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ON THE PULSATIONAL STABILITY OF THE PRE–WHITE DWARF STAR PG 1707+427

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ABSTRACT

The pulsating pre–white dwarf star PG 1707+427 has been observed with time-series photometry for more than 291 hr spanning four observing seasons (1987–1990). The principal pulsational band of periods near 448 s, found in earlier data, was observed to vary regularly in amplitude, suggesting that it was unresolved. By dividing the data into separate blocks of time, we were able to resolve this peak into two components (A and B) with periods near 447.16 and 448.95 s. Both modes appeared to be relatively constant in amplitude to the accuracy of our measurements over the observed span of time. Although we found it possible to fit our data to an emphemeris for both periods A and B, we suspect that these are not truly stable modes. The 1.3 day beat period between period A and period B is fairly consistent, suggesting that it may be related to the rotation of the star.

Subject headings: star: evolution — stars: individual (PG 1707+427) — stars: oscillations — white dwarfs

1. INTRODUCTION

Recently the study of stellar evolution has been invigorated by the idea that the pulsations of white dwarfs and their immediate progenitors can be observed to test theories of internal stellar structure, measure rates of stellar evolution, and even determine the age of the universe. In response to this stimulus, a small industry has developed. Winget et al. (1987) and Iben & Laughlin (1989) have used observations of white dwarfs and theoretical models to produce an initial estimate of the age of our part of the Galactic disk. Kawaler (1986, 1987a, b, 1988) has developed methods for using the period spacing in hot pulsating stars to determine their masses. Recently, phase shifts in stellar pulsations over time have been interpreted as representing evolutionary changes for PG 1159−035 (Winget et al. 1991) and G117-B15A (Kepler et al. 1991). However, observations of G29-38 (Winget et al. 1990) have shown some of the difficulties involved in giving a physical interpretation to changes in pulsational arrival times.

PG 1707+427 is a member of the DOV (also GW Virginis) class of pulsating white dwarf stars. The prototype of this group of stars is PG 1159−035 (GW Vir), which McGraw et al. (1979) found to be a low-amplitude multiperiodic pulsator. Wesemael, Green, & Liebert (1985) have identified a class of such stars from among the Palomar-Green survey for objects with ultraviolet excess (Green, Schmidt, & Liebert et al. 1986). Their spectral signature is the presence of He II and C IV absorption lines (with emission cores present in some instances) and the absence of Balmer lines. It is also important to note that atmospheric analyses (Wesemael et al. 1985; Werner, Herber, & Hunger 1991) showed that the DOV stars have hydrogen-poor atmospheric abundances. There are at present seven members of the spectroscopic class. In addition to PG 1159−035 itself, PG 1707+427, PG 2131+066, and PG 0122+200 also display complex nonradial *g*-mode pulsations with pulsation periods of the order of 6–10 minutes (Bond et al. 1984; Bond & Grauer 1987). Since these stars are near the point of entering the white dwarf cooling sequence at effective temperatures above 10^5 K, theory suggests that they should be evolving rapidly enough to produce a measureable evolutionary change in their pulsational periods over several observing seasons. For PG 1159−035, Winget et al. (1985, 1991) have reported $dP/dt = (-2.49 \pm 0.06) \times 10^{-11}$ s s^{-1} for the 516 s period. (Note that they have defined dP/dt more rigorously in the latter of these two papers, which is the origin of the factor of 2 difference between these two published results.)

This paper is the result of our effort to investigate the pulsational stability of PG 1707+427. Our hope was to measure its rate of evolution.

2. TIME-SERIES PHOTOMETRIC OBSERVATIONS

During the period of 1987 May through 1990 June we have obtained light curves with monitoring totaling 291.7 hr. Photometric observations were carried out with 1.5 m telescopes at Steward Observatory (1.55 m [61 inch] on Mount Bigelow and 1.52 m [60 inch] on Mount Lemmon) and the 1.3 m (50 inch) telescope at Kitt Peak National Observatory. One run was obtained by A. Hill using the 82 inch (2.1 m) telescope at McDonald Observatory. All observations were made in white light using blue-sensitive (3200–6500 Å) bi-alkali photocathode photomultiplier tubes. The effective wavelength of the combined effects of the photomultiplier tube and atmospheric transmission is slightly bluer than Johnson *B*, with a peak response occurring between 3700 and 4000 Å. In 1987, 1988, and 1989 data were taken with the UALR two-star photometer

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using an EMI 9840B photomultiplier tube for the program star and an EMI 9826 photomultiplier tube for the comparison star. In 1990 the observations were made with "Lepus," which employs three Hamamatsu R647-04 photomultiplier tubes in three identical photometers to record data from the program object, a nearby comparison star, and the sky simultaneously. Lepus also features continuous autoguiding on a third star, which eliminates guiding errors. A journal of observations is provided in Table 1.

At the beginning, during the course, and at the end of each time-series run the time base was calibrated by visually comparing the time displayed by the data acquisition computer and the broadcast WWV time signals. The error here was never more than 0.1 s. A heliocentric correction was calculated, and all times were thus converted to heliocentric Julian Dates (HJD). The correction for a leap second and the conversion to the barycenter of the solar system were not made, since these factors are an insignificant source of error over the time span considered in this paper.

Ten second integrations were used for each time-series run. Division of the sky-subtracted PG 1707+427 data stream by a second- or third-order polynomial obtained from a least-squares fit to the data removed the effects of atmospheric extinction. This light curve, consisting of counts per integration period as a function of time, was then divided by the mean number of counts and unity subtracted from the result. Thus the light curve prepared for analysis has a zero mean intensity. The 10 s integrations were then summed into 20 s bins.

