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τ PEGASI: A FOURIER REPRESENTATION OF LINE-PROFILE VARIATIONS

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ABSTRACT

The complex line-profile variations, probably caused by nonradial pulsations, in the spectrum of the rapidly rotating δ Scuti star τ Peg are analyzed with a novel two-dimensional Fourier transform technique. It resolves the oscillations in temporal frequency ν and apparent azimuthal order $\hat{m}$. The two-dimensional Fourier spectrum resembles the (l, ν) diagrams generated for the Sun. Advantages of the technique are (1) both frequency and mode of the oscillations are determined directly; (2) complex variations in the line profiles are resolved into individual modes; (3) modes of both high and low degree are resolved. For τ Peg, we identify four apparent modes having $|\hat{m}| \simeq 3$, 7, 11, and 15. After correcting for rotation, most of the modes have frequencies of $\sim$17 cycles day^{-1}. Assuming $T_{\rm eff} \simeq 8000$ K and $L \simeq 40\ L_{\odot}$, all of the modes fall within the range of frequencies theoretically predicted to be unstable.

Subject headings: stars: oscillations — stars: individual: τ Pegasi — δ Scuti

1. INTRODUCTION

Asteroseismology of δ Scuti stars offers an attractive prospect for determining the interior properties of normal nonmagnetic A-F stars on and near the main sequence. In theory, the modes of oscillation act as probes of internal structure. A seismological investigation of δ Scuti stars would greatly improve our understanding of the evolutionary changes in their structure.

To be effective, asteroseismology requires several modes of oscillation to be present and identifiable. Many δ Scuti stars are known to undergo multiperiodic nonradial oscillations. The identification of modes and frequencies in such stars has been an extensive area of research in recent years. Especially successful have been multisite photometric campaigns lasting several weeks which have provided the necessary frequency resolution to reveal multiple modes of oscillation (e.g., θ^2 Tau: Breger et al. 1989; GX Peg: Michel et al. 1990). However, even such intensive examinations as these cannot provide a complete representation of the oscillations.

Nonradial stellar oscillations are characterized by their frequency and three quantum numbers: the order n, which usually equals the number of radial nodes; the azimuthal order m, where $2|m|$ specifies the number of nodes in longitude; and the nonradial degree l, where $l - |m|$ specifies the number of nodal lines in planes parallel to the equator. The azimuthal order may take on the values $m = -l, -l+1, \ldots, l$. (For rotating stars, modes with negative orders conventionally propagate in a prograde sense.) Photometric observations are disk integrated and provide direct information only about the frequency of oscillation. Using both light and color variations it is possible to estimate the quantum numbers, but often theory must be invoked to infer *all three* quantum numbers. Moreover, due to cancellation effects only modes of low degree ($l \leq 3$) are detected photometrically. Fortunately, low-degree (p-mode) oscillations propagate deeply into a star and therefore contain important information about the star's deep interior structure.

Modes of oscillation can be determined directly from the Doppler-broadened line profiles of rapidly rotating stars observed with high spectral and temporal resolution. Low-degree modes appear as cyclical changes in the overall shape of the line. High-degree modes appear as traveling bumps, moving from blue to red through the profiles. The moving bump phenomenon was first discovered for δ Scuti stars by Yang & Walker (1986) and Walker, Yang, & Fahlman (1987). That they are a signature of nonradial oscillations is now commonly accepted. The surface variations are mapped in each line profile by the effect of "Doppler imaging" (Vogt & Penrod 1983).

