THE ASTROPHYSICAL JOURNAL, 399: L91–L94, 1992 November 1

THE CASE OF G226-29: EVIDENCE FOR A PULSATING DA WHITE DWARF WITH A THICK HYDROGEN LAYER?

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Received 1992 June 1; accepted 1992 August 13

ABSTRACT

We reexamine the question of the short periods (~109.3 s) observed in the ZZ Ceti star G226-29 by combining new period data obtained from our recent exhaustive adiabatic pulsation survey of DA white dwarfs with new spectroscopic determinations of the atmospheric parameters of that star. If, as suggested by the triplet structure of the 109.3 s peak in the Fourier spectrum of the light curve, the observed pulsations correspond to an $l = 1$ mode split by slow rotation, then we find that G226-29 must have a relatively thick hydrogen layer. Our best estimates suggest that the hydrogen layer mass in G226-29 is $\log q(\mathrm{H}) \simeq -4.4 \pm 0.2$. The constraints on the hydrogen layer mass become less severe if the observed 109.3 s complex turns out to be an $l = 2$ or 3 mode (which could occur if some frequency components were excited with amplitudes below detectability levels). For an $l = 2$ mode, we find that $\log q(\mathrm{H}) \simeq -6.6 \pm 0.2$, while no useful constraint on the hydrogen layer mass can be obtained from the period data for an $l = 3$ mode. We suggest an observational test which should settle the issue of the l value of the 109.3 s mode in G226-29.

Subject headings: stars: interiors — stars: oscillations — white dwarfs

The pulsating DA white dwarf G226-29 (LP 101−148, WD 1647+591) shows many extreme properties among the known sample of ZZ Ceti variables (nonradial pulsators of the g-type). At $V = 12.24$ (compared to an average magnitude $\langle V \rangle = 14.70$ for the ZZ Ceti population), it is, by far, the brightest object of the group. Yet it also shows the smallest amplitude, and its variability was not recognized until it was observed at the MMT (McGraw & Fontaine 1980). In addition, the Fourier spectrum of its light curve for a typical nightly run is the simplest known for a ZZ Ceti star, showing a single frequency peak centered on 9.15 mHz (109.3 s). This also makes G226-29 the pulsating white dwarf with the shortest dominant period. The amplitude of the 109.3 s peak varies from night to night, however, showing that it contains more than one frequency component and that it is unresolved on the scale of several hours. The amplitude varies typically from 2 to 6 mmag.

The careful study of Kepler, Robinson, & Nather (1983), based on some 65 hr of high-speed photometry, has established that the amplitude modulation of the 109.3 s peak in G226-29 is caused by the presence of three closely spaced components whose amplitudes and periods are highly stable in time. The components are evenly spaced in frequency with a spacing $\Delta f = 0.016144$ mHz. The two side components have nearly equal amplitudes of 3.1 mmag, and the central component at 9.150865 mHz has a lower amplitude of 1.2 mmag. Theoretically the structure of this triplet is most naturally explained in terms of an $l = 1$ g-mode of given radial order k evenly split into its 3 $(2l + 1)$ components in the frequency domain through removal of the spherical symmetry of the star caused by slow rotation. While this identification remains most suggestive, it is well to keep in mind, however, that the available observational data do not allow us to rule out completely other values of l at this stage. We note, as have others, that a period as short as 109.3 s for an $l = 1$ g-mode in a white dwarf cannot be easily accommodated in terms of standard stellar parameters. For instance, Kepler et al. (1983) estimated that, if the $l = 1$ identification is correct, G226-29 must have a mass greater than 1.0 $M_\odot$ (or, equivalently, $\log g \gtrsim 8.65$ according to Lamb & Van Horn 1975), which is much larger than the average mass for DA white dwarfs (~0.56 $M_\odot$ according to the recent determination of Bergeron, Saffer, & Liebert 1992a).