Figure 1 presents two representative light curves, the first two obtained in this study. It is obvious from the figure that the variability is dominated by what appears to be a single, simple mode of modest amplitude, as first found by Bond et al. (1984). It was not until a number of data sets had been acquired that the effects of beating began to be observed. An earlier discussion of the mode structure of this star (and a preliminary report of results from this data set) was given in Grauer, Liebert, & Green (1989).

3. THE PRIMARY PULSATIONAL PERIODS

A power spectrum was calculated for each light curve after it had been prepared for analysis. The method of Deeming (1975) was employed. Synthetic data sets containing modes of known periods, amplitudes, and noise levels (from a random-number generator) were created to have the same temporal spacing as

TABLE 1

Observing Log for PG 1707+427

Run	UT Date	Telescope	Starting UT	Run Duration (hr)
P1........	1987 May 23	UAO 1.55 m	05:45:54	3.1
P2........	1987 May 24	UAO 1.55 m	04:18:34	6.4
P3........	1987 May 31	UAO 1.52 m	06:19:50	4.3
P4........	1987 Jun 1	UAO 1.52 m	04:17:08	5.9
P5........	1987 Jun 21	UAO 1.52 m	04:56:50	5.0
P6........	1987 Jun 22	UAO 1.52 m	04:53:08	3.9
P7........	1987 Jul 1	McD 2.1 m	03:40:00	6.7
P8........	1987 Oct 15	KPNO 1.3 m	02:12:44	2.7
P9........	1987 Oct 16	KPNO 1.3 m	02:24:48	2.6
P10.......	1987 Oct 17	KPNO 1.3 m	02:13:32	2.7
P11.......	1987 Oct 19	KPNO 1.3 m	02:24:57	2.4
P12.......	1987 Oct 20	KPNO 1.3 m	02:12:48	1.5
P13.......	1988 May 8	UAO 1.55 m	04:57:00	6.1
P14.......	1988 May 9	UAO 1.55 m	04:30:00	6.4
P15.......	1988 May 10	UAO 1.55 m	04:20:00	2.6
P16.......	1988 May 10	UAO 1.55 m	07:37:00	1.7
P17.......	1988 May 11	UAO 1.55 m	03:55:00	7.0
P18.......	1988 May 12	UAO 1.55 m	04:02:00	6.9
P19.......	1988 May 13	UAO 1.52 m	09:36:00	1.3
P20.......	1988 May 14	UAO 1.52 m	06:55:00	4.0
P21.......	1988 May 15	UAO 1.52 m	06:24:59	0.9
P22.......	1988 May 15	UAO 1.52 m	10:02:00	1.0
P23.......	1988 May 16	UAO 1.52 m	07:01:00	1.8
P24.......	1988 Jun 7	KPNO 1.3 m	04:46:00	2.8
P25.......	1988 Jun 8	KPNO 1.3 m	04:32:00	2.6
P26.......	1988 Jun 9	KPNO 1.3 m	05:23:00	2.6
P27.......	1988 Jun 11	KPNO 1.3 m	06:27:00	3.6
P28.......	1988 Jun 12	KPNO 1.3 m	04:13:00	3.0
P29.......	1988 Jun 13	KPNO 1.3 m	04:05:00	3.6
P30.......	1988 Jun 14	KPNO 1.3 m	04:07:00	4.6
P31.......	1988 Jun 16	KPNO 1.3 m	05:15:00	4.3
P32.......	1988 Jun 17	KPNO 1.3 m	06:03:00	3.2
P33.......	1988 Jun 20	KPNO 1.3 m	06:22:00	3.9
P35.......	1989 Apr 29	UAO 1.52 m	09:22:00	1.9
P36.......	1989 May 2	UAO 1.52 m	06:33:00	4.6
P37.......	1989 May 8	UAO 1.55 m	05:51:30	5.1
P38.......	1989 May 9	UAO 1.55 m	05:19:00	4.5
P39.......	1989 May 10	UAO 1.55 m	05:29:00	5.6
P40.......	1989 May 12	UAO 1.55 m	07:51:00	3.1
P41.......	1989 May 13	UAO 1.55 m	08:08:30	2.9
P42.......	1989 May 14	UAO 1.55 m	07:54:00	3.0
P43.......	1989 May 15	UAO 1.55 m	08:53:00	2.0
P44.......	1989 May 26	KPNO 1.3 m	04:11:00	3.0
P45.......	1989 May 27	KPNO 1.3 m	03:54:00	3.8
P46.......	1989 May 28	KPNO 1.3 m	06:03:00	3.0
P47.......	1989 May 29	KPNO 1.3 m	03:55:00	5.3
P48.......	1989 May 30	KPNO 1.3 m	03:58:00	6.8
P49.......	1989 May 31	KPNO 1.3 m	04:02:00	6.7
P50.......	1989 Jun 1	KPNO 1.3 m	04:07:00	6.7
P51.......	1989 Jun 2	KPNO 1.3 m	04:12:00	6.5
P52.......	1989 Jun 3	KPNO 1.3 m	04:02:00	6.7
P53.......	1989 Jun 4	KPNO 1.3 m	05:36:00	5.0
P54.......	1989 Jun 8	KPNO 1.3 m	06:33:00	4.1
P55.......	1989 Jun 9	KPNO 1.3 m	07:10:00	3.5
P56.......	1989 Sep 25	UAO 1.55 m	02:46:00	2.1
P57.......	1989 Sep 28	UAO 1.55 m	02:44:00	2.0
P58.......	1989 Sep 29	UAO 1.55 m	02:40:00	3.3
P59.......	1989 Sep 30	UAO 1.55 m	02:32:00	3.3
P60.......	1989 Oct 1	UAO 1.55 m	02:21:00	3.2
P61.......	1989 Oct 2	UAO 1.55 m	02:29:00	3.2
P62.......	1990 May 21	KPNO 1.3 m	07:41:00	3.2
P63.......	1990 May 25	KPNO 1.3 m	04:59:00	6.0
P64.......	1990 May 27	KPNO 1.3 m	04:31:00	6.3
P65.......	1990 May 30	KPNO 1.3 m	04:08:00	6.6
P66.......	1990 May 31	UAO 1.55 m	06:38:00	4.2
P67.......	1990 Jun 1	UAO 1.55 m	06:08:00	4.6
P68.......	1990 Jun 2	UAO 1.55 m	04:12:00	6.7
P69.......	1990 Jun 21	UAO 1.55 m	05:18:00	5.2
P70.......	1990 Jun 22	UAO 1.55 m	04:41:00	5.6
P71.......	1990 Jun 23	UAO 1.55 m	05:10:00	5.5
P72.......	1990 Jun 25	UAO 1.55 m	04:45:00	5.8