For monoperiodic variations the mode can be identified by counting bumps in the profiles. Campos & Smith (1980), Smith (1982), and Kennelly, Walker, & Hubeny (1991) approached this problem by fitting synthetic profiles to a time series of observations by a method of trial and error. However, this is unsatisfactory for multiperiodic variations. A more reliable method for detecting multiple modes was introduced by Gies & Kullavanijaya (1988), who performed a Fourier analysis on the temporal dependence of the variations to determine the frequencies of oscillation. The modes were calculated from phase variations of the individual Fourier components across the line profile. In this paper we extend the method of Gies & Kullavanijaya (1988). With observations of τ Peg as example, we demonstrate that a *two-dimensional* Fourier analysis of line-profile variations in both time and "Doppler space" can simultaneously identify both frequencies and modes of oscillation.

τ Peg (A5 IV) is a bright δ Scuti star with projected rotational broadening $v \sin i = 150$ km s^{-1} (see § 3). Photometric investigations have identified a single frequency of oscillation at 18.4052 cycles day^{-1} (Breger 1991). However, the amplitude of the light variations has been observed to vary over the last 20 yr of observation. Breger (1991) reproduced the amplitude variations with a mathematical model invoking three very closely spaced frequencies, but the results were inconclusive. In § 3 we present observations of τ Peg which demonstrate the presence of high-degree modes as line-profile variations. Because it is bright, rotates rapidly, and exhibits multiple

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modes, τ Peg is an ideal star on which to perform a two-dimensional Fourier analysis.

2. TWO-DIMENSIONAL FOURIER TECHNIQUE

The surface velocity field generated by nonradial pulsations can be described in terms of spherical coordinates (r, θ, ϕ) and time t as

$$V(\theta, \phi, t) = \left(V_0, kV_0 \frac{\partial}{\partial\theta}, kV_0 \frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\right)P_l^m(\cos\theta)e^{i(m\phi+\nu t)} \quad (1)$$

where V_0 is the velocity amplitude of the oscillations in the radial direction, k is the ratio of the horizontal to radial velocity amplitude, ν is the frequency of oscillation, and $P_l^m(\cos\theta)$ are associated Legendre polynomials of degree l and order $m = -l, -l+1, \ldots + l$. The magnitude of k can be predicted theoretically and is determined by the boundary conditions on the pressure at the surface of the star. It is related to the pulsation constant $Q = P_{\rm osc}(\bar{\rho}/\bar{\rho}_\odot)$ by $k = 74.4\,Q^2$ (Unno et al. 1989) where $P_{\rm osc}$ is the period of oscillation and $\bar{\rho}$ is the mean density of the star. For the high-frequency oscillations associated with p modes, $k \ll 1$ and the surface velocity is primarily in the radial direction.

The azimuthal dependence of the oscillation amplitude is described by the associated Legendre functions $P_l^m(\cos\theta)$. As the line-profile variations are most sensitive to sectoral modes ($l = |m|$), we concern ourselves only with these modes. (Modes with $l \neq |m|$ suffer from cancellation from oppositely directed variations on either side of the equator; see Kambe & Osaki 1988). In this case, amplitudes are largest near the equator and decrease to zero at the poles, so that the observed variations are weighted most by the region situated near the equator of the star. (For nonmagnetic, rotating stars the axis of oscillation is aligned with the axis of rotation.) In this region, the radial velocity field is approximately

$$V = V_0\,P_l^m(0)e^{i(m\phi+\nu t)}\,. \quad (2)$$

Equation (2) lends itself to a two-dimensional Fourier analysis in time and space. The oscillations of the Sun are analyzed by a similar approach, although in observing the Sun, the radial velocities are resolved in both angular directions.

Assuming that the star is observed equator-on ($\sin i = 1$), we can interpret the time-varying component of the rotationally broadened profiles (i.e., the residuals) as Doppler images of ($l = |m|$) oscillations confined near the equator. Wavelength is mapped to velocity by the standard Doppler shift formula, and residuals are truncated at $+v \sin i$ and $-v \sin i$. The projected rotational broadening ($v \sin i$) is determined by fitting rotationally broadened synthetic profiles to the observations. Finally, residuals are mapped onto a coordinate system corresponding to stellar longitude ϕ using

$$\phi_i = \frac{1}{4}\sin^{-1}\left(\frac{v_i}{v \sin i}\right) \quad (3)$$

where v_i denotes the velocity corresponding to a given wavelength with respect to the center of the line.