The possibility of a significantly larger-than-average mass for G226-29 was examined quantitatively by Daou et al. (1990) who combined spectroscopic observations with model atmosphere calculations to estimate anew the atmospheric parameters of the star. They were unable to confirm the large gravity required by Kepler et al. (1983) to account for the 109.3 s period of the (assumed) $l = 1$ mode. Although there were some limitations in the Daou et al. analysis of G226-29 (in particular, only the lines from $H\gamma$ to $H\epsilon$ were available for their fits), their derived value $\log g \approx 8.20$ was significantly below the required value $\log g \gtrsim 8.65$. As an alternative to an unusually large mass for G226-29, Daou et al. (1990) pointed out, on the basis of the then-ongoing pulsation survey of Brassard et al. (1992b), that the period of a given g-mode in a DA white dwarf not only shortens with increasing mass but also with increasing hydrogen layer mass. Hence, the possibility that G226-29 could be explained in terms of a model with a less extreme stellar mass than anticipated by Kepler et al. (1983) but having a relatively thick hydrogen layer was invoked.

With the recent completion of the adiabatic pulsation survey of Brassard et al. (1992b), we are now in a position to reexamine the question of the 109.3 s period in G226-29. That survey is based on the new finite-element adiabatic pulsation code of Brassard et al. (1992c) and, as such, provides the most accurate periods for ZZ Ceti star models to date. At the same time, the survey is the most extensive carried out in the field so far, having explored a very large volume of parameter space provided by the equilibrium models of Tassoul, Fontaine, & Winget (1990). We also take advantage of new spectroscopic observations of G226-29, which are of much higher quality than those available to Daou et al. (1990). Likewise, we have improved considerably our capabilities at modeling in detail the spectra of ZZ Ceti stars along the lines of Bergeron, Wesemael, & Fontaine (1991, 1992b), and Bergeron et al. (1992a). In particular, we have thoroughly investigated the effects of varying the assumed convective efficiency in the atmospheres of DA white dwarfs, a topic largely ignored in the past. Furthermore, we have obtained new high signal-to-noise spectroscopy for all ZZ Ceti stars observable from the northern hemisphere. Our preliminary analysis indicates that the mass distribution of ZZ Ceti stars better matches that of hotter DA stars (Bergeron et al. 1992a) when the ML2 version of the mixing-length theory is used in the model calculations. We will report elsewhere the results of our detailed analysis.

Our best estimates of the atmospheric parameters of G226-29 are currently $T_{\rm eff} = 13{,}630 \pm 200$ K, and $\log g = 8.18 \pm 0.05$. The mass can be estimated from the evolutionary models of Wood (1990), from which we obtain $M_* = 0.70 \pm 0.03\ M_\odot$. We also use the atmospheric parameter solution to calculate the absolute visual magnitude as outlined in Bergeron et al. (1992a); we obtain a value of $M_V = 11.75 \pm 0.07$ where the uncertainty reflects the errors of the atmospheric parameters. The observed value of the trigonometric parallax is $\pi = 0''.0827 \pm 0''.0046$ (Harrington & Dahn 1990) which translates into an absolute magnitude of $M_V = 11.83 \pm 0.12$, in excellent agreement with the value derived from our spectroscopic solution.

In Brassard et al. (1992b), it was shown that the periods of g-modes with the same value of the radial order k but with different values of l generally obey a relationship such that the "normalized" periods, $P_{kl}[l(l+1)]^{1/2}$, are the same to a good approximation, irrespective of l. A single diagram is then sufficient to summarize our pulsation calculations carried out for $l = 1$, 2, and 3. Hence, Figure 1 shows a plot of the normalized period (expressed in seconds) of the $k = 1$ mode as a function of the hydrogen layer mass [$q(\mathrm{H}) \equiv \Delta M(\mathrm{H})/M_*$] for several models. Specifically, we have used the periods for the $l = 1$, $k = 1$ mode multiplied by $\sqrt{2}$. As is well known, the $k = 1$ mode has the *shortest* period for a sequence of g-modes belonging to the same value of l. A consideration of higher overtones ($k > 1$) would thus only exacerbate the problem of trying to accommodate the short observed period of 109.3 s in G226-29 with acceptable models. Hence, in the comparison which follows, we implicitly assume that the mode of interest has $k = 1$. Each set of two continuous curves in Figure 1 refers to equilibrium models with $M_*/M_\odot = 0.4$, 0.6, and 0.8. For each set, the lower (upper) curve corresponds to models with an effective temperature $T_{\rm eff} = 14{,}000$ K (12,000 K). These two values were considered to illustrate graphically the effects of the effective temperature; the former and latter are, respectively, representative of our new spectroscopic estimates and of the older results of Daou et al. (1990). Interpolation in the tables of Brassard et al. (1992b) was carried out to derive the values of the period at these specific temperatures. Note that, for the present paper, we have also carried out additional pulsation calculations to obtain the periods for the 80204L1 sequence (in the notation of Tassoul et al. 1990) which was not considered in Brassard et al. (1992b). As discussed in that paper, cooling leads to a monotonic *increase* of the period of a given mode in a white dwarf. The figure shows explicitly that the magnitude of the effect in the range of $T_{\rm eff}$ considered is not very large, however, especially for the models with the thicker hydrogen layers. This has the happy consequence here that our results will largely be independent of the precise value of the effective temperature of G226-29.