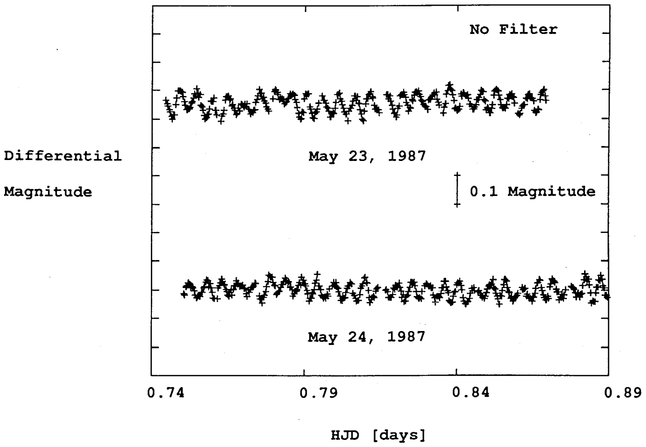

FIG. 1.—Portions of the first two light curves of PG 1707+427 obtained for this study. The first run is shown in its entirety, while only a little more than 3 hr of the 6.4 hr run obtained on 1987 May 24 is plotted. The difference in magnitude between the variable and comparison stars are plotted against heliocentric Julian Date. The amplitude of pulsations ranges from slightly greater than that observed during these two runs to nights when the star has a relatively flat light curve.

the real data sets. The absolute amplitudes in percent generated by the Fourier transforms were determined by using a normalizing constant. This procedure was checked by taking power spectra of the artificial data sets. The accuracy of an amplitude obtained from a power spectrum depends on the signal-to-noise ratio in the original light curve and is hard to quantify. Typical uncertainties in our amplitude determinations are in the 5%–10% range.

The Fourier transform of every data set is dominated by a peak near 447.3 s (2.236 mHz) whose semiamplitude (square root of the power) ranges from 0.5% to 3.3%. The study of this main peak in PG 1707+427's Fourier transform is the subject of this paper.

We have also observed a band of periods near 335 s (2.985 mHz) whose semiamplitudes are less than 1%. We have not found any modes in this band which are consistent in either period or amplitude from night to night.

Figure 2 presents the Fourier transform of the entire 6.4 hr data set obtained on 1987 May 24 (a portion of whose light curve is presented in Fig. 1). This plot is representative of our data. A large peak at 447.50 s and a smaller one at 334.46 s are the only ones which appear above the noise. As the result of using high signal-to-noise data from the Canada-France-Hawaii Telescope, several other peaks in the power spectrum of this star have been reported (Fontaine et al. 1991). Their amplitudes are all smaller than that which we have found for the 335 s band of periods in our data. It is interesting to note that the main peak and the 335 s band are nearly in a 4/3 ratio in periods.

As will be described in this and the following sections, we have found that the main peak in PG 1707+427's power spectra can be divided into two and perhaps three closely spaced components, which we have called periods A, B, and C. Periods A and B can be resolved in most blocks of data longer than two nights in duration. In the analysis to follow, there are a number of reasons to be confident that periods A and B have been correctly distinguished from their respective one-day aliases. First, these two periods are indicated for most of the blocks of nights when the transform of the particular block of nights is compared with its window function. Second, when the transform of the entire data set (Fig. 7) is compared with its window function (Fig. 8) and with the transform of synthetic data (Fig. 9), periods A and B are clearly exhibited. Third, period A at 447.16 s and period B at 448.95 s yield a beat period of 1.3 days, which is consistent with the behavior of the amplitude of the main peak. Fourth, the Whole Earth Telescope run in 1991 May (after this paper was originally submitted) provided much longer strings of continuous data which permit us to resolve the main peak into components near 447.2 and 449.0 s (Grauer et al. 1992). Period C is suggested by the behavior of the amplitude of the main peak in the power spectra and the Fourier transform of the entire data set (46,505 20 s points covering the period from 1987 May through 1990 June).

Table 2 presents the results of Fourier transforms calculated for light curves obtained in 1988, 1989, and 1990. The heliocentric Julian Date at the midpoint of the light curve, the amplitude (square root of the power), and the period of the main peak are given for each run. Fourier transforms were also calculated for subsets of long runs which have an exceptionally good signal-to-noise ratio. The letter "a" is for the first portion, while the letter "b" designates the second part of a given light curve. The absence of an appended letter indicates that the entire data set was used. The amplitude of the main peak was thus observed to change significantly in a 3 hr time span. The light curves were qualitatively graded in terms of signal-to-noise ratio with weights of 3, 2, and 1 for those rated excellent, good, and fair, respectively. A simple average for the 88 entries in Table 2 yields a mean period of 447.323 s (standard deviation 1.431 s) for the main peak in Fourier transforms of PG 1707+427's light curves.