Our Fourier method uses a routine for unequally spaced data (Matthews & Wehlau 1985) to transform the variations in time and "Doppler space" to Fourier space, where an amplitude spectrum expressed as a function of frequency and inferred azimuthal order $\hat{m}$ is calculated from the real and imaginary parts of the two-dimensional transform. Because both the temporal and spatial frequencies of the modes are resolved, the method is compatible with the identification of multiple modes. However, if the line-profile variations are not sinusoidal then harmonic terms (at integer multiples of the true frequency) will be introduced into the Fourier transform and may lead to erroneous interpretations of the data. Despite this potential drawback, the Fourier technique is attractive because it directly determines both the frequency and the mode of oscillation, plus it simplifies the otherwise complex patterns in the profile variations by decomposing them into Fourier components.

3. OBSERVATION AND ANALYSIS OF τ Peg

We obtained 38 Reticon spectra of τ Peg at the CFHT over 7.3 hr on 1990 October 05 (UT) with a reciprocal dispersion of 2.4 Å mm^{-1} (0.035 Å pixel^{-1}). The spectral region, centered on λ4490, was chosen to match that of Walker, Yang, & Fahlman (1987) where more details are given. Exposure times were $\sim$600 s, with a signal-to-noise ratio of $\sim$520 pixel^{-1} in the continuum. The data were processed with IRAF[4].

The 65 Å of spectrum is rich in spectral features. The portion presented in Figure 1*a* demonstrates the variations in the profile of the unblended Fe II λ4508, with bumps moving from blue to red through the profile. Averaged over sufficiently long times, the line profile variations cancel, leaving only the rotationally broadened spectrum. The projected rotational broadening, $v \sin i = 150$ km s^{-1}, was derived by fitting synthetic profiles to the mean profile. The mean profile was subtracted from each profile of Figure 1*a* to produce the time series of residuals shown in Figure 1*b*. A steady progression of strong features is seen early in the data but suddenly dies out midway. Could this be caused by beating between modes of oscillation?

Variations in the residuals were analyzed using the two-dimensional Fourier method outlined in § 2. A contour map of the resulting Fourier amplitude spectrum is presented in Figure 2. A series of peaks can be seen running diagonally across the center of the diagram with apparent frequencies $\hat{\nu}$ between 18 and 30 cycles day^{-1} and apparent azimuthal orders $|\hat{m}|$ between 2 and 15. We interpret these peaks as likely modes of oscillation. The values of $|\hat{m}|$ and $\hat{\nu}$ and the Fourier amplitudes corresponding to these peaks are tabulated in Table 1. The small peak at $|\hat{m}| \simeq 2.6$ and $\hat{\nu} \simeq 18.5$ cycles day^{-1} apparently corresponds to a mode previously detected by disk-integrated photometric and radial-velocity observations (Kennelly, Matthews, & Walker 1992) and is consistent with the published, photometrically determined frequency of variation at 18.4052 cycles day^{-1} (Breger 1991).

The upper limiting frequency for unevenly sampled data is determined by the generalized Nyquist frequency, $\hat{\nu}_{\rm Nyq} = 1/(2\Delta t)$, where Δt is the mean time spacing between observations. For our observations of τ Peg, $\hat{\nu}_{\rm Nyq} \simeq 60$ cycles day^{-1}. The upper limit of $|\hat{m}|$ is determined by the intrinsic resolution of the star, that is, the ratio of the intrinsic line width V_b to the projected rotational broadening. For τ Peg, $v \sin i/V_b \simeq 30$.

