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DIFFUSIVE ACCELERATION OF ELECTRONS IN SN 1987A

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ABSTRACT

We suggest that synchrotron radiation emitted by electrons undergoing diffusive acceleration at the supernova blast wave is the cause of the reappearance of radio emission from SN 1987A in 1990 July. Making reasonable assumptions concerning adiabatic losses and the magnetic field in the preexplosion stellar wind, we find the light curve and spectrum which would be produced by a constant rate of particle injection occurring in a localized region of the shock front; i.e., in a clump or knot. Observations indicate that two such clumps have been encountered to date. From the observed delay in the switch-on of the emission between 843 MHz and 4.8 GHz we find a spatial diffusion coefficient for electrons of $\kappa_\perp = 2 \times 10^{20}$ m^2 s^{-1}. This value agrees with that found for the second clump and is substantially greater than the Bohm value. Estimating the magnetic field to be $10^{-7}T$ and assuming the diffusion coefficient is constant leads to the conclusion that the shock encountered the first clump on about day 900 and the second clump on day 1190. From the observed spectrum we find the compression ratio of the shock responsible for electron acceleration to be $\sim$2.7. This implies strong modification of the shock front which may be due to the acceleration of cosmic rays.

Subject headings: acceleration of particles — shock waves — supernovae: general — supernovae: individual: SN 1987A — supernova remnants

1. INTRODUCTION

Supernova 1987A reappeared at radio frequencies in 1990 July after being undetectable for $\sim$3 yr and 4 months (Staveley-Smith et al. 1992). Other Type II supernovae have also exhibited a substantial delay before becoming visible at radio wavelengths, and this has been attributed to the time taken by the supernova shock front to penetrate the thick absorbing screen expelled in the stellar wind of the progenitor (Chevalier 1982b). However, in the case of SN 1987A, this explanation is inadequate. Not only does the spectrum show no sign of the low-frequency turnover characteristic of absorption, but the shock front was visible at radio wavelengths as early as 2 days after explosion (Turtle et al. 1987). Chevalier (1992) has suggested that it is the encounter of the shock front with the termination shock of the blue giant wind which is responsible for the radio emission, but has not proposed a detailed model of the particle acceleration.

We advance an explanation of the radio emission based on the diffusive acceleration of electrons at the outer shock front of the supernova (Ball & Kirk 1992). To keep the model as simple as possible, we assume the shock front remains spherically symmetric and propagates at constant speed. After a certain time t_a, electrons begin to be injected into the diffusive acceleration process at some point on the shock front. We have in mind that the shock encounters a region of enhanced density—a clump or knot of material—which, however, is not massive enough to distort the front significantly. Once the shock enters a clump, we assume a constant input of electrons into the diffusive acceleration mechanism. As these electrons gain energy, they begin to emit synchrotron radiation in the compressed, azimuthal magnetic field swept up with the stellar wind material. In the nature of the diffusive process, there is a steady leakage of accelerated particles swept out of the immediate vicinity of the shock front by the downstream flow of plasma. These particles continue to emit synchrotron radiation, but suffer adiabatic losses in the overall expansion, which is a result of the motion imparted to the clump material by the shock. To calculate the time dependence we neglect the effects of the finite light travel time across a clump. This is justified if the clump is small, as is indicated by the observed short time-scale variability of the total flux (Staveley-Smith et al. 1992).

2. THE MODEL

Our model is based on a division of each emitting knot into two parts: an acceleration region close to the shock front and an expansion region downstream of it. We assume the electrons in the acceleration region behave as if they were encountering a plane shock. Once they escape into the expansion region, they cease to diffuse and remain frozen into the fluid flow, suffering only adiabatic losses. The philosophy of this approach is similar to that of the "onion-skin" models introduced by Bogdan & Völk (1983). We assume the shock to be traveling in the stellar wind of the progenitor, so that the magnetic field should have the form of a Parker spiral. This means the shock front is always quasi-perpendicular, so that the relevant diffusion coefficient $\kappa_\perp$ relates to transport across the ambient field. If the clumps arise from variations of the local mass-loss rate in the wind, then we may assume that B varies as r^{-1} everywhere. If, on the other hand, clumps are primarily a result of fluctuations in the stellar wind velocity, then we still have $B \propto 1/r$ within each clump, but must apply a different normalization from clump to clump. For simplicity, we adopt the former prescription.

