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DWARF SPHEROIDAL GALAXIES AND NON-NEWTONIAN GRAVITY

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ABSTRACT

We derive a virial theorem and an analog for King's formula in modified Newtonian dynamics (MOND; Milgrom 1983) and use these to estimate the mass-to-light ratios (M/Ls) in MOND of the dwarf spheroidal galaxies in the Local Group. We find that the low-velocity dispersion observed in the Fornax dwarf galaxy implies low values for its M/L in MOND: 0.3–1. In particular the derived value in the core of Fornax (0.3) is much lower than expected for a normal old stellar population. Conversely, the velocity-dispersion measurements in Draco and Ursa Minor appear to still require dark matter in MOND. We show that M/L must vary between the dwarf spheroidals around the Galaxy by a factor of order at least 20, even in MOND.

Subject headings: dark matter — galaxies: elliptical and lenticular, cD — galaxies: kinematics and dynamics — gravitation

1. INTRODUCTION

The strong observational evidence for dark matter in many galaxies and clusters of galaxies suggests either that most of the mass in the universe is dark or that the standard Newtonian theory of gravity and dynamics is not valid on galactic scales (e.g., Milgrom 1983; Sanders 1984; Fahr 1990). Of the proposed modified gravity theories, Milgrom's (1983) modified Newtonian dynamics (MOND) has been surprisingly successful (Sanders 1990). Begeman, Broeils, & Sanders (1991) claim that one-parameter MOND models can fit the data better than three-parameter disk plus dark halo models.

Dwarf galaxies, because of their weak internal accelerations and large spread of observed mass-to-light ratios (M/Ls), are excellent laboratories for testing such alternative theories of gravity and for constraining candidates for the galactic dark matter. Lake (1989) emphasized the discrepancies between MOND and rotation curves of dwarf irregular galaxies (but see the counterarguments of Milgrom 1991). In this paper, we focus on dwarf spheroidals in the Local Group and discuss in some detail the constraints placed on MOND by the absence of significant observed dark matter in Fornax. We then discuss Draco, UMi, and Carina, for which very large M/Ls are inferred from their observed velocity dispersions (Aaronson & Olszewski 1988; Pryor 1991). We find that some of these dwarfs probably contain some dark matter even in MOND, and that in any case a large spread of intrinsic M/L is also required in MOND for the Local Group dwarf spheroidals.

2. VIRIAL THEOREM AND MASS-TO-LIGHT RATIOS IN MOND

Milgrom (1983) has suggested that a modification of Newton's laws in the regime of weak acceleration can obviate the need for large amounts of dark matter in galaxies and clusters. Subsequently, Bekenstein & Milogrom (1984) constructed a classical field theory for MOND, in which Newton's first law, $m\ddot{\boldsymbol{r}} = m\boldsymbol{g} = \boldsymbol{F}$, is retained, while modifying the relationship between the gravitational potential and mass distribution in the limit of weak gravity:

$$\nabla \cdot [\mu(|\boldsymbol{g}|/a_0)\boldsymbol{g}] = 4\pi G\rho \,. \tag{1}$$

The transition from Newtonian to modified gravity (MOND) occurs at $a_0 \simeq 1\text{–}3 \times 10^{-8}$ cm s^{-2}; see below. For $|\boldsymbol{g}| \gg a_0$, $\mu \to 1$, and MOND approaches Newtonian gravity. For $|\boldsymbol{g}| \ll a_0$, $\mu \to |\boldsymbol{g}|/a_0$. There is currently no cosmological formulation of MOND, so that the long-range cutoff ($r \to c/H_0$) is not defined in the theory.

In the weak acceleration limit in MOND, the gravitational acceleration due to an isolated spherical system is proportional to $M(r)/r$ (see also below). Because of the $1/r$ falloff at large radii MOND can explain flat rotation curves without the need to resort to dark matter. From fitting rotation curve shapes for a sample of spiral galaxies, Kent (1987), Milgrom (1988), and Begeman et al. (1991) find a mean $a_0 \sim (1\text{–}1.3) \times 10^{-8}\ h_{50}$ cm s^{-2}. The range in a_0 from best fits corresponds to a factor 3, after including contributions from H I gas. With a_0 fixed at its median value and adjusting M/L, one obtains reasonably good fits. The asymptotic rotation velocity in the weak-field regime of MOND depends only on a_0 and mass; applying this to the Galactic rotation curve yields

$$v_\infty = 220\left(\frac{M_{\rm MW}}{6 \times 10^{10} M_\odot}\right)^{1/4}\left(\frac{a_0}{3 \times 10^{-8}}\right)^{1/4} \text{km s}^{-1} \,. \tag{2}$$