Inspection of the amplitudes and periods listed in Table 2 suggested to us that the dominant pulsational mode in PG 1707+427 is unresolved by one night's data. Figure 3 is a plot of the mean period of a given run versus the amplitude (two columns of data from Table 2). There is a suggestion of a slight S-shape in this plot. However, we do not have enough information to come to a definite conclusion about the significance of this graph.

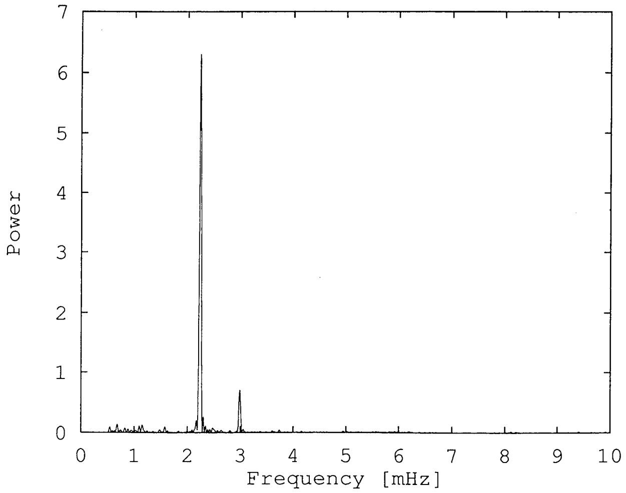

FIG. 2.—Fourier transform of the entire 6.4 hr data set obtained on 1987 May 24. The power (square of the amplitude in percent) is plotted against the frequency in millihertz. Only two peaks, at 447.50 s and 334.46 s, are well above the noise in these data.

TABLE 2

THE PRIMARY PEAK IN PG 1707+427'S SINGLE NIGHT FOURIER TRANSFORM

Run	Weight	HJD 2,440,000+ (days)	Amplitude	Period of Peak (s)	Run	Weight	HJD 2,440,000+ (days)	Amplitude	Period of Peak (s)
P13	2	7289.83467	2.141	447.2	P45	2	7673.74356	1.229	447.7
P13a	2	7289.77147	2.374	448.6	P46	2	7674.81794	1.988	445.4
P13b	2	7289.89751	1.919	447.9	P48	3	7676.80926	1.439	447.6
P14	2	7290.82357	2.992	447.9	P48a	3	7676.73765	1.105	448.0
P14a	2	7290.75632	2.942	447.6	P48b	3	7676.87997	1.781	447.7
P14b	2	7290.89000	3.048	448.3	P49	2	7677.81016	0.592	445.1
P15	2	7291.73643	1.951	448.1	P49a	2	7677.74048	0.848	449.3
P16	1	7291.88886	2.748	448.7	P49b	2	7677.88007	0.505	447.7
P17	3	7292.81214	0.920	445.1	P50	1	7678.81253	1.647	447.6
P17a	1	7292.73876	0.926	445.6	P50a	1	7678.74343	1.850	446.7
P17b	1	7292.88528	0.915	445.2	P50b	1	7678.88186	1.454	447.1
P18	2	7293.81331	2.125	447.8	P52	2	7680.80932	1.787	448.2
P18a	2	7293.74178	2.526	448.1	P52a	2	7680.74011	1.632	449.5
P18b	2	7293.88460	1.731	448.0	P52b	2	7680.87888	1.958	447.1
P19	1	7294.93022	2.894	449.4	P53	1	7681.84063	0.992	443.4
P20	2	7295.87409	3.188	448.5	P54	1	7685.86082	0.646	446.0
P23	1	7297.83337	1.782	450.4	P55	1	7686.87400	1.737	445.4
P24	1	7319.76026	2.238	448.3	P56	1	7794.65748	2.215	450.9
P25	1	7320.74555	2.843	447.7	P57	1	7797.65528	2.294	447.6
P26	1	7321.78094	1.707	446.7	P58	1	7798.67892	2.279	447.0
P27	1	7323.84554	1.562	447.7	P59	1	7799.67489	0.943	446.6
P28	1	7324.74087	2.633	447.7	P60	1	7800.66715	1.513	447.3
P29	1	7325.74758	2.601	447.8	P61	1	7801.67146	2.865	446.7
P30	1	7326.76921	1.321	446.9	P63	3	8036.83516	2.507	446.8
P31	1	7328.80935	2.455	448.4	P63a	3	8036.77225	2.020	447.8
P32	1	7329.82113	3.275	449.4	P63b	3	8036.89679	3.025	446.0
P33	1	7332.84898	2.014	448.6	P64	2	8038.82196	0.827	446.9
P35	1	7645.93191	1.799	449.6	P64a	2	8038.75200	0.931	446.7
P36	1	7648.87074	0.858	446.0	P64b	2	8038.88961	0.773	447.8
P36b	2	7648.91866	0.933	446.5	P65	2	8041.81224	1.437	448.5
P37	3	7654.85359	2.141	447.1	P65a	2	8041.74320	1.009	445.0
P37a	3	7654.80023	2.316	446.6	P65b	2	8041.87989	1.931	448.0
P37b	3	7654.90729	1.998	447.3	P66	1	8042.86641	0.877	442.2
P38	3	7655.81691	2.310	447.7	P67	1	8043.85391	1.718	447.6
P38a	3	7655.77050	2.115	448.2	P68	1	8044.81710	2.729	447.6
P38b	3	7655.86344	2.475	446.8	P68a	1	8044.74736	2.782	444.6
P39	2	7656.84719	1.129	446.4	P68b	1	8044.88753	2.868	446.8
P39a	2	7656.78909	1.080	444.3	P69	2	8063.83150	1.869	448.3
P39b	2	7656.90541	1.209	445.2	P69a	2	8063.79487	2.319	447.1
P40	2	7658.89351	2.719	446.5	P69b	2	8063.86779	1.324	448.7
P41	2	7659.90116	2.824	449.0	P71	3	8065.83217	2.066	447.1
P42	2	7660.89336	1.233	448.8	P71a	3	8065.79438	2.219	446.7
P43	1	7661.91449	1.196	448.4	P71b	3	8065.87135	2.001	446.6
P44	2	7672.73807	0.756	449.0	P72	1	8067.82102	1.148	447.0

