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ON THE ABSOLUTE MAGNITUDE OF RR LYRAE STARS: UU CETI, RV PHOENICIS, AND W TUCANAE[1]

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ABSTRACT

We present infrared *JHK* light curves for the RRab Lyrae stars UU Ceti, RV Phoenicis, and W Tucanae. These stars have similar periods, $\sim 0^{\rm d}.6$, and metallicities [Fe/H] ~ -1.00, -1.50, and -1.35, respectively. The infrared data, together with *BVRI* photometry and CORAVEL radial velocity data previously published by two of us, and Walraven photometry by Lub, are used to derive absolute magnitudes for the stars using two formulations of the Baade-Wesselink method: (1) the infrared flux version and (2) the surface brightness version. The two methods are directly compared and their respective advantages and shortcomings are discussed. Finally, a comparison is made with previous results on the absolute magnitude of RR Lyrae variables.

Subject headings: stars: fundamental parameters — stars: oscillations — stars: variables: other (RR Lyrae)

1. INTRODUCTION

In the last decade a great effort has been devoted by many authors to the determination of the absolute magnitude of RR Lyrae stars using the intrinsic physical parameters of the variables as derived from their light and radial velocity curves. Besides the original Baade-Wesselink (B-W) method (Baade 1926; Wesselink 1946, 1969), which first used the comparison of the angular diameter derived from the light and color curves with the linear diameter derived from the radial velocity curve to determine the distance and absolute magnitude of the star, a number of revised or new methods have been developed whose main differences are in the portion of the photometric continuum used in the analysis, and in its progressive extension toward the red and infrared region (see, for example, Jones, Carney, & Latham 1988; Liu & Janes 1990; Cacciari, Clementini, & Buser 1989a, [hereafter CCB], Fernley et al. 1989, and references therein).

In this paper we present *JHK* photometry for three ab-type RR Lyrae stars, namely UU Ceti, RV Phoenicis, and W Tucanae, and use these data along with the available *BVRI* and Walraven photometry and CORAVEL radial velocities, to derive the absolute magnitudes M_V(RR) of the three stars. The methods we have used are (1) a combination of the B-W and the infrared flux (IF) method, as it has been used in previous papers (Fernley et al. 1989; Skillen et al. 1989; Fernley et al. 1990a, b); and (2) the B-W method in the surface-brightness (SB) version, as it was used by Cacciari et al. (1989b) and CCB with the additional modifications described in § 3.2. The application to the same stars of the two different methods of analysis makes it possible to confront directly the two techniques and to assess the reliability and accuracy that can be attained in the RR Lyrae absolute magnitude determinations.

Both methods are in principle capable of determining M_V(RR) with a realistic error of approximately ± 0.2 mag. Although this degree of accuracy is not sufficient to resolve some of the important questions related to the variation if M_V(RR) with period and metallicity (e.g., the age of globular clusters with a smaller uncertainty than the current 2–3 Gyr), it can be substantially improved by enlarging the statistics, i.e., applying the methods to as many stars as possible (see Sandage 1989, 1990, and Sandage & Cacciari 1990, for a review and detailed discussion of this and related problems).

The infrared light curves for the three stars are presented in § 2. In § 3, we derive distances and absolute magnitudes using the IF and the SB methods, and in § 4 we compare the results obtained from the two methods, and discuss and compare them with previous results on RR Lyrae variables.

2. OBSERVATIONS

2.1. *Ephemerides*

In this paper we use basically three data sets for each star; first, Walraven photometry from Lub (1977); second, *BVRI* and radial velocity from either Cacciari et al. (1987a) & CCB or Clementini, Cacciari, & Lindgren (1990, hereafter CCL); and third, our own infrared photometry. To phase these different data sets we need accurate and reliable ephemerides of the three stars. From the references listed in the CDS Databank we find for the three stars the times of maximum light and periods shown in Table 1. The data in Table 1 would suggest the periods may be changing; however, as there are only two or three epochs for each star (the epochs given by Kukarkin et al. 1970 have not been used in deriving the adopted ephemerides, since they are often based on visual estimates and are consequenty less accurate), a proper analysis of period variation is not possible, and thus we have only fitted linear ephemerides. These are shown below and have been used to phase all the data presented in the remainder of this section and all the previously published data that we use in the analysis. The *JHK* photometry being quite close in time to the *BVRI* photometry, any phasing error due to period changes is minimized by using Cacciari et al. (1987a), CCB, CCL epochs of maximum light:

$$\text{UU Cet HJD(max)} = 47054.775 + 0.606075N$$

$$\text{RV Phe HJD(max)} = 46305.708 + 0.596411N$$

$$\text{W Tuc HJD(max)} = 47490.719 + 0.642235N \ ,$$

where N is the number of full cycles the star has gone through. The periods previously determined for the three stars differ

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TABLE 1

EPOCHS OF MAXIMUM LIGHT AND PERIODS FOR THE PROGRAM STARS

Star	HJD(+2400000)	Period	Source
UU Cet	30934.392	0.606081	Kukarkin *et al.* (1970)
	40157.640	0.606	Bookmeyer *et al.* (1977)
	41208.576	0.606081	Lub (1977)
	47054.775	0.606081	Clementini et al (1990)
RV Phe	36847.188	0.596413	Kukarkin *et al.* (1970)
	41915.525	0.5964182	Lub (1977)
	46305.708	0.5964137	Cacciari et al (1987,1989b)
W Tuc	28064.433	0.6422299	Kukarkin *et al.* (1970)
	41242.414	0.6422299	Lub (1977)
	47490.719	0.642237	Clementini et al (1990)

from the adopted ones by typically $\leq 6 \times 10^{-6}$ day: since the number of cycles occurred between our *JHK* photometry and the adopted epochs is ~ 2000, the phasing error produced by this period variation is ≤ 0.017.

2.2. *Infrared Photometry*

Between 1990 October 19 and 28, the three program stars were observed at *J*, *H*, and *K* using the 1.0 m telescope at the European Southern Observatory, La Silla, Chile. The photometry was differential with respect to the same comparison stars used for the *BVRI* photometry by Cacciari et al. (1987a), CCB, and CCL.

Seventeen ESO standard stars were observed during the run, for a total of 30 measurements. The list of the standard stars is given in Table 2. Bouchet, Manfroid, and Schmider (1991) summarize more than 10 years of infrared observations made at La Silla and their work supercedes that of Engels et al. (1981) and Wamsteker (1981) in terms of defining the ESO system. They also provide a detailed comparison with other photometric systems, i.e., SAAO, AAO, MSSO, and CTIO. Since in § 3.2 we will use $V-K$ colors in the Johnson photometric system in order to derive the stellar temperatures, our $V-K$ data have been converted from the ESO to the Johnson system via the relation

$$(V - K)_J = (V - K)_{ESO} - 0.013 - 0.024(J - K)_{ESO}$$

which was derived on the basis of nine stars in common between Bouchet, Manfroid, and Schmider (1991) and Fernley (1989) who provides the color-temperature calibration.

The mean *JHK* magnitudes in the ESO system for the program variable stars, averaged in intensity over the pulsation cycle, and for the corresponding comparison stars are listed in Table 3. The photometry is listed in Tables 4–6 and the *J*, *H*, and *K* light curves are illustrated in Figure 1.