The mass of stars and interstellar matter in the Galaxy is not very well determined (van der Kruit 1989). In the Bahcall-Soneira model (Bahcall & Soneira 1980) the total disk mass is $\sim 6 \times 10^{10}\ M_\odot$, including $\sim 40\%$ "disk dark matter" (which ought not to be counted in MOND); however, this value depends on the assumed radial scale length of the disk. In van der Kruit's model the total disk mass including "disk dark matter" is $\sim 7 \times 10^{10}\ M_\odot$, whereas the bulge contributes $\sim 1 \times 10^{10}\ M_\odot$. Thus to explain the Galactic rotation curve in MOND we would require that $a_0 \simeq 3 \times 10^{-8}$ cm s^{-2}. Finally, The & White (1988) obtain $a_0 \gtrsim 2.0 \times 10^{-8}\ h_{50}^{2.5}$ cm s^{-2} from an analysis of the X-ray gas and galaxies in the Coma Cluster. Because of uncertainties in the value of the Hubble constant and the differences in the various methods, a_0 is therefore uncertain: $a_0 \simeq 1\text{–}3 \times 10^{-8}$ cm s^{-2}. Hereafter, we will use $a_0 = 2 \times 10^{-8}$ cm s^{-2} and indicate the dependence of the various quantities on its value.

We can derive a virial theorem in MOND by differentiating the trace of the moment of inertia tensor,

$$0 = \frac{d^2}{dt^2} \int_V d^3r\, \rho(r) |\boldsymbol{r}|^2 , \tag{3}$$

where the integration is over an arbitrary volume V and the density distribution is assumed invariant in the differentiation. Thus

$$\int_V d^3r\, \rho |\dot{\boldsymbol{r}}|^2 = -\int_V d^3r\, \rho(r) \boldsymbol{r} \cdot \boldsymbol{g} . \tag{4}$$

The first term is the kinetic energy, which we can relate directly to the system's luminosity, $L_{\rm tot}$, M/L, ζ, and internal three-dimensional rms velocity, $v_{\rm rms}$. The second term cannot be transformed to yield a total potential energy as in Newtonian r^{-2} gravity; this concept is not well-defined in MOND.

2.1. *Isolated Weak Field Limit*

For an isolated spherical system in the weak acceleration limit, the gravitational acceleration in MOND can be calculated from equation (1):

$$g^{\rm iso} = \frac{|G a_0 M(r)|^{1/2}}{r} . \tag{5}$$

Using this equation, the second term in equation (4) can be calculated and, in terms of the measured luminosity density profile, $\nu(r)$, and M/L, ζ, we have

$$\zeta L_{\rm tot} v_{\rm rms}^2 = a_0^{1/2} G^{1/2} \zeta^{3/2} \int_V d^3r\, L(r)^{1/2} \nu(r) , \tag{6}$$

where $L(r)$ is the luminosity within radius r. Solving for the M/L yields

$$\zeta^{\rm iso} = \frac{v_{\rm rms}^4}{a_0 G F^2 L_{\rm tot}} , \tag{7}$$

where in the absence of rotation $v_{\rm rms}$ is the rms three-dimensional velocity dispersion, and the constant F is given by

$$F = \frac{1}{L_{\rm tot}^{3/2}} \int d^3r\, L(r)^{1/2} \nu(r)$$
$$= \frac{1}{L_{\rm tot}^{3/2}} \int_0^\infty dr \frac{2}{3} \frac{d}{dr} \left[\left[\int_0^r 4\pi r^2 dr\, \nu(r) \right]^{3/2} \right] = \frac{2}{3} . \tag{8}$$

Thus the total mass of an isolated system in the weak field regime of MOND is $M = 9 v_{\rm rms}^4 / 4 a_0 G$. Milgrom (1984) found this relation for the low-density isothermal spheres when solving the MOND structure (Jeans) equation with the assumption that the velocity dispersions are independent of radius; but the result is general.

2.2. *Quasi-Newtonian Weak Field Limit*

In MOND, unlike Newtonian gravity, there is a nonlinear relationship between acceleration and mass. The gravitational acceleration produced by a satellite galaxy, moving in the gravitational field of a central galaxy, is altered by the external field. Milgrom (1986) solved the MOND equation in the limit that the acceleration due to the central galaxy, g_c, is much larger than the internal acceleration of the satellite. In this case the effective gravitational field strength that governs the kinematics of stars in the satellite is

$$g^{\rm QN} = \frac{GM(r)}{\mu(g_c/a_0) r^2} . \tag{9}$$

In this so-called quasi-Newtonian limit, the effective mass in MOND is related to the Newtonian mass by a factor μ^{-1}, and the internal gravitational field predicted by MOND is elongated in the direction of the external field, by a factor of at most $2^{1/2}$ for $\mu(x) = x$.