We consider a plane shock front of compression ratio $\rho = v_1/v_2$ into which plasma flows along the normal at speed v_1 and exits also along the normal at speed v_2. This shock accelerates test particles which diffuse in the background plasma with a spatial diffusion coefficient across the field of $\kappa_\perp$, and which enter the acceleration process at the rate Q with momentum p_0. The distribution function of such particles can be found as a function of momentum, time, and space (Toptygin 1980; Drury 1991), but it suffices for our purposes to use a spatially averaged model (Axford 1981; Bogdan & Völk 1983): a test particle is presumed to undergo continuous acceleration while in the vicinity of the shock, such that its momentum p increases at a rate $p\Delta/t_c$, where $\Delta = 4(v_1 - v_2)/3v$ is the average fractional momentum gain per shock crossing/recrossing (with v the test particle's velocity) and $t_c = 4\kappa_\perp(1/v_1 + 1/v_2)/v$ is the average time taken to perform such a cycle (Bell 1978; Drury 1983). In addition, these particles escape from the shock region at a rate which is just the escape probability per cycle $P_{\rm esc} = 4v_2/v$ divided by the average cycle time. Writing $N(p, t)dp$ for the differential number of particles in the acceleration region with momentum between p and $p + dp$, we find:

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial p}\left(\frac{p\Delta}{t_c} N\right) + \frac{P_{\rm esc}}{t_c} N = Q\delta(p - p_0) . \quad (1)$$

Assuming t_c independent of p, and Q independent of t for $t \geq t_a$, the solution satisfying the boundary condition $N(p, t_a) = 0$ is

$$N(p, t) = \frac{t_c Q}{p_0 \Delta}\left(\frac{p}{p_0}\right)^{-2\alpha-1}\{H(p - p_0) - H[p - p_0 e^{(t-t_a)\Delta/t_c}]\} , \quad (2)$$

where $\alpha = P_{\rm esc}/(2\Delta)$ and $H(x)$ is a Heaviside step function. The number of particles *leaving* the acceleration region per second is $N(p, t)P_{\rm esc}/t_c$, and, since plasma leaves this region at speed v_2, the distribution function of particles advected with it is given by

$$\int dA f_s(\Omega, p, t) = \frac{1}{4\pi p^2} N(p, t) \frac{P_{\rm esc}}{t_c v_2} , \quad (3)$$

where $\int dA$ is the area of the surface through which the plasma leaves the acceleration region and Ω represents the coordinates of a point on this surface. We now move to the rest frame of the upstream plasma and assume the shock front to be spherical and moving at constant speed v_s, so that $dA = (v_s t)^2 d\Omega$, where $d\Omega$ is a differential solid angle. Particles which have left the shock are frozen into the downstream plasma, so that the equation satisfied by the distribution function is

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla f - \frac{1}{3} \nabla \cdot \boldsymbol{v} p \frac{\partial f}{\partial p} = 0 . \quad (4)$$

We assume the downstream plasma moves radially with (constant) speed $|\boldsymbol{v}| = v_d = v_s(\rho - 1)/\rho$, as in the spherically symmetric similarity solutions of Chevalier (1982a). Consequently, in equation (4) we have $\boldsymbol{v} \cdot \nabla = v_d(\partial/\partial r)$ and in the adiabatic loss term $\nabla \cdot \boldsymbol{v} = 2v_d/r$. The resulting equation is easily solved using the Lagrangian (comoving) coordinate $R = r - v_d t$, where $t = 0$ is the time of explosion. The general solution is an arbitrary function of R and of the combination $pr^{2/3}$, as well as the two polar angles represented by Ω. The boundary condition is that the distribution at the position of the shock front is given by equation (3). Noting that the radius at which the fluid element labeled by R passed through the shock is ρR, one finds the solution:

$$f(\Omega, R, p, t) = f_s(\Omega, xp, \rho R/v_s) , \quad (5)$$

where $x = [r/(\rho R)]^{2/3}$ is the factor by which adiabatic losses reduce the particle momentum. Since we are not interested in the dependence of these quantities on Ω we integrate equation (5) over all angles, and use equation (3) to obtain

$$\int d\Omega f(\Omega, R, p, t) = \frac{1}{4\pi(xp)^2(\rho R)^2} N\left(xp, \frac{\rho R}{v_s}\right) \frac{P_{\rm esc}}{t_c(v_s/\rho)} . \quad (6)$$

To simplify the calculation of synchrotron emission we approximate the emissivity of a single particle by $j_\nu(p) = a_0(p/mc)^2 B^2 \delta[\nu - a_1(p/mc)^2 B]$ with $a_0 = 1.6 \times 10^{-14}$ W Hz^{-1} T^{-2} and $a_1 = 1.3 \times 10^{10}$ Hz T^{-1}. Integrating over the entire downstream electron population then leads to the predicted emission from particles behind the shock. According to the frequency, different spatial regions contribute to the emission, but these are easily located when $\rho > 7/4$, which covers the cases of interest to us. Introducing the dimensionless variables $\hat{t} = t/t_0$ and $\hat{\nu} = \nu/(a_1 B_0 p_0^2)$, where B_0 is the magnetic field immediately behind the shock at the (arbitrary) time t_0, i.e., $B = B_0 v_s t_0/r$, we find that the effects of electrons of momentum close to that of injection can be ignored provided $\hat{\nu} > 1/\hat{t}$. Then the emission occurs up to a cutoff given by

$$\hat{\nu}_{\rm max}(\hat{t}) = \hat{t}^{-1} \exp[2(\hat{t} - \hat{t}_a)/\rho\eta] , \quad (7)$$

where $\eta = t_c/(\Delta\rho t_0)$ is a dimensionless parameter roughly equal to the acceleration time scale in units of t_0.

For frequencies $\hat{t}^{-1} < \hat{\nu} < \hat{\nu}_{\rm max}$ contributions to the emission arise not only from particles which have left the shock front but also from those still engaged in the acceleration process. These particles are described by equation (2) and radiate primarily in the magnetic field just downstream of the shock front. With these simplifications we obtain the total flux density emitted by a single clump in closed form:

$$F(\nu, t) = C(\hat{\nu}\hat{t})^{-\alpha}\left\{\left(\frac{\rho}{\rho - 1}\right)^{\alpha}\left[B_{y_1}\left(\alpha, 1 + \frac{4\alpha}{3}\right) - B_{y_2}\left(\alpha, 1 + \frac{4\alpha}{3}\right)\right] + \frac{\eta}{2\alpha\hat{t}} \rho^{-4\alpha/3}\right\} , \quad (8)$$

where $B_y(a, b)$ is the incomplete beta function (Abramowitz & Stegun 1972), the constant C is given by $C = a_0 Q B_0 t_0 \alpha \rho^{1+4\alpha/3}/(4\pi D^2 a_1)$, where D is the distance to SN 1987A, and where $y_1 = (\rho - 1)\hat{t}/[\rho\hat{R}_1 + (\rho - 1)\hat{t}]$ and $y_2 = (\rho - 1)/\rho$ with $\hat{R}_1$ the solution of the equation

$$\hat{\nu}(\rho\hat{R}_1)^{-4/3}[\hat{R}_1 + \hat{t}(\rho - 1)/\rho]^{7/3} = \exp[2(\hat{R}_1 - \hat{t}_a/\rho)/\eta] . \quad (9)$$