2.3. *Optical Photometry and Radial Velocities*

Extensive Walraven photometry is available for the three program stars (Lub 1977). *BVRI* photoelectric photometry and CORAVEL radial velocities of the three stars have been published by Cacciari et al. (1987a), CCB, and CCL. The *RI*

TABLE 2

INFRARED STANDARD STARS

Star	J	H	K
HR0033	3.943	3.679	3.617
HR0077	3.181	2.885	2.818
HR0088	5.322	5.004	4.942
HR0180	2.989	2.480	2.375
HR0448	4.691	4.379	4.305
HR0591	2.320	2.194	2.138
HR0721	4.489	4.559	4.596
HR1006	4.423	4.088	4.016
HD2811	7.207	7.131	7.108
HR6736	5.798	5.790	5.792
HR7120	2.893	2.221	2.053
HR7330	5.379	5.082	5.014
HR8658	5.523	5.211	5.134
HR8700	5.068	4.741	4.721
HR8701	4.980	4.676	4.598
HR8709	3.174	3.145	3.078
HR8848	3.289	3.083	3.008

TABLE 3

INFRARED MAGNITUDES OF THE VARIABLE AND COMPARISON STARS

Variable	⟨*J*⟩	⟨*H*⟩	⟨*K*⟩	Comparison	*J*	*H*	*K*
UU Cet ...	11.162	10.900	10.850	Unnamed	9.572	9.320	9.268
RV Phe ...	11.038	10.781	10.721	HD 220710	9.443	9.169	9.115
W Tuc	10.643	10.406	10.354	HD 5648	9.554	9.437	9.398

photometry is on the Cousins photometric system (Menzies, Banfield, & Laig 1980). A detailed description of the data taking and handling is provided in those papers, to which the reader is referred for further information. Here we only point out that the accuracy of the *BVRI* photometry is approximately 0.01–0.02 mag per single observation, which becomes better than 0.01 mag for the magnitude and color average values. W Tuc and UU Cet have very "clean" light and color curves (see Fig. 1 of CCL), while RV Phe shows larger scatter (Cacciari et al. 1987a), which is also seen in the infrared data in Figure 1. The radial velocities were obtained with the CORAVEL photoelectric scanner (Baranne, Mayor, & Poncet 1977, 1979; Mayor 1985), and the zero-point of the CORAVEL system was checked by measuring three or more IAU radial velocity standard stars each night. The accuracy of the CORAVEL radial velocities of an RR Lyrae star depends on the metallicity, the average color of the star, and the phase at which v_r is measured, the correlation dip being shallower and less well-defined when the star has earlier spectral type (i.e., at maximum light). Typically the accuracy is ± 1–2 km s^{-1}. Barycentric stellar velocities were derived for the three stars by integrating the radial velocity curves obtained from fitting the radial velocity data points to eighth-order Fourier series and are $\gamma = +98.88$, -114.48, and $+64.61$ km s^{-1} for RV Phe, UU Cet, and W Tuc, respectively. Note the difference of ~0.2 km s^{-1} with the value previously determined with a different fitting procedure for RV Phe (Cacciari et al. 1987a), which has led us to reanalyze this star with the visual colors as well (see § 3.2.1).

2.4. *Reddening and Metallicity*

Jones (1973) and more recently Lub (1979) both made detailed photometric studies of a large number of RR Lyraes from which individual reddenings were determined. An entirely independent method is provided by Burstein & Heiles (1982) who used H I column densities and galaxy counts to determine the average reddening as a function of Galactic latitude and longitude. The different authors are in reasonable agreement that all three RR Lyraes are only slightly reddened; however, whereas Burstein & Heiles give $E(B-V)$ as zero for all three stars assuming a reddening between 0.01 and 0.02 mag at the south Galactic pole, Jones & Lub both claim nonzero values assuming, respectively, 0.03 and 0.015 mag for the reddening at

TABLE 4

JHK PHOTOMETRY FOR UU CETI

HJD−2440000	φ	J	H	K	HJD−2440000	φ	J	H	K
8184.6360	0.226	11.065	10.795	10.735	8189.7175	0.611	11.216	10.918	10.903
.6590	0.264	11.094	10.807	10.753	8190.5155	0.927	11.173	10.972	10.916
.6775	0.295	11.090	10.810	10.753	.5273	0.947	11.148	10.947	10.900
.7016	0.335	11.115	10.823	10.785	.5438	0.974	11.076	10.901	10.849
.7228	0.370	11.110	10.836	10.798	.5549	0.992	11.070	10.889	10.851
.7456	0.407	11.117	10.837	10.807	.5719	0.020	11.036	10.859	10.832
8188.5182	0.632	11.222	10.938	10.895	.5842	0.041	11.049	10.868	10.827
.5322	0.655	11.240	10.960	10.931	.6164	0.094	11.072	10.852	10.821
.5518	0.687	11.249	10.965	10.920	.6273	0.112	11.065	10.849	10.790
.5643	0.708	11.261	10.986	10.910	8191.5275	0.597	11.214	10.910	10.851
.5815	0.736	11.285	11.011	10.902	.5409	0.619	11.233	10.931	10.867
.5993	0.766	11.288	11.024	10.939	.5680	0.664	11.252	10.954	10.895
.6311	0.818	11.397	11.080	11.051	.5802	0.684	11.250	10.959	10.901
.6508	0.851	11.364	11.063	11.019	.5982	0.714	11.284	10.991	10.909
.6776	0.895	11.233	10.985	10.923	.6104	0.734	11.294	11.010	10.982
.6957	0.925	11.189	10.985	10.883	.6290	0.765	11.298	11.031	10.941
.7139	0.955	11.145	10.940	10.883	.6424	0.787	11.347	11.039	10.995
.7293	0.980	11.086	10.882	10.863	.6628	0.820	11.349	11.064	11.005
.7466	0.009	11.055	10.899	10.870	.6763	0.843	11.361	11.072	11.048
8189.5336	0.307	11.101	10.814	10.796	.6979	0.878	11.288	11.046	10.956
.5472	0.330	11.097	10.821	10.798	.7112	0.900	11.238	10.988	10.932
.5678	0.364	11.114	10.818	10.745	.7309	0.933	11.171	10.974	10.890
.5805	0.385	11.147	10.824	10.783	.7427	0.952	11.144	10.931	10.905
.5981	0.414	11.144	10.847	10.776	.7610	0.982	11.091	10.896	10.830
.6099	0.433	11.140	10.843	10.803	.7727	0.002	11.069	10.909	10.870
.6279	0.463	11.152	10.843	10.809	8193.6476	0.095	11.056	10.834	10.798
.6405	0.484	11.178	10.847	10.837	.6594	0.115	11.049	10.839	10.814
.6726	0.537	11.199	10.893	10.830	.7237	0.221	11.080	10.832	10.789
.6844	0.556	11.191	10.903	10.842	.7348	0.239	11.063	10.822	10.747
.7057	0.591	11.196	10.902	10.846					