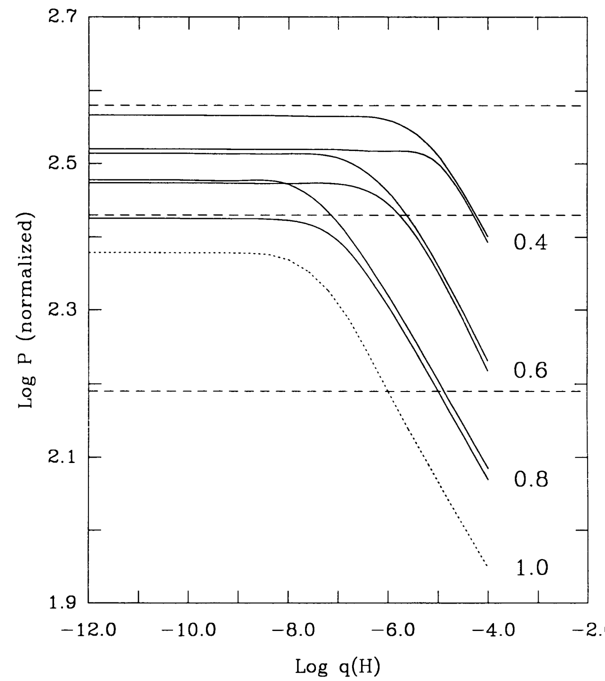

FIG. 1.—Normalized period, $P_{1l}[l(l+1)]^{1/2}$, of the $k = 1$ g-mode as a function of the fractional hydrogen layer mass [$q(\mathrm{H}) \equiv \Delta M(\mathrm{H})/M_*$] for DA models with different massses and effective temperatures. Each set of two continuous curves refers to a different mass ($M_*/M_\odot = 0.4$, 0.6, and 0.8), and the lower (upper) curve corresponds to an effective temperature $T_{\rm eff} = 14{,}000$ K (12,000 K). The dotted curve gives the periods for 14,000 K models with $M_* = 1.0\ M_\odot$, obtained by extrapolation. These curves are valid for any value of l. In contrast, the dashed horizontal lines only apply to specific values of l and give the normalized periods for the observed 109.3 s pulsation in G226-29 assuming that the mode has $l = 1$ (*lower line*), $l = 2$ (*middle line*), and $l = 3$ (*upper line*).

For each mass and effective temperature the behavior of the period of the $k = 1$ mode is qualitatively the same in terms of the hydrogen layer mass: the period is independent of $q(\mathrm{H})$ for thin hydrogen layers, and then decreases with increasing $q(\mathrm{H})$ for sufficiently thick hydrogen layers. Although these trends have been explained in detail in Brassard et al. (1992b), we recall briefly that the period of a g-mode is roughly inversely proportional to $\int_0^R |N|/r\,dr$, where N is the Brunt-Väisälä frequency (Tassoul 1980). It has been pointed out in Tassoul et al. (1990) that the integral of the Brunt-Väisälä frequency in a white dwarf is particularly sensitive to a change of material with a different electronic mean molecular weight μ_e in presence of degeneracy. In particular, if the hydrogen layer is thick enough to reach below the degeneracy boundary in a white dwarf, the overall integral $\int_0^R |N|/r\,dr$ is increased (see Fig. 41 of Tassoul et al. 1990), and the period of a given mode is decreased. The effect is larger for larger hydrogen layer masses where the changeover in composition occurs in deeper, more degenerate regions. In contrast, if the hydrogen layer is so thin that its base does not reach into the degenerate core of a model, the quantity $\int_0^R |N|/r\,dr$ is no longer sensitive to a variation of $q(\mathrm{H})$. Hence, it is no surprise that the change of regime exhibited by each curve in Figure 1 roughly corresponds to the location of the degeneracy boundary in the models. The well-known increased global state of degeneracy in a more massive white dwarf then accounts for the higher location [smaller values of $q(\mathrm{H})$] of the turnover in the curves.