Inserting equation (9) into the virial theorem (eq. [4]) results in the standard Newtonian virial mass multiplied estimate reduced by the factor μ^{-1}:

$$\zeta^{\rm QN} = \frac{2\mu(g_c/a_0) v_{\rm rms}^2 r_V}{G L_{\rm tot}} , \tag{10}$$

where r_V is the virial radius for the respective luminosity density profile.

2.3. *Central Mass-to-Light Ratios in MOND*

It is often more reliable to determine a central M/L than to use uncertain global parameters. In Newtonian theory, assuming that the stellar velocity distribution is isotropic, we may use King's formula (King & Minkowski 1972; Richstone & Tremaine 1986):

$$\zeta_c^N = \eta \frac{9\sigma_0^2}{2\pi G I(0) r_c} . \tag{11}$$

Here σ_0 is the central projected line-of-sight velocity dispersion, r_c is the core (half-brightness) radius, and η is a constant very close to unity for a variety of mass models. Merritt (1988) discussed the main limitation of this formula—if the stars are on primarily radial orbits, the M/L ratio can be overestimated by up to a factor $\sim$2–3, while if the stars are on primarily tangential orbits M/L is underestimated. In the quasi-Newtonian regime, we obtain the MOND value immediately:

$$\zeta_c^{\rm QN} = \mu(g_c/a_0) \zeta_c^N . \tag{12}$$

In the isolated weak-field limit of MOND, we can derive a similar formula by fitting to the isotropic low surface density isothermal sphere solution given in Milgrom (1984), whose volume density distribution can be written as

$$\rho = \rho_0 \left[1 + \left(\frac{r}{r_0} \right)^{3/2} \right]^{-3} , \tag{13}$$

where

$$r_0^3 = \frac{3^5 \sigma^4}{16\pi a_0 G \rho_0} . \tag{14}$$

Here σ is the (isotropic and constant) velocity dispersion, equal in this case to the central σ_0. The (scaled) surface density distribution of this model is fairly similar in the central parts to the King models we will use below. For the core radius and central surface density we find $r_c = 0.582 r_0$ and $\Sigma_0 = 0.923 \times 2\rho_0 r_c$. Dividing by the observed surface brightness and using the expression for r_0 and $\sigma_0 = \sigma$ then gives

$$\zeta_c^{\rm iso} = 1.76 \frac{\sigma_0^4}{a_0 G I(0) r_c^2} = 1.23 \frac{\sigma_0^2}{a_0 r_c} \zeta_c^N . \tag{15}$$

This has the same scaling as the global virial relation (7), as expected.

2.4. *Estimate from Tidal Radius*

The tidal radius of a dwarf galaxy in the gravitational field of the Galaxy in principle yields yet another estimate of its mass in MOND. The tidal field of the Galaxy is $v_\infty^2 r_t/D_{\rm dw}^2$, where r_t is the tidal radius and $D_{\rm dw}$ the distance of the dwarf galaxy from the Galactic center. At r_t this must balance the internal MOND gravitational field, $(a_0/g_{\rm MW})GM_{\rm dw}/r_t^2$ (in the quasi-Newtonian limit). Equating the two accelerations gives

$$\zeta_t^{\rm QN} = v_\infty^4\left(\frac{r_t}{D_{\rm dw}}\right)^3 (Ga_0 L)^{-1} . \tag{16}$$

In the isolated limit of MOND, the corresponding mass estimate is reduced from the quasi-Newtonian value by an extra factor $r_t/D_{\rm dw}$, but this will generally not be appropriate near r_t. While this tidal estimate is subject to obvious uncertainties, it does not depend on errors in either the velocity measurements or the distance to the satellite galaxy.

3. THE FORNAX DWARF: CONSTRAINTS ON MOND

Fornax, the most luminous and one of the most distant dwarf ellipticals in the Local Group, appears to be a good case for testing MOND. Buonnano et al. (1985) estimate that its distance is 131 ± 13 kpc. Then, based on photoelectric photometry by de Vaucouleurs & Ables (1968), which extends to about $r \sim 50'$, they obtain Fornax's luminosity as $7.0^{+2.0}_{-1.5} \times 10^6\ L_{\odot,V}$. This may be a lower limit: From the measured surface brightness and the King models fitted to the star counts one obtains about 1.5–3 times this value. Hodge (1971) finds that Fornax's star count profile is well fitted by a King (1966) model with concentration parameter, $c = \log (r_t/r_c) = 0.5$, and King radius $r_0 = 17'.7 = 675$ pc. This corresponds to a core (half surface brightness) radius of $r_c = 11'.2 = 430$ pc. More recent star counts (Eskridge 1988) result in a somewhat larger tidal radius: $r_0 = 16'.7 = 640$ pc; $r_t = 108' = 4100$ pc; $c = 0.8$; $r_c = 13'.8 = 530$ pc.