TABLE 5

JHK PHOTOMETRY FOR RV PHOENICIS

HJD−2440000	ϕ	J	H	K	HJD−2440000	ϕ	J	H	K
8185.5210	0.875	11.223	10.966	10.874	8187.6169	0.389	11.013	10.705	10.630
.5379	0.904	11.092	10.868	10.807	.6488	0.443	11.014	10.730	10.635
.5551	0.932	11.069	10.876	10.799	.6658	0.471	11.050	10.725	10.696
.5733	0.963	11.012	10.829	10.742	.6857	0.505	11.053	10.756	10.733
.6151	0.033	10.959	10.770	10.690	.7049	0.537	11.081	10.765	10.691
.6339	0.064	10.911	10.752	10.683	.7237	0.568	11.086	10.787	10.711
.6515	0.094	10.931	10.742	10.681	.7380	0.592	11.105	10.820	—
.6683	0.122	10.932	10.712	10.641	8192.5803	0.711	11.139	10.858	10.783
.6871	0.154	10.914	10.718	10.663	.5925	0.732	11.162	10.856	10.815
.7052	0.184	10.946	10.693	10.641	.6130	0.766	11.175	10.903	10.863
.7225	0.213	10.941	10.676	—	.6244	0.785	11.179	10.900	10.870
.7436	0.248	10.955	10.696	10.645	.6458	0.821	11.227	10.958	10.885
8186.5169	0.545	11.062	10.779	10.731	.6576	0.841	11.218	10.943	10.884
.5487	0.598	11.117	10.827	10.730	.6812	0.881	11.213	10.964	10.849
.5712	0.636	11.122	10.808	10.774	.6960	0.905	11.091	10.880	10.823
.5888	0.666	11.095	10.826	10.785	8193.4999	0.253	10.975	10.699	—
.6101	0.701	11.138	10.869	10.820	.5129	0.275	10.953	10.677	10.644
.6481	0.765	11.190	10.908	10.835	.5306	0.305	10.970	10.684	10.630
.6666	0.796	11.197	10.920	10.905	.5407	0.322	10.966	10.678	10.653
.6800	0.818	11.214	10.946	10.896	.5566	0.348	10.982	10.675	10.623
.7117	0.872	11.203	10.942	10.908	.5707	0.372	11.005	10.718	10.645
.7244	0.893	11.161	10.868	10.834	.5845	0.395	11.000	10.695	10.614
.7442	0.926	11.051	10.949	10.777	.5958	0.414	11.007	10.687	10.640
.7558	0.946	10.989	10.812	10.761	.6111	0.440	10.996	10.703	10.636
8187.5149	0.218	10.952	10.702	10.642	.6230	0.460	11.035	10.731	10.690
.5333	0.249	10.939	10.697	10.635	.6820	0.559	11.054	10.769	10.722
.5511	0.279	10.945	10.694	10.634	.6959	0.582	11.095	10.784	10.734
.5683	0.308	10.975	10.697	10.624	.7622	0.693	11.128	10.845	10.791
.5900	0.344	10.993	10.702	10.633	.7736	0.712	11.136	10.853	10.800
.6009	0.363	10.988	10.692	10.632					

the SGP. Another possible method of reddening determination is by applying Sturch's (1966) formulation, which assumes 0.03 reddening at the SGP, along with ΔS and the $(B-V)$ colors at minimum light from the photometric data by Clube, Evans, & Jones (1969), Cacciari et al. (1987a), CCB, & CCL. For the purposes of the present paper, it is adequate to take the mean values of all these determinations, which are $E(B-V) = 0.025$, 0.015, and 0.005 for UU Cet, RV Phe, and W Tuc, respectively, on the assumption that the amount of reddening at the SGP is 0.015 mag. The typical error of such determinations is ~0.02–0.03 mag, including both internal rms errors and the uncertainties on the zero point. Incidentally, we notice that a very recent study on the reddening of ab-type RR Lyrae stars by Blanco (1992) finds consistent values of reddening, i.e., $E(B-V) = 0.01$ (UU Cet), 0.03 (RV Phe), and 0.01 (W Tuc). The reddening ratios we have adopted in the following section are $E(V-R) = 0.56E(B-V)$, $E(V-I) = 1.24E(B-V)$, $E(V-K) = 2.80E(B-V)$, $A_V = 3.2E(B-V)$, and $A_K = 0.11A_V$.

Metallicities are required as an input parameter to the model atmospheres from which we derive the theoretical infrared flux used in the IF method and the surface brightness used in the SB method. The metal abundances of the three stars can be derived from their ΔS values. In the literature we find $\Delta S = 4$ (Preston 1959; Clube et al. 1969), $\Delta S = 8$ and $\Delta S = 7$ (Jones 1973; Lub 1977) for UU Cet, RV Phe, and W Tuc respectively, and from Butler's (1975) [Fe/H]-ΔS calibration we obtain [Fe/H] = −0.87, −1.50, and −1.35 respectively. The metallicity of UU Cet has also been derived by Jones (1973) and Lub (1979) from photometric indices. Both authors provide a calibration of their indices in terms of ΔS and then derive the metallicity [Fe/H] using Butler's (1975) relation. From Jones (1973) index $(k-b)_2 = 0.08$ UU Cet has $\Delta S = 6.5$ (see also eq. [8] of Butler 1975), while from Lub's (1979) $\Delta(B-L) = 0.027$ UU Cet has $\Delta S = 7$, corresponding to [Fe/H] = −1.27 and [Fe/H] = −1.35, respectively. The discrepancy with the spectroscopic metal abundance determination is rather strong. A recent spectroscopic analysis of this star based on a new Ca-metallicity calibration (Clementini, Tosi, & Merighi 1991b) suggests a value [Fe/H] = −0.92, thus consistent with Preston's estimate. We feel that the metal abundances derived from spectroscopic techniques should be given higher weight, since these methods measure the metallicity of the object in a more direct way. Therefore we adopt for UU Cet the metallicity [Fe/H] = −1.00, as the mean of the four previous estimates giving half-weight to the photometric determinations. Since the infrared fluxes have little sensitivity to metallicity, the results of the IF method and of the SB method with infrared colors will not be much affected by possible inaccuracies in the adopted metallicity values, which will have a stronger impact on the results of the SB method with optical colors. The input parameters used in the following analyses are summarized in Table 7.