The intensive observations performed in 1988 May and June suggested that the 447 s "mode" has several components. At first the pattern was not clear. On some nights this peak would be large, while on others it was barely detectable. Figure 4 is a plot of all of the data obtained in one dark-of-the-Moon observing run. The main peak, near 447.3 s (2.236 mHz), is seen to be present in the same place each night with a different amplitude.

If the observed changes in the main peak of the power spectrum for this star are completely due to the beating of two closely spaced periods, then the amplitude of the peak (square root of the power) should fit a sine wave over time. The difficulty in testing this possibility is that temporal gaps in the data create aliases which are impossible to distinguish from the real beat period with absolute certainty. In order to narrow the range of solutions to this problem, the data from Table 2 (amplitude of the main peak versus time) were fitted to sine waves of various periods, amplitudes, phases, and *y* offsets. These curves were then compared with the data points. A simple plot of the night-to-night amplitudes suggests a 4 day beat period. However, no period in this range was found which will fit both the direction and the magnitude of the changes between the "a" and "b" entries of Table 2 very well. On the other hand, several periods near 1.3 days fit both night-to-night and during the night changes to a greater degree than those near 4 days. We were able to create several ephemerides that crudely fitted the amplitude of the main peak over a time span of several years.

The light curve obtained on each night, as well as light curves obtained by combining two or more adjacent nights into a single data set, were analyzed. Fourier transforms and nonlinear least-squares fitting to a sine wave were employed to

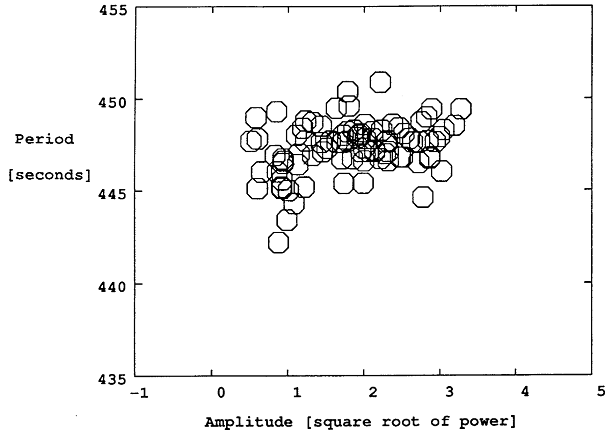

FIG. 3.—All of the data from the last two columns of Table 2 for the main peak in PG 1707+427's Fourier transform, are plotted, with the period in seconds on the vertical axis and amplitude in percent on the horizontal axis. There is no apparent relationship between these two parameters.

find and separate the real and apparently stable periods from aliases created by gaps in the data sets. By using these methods of analysis on larger and larger blocks of data, we were able to find consistent periods for each run and to bridge the gaps between runs and observing seasons. At each step we tested our methods on realistic artificial data sets which have periods of known frequency and amplitude and include numerical noise from a random number generator.

The rough fit of a "slow" sine wave to the amplitude of the main peak in the nightly Fourier transform data suggested that we should explore the possibility that this is a beat frequency between two closely spaced periods. To improve the frequency resolution over that of a single night's data, we divided the time-series runs into blocks of time, as shown in the first columns of Tables 3, 4, and 5. A Fourier transform was calculated for each block which resolved the nightly main peak into two peaks with periods A and B. Figure 5 shows a typical result. A window function, which is the Fourier transform of a single sine wave sampled at the same times as the real data, was also calculated for each block. Figure 6 is the window function

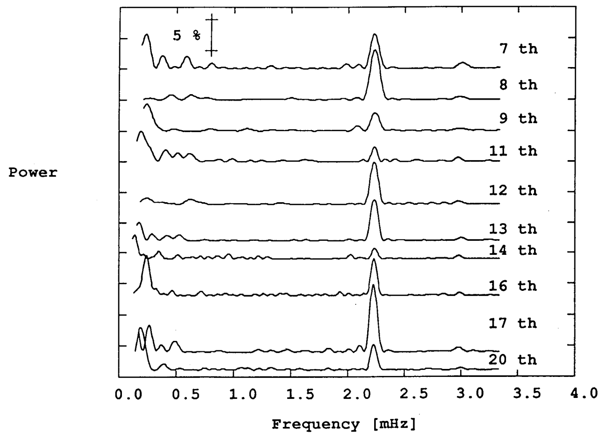

FIG. 4.—Fourier transforms of all of the light curves obtained for PG 1707+427 during an entire dark run in 1988 June. The power in percent is plotted against the frequency in millihertz. The peak near 447.3 s (2.236 mHz) is seen to be present with a different amplitude on each night. The peaks with frequencies less than 0.8 mHz are probably not real.

