.m. The two dashed lines indicate the allowed range of observational values for the same quantity.

servative indetermination of ± 0.02 . As a result, Figure 4 shows that an upper limit of the order of $10^{-12} \mu_B$ can be safely assumed, supporting in this way the previous indication by Raffelt. Before leaving the argument, it is important to note that the quoted analysis is relatively unaffected by assumptions about suggested non-solar-scaled abundances, as recently discussed by Bencivenni et al. (1991).

One may note that in the analysis so far performed we implicitly assumed a zero mass for the three species of neutrinos. If, on the contrary, according to recent suggestions (see, e.g., Hime & Jelley 1991 and references therein), the μ neutrino has a 17 keV mass, the production of plasma neutrinos due to μ_v can occur only in a very small region around the stellar center, where the plasma frequency is greater than the rest mass of the produced neutrinos, so that plasmon decays are kinematically allowed. This evidently decreases the influence of the plasma neutrino losses on stellar evolution leading to a greater value for an estimated upper limit on n.m.m. Following such an approach, preliminary evaluations seem to indicate an upper limit for μ_v greater than $10^{-9} \mu_B$.

4. NEUTRINO MAGNETIC MOMENT AND GALACTIC CLUSTER STARS

The previous analysis, as performed for old GGCs, can be usefully extended to more massive clusters, near the "RG transition." As already discussed, the value of μ_v governs whether or not the He core becomes degenerate before the onset of the 3α reactions, with substantial consequences on the structure of the evolving stars. Following such an approach, we studied the influence of n.m.m. on a $2.3 M_\odot$ star, whose canonical evolution describes the evolving stars in both Praesepe and Hyades clusters, as shown in Figure 5. Note that the mass of evolving stars is constrained by the turnoff luminosity, which is not affected by neutrino physics because the plasma neutrino losses in the main-sequence phase are negligible. We found that if

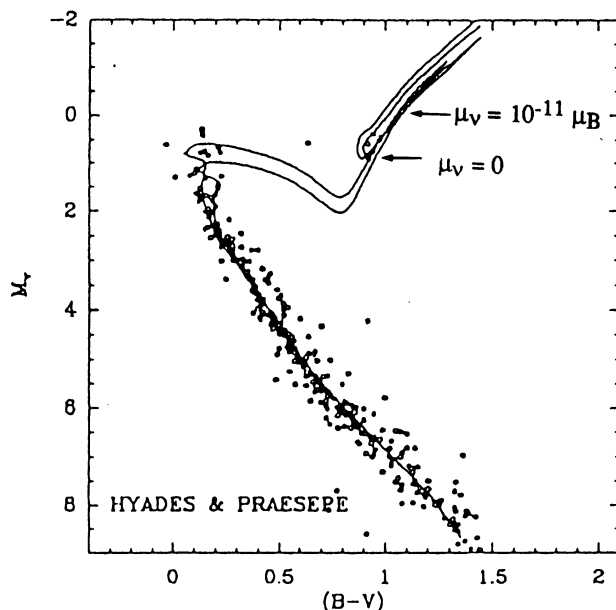


FIG. 5.—The color-magnitude diagram of stars in the Hyades and Praesepe with, superposed, the 700 and 900 Myr isochrones for $\mu_v = 0$. The figure shows theoretical expectations about the lower luminosity of the clump for $\mu_v = 0$ and $\mu_v = 10^{-11} \mu_B$.

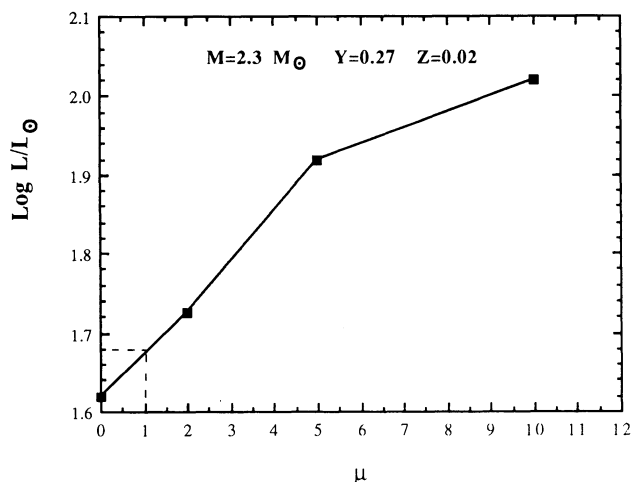


FIG. 6.—Theoretical expectations about the luminosity of the clump for different assumptions on n.m.m. compared with the upper value of luminosity allowed by observations (dashed line).

$\mu_\nu = 10^{-11} \mu_B$, a $2.3 M_\odot$ star, with solar composition, should develop an electron degenerate core, igniting He through a rather violent flash; as a consequence, the luminosity of the RG tip is enhanced by about 4 mag compared with the observational value and the cluster should show a well-developed RG branch, contrary to what is observed. Again $\mu_\nu = 10^{-11} \mu_B$ is clearly ruled out by observation.