TABLE 6

JHK PHOTOMETRY FOR W TUCANAE

HJD−2440000	ϕ	J	H	K	HJD−2440000	ϕ	J	H	K
8184.7823	0.700	10.757	10.471	10.405	8189.8363	0.569	10.722	10.438	10.388
.8030	0.732	10.801	10.519	10.430	.8522	0.594	10.732	10.443	10.370
.8204	0.759	10.846	10.551	10.488	.8629	0.611	10.760	10.450	10.399
.8420	0.793	10.853	10.600	10.512	8190.6521	0.840	10.939	10.640	10.542
.8615	0.823	10.926	10.634	10.589	.6612	0.854	10.949	10.657	10.583
8185.7828	0.258	10.556	10.299	10.220	.6764	0.877	10.944	10.649	10.616
.8040	0.291	10.561	10.301	10.262	.6862	0.893	10.911	10.647	10.553
.8209	0.317	10.563	10.312	10.254	.7039	0.920	10.807	10.546	10.486
.8418	0.350	10.593	10.329	10.272	.7130	0.934	10.671	10.492	10.430
8186.8458	0.913	10.871	10.634	10.511	.7267	0.956	10.563	10.435	10.343
.8565	0.930	10.718	10.501	10.442	.7351	0.969	10.508	10.388	10.328
.8695	0.950	10.612	10.459	10.385	.7494	0.991	10.424	10.312	10.275
8187.7785	0.365	10.583	10.323	10.288	.7571	0.003	10.418	10.317	10.293
.7886	0.381	10.625	10.343	10.293	.7697	0.023	10.414	10.320	10.272
.8045	0.406	10.622	10.323	10.290	.7783	0.036	10.418	10.301	10.264
.8192	0.429	10.621	10.340	10.287	.8048	0.077	10.448	10.296	10.254
.8312	0.447	10.663	10.362	10.288	.8133	0.091	10.451	10.305	10.263
.8504	0.477	10.673	10.381	10.337	.8286	0.114	10.450	10.301	10.241
.8609	0.494	10.663	10.404	—	.8371	0.128	10.485	10.290	10.259
8188.7668	0.904	10.876	10.608	10.538	.8535	0.153	10.486	10.305	10.277
.7799	0.924	10.777	10.528	10.470	.8612	0.165	10.482	10.314	—
.8074	0.967	10.503	10.374	10.332	8191.7972	0.623	10.752	10.467	10.403
.8169	0.982	10.462	10.366	10.349	.8075	0.639	10.776	10.455	10.374
.8341	0.009	10.418	10.335	10.256	.8230	0.663	10.750	10.465	10.402
.8485	0.031	10.424	10.309	10.282	.8321	0.677	10.762	10.486	10.435
.8638	0.055	10.439	10.314	10.263	.8467	0.700	10.777	10.504	10.440
8189.7369	0.415	10.615	10.348	10.283	8192.7882	0.166	10.489	10.304	10.278
.7466	0.430	10.646	10.360	10.288	.7960	0.178	10.479	10.298	10.271
.7631	0.455	10.638	10.355	10.308	.8096	0.199	10.506	10.302	10.282
.7727	0.470	10.651	10.350	10.309	8193.7986	0.739	10.813	10.523	10.488
.8007	0.514	10.659	10.384	10.320	.8096	0.756	10.841	10.538	10.477
.8089	0.527	10.676	10.389	10.325	.8247	0.780	10.834	10.547	10.523
.8249	0.552	10.702	10.420	10.368	.8351	0.796	10.892	10.580	—

3. ANALYSIS

In this section we analyze the program variable stars using the IF method which has been previously described and discussed in detail by Fernley et al. (1990a), and the SB version of the B-W method described by Cacciari et al. (1989b) with further modifications discussed in § 3.2. A few problems common to both methods are discussed below:

1. The factor p used to convert from radial to pulsational velocities when deriving the linear radius displacement has been taken as 1.36, which is the appropriate value for CORAVEL radial velocities (Burki, Mayor, & Benz 1982).

TABLE 7

INPUT PARAMETERS FOR THE PROGRAM VARIABLES

Parameter	UU Cet	RV Phe	W Tuc
P (days)	0.6061	0.5964	0.6422
$\langle m_V \rangle$	12.085	11.920	11.460
$\langle m_K \rangle$	10.850	10.721	10.354
$E(B-V)$	0.025	0.015	0.005
[Fe/H]	−1.00	−1.50	−1.35

2. The stellar gravity has been assumed to be constant over that part of the pulsation cycle where we apply our analysis, and the value has been taken as $\log g = 2.75$. It has been questioned (B. W. Carney 1991, private communication) whether the assumption of a constant gravity for all RR Lyrae stars will not introduce some systematic bias. The constancy of the gravity over most of the pulsation cycle, except a sudden increase on a small interval (≤ 0.2 in phase) around maximum light, had been already shown by Lub (1977) and later confirmed by many authors. Its absolute value may vary slightly from star to star, but assuming the plausible range of values for masses and radii of RR Lyraes are 0.5–0.7 $M_\odot$ and 4.5–6.5 $R_\odot$, then $\log g$ is between 2.5 and 3.0. This is confirmed by the data of Table 9 in Jones et al. (1992, hereafter J92), which lists the results of the most recent and supposedly accurate B-W reanalysis of field RR Lyrae stars, and from which one can derive an average value for the stellar mean gravity $\log g = 2.83 \pm 0.11$. The only reason $\log g$ is needed in the IF method is to determine the infrared fluxes at $H = 16500$ Å and $K = 22000$ Å. From Table 10 of Fernley et al. (1989), it can be seen that for the temperature range of RR Lyraes an increase from 2.5 to 3.0 in $\log g$ causes ~0.5% increase in flux. From the error discussion in the same paper, § 5.1.5, such an increase