To investigate the possibility envisioned by Kepler et al. (1983) of a mass larger than 1.0 $M_\odot$ for G226-29, we have also added the dotted curve in Figure 1, which corresponds to the expected values of the normalized period of the $k = 1$ mode for 14,000 K models with such a large mass. These values are based on an *extrapolation* from the results of Brassard et al. (1992b) which are restricted to three masses: $M_*/M_\odot = 0.4$, 0.6, and 0.8. To our knowledge, no recent pulsation calculations have been carried out for white dwarf models with masses larger than 0.8 $M_\odot$. Nevertheless, we are quite confident that our extrapolation technique gives reasonable results. As a measure of this, we point out that the rough semianalytic approach developed by Brassard et al. (1992a; eq. [28]) suggests that the period of a (trapped) mode scales as $P \propto M_*^{-1.079}$. Ignoring the fact that the $k = 1$ mode considered here is generally not trapped, this scaling relationship suggests that its normalized period in the 1.0 $M_\odot$ model is roughly $(1.0/0.8)^{1.079} \approx 1.27$ times smaller than in the $0.8/M_\odot$ model. This is between the values for the two regimes (ignored in the semianalytic approach) illustrated in Figure 1 which show that the period ratio is ~ 1.12 for thin hydrogen layers and ~ 1.35 for thick layers. Thus, there is at least consistency between the two approaches.

A comparison of the period data in Figure 1 with the normalized period of G226-29 for an $l = 1$ mode ($109.3 \times \sqrt{2}$ s; represented by the lower dashed horizontal curve in the figure) clearly reveals that a consistent solution can only be obtained for "thick" hydrogen layers (in the sense of Winget et al. 1982; see below). Adopting our best estimates for the mass of G226-29 ($M_*/M_\odot = 0.70 \pm 0.03$), we find that the hydrogen layer mass is comprised in the range $\log q(\text{H}) = -4.4 \pm 0.2$. Note that these inferred values underestimate somewhat the true value of q(H) in G226-29 because they are based specifically on the 14,000 K models (giving rise to the shorter periods) and that the true effective temperature is somewhat smaller according to our spectroscopic analysis. As indicated previously, however, the effect is quite small. Likewise, changing other parameters in the models such as decreasing the helium layer mass or increasing the assumed convective efficiency [Fig. 1 refers to models with the maximum possible helium layer mass, $\log q(\text{He}) = -2$, and with the standard ML1 treatment of convective transport] can only lead to an increase of the period of a given mode as explained in Brassard et al. (1992b), and, therefore, to an increase in the estimated q(H) for G226-29. Note also that, even if we consider an unlikely mass as large as 1.0 $M_\odot$ for G226-29 (as suggested by Kepler et al. 1983), we still are forced into concluding that the star must have a thick hydrogen layer, $\log q(\text{H}) \gtrsim -6.0$.

We emphasize the fact that the periods used here are quite accurate. Even if the equilibrium models were to be improved in the future, we are quite confident that the revised periods would not be changed to the point of reversing the conclusions reached in this paper (the periods would have to be decreased by very large factors in the range ~ 1.6 to ~ 2.0). This is because period data are particularly robust for white dwarfs as they reflect primarily the mechanical structure of these stars which is specified by the well-known degenerate electron pressure. Thus, *if the* 109.3 s *pulsation in G226-29 is indeed an* $l = 1$ *mode, it appears inescapable that this star has a relatively thick hydrogen layer.*

Can g-mode pulsations be excited in a ZZ Ceti star with such a massive hydrogen envelope? There are currently two points of view on this problem. On the one hand, Winget (1981), Winget et al. (1982), and Winget & Fontaine (1982) have found a correlation between the effective temperature at the blue edge of the theoretical ZZ Ceti instability strip and the mass of the outer hydrogen layer in a DA white dwarf. By matching their theoretical results with the blue edge temperature actually observed, they found that ZZ Ceti stars should have hydrogen layer masses in the range $-12 \lesssim \log q(\text{H}) \lesssim -8$. Additional unpublished calculations by Winget & Fontaine pushed the upper limit to $\log q(\text{H}) \approx -7$. Models with thicker hydrogen layers were also found to be unstable against g-mode pulsations, but only at effective temperatures too low to be consistent with the observations; such models were therefore rejected. Coupled with the suggestion that most, if not all, DA white dwarfs evolve to become ZZ Ceti pulsators (Fontaine et al. 1982; Greenstein 1982), the implication of the Winget et al. results is that DA white dwarfs, as a class, should have retained only "thin" hydrogen layers in the range $-12 \lesssim \log q(\text{H}) \lesssim -7$ by the time they enter the ZZ Ceti instability strip.