King's formula yields Fornax's Newtonian central M/L for an isotropic velocity distribution:

$$\zeta_c^N = 5.6\left(\frac{\sigma_0}{10\ \rm km\ s^{-1}}\right)^2\left(\frac{r_c}{500\ \rm pc}\right)^{-1} \frac{M_\odot}{L_{\odot,V}} . \tag{17}$$

Here we have used a central surface brightness of 14.8 $V_{\rm mag}$ arcmin^{-2} (Pryor 1992) = 11.8 $L_{\odot,V}$/pc^2. As discussed above, the uncertainty in this estimate may be as large as a factor ~2–3, depending on the anisotropy of the stellar velocity distribution.

Paltoglou & Freeman (1987, 1991) based on observations of 80 K giants, found a central velocity dispersion of $9.4^{+1.5}_{-1.4}$ km s^{-1}. Mateo et al. (1991) report a velocity dispersion of 9.9 ± 1.7 km s^{-1} for their central field, while for their outer field, they find 12.0 ± 2.8 km s^{-1}. Aaronson & Olszewski (1986) measured a dispersion of 7.8 ± 3.2 km s^{-1} from three globular clusters. Some of these measurements refer to different Fornax-intrinsic radii, which may account for the small differences. Paltoglou & Freeman (1987) also report a velocity difference of 3.4 ± 2.4 km s^{-1} between their two equidistant fields on either side of Fornax. The implied rotation velocity of ~1.7 km s^{-1} is probably not dynamically important; see below. Using the measured dispersions in equation (17), we find that Newtonian gravity predicts a central M/L of ~5 in Fornax, consistent with an old stellar population and little missing mass.

The *global* Newtonian M/L can be obtained from equation (10) without the factor μ, but depends on uncertain global parameters. As the limiting radius of the de Vaucouleurs & Ables (1968) photometry is similar to the tidal radius of Hodge's (1971) star counts, we use the $c = 0.5$ King model to estimate the virial radius $r_V = 800$ pc corresponding to that luminosity. Then with $v_{\rm rms}^2 = 3\sigma_0^2$ we find ζ^N of order 15, but this may still be an overestimate.

We now derive the analogous result in MOND. At the distance of Fornax the MOND acceleration due to the Milky Way, $g_{\rm MW} \simeq v_{\rm circ}^2/r \simeq 1.2 \times 10^{-9}$ cm s^{-2} is much less than a_0. We then need to determine whether Fornax is an isolated system in MOND, $a_0 \gg g_{\rm Fornax} \gg g_{\rm MW}$, or whether it is in the quasi-Newtonian regime, $a_0 \gg g_{\rm MW} \gg g_{\rm Fornax}$. We can roughly estimate the internal acceleration in Fornax by $g_{\rm Fornax} = f\sigma_0^2/r_c$, where f depends on radius and includes the effects of anisotropy. If the system is isotropic, f can be determined from the light profile and assuming the M/L to be constant and given through either equation (12) or equation (15). In the quasi-Newtonian case we combine equations (9) and (12) to get

$$g^{\rm QN} = \frac{9L(r)}{2\pi I(0) r^2}\frac{\sigma_0^2}{r_c} , \tag{18}$$

while in the isolated weak-field limit we use equations (5) and (15) to find

$$g^{\rm iso} = \left[\frac{1.76 L(r)}{I(0) r^2}\right]^{1/2} \frac{\sigma_0^2}{r_c} . \tag{19}$$

The two dimensionless functions of radius in equations (18) and (19) are plotted in Figure 1 for three King models with concentration $c = 0.5, 0.8, 1.0$. They all peak near $r = r_c$ and the maximum value for f is about 1.7 in the quasi-Newtonian

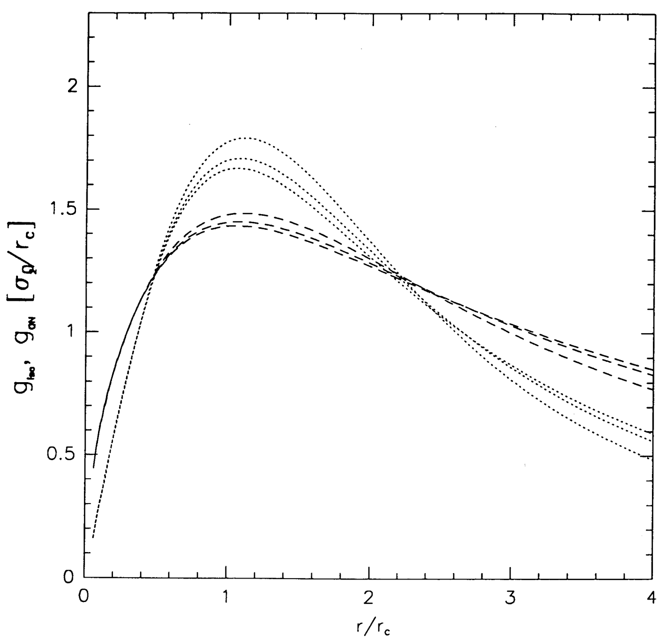

FIG. 1.—Dimensionless internal accelerations for King models with $c = 0.5, 0.8, 1.0$ (from top). Dashed lines show weak field isolated regime of MOND, and dotted lines quasi-Newtonian regime. See eqs. (18) and (19).