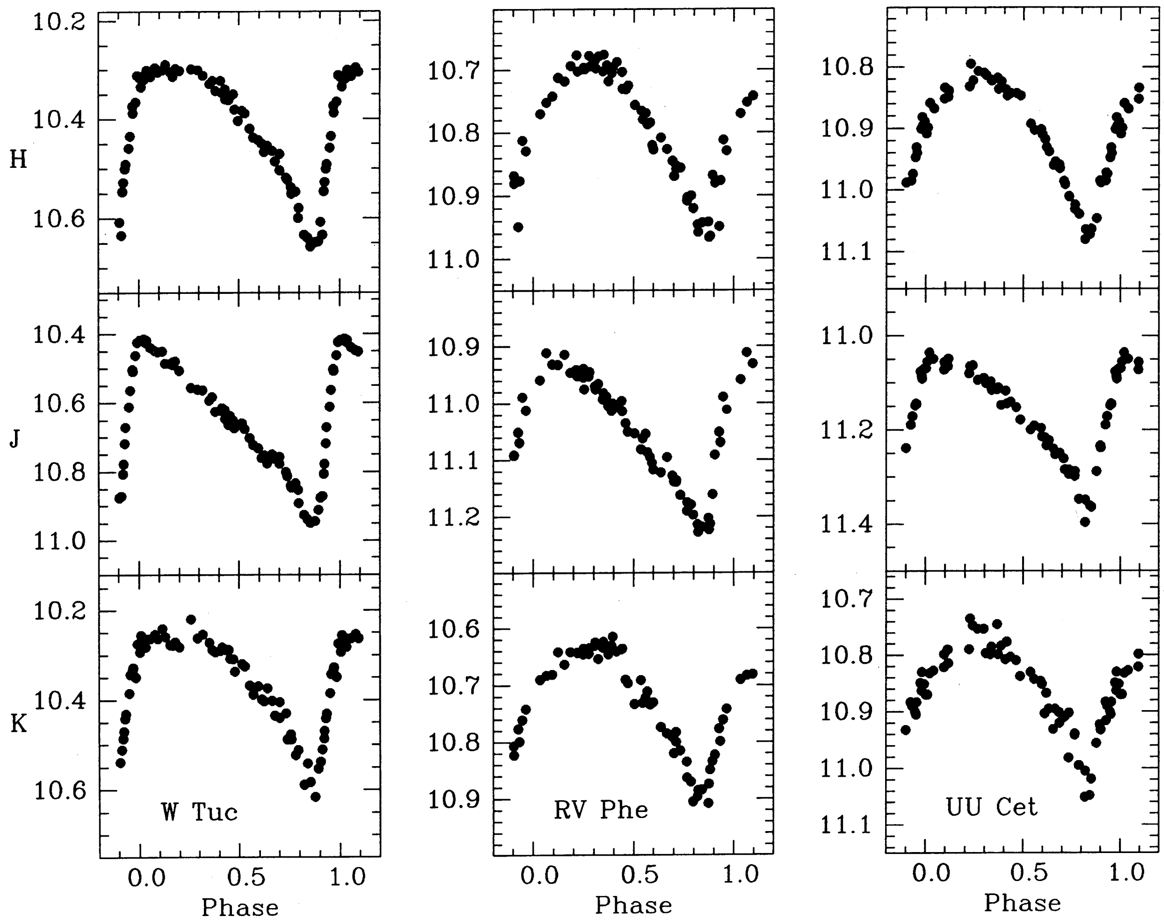

FIG. 1.—H, J, and K light curves for W Tuc, RV Phe, and UU Cet

in infrared flux changes the derived temperatures by $\sim$10 K which is equivalent to less than 0.01 mag in M_V. In the SB method, the gravity is used along with the metallicity in the selection of the most appropriate color-temperature relation, which is insensitive to a 0.1 dex error in the adopted value of the gravity; moreover, the physical parameters obtained from the SB analysis are then used to determine a new value of the stellar mean gravity, and the analysis is repeated iteratively, if necessary, until the initial and final values agree within 0.1 dex. It can be argued that the large outward accelerations in the atmosphere just before maximum light mean that the effective gravity ($=GM/R^2 + d^2r/dt^2$) is significantly different from the static gravity ($=GM/R^2$). Thus, if effective gravity is considered, there is a phase-dependent variation in the gravity. However, as will be argued in the next part of this section, the phase region around maximum light should not be used in B-W analyses, so the point is irrelevant.

3. In the IF method the effective temperatures, along with the angular radii, are determined from integrated plus infrared fluxes, taken from Kurucz (1979). In the SB method the temperatures are obtained from the $V-R$, $V-I$, and $V-K$ colors, using Kurucz (1979) unpublished model atmospheres for the visual colors, and the semiempirical calibration of Fernley (1989) for the infrared colors. The latter calibration, which is appropriate for $V-K$ colors in the Johnson system, is based on an empirical $(V-K)$–T_e relation from 38 Population I main-sequence stars ([m/H] = 0.0, log $g = 4.0 \pm 0.1$), and on a grid of synthetic colors calculated from Kurucz models, which define the color offsets with respect to the synthetic Population I main-sequence relation. The shape and zero point of the adopted calibration are therefore empirical, while the relative behavior at different values of gravity and metallicity is taken from model atmospheres for lack of accurate effective temperature determinations of low-gravity low-metallicity stars.

Referring now to the problem of defining correctly the equilibrium temperature T_e for a variable star, i.e., the temperature the star would have were it not pulsating, this has been discussed by many authors, the most recent summary being that by Carney, Storm, & Jones (1992). A direct time estimate over the pulsation cycle, as suggested by Davis & Cox (1980), is not strictly correct since it averages roughly LTE with non–LTE temperatures. The temperatures around maximum light, in fact, are affected by two problems acting in the opposite sense: (1) they may be overestimated because of the presence of shock waves in the atmosphere, this effect being stronger for the bluer colors, and (2) they may be underestimated in analogy with the theoretical optical color-temperature calibration for solar metallicity dwarfs, which lies systematically below the empirical one at temperatures around $\sim$7000 K and hotter (Cacciari et al. 1989b). It is very difficult to quantify the net effect, especially because the effect of shocks on colors is not well known. The use of infrared photometry should in principle provide more accurate temperatures since first, it minimizes the problem of shock-induced temperature distortions, and second, it uses a color-temperature calibration whose shape and zero point are based on empirical data. However, the B-W method provides us with an estimate of the star mean luminosity and radius; therefore, the equilibrium temperature can be obtained by definition from the Stefan-Boltzmann law.

4. As discussed in detail by J92, only part of the pulsation cycle is suitable for the application of the B-W method. There are two issues that determine the most appropriate phase interval to use for B-W analyses. First, numerical considerations favor using as large a phase interval as possible in order to minimize the effect of random errors, particularly in the infrared light curves. Second, physical considerations require only the use of phases where the star is well represented by a series of static, LTE model atmospheres. In this regard, several phase intervals are problematic:

1. In the phase interval 0.8–1.0, there is evidence of shock waves in the atmospheres of many RR Lyraes (e.g., Preston & Paczyński 1964; Gillet & Crowe 1988; Clementini et al. 1991a and references therein) as outward-moving material from the new pulsation collides with infalling material from the old pulsation. Emission from shock-heated material is clearly a non–LTE radiation source that is not included in the model atmospheres.

2. In the phase region around 0.65–0.70, many RR Lyraes show a noticeable "bump" on their light curve. The mechanism responsible for this "bump" is unclear but may well be an echo as the original inward moving pulsation wave bounces off the stellar core (Christy 1966). Whatever the mechanism, there is evidence that again there is a shock wave in the atmosphere (Gillet & Crowe 1988), though a much weaker one than that just preceding maximum light.

3. In the phase interval 0.0 to approximately 0.4, B-W analyses using optical colors to determine temperatures show a poor fit between the photometric and spectroscopic angular radii (e.g., Carney & Latham 1984; Cacciari et al. 1989b; CCB). A detailed analysis of this problem by Jones (1988) suggested there was a general problem in this phase region with the model atmospheres predicting too little flux at B and too much flux at R_C and I_C compared to the observed values (see also § 3.1.1 for a more detailed discussion). This problem, however, does not seem to affect significantly the infrared K fluxes, as the photometric angular radii derived from $V-K$ colors or K fluxes appear to match the spectroscopic radii quite well, at least for $\phi \geq 0.2$ (Jones et al. 1988; Liu & Janes 1990; Fernley et al. 1990a and references therein). The phase interval that can be used with $V-K$ colors may therefore be significantly larger than the intervals used with the optical colors, and the infrared results are correspondingly more accurate and reliable.

3.1. *The IF Method*

The IF method (Blackwell & Shallis 1977) requires light curves at ultraviolet, optical, and infrared wavelengths in order to evaluate the integrated flux at all phases of the pulsation cycle. The integrated flux plus an infrared flux are then used to determine both the effective temperature and angular radius of the RR Lyrae as a function of phase. By matching the angular radius and the integrated radial velocity curve the mean radius and distance are then determined by conventional B-W analysis. The method and the procedures are exactly the same as those used and described in detail in Fernley et al. (1990a), to which the interested reader is referred. It was mistakenly stated in Fernley et al. (1990a) that the model atmosphere H and K fluxes were from MARCS models (Gustafsson et al. 1975). The fluxes in that paper, as in this paper and previous ones, are from Kurucz (1979).