TABLE 3

ANALYSIS OF BLOCKS OF DATA: PERIOD A

Runs	Period[a] (s)	Amplitude[b]	HJD 2,440,000+ (days)
P1, P2, P3, P4	447.171	0.027	6938.768451
P5, P6, P7	447.121	0.028	6967.709586
P8, P10, P11, P12	447.147	0.018	7087.604910
P13, P14, P15, P16, P17	447.028	0.022	7289.714794
P17, P18, P19, P20, P20, P21, P22, P23	447.151	0.018	7292.706044
P13, P14, P16, P17, P18, P19, P20, P21, P22, P23	447.160	0.021	7289.719701
P24, P25, P26, P27, P28	447.173	0.022	7320.700048
P28, P29, P30, P31, P32	447.175	0.025	7324.819400
P24, P25, P26, P27, P28, P29, P30, P31, P32	447.150	0.023	7325.870199
P37, P38, P39, P40, P41, P42, P43	447.221	0.018	7654.754539
P44, P45, P46, P47, P48, P49	447.097	0.013	7672.690824
P44, P45, P46, P47, P48, P49, P50, P51, P52, P53	447.161	0.015	7672.690535
P50, P51, P52, P53	447.060	0.018	7678.865245
P57, P58, P59, P60, P61	447.289	0.020	7797.625176
P62, P63, P64, P65, P66, P67, P68	447.219	0.016	8032.852051
P63, P64, P65, P66, P67, P68	447.216	0.017	8036.744551
P64, P65, P66, P67, P68	447.181	0.015	8038.696345
P65, P66, P67, P68	447.246	0.018	8041.677109
P69, P70, P71	447.000	0.020	8063.728486

[a] Average period = 447.156 s; number measured = 19; $\sigma = 0.070$ s.
[b] Average amplitude = 0.020; number measured = 19; $\sigma = 0.004$.

TABLE 4

ANALYSIS OF BLOCKS OF DATA: PERIOD B

Runs	Period[a] (s)	Amplitude[b]	HJD 2,440,000+ (days)
P13, P14, P15, P16, P17, P18, P19, P20, P21, P22, P23	448.956	0.014	7289.720129
P24, P25, P26, P27, P28	448.937	0.018	7319.706935
P28, P29, P30, P31, P32	448.905	0.019	7325.677780
P24, P25, P26, P27, P28, P29, P30, P31, P32	448.954	0.017	7322.694840
P37, P38, P39, P40, P41, P42, P43	448.973	0.015	7654.754956
P44, P45, P46, P47, P48, P49, P50, P51, P52, P53	449.052	0.009	7672.702491
P57, P58, P59, P60, P61	448.913	0.015	7797.620708
P65, P66, P67, P68	448.838	0.015	8043.795523
P69, P70, P71	448.137	0.012	8063.909921

[a] Average period = 448.852 s; number measured = 9; $\sigma = 0.258$ s.
[b] Average amplitude = 0.015; number measured = 9; $\sigma = 0.003$

TABLE 5

YEAR BLOCKS OF DATA

Year	Period (s)	Amplitude	HJD 2,440,000+ (days)
	Period A		
1987.......	447.1451	0.025	6967.709033
	(0.0011)	(0.004)	(0.000014)
1988.......	447.1591	0.022	7320.699948
	(0.0037)	(0.003)	(0.000017)
1989.......	447.2153	0.015	7678.864959
	(0.0018)	(0.003)	(0.000020)
1990.......	447.2213	0.017	8041.677367
	(0.0177)	(0.005)	(0.000033)
	Period B		
1987.......	448.9213	0.021	6967.725619
	(0.0014)	(0.004)	(0.000014)
1988.......	448.9442	0.015	7322.694739
	(0.0060)	(0.004)	(0.000030)
1989.......	448.9721	0.009	7654.754890
	(0.0031)	(0.003)	(0.000041)
1990.......	448.9722	0.013	8043.795399
	(0.0156)	(0.005)	(0.000042)

for the block of data whose transform is presented in Figure 5. It shows the signature of a single period in this data set. In Figure 5, neither period A (447.241 s) and its 1 day alias (449.568 s) nor period B (448.976 s) and its 1 day alias (446.655 s) can be unambiguously distinguished from each other by their amplitudes. However, the four reasons listed previously in this section of this paper give us confidence that we have made the correct identifications.

A Fourier transform was used to determine periods A and B in each block of data. These periods, along with an estimated amplitude and time of zero crossing, were used as the starting parameters for a nonlinear least-squares fit of each block of data to two sine waves, one for period A the other for period B. The result of these calculations is a period, amplitude, and HJD for the positive-going zero crossing of the fitted sine wave for each block of data. These parameters are presented in Tables 3–5. In addition, their simple mean values are listed at the end of each of Tables 3 and 4. Period B could not be reliably extracted from the blocks of data obtained in 1987. The errors from the least-squares fit of a sine wave to the shorter blocks of data appeared to be much too optimistic and are not included in Tables 3 and 4. The standard deviations from the averages for the periods and amplitudes enumerated at the end of Tables 3 and 4 provides a clue as to the accuracy of the individual entries. Further, the heliocentric Julian Dates from these fits, which are presented in Tables 3 and 4, are probably good to ±20 s (0.000231 days).