case and 1.5 in the isolated case. Then we find the peak value of the internal $g_{\rm Fornax} \sim 1.6\sigma_0^2/r_c \simeq 8 \times 10^{-10}$ cm s^{-2}, somewhat less than the external acceleration due to the Galactic field. If the system is radially (tangentially) anisotropic then using the central σ_0 will overestimate (underestimate) ζ_c and hence $g_{\rm Fornax}$ by a factor of order 2. Thus Fornax is not clearly in either the quasi-Newtonian or isolated regime and, given the current errors in the velocity measurements, the ratio of internal to external acceleration is sufficiently close to one that we will consider both limits separately.

We begin by assuming that Fornax is in the isolated system regime. Using equation (7), we determine a corresponding global mass-to-light ratio:

$$\zeta^{\rm iso} \simeq 0.85\left(\frac{\sigma_0}{9.4\ {\rm km\ s^{-1}}}\right)^4\left(\frac{D_{\rm Fornax}}{131\ {\rm kpc}}\right)^{-2}\left(\frac{a_0}{2\times10^{-8}}\right)^{-1}\frac{M_\odot}{L_{\odot,V}} \tag{20}$$

and from equation (15) a central value

$$\zeta_c^{\rm iso} \simeq 0.18\left(\frac{\sigma_0}{9.4\ {\rm km\ s^{-1}}}\right)^4\left(\frac{r_c}{500\ {\rm pc}}\right)^{-2}\left(\frac{a_0}{2\times10^{-8}}\right)^{-1}\frac{M_\odot}{L_{\odot,V}}. \tag{21}$$

On the other hand, if we assume that Fornax is in the quasi-Newtonian (QN) regime, then MOND predicts accelerations that are a factor $\mu^{-1}(x) \simeq x^{-1} = a_0/g_{\rm MW} = 0.06^{-1}$ greater than they would be in the Newtonian case for the same mass distribution; cf. equation (9). Thus, applying MOND to Fornax in the QN regime yields

$$\zeta^{\rm QN} \simeq \mu\zeta^N \simeq \frac{g_{\rm MW}}{a_0}\zeta^N$$
$$\simeq 0.85\left(\frac{\sigma_0}{9.4\ {\rm km\ s^{-1}}}\right)^2\left(\frac{r_V}{800\ {\rm pc}}\right)^{-1}\left(\frac{D_{\rm Fornax}}{131\ {\rm kpc}}\right)^{-1}$$
$$\times\left(\frac{v_\infty}{220\ {\rm km\ s^{-1}}}\right)^2\left(\frac{a_0}{2\times10^{-8}}\right)^{-1}\frac{M_\odot}{L_{\odot,V}}. \tag{22}$$

As in the Newtonian case, the global QN value requires specifying the virial radius, rms velocity dispersion, and total luminosity. From equation (12) we find the corresponding core value:

$$\zeta_c^{\rm QN} \simeq 0.34\left(\frac{\sigma_0}{9.4\ {\rm km\ s^{-1}}}\right)^2\left(\frac{D_{\rm Fornax}}{131\ {\rm kpc}}\right)^{-2}\left(\frac{v_\infty}{220\ {\rm km\ s^{-1}}}\right)^2$$
$$\times\left(\frac{a_0}{2\times10^{-8}}\right)^{-1}\frac{M_\odot}{L_{\odot,V}}. \tag{23}$$

Finally, the tidal radius of Fornax yields a last estimate of its mass in MOND. From equation (16) we find a total mass of $2.7 \times 10^6\ M_\odot$ and

$$\zeta_{t,{\rm MOND}}^{\rm QN} = 0.39\left(\frac{v_\infty}{220\ {\rm km\ s^{-1}}}\right)^4\left(\frac{a_0}{2\times10^{-8}}\right)^{-1}, \tag{24}$$

where we have used Eskridge's (1988) larger tidal radius in order to favor MOND and divided by the photometric luminosity. This gives an upper limit to the M/L; to be fair, we should rather have used the total luminosity predicted from the measured central surface brightness and the King model with *that* tidal radius, $L_K \sim 2.4 \times 10^7\ L_{\odot,V}$, which would have reduced ζ by a factor 4. While this tidal estimate is uncertain, it is independent of any errors in either the velocity measurements or the distance to Fornax. The result is comparable to that derived above from the core parameters.