3.1.1. *Comparison between Observed and Model Fluxes*

Twelve RRab Lyraes have now been analyzed using the IF method, X Ari (Fernley et al. 1989); DX Del (Skillen et al. 1989); V445 Oph, VY Ser, and SS Leo (Fernley et al. 1990a); UU Cet, RV Phe, and W Tuc (this paper); WY Ant, BB Pup, W Crt, and RV Oct (Skillen et al. 1992). For each of these stars we have light curves in the Walraven W, Johnson $BVJHK$, and Cousins RI filters. All these light curves have been converted from magnitudes to monochromatic fluxes at the effective wavelengths of the filters (the flux conversions and effective wavelengths are given in Table 8 of Fernley et al. 1989) and the resulting monochromatic flux curves have been fitted to Fourier series. Thus for all 12 RRab Lyraes, we have at any phase the observed flux at eight wavelengths from the ultraviolet through the optical and into the infrared. We have extracted from Table 6 of Kurucz (1979) the model fluxes at the same wavelengths at values of $\log g = 2.75$, $[M/H] = 0, -1$, and -2, and $T_e = 5500\ (500)\ 8500$. Interpolating in this grid in the [M/H] appropriate to each of the 12 RR Lyraes, we then have, for all 12 stars and eight wavelengths, the model fluxes as a function of temperature. The IF method was run for each of the RR Lyraes to give temperatures and angular radii as a function of phase. These temperatures and angular radii were also fitted to Fourier series. Thus at any phase we can recall the temperature and angular radius of each RR Lyrae and hence calculate the ratio of model to observed flux at all wavelengths.

Because of the way the IF method works, the ratio of the model to the observed flux at the effective wavelength of the K filter (22,000 Å) is unity for all the RR Lyraes at all phases. For the remaining wavelengths the difference in this ratio from unity is a measure of how well the models match the real stars at that phase. We have looked at these ratios for all 12 stars; however, the interpretation for individual stars is complicated by the effects of random errors in both $E(B-V)$ and the absolute calibration of the photometry and, more seriously, by "wiggles" in the Fourier series due to uneven spacing of the data points. In order to smooth these random errors and bring out the systematic effects (which are what we are interested in), we have combined the ratios at each wavelength and each phase from all 12 stars. The results are illustrated in Figure 2 where we plot $R(\lambda) =$ model flux/observed flux as a function of phase for the following wavelengths: 3255 (Walraven W), 4445 and 5540 (Johnson B and V), 6440 and 7940 (Cousins R and I), 12500 and 16500 (Johnson J and H).

Several systematic effects can be clearly seen in Figure 2. Most strikingly we note the sudden decrease in the ratio at W and an increase in the ratio at the optical wavelengths around phase 0.9. The infrared wavelengths are unaffected. This is probably a combination of two effects: first an increase in effective gravity due to the large atmospheric accelerations as the star changes from contraction to expansion, and second, emission as a shock wave travels outward through the upper atmosphere. Considering first the gravity, we have assumed a value $\log g = 2.75$ throughout the pulsation cycle. However, if this were increased around phase 0.9, the model fluxes, and hence the ratios, would increase at W and decrease in the optical, the effect being much larger at W. Considering now the shock wave, as collisionally ionized hydrogen recombines into the $n = 2$ level, there will be radiation at wavelengths short-

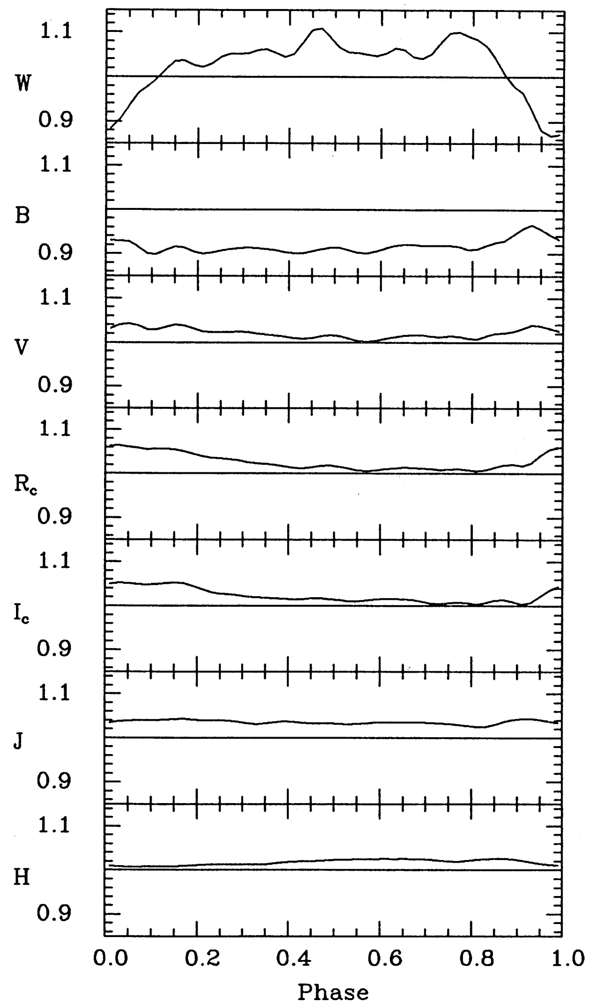

FIG. 2.—The ratios of model flux/observed flux at the effective wavelengths of the Walraven *W* filter, the Johnson *BVJH* filters, and the Cousins *R* and *I* filters vs. phase. The ratios are the average values for the 12 RRab Lyraes listed in the text and are normalized to the Johnson *K* filter.

ward of the Balmer discontinuity. In terms of the IF method this radiation will increase the integrated flux and hence the temperature, but it will not increase the temperature enough to account for the observed *W* flux. On the other hand, it increases the temperature too much to account for the observed fluxes in the other filters since these are not significantly affected by the emission from the shock-heated material. However, both the shock emission and the change in effective gravity should be short-lived effects, whereas it can be seen in Figure 2 that the model flux deficiency at *W* and excess in the optical, notably R_C and I_C, continue until about phase 0.35 where all the flux ratios settle down to an approximately constant value until about phase 0.85. The phase range 0.00–0.35 is when the star is expanding and cooling rapidly; the phase range 0.35–0.85 is when the star contracts nearly isothermally.