Seventy-one times-series runs were placed in one block of data containing 46,505 20 s points. Figure 7 is a transform of this data set. Figure 8, the window function, is a 447.16 s period sine wave with a semiamplitude (square root of the power) of 2.0% which has been sampled at exactly the same times as the real data. It is dominated by the 1 day gaps in the original data set. In Figure 7 the overlapping alias patterns of periods A and

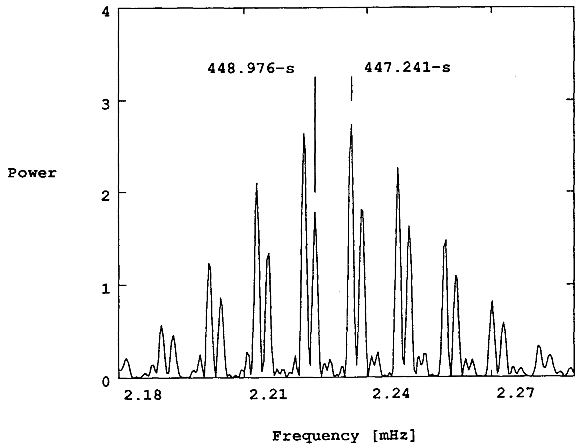

FIG. 5.—Power is plotted as a function of the frequency in millihertz for PG 1707+427 for a typical block of nights: 1989 May 8, 9, 10, 12, 13, 14, 15. The overlapping alias pattern of periods A and B are clearly present. For this block of data the values of these are 447.241 and 448.976 s, respectively.

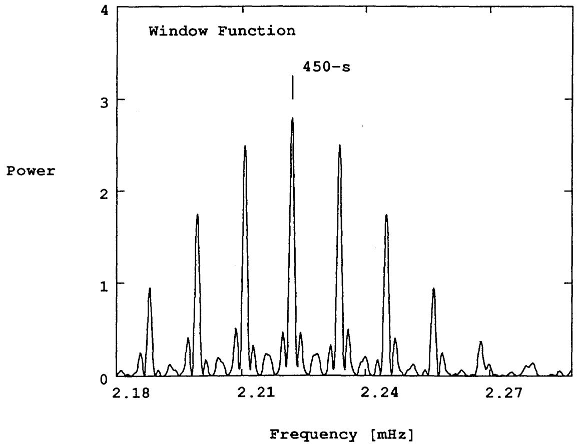

FIG. 6.—Fourier transform of a 450 s period sine wave sampled at exactly the same times as was PG 1707+427 during the block of nights whose power spectrum is plotted in Fig. 5. This is the alias pattern of a single period.

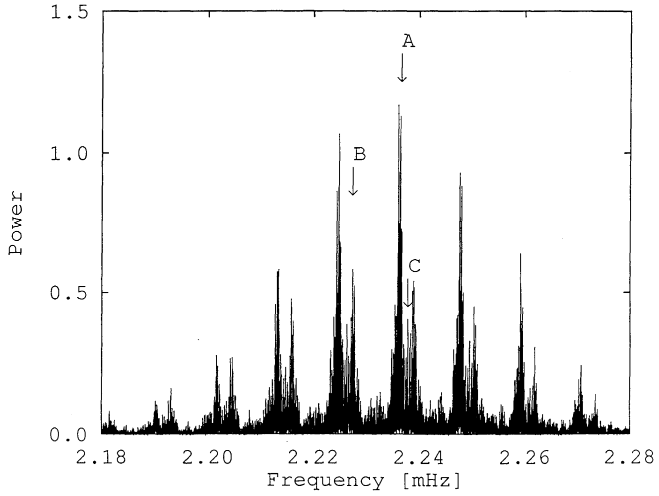

FIG. 7.—Fourier transform of the entire data set, from 1987 May 23 to 1990 June 23 (Table 1, 46,505 20 s points). The frequency in millihertz is plotted against the power for the region of the main peak in PG 1707+427's power spectra (2.18 mHz [458.7 s] to 2.28 mHz [438.6 s]). Periods A and B are marked along with the suggested location of period C.

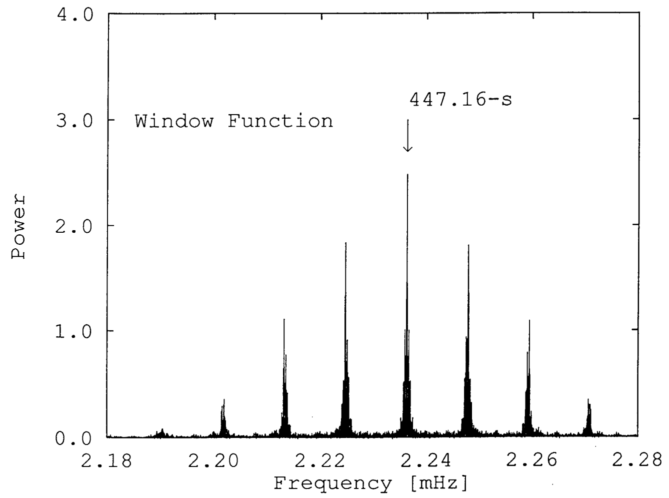

FIG. 8.—Fourier transform of a 447.16 s period sine wave with a 2.0% semiamplitude (square root of the power). It was sampled at 46,505 points at exactly the same times as was the real PG 1707+427 data set whose power spectrum is presented in Fig. 7. This plot is the signature of a single period in the entire PG 1707+427 data set. Note that the 1 day gaps in the original data dominates the alias pattern.

B are clearly present. Note that the 1 day alias structure dominates in this plot just as it did in the spectral window of Figure 8. A careful inspection of Figure 7 reveals that there is also the hint of the alias pattern of a weaker period C in this transform. The identification of the real periods among the aliases in Figure 7 is difficult, and there are a variety of reasons why an alias may have a greater amplitude than the real period. Figure 9 is the transform of the synthetic data set which has the same number of points (46,505) and data spacings as the real PG 1707+427 data set. This is a realistic simulation, in that periods and amplitudes close to those found in PG 1707+427 have been introduced. Noise from a random number generator has also been placed in this data set. Comparison of this transform with that of the real data in Figure 7 and the spectral window of Figure 8 confirms the existence of periods A and B in the entire data set and supports the suggestion that there is also a period C. The probable location for period C is marked, although it should be noted that we do not have enough information to discriminate the real period from one of its aliases. Period C either was not present in 1991 May or is an artifact of our data sampling, since it was not found with the Whole Earth Telescope (Grauer et al. 1992). It should also be stated that we cannot rule out the possibility of other lower amplitude frequency components. Thus our suggestion that the existence of period C can explain away all of the inconsistencies which we have observed in the "beating" of periods A and B may be an over simplification of the actual light curve.