On the other hand, Cox et al. (1987) have presented another set of nonadiabatic calculations from which they concluded that the onset of g-mode instability in DA white dwarfs is independent of the hydrogen layer mass. In that picture, there is no constraint on the thickness of the hydrogen layer, and even the DA stars with the largest possible amount of hydrogen allowed by pre–white dwarf evolution [$\log q(\text{H}) \approx -4$] should become ZZ Ceti pulsators. As emphasized elsewhere, the results of Cox et al. (1987) have remained suspect on at least two accounts. First, these calculations fail to reproduce the observed blue edge: according to Cox et al., no conceivable change in the constitutive physics of their models is able to push their theoretical blue edge beyond 11,000–11,500 K, which is about 2000 K cooler than the actual observed blue edge (see, e.g., Wesemael et al. 1991). Second, and more fundamentally, Brassard et al. (1989, 1991) have demonstrated that the period structure of all the models of Cox et al. (1987) is unreliable and, in large part, unphysical. Because their periods are wrong, their study fails at the most fundamental level of astroseismology, and their conclusions based on nonadiabatic considerations remain questionable.

Our own recent experiments with various pulsation codes have convinced us that past nonadiabatic investigations of the pulsation properties of white dwarf stars may well be unreliable in their details, although they remain certainly useful as broad indicators of instability. These past studies indeed appear to be quite sensitive to numerical noise. Because of these difficulties, we are currrently unable to answer the question posed above, and more generally that of the relationship between the onset of pulsational driving and the thickness of the hydrogen layer. This problem clearly deserves to be reexamined once more potent numerical tools for solving the nonadiabatic equations become available. In this connection, the case of G226-29 discussed here may pose an interesting challenge.

In conclusion we point out that the constraints on the hydrogen layer mass in G226-29 would become less severe should the 109.3 pulsation turn out to be a mode with $l = 2$ or 3 instead of $l = 1$ as assumed in this paper. This is because the period of a g-mode decreases with increasing spherical harmonics index l for a fixed value of the radial order k. Since Figure 1 is expressed in terms of normalized periods, it can be used directly in these cases also. In this context, we point out that we have explicitly verified in all our models that the normalization relation between the periods of modes with $k = 1$ and $l = 1$, 2, and 3 indeed holds to sufficient accuracy. For instance, the normalized periods for the $k = 1$ modes are the same to better than 1% for all of our 0.6 and 0.8 $M_\odot$ models, and the agreement is only slightly worse in our 0.4 $M_\odot$ models.

The middle (upper) horizontal dashed line in Figure 1 corresponds to the normalized period of the 109.3 s pulsation in G226-29 assuming that it is a mode with $l = 2$ (3). In a way similar to the case already discussed, we find by using our spectroscopic estimate of the mass of G226-29 and by interpolating in Figure 1 that the hydrogen layer mass in that star is in the range $\log q(\mathrm{H}) = -6.6 \pm 0.2$ if the observed mode has $l = 2$. By contrast, as indicated by the upper dashed line, any value (small or large) of the hydrogen layer mass is compatible with the observed 109.3 s period if the mode has $l = 3$.

As alluded to above, the structure of the observed frequency triplet in the Fourier spectrum of the light curve of G226-29 remains quite suggestive of an $l = 1$ mode, but the available observations do not allow us to rule out completely other values of l. It is possible, for instance, that some m components are only excited with amplitudes below current detectability limits. Fortunately, a new type of observations may be helpful to settle this issue, as Brassard, Wesemael, & Fontaine (1987, 1992d) have demonstrated that g-modes characterized by different values of l leave quite distinct signatures in a two-color diagram during a pulsation cycle. Hence, multicolor photometry of G226-29 should allow us to determine without ambiguity the l-value of the 109.3 s mode, and we are currently planning such observations.

This research has been supported in part by the NSERC Canada and by the Fund FCAR (Québec). G. F. also thanks the Council of Arts of Canada for a Killam Research Fellowship.

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