All of these estimates give low values for the M/L of Fornax in MOND. Moreover, most of the possible systematic errors in the velocity dispersion (e.g., the presence of binaries or contamination due to atmospheric motions) would tend to cause observers to overestimate the velocity dispersion. Further, we believe that the core values are the more reliable, for the reasons stated. These come out low, with $M/L \lesssim 0.3$ in both the isolated or quasi-Newtonian regime of MOND, so that the fact that Fornax is not clearly in either regime is fortunately not problematic. Such a small M/L is plausible only for a peculiar stellar system consisting mostly of young, very massive stars, while Buonnano et al. (1985) find that the stellar population in Fornax is devoid of very massive stars and is quite similar to galactic clusters. Typical global M/L ratios in globular clusters are 1.5–3 (Pryor et al. 1988). Equations (20)–(24) show how the observational parameters would have to change if the MOND M/L for Fornax is to reach this value.

M. Milgrom (private communication) has suggested that Fornax could be a small S0–like galaxy supported substantially by rotation, with its angular momentum vector nearly parallel to the line of sight. Such an effect might be expected in MOND. This hypothesis would require that Fornax appear nearly round on the night sky. Fornax, however, is significantly flattened with an ellipticity of 0.3, and this, together with the Paltoglou-Freeman measurement, is sufficient to show that Fornax is not a rotating disk (Pryor 1992). Even if Fornax were an isotropic oblate rotator, we would expect a mean rotation velocity of $\sim 0.65\sigma_0 \simeq 6$ km s^{-1} for the apparent axis ratio, which is somewhat greater than, but perhaps not inconsistent with, the Paltoglou-Freeman value. In this case, depending on the inclination angle, rotation would add $\sim\sigma_0^2$ and at most $\sim 2\sigma_0^2$ to the total kinetic energy and therefore change the derived masses by only of order 50%.

4. DWARF SPHEROIDAL GALAXIES IN THE LOCAL GROUP

The other dwarf satellites of the Milky Way for which velocity dispersion measurements are available are Draco, Ursa Minor, Sculptor, and Carina. Table 1 lists some observed quantities for these dwarfs, taken from Table 4 of Pryor (1992), and some quantities derived from these data. The virial radius (col. [7]) is computed from the fitted King model (cols. [5]–[6]); for all these models $v_{\rm rms} = 1.46\sigma_0$. Some of the numbers for Fornax differ slightly from those used in the previous section as we have kept strictly to Pryor's conventions for a consistent comparison.

Comparing the respective external acceleration (col. [2]) in the gravitational field of the Milky Way with the internal acceleration ($\sim 1.6\sigma_0^2/r_c$; see § 3) shows that the first three of these dwarfs are not clearly in either the quasi-Newtonian or isolated weak-field limit of MOND, while Draco and UMi appear to lie on the isolated side. Thus we have again computed central and rms M/Ls in the two regimes, both of which show the same trends. Again the central values seem more reliable. The central Newtonian ζ_c^N (col. [9]) is taken directly from Pryor (1992); the corresponding MOND values are computed from equations (12) and (15). In the former case we have used $\mu(x) = x$. To calculate the rms Newtonian M/L (col. [12]) and the quasi-Newtonian MOND values we have used the above $v_{\rm rms}$, i.e., used the observed King profile as the radial profile for

TABLE 1

OBSERVABLES AND MASS-TO-LIGHT RATIOS IN DWARF SPHEROIDAL GALAXIES

Galaxy (1)	D kpc (2)	$v_\infty^2 D^{-1}$ 10^{-9} cm s^{-2} (3)	$1.6\sigma_0^2 r_c^{-1}$ 10^{-9} cm s^{-2} (4)	σ_0 km s^{-1} (5)	r_c pc (6)	c (7)	r_V pc (8)	L $L_{\odot,V}$ (9)	ζ_c^N (10)	ζ_c^{QN} (11)	ζ_c^{iso} (12)	ζ_{rms}^N (13)	ζ_{rms}^{QN} (14)	ζ_{rms}^{iso} (15)
Fornax	145	1.1	1.1	10	480	0.8	1100	$8.0\cdot \times 10^6$	5.7	0.31	0.24	13.5	0.74	0.95
Sculptor	78	2.0	1.8	7.0	140	1.0	380	3.1×10^6	11	1.1	0.77	5.9	0.59	0.59
Carina	92	1.7	2.5	8.8	160	0.7	330	4.1×10^5	53	4.5	5.1	61	5.2	11
Draco	75	2.1	4.8	10.5	120	0.7	250	3.1×10^5	94	9.9	17	88	9.2	30
Ursa Min	69	2.3	4.4	10.5	130	0.4	225	3.1×10^5	83	9.5	14	80	9.2	30

NOTE.—Observables taken or converted from Table 4 of Pryor 1992: col. (2): distance; col. (5): central velocity dispersion; col. (6): core radius; col. (7) concentration index; col. (8): virial radius; col. (9): absolute V luminosity. External acceleration of the Galaxy (col. [2]) uses $v_c = 220$ km s^{-1}. Col. (3) is approximate internal acceleration. Cols. (10)–(15) give central and rms mass-to-light ratio in Newtonian, MOND quasi-Newtonian, and MOND isolated weak-field dynamics.

the dominant component of the mass. In the isolated limit of MOND we have used the virial equation (7), but this time with $v_{rms}^2 = 3\sigma_0^2$, as this is more appropriate in this case.