The term in square brackets in equation (8) is the contribution of particles which have left the acceleration region. Clearly, for small η the emission is in general dominated by such particles. However, close to the cutoff $\hat{\nu}_{\rm max}$ the contribution of particles in the shock can be comparable, since then $y_1 \approx y_2$ and the two β functions almost cancel. The basic features of the predicted emission from a single clump are illustrated in Figures 1 and 2, both plotted with t_0, ρ, η, and $\hat{t}_a$ chosen to fit the emission from the first clump encountered by the blast wave of SN 1987A, as discussed below. A power law of index α is obtained extending from $\hat{\nu} = 1/\hat{t}$ up to close to $\hat{\nu}_{\rm max}$. The emission at a fixed frequency first rises as particles are accelerated up to the required

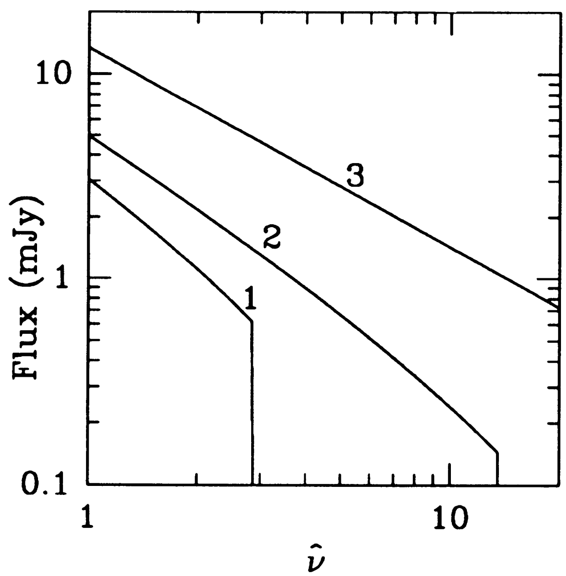

FIG. 1.—The predicted emission spectra from a single clump, at fixed times denoted by the labels 1:1223 days, 2:1260 days, and 3:1500 days.

energy and then falls as adiabatic losses set in and the freshly injected particles, which feel a weaker magnetic field, are no longer able to compensate. The peak and subsequent decay of the emitted flux could also come about in other ways: the contribution of an individual clump could switch off once the shock has passed right through it, the adiabatic losses could be faster than they are in our model, or the magnetic field could fall off faster than r^{-1}, so the precise form of the decay is not unique to our particular model.

3. IMPLICATIONS

When the radio emission from the remnant of SN 1987A was first detected, the flux at both 843 MHz and 4.8 GHz rose essentially linearly in time. Extrapolating back to zero flux, it

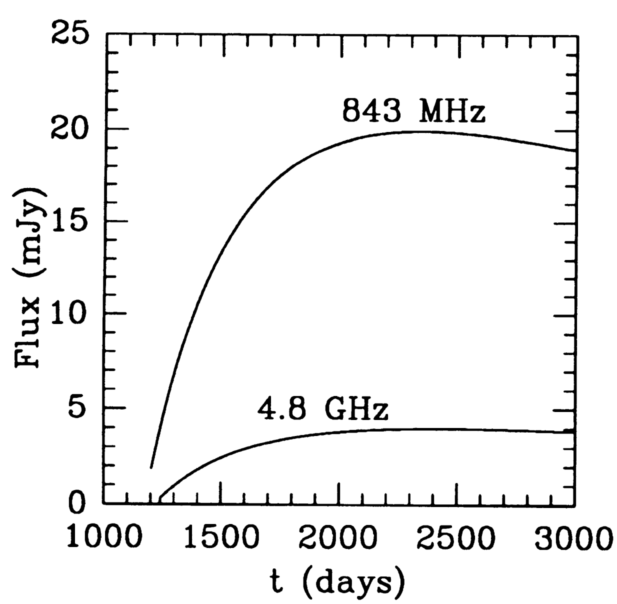

FIG. 2.—The predicted light curves at 843 MHz and 4.8 GHz, from a single clump.

can be seen that the emission at 843 MHz systematically precedes that at 4.8 GHz by ~30–60 days. This is the most important datum for the model; it determines the rate at which electrons are accelerated, leading to an estimate of their diffusion coefficient and of the time at which they began to be diffusively accelerated.