In order to understand this behavior, we have looked at the standard star Procyon = α CMi = BS 2943. The star has magnitudes, on the Johnson system unless otherwise stated, $B = 0.79$, $V = 0.37$, $R_C = 0.09$, $I_C = -0.14$ (Johnson et al. 1966, with *R* and *I* transformed to the Cousins system using the relations given by Bessell 1983), $J = -0.40$, $H = -0.60$, and $K = -0.65$ (Koornneef 1983 and references therein). The star has no Walraven photometry; however, it was observed in the 13 color system of Johnson & Mitchell (1975). These authors give a flux calibration for their system from which we obtain the flux at $\lambda_{\text{eff}} = 3370$. Examination of the Kurucz models appropriate to Procyon ($T_e = 6500$, log $g = 4.0$, [M/H] = 0 from Steffen 1985) and the filter characteristics of Johnson & Mitchell and Walraven implies that at $\lambda_{\text{eff}} = 3255$ of the Walraven *W* filter the Johnson & Mitchell flux should be reduced by ~3%. For the other filters, the effective wavelengths and conversions from magnitudes to fluxes use the data given in Table 8 of Fernley et al. (1989). Using these observed fluxes and the same program and procedures used for the RR Lyraes, we obtain $T_e = 6586$. Going back into the Kurucz models, we find the predicted fluxes at $T_e = 6586$, log $g = 4.0$, [M/H] = 0, and thus the ratios model flux/observed flux. After normalizing to unity at *K* these ratios are *R* = 1.09 (*W*), 0.93 (*B*), 1.03 (*V*), 0.97 (*R*), 0.99 (*I*), 1.02 (*J*), 1.02 (*H*), and 1.00 (*K*). Comparison of these ratios with Figure 2 shows that during the isothermal contraction from ~0.35 to ~0.85, the RR Lyraes and Procyon have a similar behavior, i.e., a model flux excess at *W* and deficiency at *B* with the other filters being close to unity. There are only two possible explanations for this difference: either there is an error in the conversion of magnitudes to fluxes or there is an error in the models. Looking first at the flux conversions we note that the fluxes listed in Table 8 of Fernley et al. (1989) are for Vega except in the case of the *W* filter for which it is for another A0 V standard star, 109 Vir. Tüg, White, & Lockwood (1977) list the observed monochromatic fluxes for both 109 Vir and Vega, and using these data we transform the flux conversion of *W* to Vega. Adopting $T_e = 9660$, log $g = 4.0$ and [M/H] = 0 for Vega (Code et al. 1976), we interpolate in the Kurucz models to obtain the model flux for this star, and once again normalizing at *K* we obtain the following ratios: 1.01 (3255), 1.02 (4445), 1.03 (5540), 1.03 (6440), 1.02 (7940), 1.01 (12,500), 0.99 (16,500), and 1.00 (22,000). It can be seen that there is close agreement between the model and observed fluxes for Vega, which implies the flux conversion is correct. Of course for a different spectral type the ratio of monochromatic flux, $F(\lambda)$, to a magnitude, $\int F(\lambda)S(\lambda)d\lambda$, where $S(\lambda)$ is the filter response function, will change. This effect was investigated in Fernley et al. (1989) and Skillen et al. (1989), and correction factors appropriate to RR Lyrae stars were calculated in these papers. These correction factors are typically a few percent and have been applied both to the RR Lyraes and Procyon. We thus conclude that there is a temperature-dependent error in the models which leads to a systematic underestimate of flux at *B* and overestimate at *W*, with some effect also on the other optical filters (see Fig. 2). A detailed analysis on the nature of this error is beyond the scope of this paper; we only recall here that theoretical UV fluxes in Population II stars with $T_e \sim 6000$–7000 K have been found to be higher than the corresponding observed ones, as some sources of opacity (e.g., Si I and Mg I lines, continuous opacity of Fe I) were not included in Kurucz (1979) model atmospheres (Bonnell & Bell 1985; Magain 1987; Cacciari et al. 1987b). This temperature-dependent error in the models is also confirmed by the relative behavior of the temperatures estimated from optical colors with respect to those derived from $V-K$ colors at different phases of the RR Lyrae pulsation cycle. For example, while at phases 0.50–0.80 (i.e., at minimum light and temperature) the temperatures derived from $B-V$, $V-R$, and $V-I$ colors are systematically higher than those derived from

TABLE 8

THE RADII AND DISTANCES OF THE STARS

		Using H Flux[a]				Using K Flux[a]			
Star	Phase Interval	$R/R_{\odot}$	d(pc)	N^{b}	σ^{c}	$R/R_{\odot}$	d(pc)	N^{b}	σ^{c}
UU Cet	0.35–0.85	5.63	1864	30	0.51	5.88	1911	29	1.05
RV Phe	0.35–0.85	5.36	1681	31	0.73	4.86	1490	29	0.96
W Tuc	0.35–0.85	5.70	1527	32	0.61	6.07	1582	30	0.86

NOTE.—The outlier rejection removes a data point if its standard deviation is $>2\sigma$ (defined in footnote c below) and $>2.5\%$.

[a] H and K fluxes refer to the infrared flux in eq. (2) of Fernley et al. 1990a.

[b] Number of infrared data points in the solution.

[c] Defined as $\sum_{i=1}^{N} |\theta_{\rm spec}^{i} - \theta_{\rm phot}^{i}|/\theta_{\rm spec}^{i}$, where $\theta_{\rm phot}$ and $\theta_{\rm spec}$ are given by eqs (1) and (2) and (5) and (6), respectively, in Fernley et al. 1990a.

$V-K$ colors by approximately 30, 40, and 20 K, respectively, which can be seen as a zero-point difference in the various color-temperature conversion scales, these differences become approximately 220, 190, and 90 K at $\phi = 0.10$, and 80, 120, and 60 K at $\phi = 0.40$. We recall here that an error of 100 K in the temperature of the star leads to an uncertainty of approximately 3% in the integrated flux, which results in an error of ~0.06 mag in the final absolute magnitudes (Fernley et al. 1990a). From the point of view of B-W analysis, the existence of this error suggests that the phase interval ~0.35 to ~0.85, where the temperature variation is relatively small and radius variation relatively large, should be preferred. Finally we note that in the phase region around 0.65–0.70, where there is a pronounced "bump" on many RRab Lyrae light curves and where Gillet & Crowe (1988) observed evidence of shock wave activity (weak Hα emission), there is no anomalous flux distribution evident in Figure 2.

3.1.2. *Results*

The results of the analysis are given in Table 8 and illustrated in Figures 3 and 4. In Figure 3 we show the temperatures of the three stars during the pulsation cycle. It can be seen that the temperature amplitude of W Tuc is appreciably larger than for the other two stars, and also that the temperatures derived using the H flux (in eq. [2] of Fernley et al. 1990a) are larger by typically ~60 K than those derived using the K flux. This effect has been seen in all the RR Lyrae stars analyzed with the IF method (Fernley et al. 1989, 1990a, b) and is also present in other applications of the method to A-type and later stars, e.g., Leggett et al. (1986) and references therein. Clearly there is a relative error between H and K of ~3%, this error being in either the absolute calibration of infrared photometry or the model atmosphere infrared fluxes. This effect will be discussed more fully in § 4.1.

In Table 8 we list the derived values of the radii and distances of the stars again using both the H and K fluxes and the phase interval 0.35–0.85. The radii and distances are estimated using an iterative least-squares fitting routine that was described in Fernley et al. (1989). Finally in Figure 4 a visual display of the results from Table 8 is shown. Here we plot the "photometric" and "spectroscopic" angular radii as a function of phase. The plot is analogous to Figure 11 of Fernley et al. (1990a) where the two forms of angular radius are defined. It can be seen in Figure 4 that in the phase region 0.35–0.85 the fits for UU Cet and W Tuc are satisfactory but less so for RV Phe. As discussed in § 3.2.1 this star may suffer from phasing problems. It can also been seen in Figure 4 that, recalling the discussion in § 3.1.1, the points in the phase interval 0.85–1.00 lie systematically above the curve for all three stars and for W Tuc, which has the largest amplitude, the points in the phase interval 0.00–0.15 continue to fall above the line.