The most important point is that, under the above quoted hypothesis, the increased mass of the He core drives a substantial increase in the luminosity of He-burning stars populating the clump of red giants, which is an observational parameter well detected in both the quoted clusters. The distance modulus of the clusters is well constrained by the MS location, so that the good agreement between observation and theory reported in Figure 5 shows that available observational data do not contradict the assumption $\mu_\nu = 0$. Figure 6 shows the theoretical expectations about the luminosity of the clump (L_{clump}) for different assumption on n.m.m., as compared with the estimate of the larger value of luminosity allowed by obser-

vations. Again one finds that $\mu_\nu = 10^{-12} \mu_B$ appears as a safe upper limit for the n.m.m.

5. CONCLUSION

In this paper we have discussed astrophysical parameters which can give information about the value of a neutrino magnetic moment. The general approach to the problem is rather simple: a nonvanishing n.m.m. increases the production of plasma neutrinos, enhancing stellar cooling under electron degenerate conditions and, thus, the mass of the He core at the onset of the He flash. Following such an approach, we studied the influence of n.m.m. on selected models which appear adequately representative for globular cluster RG stars and the giants around the “transition phase” in the galactic clusters Hyades and Praesepe.

For the low-mass stars we found that for μ_ν in the range 0 – $10^{-11} \mu_B$ (Bohr magnetons), M_c appears linearly dependent on the value of μ_ν . However, pushing this investigation toward larger stellar masses, one finds that giants around the so-called transition phase show a much larger nonlinear dependence of M_c on n.m.m., because the value of μ_ν governs whether or not the He core becomes degenerate before the onset of the 3α reactions. We noted that the most promising way to put a firm upper limit on n.m.m. is to study the influence of μ_ν on the evolutionary parameters in the He-burning phase. We found that pulsational properties of RR Lyrae pulsators in galactic globular clusters put an upper limit to n.m.m. of the order of $\mu_\nu = 10^{-12} \mu_B$. The same upper limit is derived by considerations of the luminosity of the He burning giants in old galactic clusters.

As a result, the previous result by Raffelt (1990) appears more and more supported by independent theoretical constraints. In conclusion, at present, astrophysical considerations lower the laboratory upper limit on n.m.m. by about two orders of magnitude. It is our hope that this result will stimulate further investigations of a similar nature.

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REFERENCES

- Beaudet, G., Petrosian, V., & Salpeter, E. E. 1967, *ApJ*, 150, 979
 Bencivenni, D., Caputo, F., Maintega, M., & Quarta, M. L. 1991, *ApJ*, 380, 484
 Blinnikov, S. I. 1988, ITP preprint, 19
 Buzzoni, A., Fusi Pecci, F., Buonanno, R., & Corsi, C. E. 1983, *A&A*, 128, 94
 Caputo, F., Martinez Roger, C., & Paez, E. 1987, *A&A*, 183, 228
 Castellani, M., & Castellani, V. 1992, in preparation
 Castellani, V., Chieffi, A., & Pulone, L. 1991, *ApJS*, 76, 911
 Castellani, V., Chieffi, A., & Straniero, O. 1992, *ApJS*, 78, 517
 Chieffi, A., & Straniero, O. 1989, *ApJS*, 71, 47
 Degl'Innocenti, S. 1991, tesi di laurea, Univ. of Pisa
 Faulkner, J. 1991, preprint
 Faulkner, J., & Swenson, F. J. 1990, in *Challenges to Theories of the Structure of Moderate Mass Stars* (Proc. of 1990 Summer Conf., ITC, Santa Barbara) (Berlin: Springer-Verlag), p. 359
 Frogel, J. A., Cohen, J. G., & Persson, S. E. 1983, *ApJ*, 275, 773
 Fukugita, M., & Yazaki, S. 1987, *Phys. Rev. D*, 36, 3817
 Hime, A., & Jelley, N. A. 1991, *Phys. Lett. B*, 257, 441
 Itoh, N., Adachi, T., Nakagawa, M., & Kohyama, Y. 1989, *ApJ*, 339, 354
 Kyudjiev, A. V. 1984, *Nucl. Phys. B*, 243, 387
 Raffelt, G. 1990, *ApJ*, 365, 559
 Raffelt, G., & Dearborn, D. 1988, *Phys. Rev. D*, 37, 549
 Raffelt, G., Dearborn, D., & Silk, J. 1989, *ApJ*, 336, 64
 Rood, R., & Renzini, A. 1989, in *Astronomy, Cosmology, and Fundamental Physics*, ed. M. Caffo, R. Fanti, G. Giacomelli, & A. Renzini (Proc. 3rd ESO-Cern Symposium), 287
 Sandage, A. 1990, *ApJ*, 350, 603
 Steigman, G. 1989, in *Proc. of the Second International Symposium for the 4th Family of Quarks and Leptons*, in press
 Steigman, G., Gallagher, J. S., III, & Schramm, D. N. 1989, *Comments Astrophys.*, 14, 106
 Sweigert, A. V. 1987, *ApJS*, 65, 95
 Vanderberg, D. A., & Durrell, P. R. 1990, *AJ*, 99, 221
 Voloshin, M. B., Vysotskii, M. I., & Okun', L. B. 1986a, *Soviet J. Nucl. Phys.* 44, 440 (1986, *Yad. Fiz.*, 44, 845)
 ———. 1986b, *Soviet Phys.—JETP*, 64, 446 (1986, *Zh. Eksper. Teoret. Fiz.*, 91, 754)
 ———. 1987, *Soviet Phys.—JETP*, 65, 209 (E) (1986, *Zh. Eksper. Teoret. Fiz.*, 92, 368)