We assume the start of emission from a clump, at a given frequency $\hat{\nu}$, occurs when $\hat{\nu} = \hat{\nu}_{max}(\hat{t})$. Using $t_0 = 1200$ days so that switch-on at the lower frequency occurs at $\hat{t} = 1$, we find using equation (7), $\eta = 0.038/\rho$, which is sufficiently small to guarantee the validity of the planar shock approximation. From η we find the average diffusion coefficient of the particles $\kappa_\perp = 2.0 \times 10^{20}$ m^2 s^{-1}, where we have assumed $\rho = 2.7$ (see below) and $v_s = 30{,}000$ km s^{-1}. The quantity $\hat{t}_a$ determining the time at which the shock encountered the first clump enters only in the factor $\exp(\hat{t}_a/\rho\eta)$ scaling the dimensionless frequency (eqs. [7] and [9]), and it therefore depends on the magnetic field B_0 and the injection momentum p_0. Extrapolating estimates of the magnetic field at the site of the radio flare on day 2 (Storey & Manchester 1987; Ball & Kirk 1992; Kirk & Wassmann 1992) we arrive at a value of the order of 10^{-7} T (1 mG), so that

$$t_a = [897 + 23 \ln(\gamma_0^2 B_{-7})] \text{ days} , \qquad (10)$$

where B_{-7} is the value of B_0 expressed in units of 10^{-7} T (milligauss) and $\gamma_0 = p_0/mc$ is the Lorentz factor of injected electrons (provided these are relativistic). The Lorentz factor of those electrons emitting at 4.8 GHz is then ~2000 so that the Bohm limit of the diffusion coefficient is 3×10^{15} m^2 s^{-1}, nearly five orders of magnitude below the value we find for $\kappa_\perp$. Resonant scattering is unlikely to produce this cross-field diffusion, which could, however, be the result of a nonresonant effect such as field line wandering (Jokipii 1971; Chuvilgin, Dorman, & Ptuskin 1990). Thus, in this picture the shock hit the first clump 2.5 yr after the explosion. The implied position of the clump is well inside the radius R_{ring} of the ring observed with the *Hubble Space Telescope* (*HST*) (Jakobsen et al. 1991), with $r_{c1}/R_{ring} \approx 0.37\% \pm 10\%$ where the uncertainty is due to light travel time effects.

Another important property of the emission is its spectrum. During the period when SN 1987A was detectable at 843 MHz and not at 4.8 GHz the spectrum was steeper than $\nu^{-1.6}$. After switch on at 4.8 GHz it quickly flattened from a power law of spectral index $\alpha \sim 1.3$ to one with $\alpha = 0.8 - 1$. Diffusive acceleration predicts a rapid flattening of the spectrum to a value $\alpha = 3/[2(\rho - 1)]$, and our best fit to the observations indicates $\rho = 2.7$ rather than the value 4 expected of a strong shock front in a gas whose ratio of specific heats is 5/3. We do not propose an explanation of how the shock has weakened. However, it is interesting to speculate that this might be the result of the back reaction of cosmic rays (other than electrons) which the shock has presumably been accelerating. Arguments along these lines have been used by Ellison & Reynolds (1991) in applying diffusive acceleration to older supernova remnants.