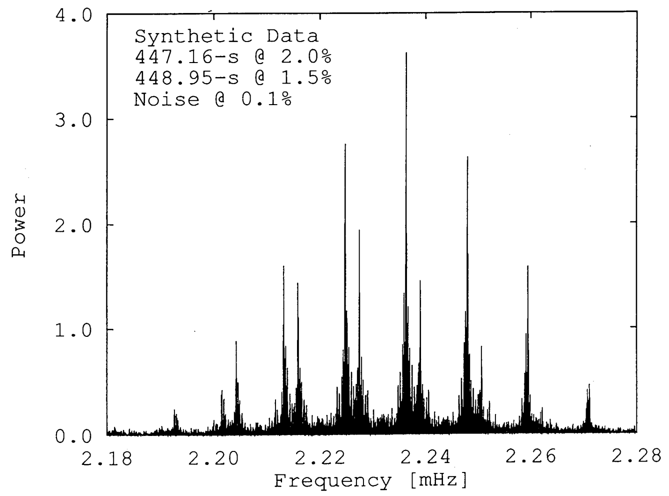

FIG. 9.—Fourier transform of a synthetic data set with the same number of data points (46,505) and spacing as that for the entire data set whose transform is given in Fig. 7. The overlapping alias patterns of two periods, 447.16 s (2.2363 mMz) and 448.95 s (2.2274 mHz) are clearly evident. Note that there is no alias pattern for period C in this plot.

4. ANALYSIS OF PERIODS A AND B

We attempted to find rates of change for periods A and B by analyzing the blocks of data listed in Tables 3–5. We also used a nonlinear least-squares fit of a sine wave with a variable period to the entire data set.

A least-squares technique was used to find the best constant period and the times of zero crossing listed in Tables 3–5. The starting point for the fit was determined from the light curve and Fourier transform of the data set in question. The errors below the parameters in Table 5 are those generated by a least-squares fit to a sine wave. The times of zero crossing were used to calculate cycle and $O-C$ values for each mode. We used the methods of Bevington (1969) and Press et al. (1986) to find the parameters required to construct an ephemeris for both period A and period B. Using the "test of additional term" (Bevington 1969), we calculated at dP_0/dt of approximately -5×10^{-11} s s^{-1}, which is not zero at a 99.9% confidence level for period A and at a 75% confidence level for period B. Ephemerides were also determined by using a nonlinear least-squares fit of a sine wave with a variable period to the entire data set (46,505 20 s points. We obtained results using these two methods which were in beautiful agreement with each other. We submitted a manuscript to the Astrophysical Journal. The referee challenged our selection of the 1 day aliases for periods A and B and suggested that we demonstrate that the results of our least-squares analysis match the Fourier transform of the entire data set to the high degree of accuracy. We found that we could satisfy these concerns and show that our ephemerides with the dP/dt terms are indeed in excellent agreement with our light curves and Fourier transforms of our data. We could easily have presented these conclusions in an authoritative manner; however, all of this scrutiny resurrected some of our latent concerns, namely, (1) Why is the amplitude of the main peak in the Fourier transform not a better fit to a sine wave? (2) Why do the amplitudes of the peaks in the Fourier transform decrease as more and more data are blocked together? (3) Why is it impossible to distinguish between various ephemerides all of which appear to fit the data more or less equally well? and (4) Is the period jitter observed in the year blocks of data (Table 5) the result of our sampling, or does it reflect some real period changes in the star?

The periods listed in Table 5 were determined by using the window function to identify the correct period unambiguously from among the aliases in the Fourier transform. Using synthetic data sets to test our methods, we have come to the conclusion that the changes in the period from year to year (Table 5) represent real changes in the star. For reference,

TABLE 6

DUAL-PERIOD WINDOW ANALYSIS

	PERIOD 1		PERIOD 2	
YEAR	Period (s)	Amplitude	Period (s)	Amplitude
1987.......	447.1594 (0.00007)	0.018 (0.0002)	448.9511 (0.0001)	0.016 (0.0002)
1988.......	447.1632 (0.0002)	0.017 (0.0001)	448.9441 (0.0003)	0.012 (0.0002)
1989.......	447.1602 (0.00004)	0.015 (0.0001)	448.9493 (0.0001)	0.010 (0.0001)
1990.......	447.1558 (0.0003)	0.017 (0.002)	448.9581 (0.0005)	0.013 (0.0002)

Table 6 presents the results obtained when we analyzed a dual-period window function (447.16 s at 2.0% amplitude and 448.95 s at 1.5% amplitude) for each year's block of data. The dual-period window function has the same sampling as does the real data.

It should be noted that in Table 5 periods A and B more or less track together with an average beat period of 1.315 days.

5. DISCUSSION

In spite of our best efforts, we are unable to demonstrate conclusively that PG 1707+427 is a stable pulsator with a unique ephemeris.

As can be seen from an inspection of Table 5, the 1.3 day beat period between period A and period B is fairly consistent, suggesting that it may be related to the rotation of the star.

The data obtained on PG 1707+427 with the Whole Earth Telescope (Nather et al. 1990) in 1991 May, when analyzed, should allow identification of the pulsational modes, resolve the 335 s band, and determine more about the behavior of the low-amplitude peaks reported by Fontaine et al. (1991).

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