The frequency resolution is proportional to the inverse of the total time covered by the observations. For our observations of τ Peg, $\Delta\hat{\nu} = 3.3$ cycles day^{-1} FWHM (full width at half-maximum). In the spatial domain, resolution is limited by

[4] IRAF is distributed by the National Optical Astronomy Observatories, which is operated by the Association of Universities for Research in Astronomy (AURA), Inc., under cooperative agreement with the National Science Foundation.

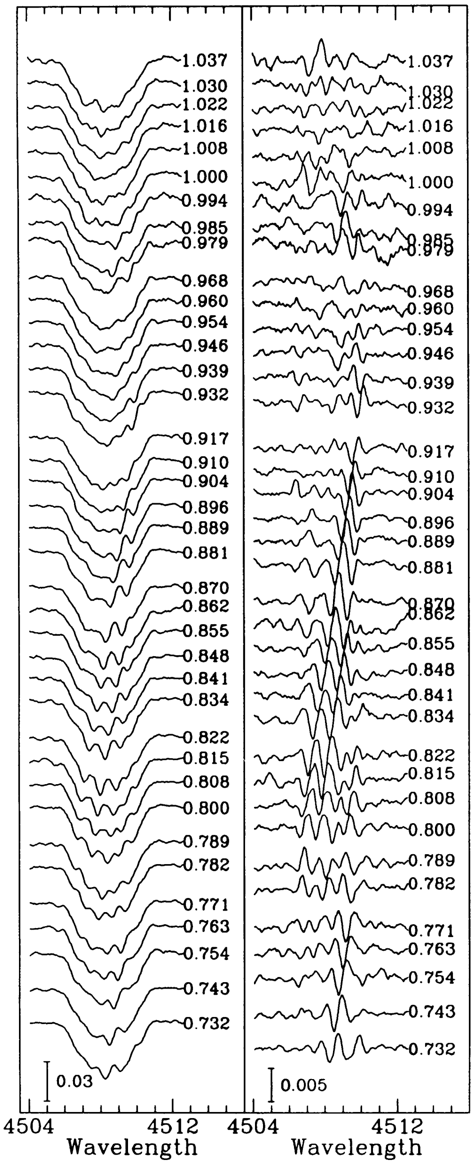

FIG. 1.—Time series of τ Peg spectra. (*a*) Variations in Fe II λ4508 (*left*). (*b*) The profile variations in (*a*) are more clearly seen as residuals from the mean (*right*). The times of observation are indicated on the right-hand side of the plot in days relative to heliocentric JD 2,448,169. The vertical scales are in units of the continuum intensity.

the fact that only half the stellar circumference can be observed and is at best $\Delta\hat{m} = 2$ FWHM. In practice, spatial frequency resolution of observed line-profile variations will be somewhat worse because of foreshortening in ϕ that occurs in the line wings.

The pattern of aliases in the observations was explored by generating a spectral window function for an $l = -m = 6$ mode with an apparent oscillation frequency of $\hat{\nu} = 22.3$ cycles day^{-1}. A computer simulation of nonradial pulsations was used to construct a time series of model line profiles with the same time (and wavelength) sampling and duration as the observations. The projected rotational broadening used in the model was $v \sin i = 150$ km s^{-1} with $i = 90°$. The velocity amplitude of the oscillations was 4 km s^{-1}. Details of a similar model may be found in Kennelly, Walker, & Hubeny 1991. Figure 3 illustrates the Fourier transform of the modeled varia-