The only remaining free parameter in the model is the injection rate Q which can be determined by normalizing the predicted flux density to that observed. The flux of 10 mJy observed at 843 MHz on day 1386 (Staveley-Smith et al. 1992) implies that the supernova shock is picking up electrons from the first clump at the rate $\sim 4 \times 10^{44}$ s^{-1}. Assuming the clump covers a fraction δ of the area of the shock front, this corresponds to a "pick-up density" in front of the shock of $\sim 10^5 \delta^{-1}$ m^{-3}. For comparison, the density observed in the circumstellar ring surrounding SN 1987A (Fransson et al. 1989) is $\sim 2 \times 10^{10}$ m^{-3}, whereas that in the undisturbed blue giant wind is $\sim 4 \times 10^6$ m^{-3}, assuming a mass-loss rate of 10^{-5} $M_\odot$ yr^{-1} and a wind speed of 550 km s^{-1} (Chevalier & Fransson 1987).

The single clump model provides a good fit to the data up to about day 1500, when the light curve at 843 MHz steepened (Staveley-Smith et al. 1992), suggesting the onset of emission from a second clump. This is confirmed by later measurements at 4.8 GHz (Manchester & Staveley-Smith 1992). The time delay between the event at the two frequencies is not significantly different to that at switch-on of the first component. Taking the same values for ρ and $\kappa_\perp$ we find a good fit with an injection rate of 7×10^{44} electrons per second starting when the shock encountered the second clump on about day 1190. The implied position of this clump is $r_{c2}/R_{ring} \approx 0.49\% \pm 10\%$. The required injection rate would be somewhat lower if the magnetic field increased on entering the second clump. Figure 3 shows the light curves predicted for the two clump model, superposed on measurements at 843 MHz from the MOST instrument and at 4.8 GHz from the Australia Telescope (Staveley-Smith et al. 1992; Campbell-Wilson, Crawford, & Turtle 1992; Manchester & Staveley-Smith 1992).

In summary, the two-component model fits the data well. Detailed images, particularly of polarized emission may soon confirm clumping in the radio source, and analysis of the centroid of the radio emission may also reveal important clues as to its geometry. The spectral fit indicates that the shock responsible for the acceleration has a compression ratio of 2.5–2.9, and may thus be the gas subshock embedded in a structure strongly modified by accelerated cosmic rays. The delay in response at high frequency compared with low frequency leads to an estimate of the spatial diffusion coefficient across the magnetic field for relativistic electrons which is presumably due to nonresonant scattering or magnetic field line wandering. A good fit is obtained using the same value of ρ and $\kappa_\perp$ for each clump; if these conditions persist in future clumps, in particular in the ring observed with the *HST* and if $\kappa_\perp$ remains roughly constant down to nonrelativistic electron energies, then the predicted sharp rise in X-ray emission (Luo & McGray 1991) should precede a strong upturn in the radio flux by several months.

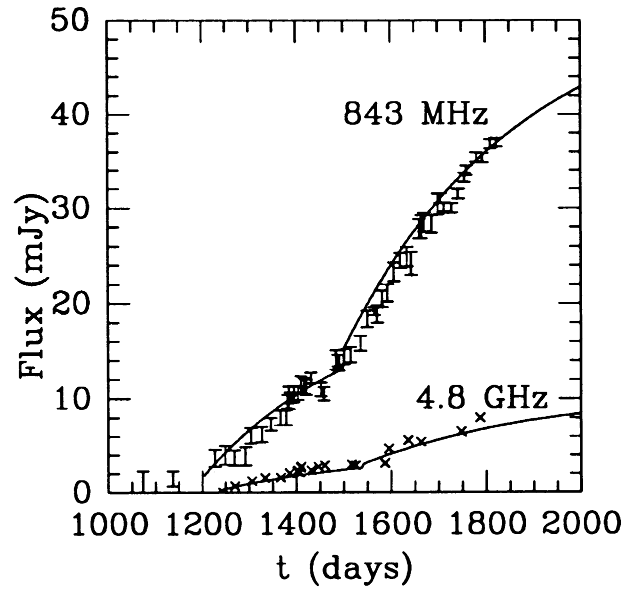

FIG. 3.—The predicted light curves 843 MHz and 4.8 GHz, from two clumps of material, superposed on the observations. For clarity, only a subset of the data at 843 MHz is shown. The errors on the 4.8 GHz are are comparable to the size of the crosses.

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