We first consider the MOND quasi-Newtonian limit. In this limit, MOND predicts a linear relationship between galactocentric distance and the M/L inferred by an observer using Newtonian theory. Here, we assume that the true M/L ratio of the dSph are all roughly the same. The solid lines in Figure 2 show the predicted relationship between ζ_c^N and galactocentric radius. The points in the figure show the Newtonian M/L; from Table 1. These points do not follow the trend predicted by MOND, but rather show the opposite effect. Fornax, the most distant galaxy, has little inferred dark matter. On the other hand, Draco and Ursa Minor have the highest mass-to-light ratios.

In the isolated regime of MOND, which is the appropriate one for Draco and Ursa Minor, both these dwarfs show evidence for dark matter. Measurements of individual stellar velocities find $\sigma \simeq 10.5$ km s^{-1} in both systems (Aaronson 1983; Aaronson & Olszewski 1988), implying the quoted Newtonian

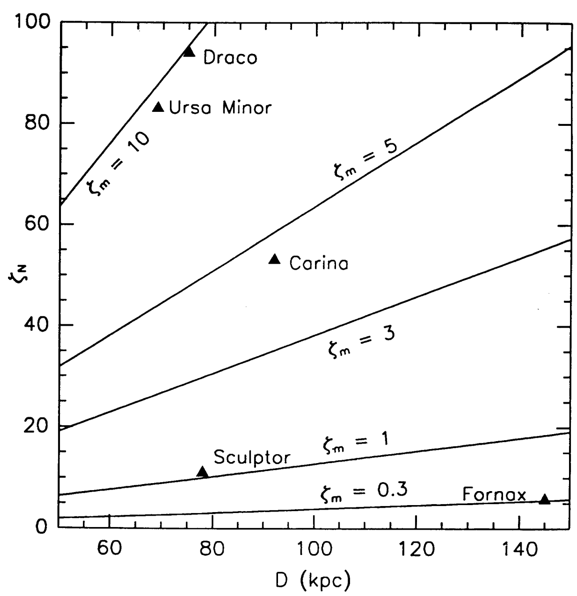

FIG. 2.—Inferred Newtonian M/Ls for the Milky Way's dwarf satellites, compared to the quasi-Newtonian MOND prediction for constant intrinsic M/L. Values for Draco, Ursa Minor, Carina, Sculptor, and Fornax were computed with King's formula (11), and the MOND prediction with eq. (12). The galactocentric distances, velocity dispersions, and core radii are from Pryor (1992). Since the dwarfs are old stellar systems, they should have similar M/Ls. The discrepancy between constant ζ^{MOND} and the observations is independent of the value of a_0.

M/Ls $\zeta^N \sim 80$–90. Aaronson & Olszewski (1988) argue that long-term monitoring of these dwarfs has reduced the possibility that contamination of the sample by binaries has spuriously raised these dispersions. The precise values of the M/L depends on the assumed core radii of the dark material relative to those for the stars, and the possible anisotropy of the stellar orbits (see Pryor 1991 for a review). In order to bring the MOND M/L, of Draco and UMi down to ~ 1 we would require that the observed σ_0 overestimates the true one-dimensional dispersion in the core by a factor of 2. For the observed stellar density distribution such a strong radial anisotropy is not feasible in Newtonian theory, and seems unlikely in MOND. Thus, unless we make strong assumptions about anisotropy and core radii, MOND does not appear to solve the dark matter problem.

Furthermore, for a set of galaxies with constant M/L and dynamics governed by the isolated limit of MOND, an observer using Newtonian dynamics should find the product of the Newtonian M/Ls and $\sigma^2/a_0 r_c$ to be a constant proportional to the true M/L (eq. [15]). The derived M/L values from Table 1 are compared in Figure 3 to this prediction of MOND; the data show the opposite trend. This effect is independent of the value of a_0.

Table 1 shows that there is a large spread in the derived M/Ls between the five dwarfs, independent of whether the central or global ratio is used, in Newtonian or MOND dynamics, and in the isolated and quasi-Newtonian regimes of MOND. Yet the stellar populations of all of these systems are similar, consisting of old metal-poor stars.