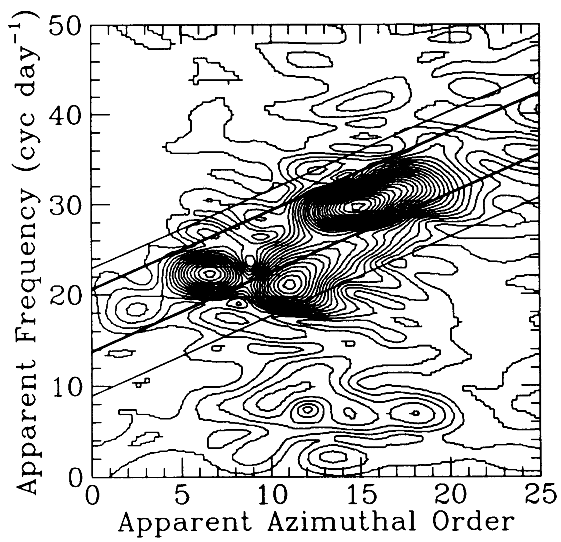

FIG. 2.—The two-dimensional Fourier spectrum of the line-profile variations of τ Peg. The variations were transformed in both space and time to produce this contour map of the apparent azimuthal orders and frequencies of oscillation. The four peaks which form a ridge in the center of the diagram are interpreted as indicating modes of oscillation. Theoretical limits to the pulsation frequencies, adopted from the calculations of Dziembowski (1990), are plotted for the standard He abundance model (*thick solid lines*) and He-rich model (*thin solid lines*).

tions. A dominant peak appears at $|\hat{m}| = 6.6$ and $\nu = 22.2$ cycles day^{-1}, in agreement with one of the peaks listed in Table 1. The two-dimensional FWHM of this peak is $\Delta\hat{m} = 4$ and $\Delta\hat{\nu} = 3.9$ cycles day^{-1}. The pattern of aliases is prominent in frequency but not in azimuthal order; foreshortening in the wings of the line profile reduces the amplitude of the sidelobes at the expense of increasing the width of the central peak. The secondary peak at low amplitude appearing at approximately twice the value of $|\hat{m}|$ and $\hat{\nu}$ is a harmonic which results from the nonsinusoidal shape of the line-profile variations.

Although many of the peaks identified in Table 1 are separated in frequency by amounts less than the calculated resolution of 3.9 cycles day^{-1}, we can identify these peaks as separate features because they are resolved in azimuthal order.

4. DISCUSSION

One must be cautious in translating the two-dimensional transform directly into mode identifications because the temporal observing window, temporal and spatial harmonics, and mode combinations may produce spurious peaks not associated with modes. Parallel analytical and numerical investiga-

TABLE 1

FOURIER ANALYSIS OF THE LINE-PROFILE VARIATIONS OF τ Peg[a]

Apparent Azimuthal Order $\lvert\hat{m}\rvert$	Apparent Frequency $\hat{\nu}$ (c/day)	Amplitude (continuum = 1)
2.6	18.5	0.0010
6.6	22.3	0.0027
11.1	21.0	0.0030
14.9	29.7	0.0044

[a] An uncertainty of ± 2 in $|\hat{m}|$ and ± 1.9 cycles day^{-1} in $\hat{\nu}$ were determined from the half width at half maximum of the dominant peak in the two dimensional Fourier transform resulting from our simulations. The two-dimensional analysis allows the separation of modes with similar $\hat{\nu}$ or $|\hat{m}|$.

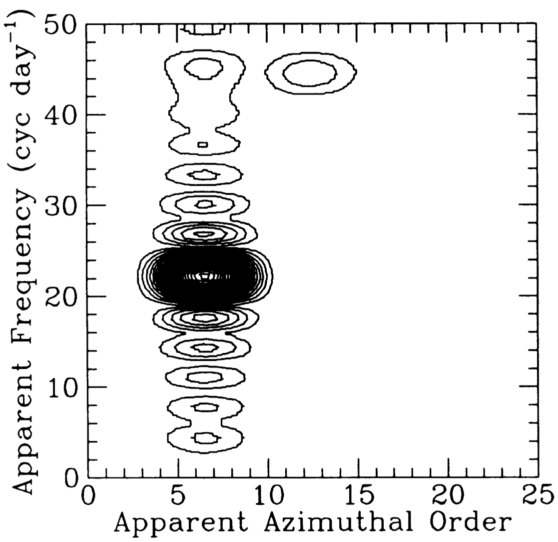

FIG. 3.—The two-dimensional Fourier spectrum generated from line profiles resulting from a nonradial pulsation model with $l = -m = 6$ and $\hat{\nu} = 22.3$ cycles day^{-1}. The pattern of aliases in frequency and azimuthal order is illustrated. The two-dimensional (FWHM) resolution is $\Delta\hat{m} = 4$ and $\Delta\hat{\nu} = 3.9$ cycles day^{-1}.

tions are being directed toward a better understanding of these effects (Merryfield & Kennelly 1992; Kennelly & Merryfield 1992). For now, we shall take the mode identifications at face value, and proceed to compare our results with theory.

To derive the stellar characteristics of τ Peg, we have adopted the Strömgren photometry published in Hauck & Mermilliod (1980) and used the calibrations and suggested uncertainties of Crawford (1979) and Philip & Relyea (1979). We find $M_V = 0.66 \pm 0.3$, $T_{\rm eff} = 8000 \pm 200$ K and $\log g = 3.7 \pm 0.1$, and from these values calculate $L = 43\ L_\odot$, $R = 3.4\ R_\odot$ and $M = 2.2\ M_\odot$. Note that our results differ from those of Breger (1991), who finds τ Peg to be less luminous and hotter.

The apparent frequencies of oscillation obtained from our Fourier analysis include the effect of rotation. Ignoring the corrections due to Coriolis terms, lines of constant rotational velocity on the $|\hat{m}| - \hat{\nu}$ plane are given by

$$\hat{\nu} \simeq \nu_0 - (0.01976)\,\frac{\hat{m}v \sin i}{R \sin i} \tag{4}$$

where ν_0 is the oscillation frequency in cycles day^{-1} in the corotating frame of the star, R is the stellar radius in $R_\odot$ and $v \sin i$ is the projected rotational broadening in km s^{-1}. We assume an inclination of $i \simeq 90°$.

Dziembowski (1990) examined pulsational instabilities for envelope models of δ Scuti stars having three different luminosities and masses. The most luminous of these, which has $L = 40\ L_\odot$ and $M = 2\ M_\odot$, closely resembles our inferred characteristics for τ Peg. While he computed the range of unstable frequencies for radial modes, they can be adopted for nonradial modes of sufficiently small l because the radial variation of the eigenfunctions in the region where opacity mechanisms excite the mode is then only weakly l-dependent. The theoretical limits, adjusted for rotation using equation (4), are superposed on the observed Fourier map of τ Peg in Figure 2. The thick solid lines denote the boundaries for a model of standard He abundance ($Y = 0.28$) and the thin solid lines denote the boundaries for a He-rich model ($Y = 0.38$). The correspondence between these models and the observations is striking. Even though the allowed region is less than 7 cycles day^{-1} in width, three of the four inferred modes fall within the band of unstable frequencies of the standard helium model. Each of these modes has nearly the same oscillation frequency in the corotating frame of the star, about 17 cycles day^{-1}. The remaining mode is indicated to be unstable in the He-rich model.

Unfortunately, the uncertainties in both the observed properties of the star and the theoretical calculations are too large to draw any firm conclusions about the importance of helium abundance to pulsation. The error in effective temperature alone, for example, is equivalent to a 6 cycles day^{-1} uncertainty in the predicted unstable frequencies. Uncertainties in $v \sin i$ affect both the values of the derived frequencies and modes of oscillation. Furthermore, the bands of unstable frequencies computed by Dziembowski (1990) were considered valid only for low-degree modes. Progress in the asteroseismology of δ Scuti stars requires improvements in both observations and theory. Accurate mode determinations and frequencies must be obtained. The method demonstrated here is capable of identifying, in a straightforward way, modes and frequencies of both low and high degree. Theoretical models of stellar pulsation which include high-degree modes are now needed for comparison with these new observations.

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