While dwarf spheroidals do not show any correlation between ζ^N and galactocentric distance, they do show a very clear correlation between luminosity, metallicity, and M/L (Djorgovoski & De Carvalho 1991). The most luminous dwarf spheroidal, Fornax, has the highest metallicity and the lowest M/L, while Draco and Ursa Minor have low luminosity, low metallicities, and high M/Ls. This correlation would suggest that Leo I and Leo II, which are more luminous than Fornax, should have low Newtonian M/Ls. On the other hand, MOND would predict that these systems, which are at roughly twice Fornax's galactocentric distance, should have high Newtonian M/Ls in the quasi-Newtonian regime. Kinematic observations of these galaxies will provide interesting additional input to these arguments.

5. DISCUSSION AND CONCLUSION

5.1. *Sanders's Finite Length Scale Antigravity*

As noted by Sanders (1990) himself, the existence of small galaxies with large mass discrepancy rules out theories of

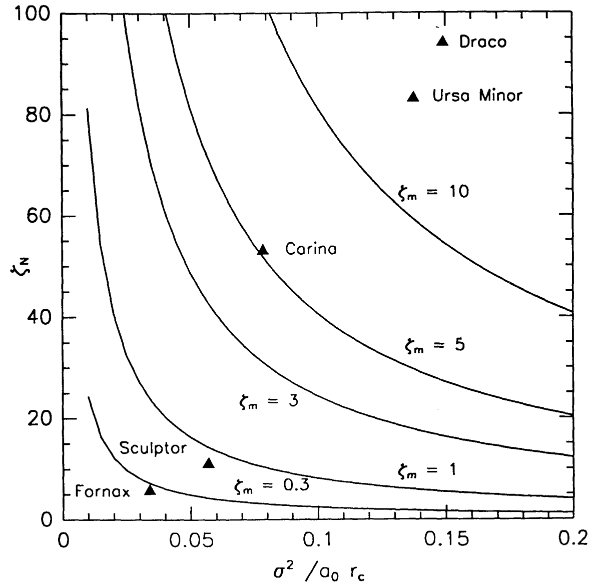

FIG. 3.—Inferred Newtonian M/Ls for the Milky Way's dwarf satellites, compared to the MOND prediction in the isolated regime, assuming that all satellites had the same intrinsic M/L. Values for Draco, Ursa Minor, Carina, Sculptor, and Fornax were computed with King's formula (11); for the MOND prediction, eq. (15) was used. In MOND, $\sigma^2/a_0 r_c \gg 1$ corresponds to the Newtonian limit. The velocity dispersions and core radii are from Pryor (1992). Since the dwarfs are old stellar systems, they should have similar M/Ls. The discrepancy between constant $\zeta^{\rm MOND}$ and the observations is independent of the value of a_0.

gravity in which the discrepancy is explained in terms of a simple modification of the r-dependence of the gravitational force, such as FLAG (finite length scale antigravity; Sanders 1984). In FLAG, the gravity of a point particle is weakened by a repulsive Yukawa term on large-galaxy scales. The corresponding length scale $r_{\rm FLAG}$ is found from fitting observations of rotation curves in well-studied spiral galaxies (Sanders 1986): $r_{\rm FLAG} = 24\ h_{75}$ kpc $\pm$ 4 kpc. The need for dark matter on scales of $\sim$100 pc in Draco and Ursa Minor, where the FLAG M/L should differ little from the Newtonian M/L, emphasizes the earlier point.

5.2. *MOND*

The kinetic properties of the Milky Way's satellites are problematic for MOND. We have used several approaches to estimate Fornax's MOND M/L. All of these techniques yield M/Ls less than one and the most reliable estimates are not consistent with the old stellar population seen in Fornax. Despite the similar stellar populations of the dwarf spheroidals around the Milky Way MOND predicts a large spread in their M/Ls and Draco and Ursa Minor appear to still require dark matter even in MOND.

Also, MOND predicts that a Newtonian observer should always find missing mass in systems with weak accelerations ($\sigma_0^2/a_0 r_c \ll 1$), and the prediction is such that an observer using Newtonian dynamics to determine the apparent M/L should see distinct trends in the isolated as well as quasi-Newtonian regimes. The dwarf spheroidals do not obey either of the predicted MOND scalings.

5.3. *Conclusion*

We conclude that both non-Newtonian gravity theories face difficulties in accounting for the observed kinematic properties of the dwarf spheroidal galaxies around the Milky Way. Any modified gravity theory, whether the modifications depend on length scales or accelerations, will find it difficult to explain Newtonian M/Ls of $\sim$5 in Fornax and the abundance of dark matter in Draco and Ursa Minor on the same scales of a few 100 pc.

Baryonic or cold dark matter, together with Newtonian gravity, appear to be better explanations for the halos of Draco and Ursa Minor. Yet, even then, galaxy formation theories still have the challenge of explaining why there is a lot of dark matter in some dwarf galaxies and none in others.

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