A summary of the derived quantities for the three stars using the IF method is given in Table 9. To derive the luminosities we assume that the phase of $\langle M_{\rm bol}\rangle$ is the same as the phase of $\langle M_V\rangle$. From Figure 3 we then obtain the temperature at this phase and from Table 7 of Liu & Janes (1990) we obtain the BC_V at the appropriate temperature and metallicity of each star and again assuming log $g = 2.75$ for all the stars. Finally we take $M_{\rm bol}(\odot) = 4.75$ (Allen 1976). The masses have been derived from the pulsation equation in van Albada & Baker

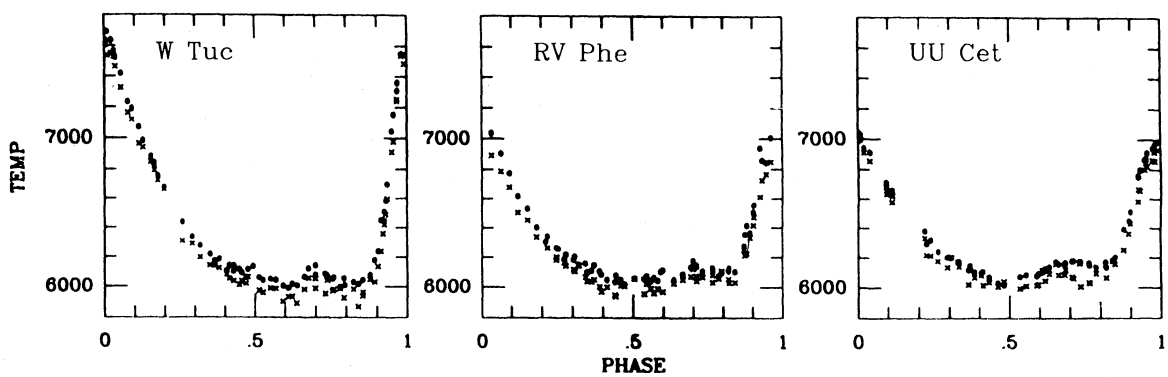

FIG. 3.—Effective temperatures for W Tuc (*left*), RV Phe and UU Cet (*right*). *Filled circles*: H as the infrared flux; *crosses*: K as the infrared flux.

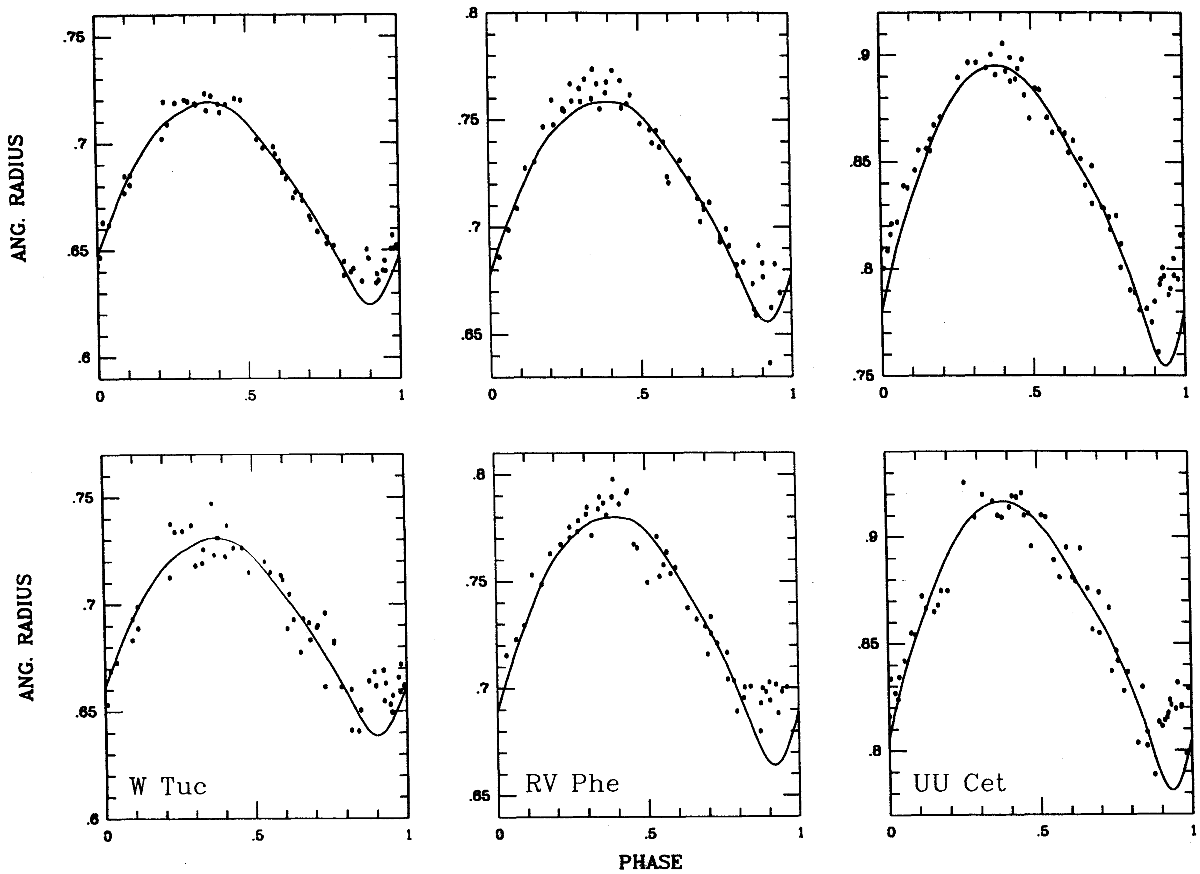

FIG. 4.—Photometric (*filled circles*) and spectroscopic (*solid line*) angular radii for W Tuc (*left*), RV Phe, and UU Cet (*right*). Upper panel uses H as the infrared flux, and lower panel uses K.

(1971) where, following the discussion in Carney et al. (1992), we use the equilibrium temperature defined from the Stefan-Boltzmann law.

An extensive error analysis is given in Fernley et al. (1990a), and this is applicable to the present work with the additional comments that the error in reddening $E(B-V)$ is certainly small for these stars, and the "fitting" error must include some allowance for the extra uncertainty due to the phasing problems discussed in § 2.1. Overall we believe $\Delta M_V = 0.17$.

3.2. *The SB Method*

In this paper we have applied two slightly different versions of the SB method: (*a*) the procedure described and applied by Cacciari et al. (1989a) and CCB, where both V magnitudes and $V-R$ and $V-I$ colors are read at evenly spaced phase intervals on the corresponding smoothed curves (in the following we shall call it "first procedure" for sake of simplicity); and (*b*) a procedure similar to the IF method, where the R, I, and K observed data points are used to derive the colors along with the V magnitudes read on the smoothed V light curves at the corresponding phases (hereafter called "second procedure").

From the colors thus derived the temperatures and bolometric corrections (hence surface brightnesses S_V) are calculated using the appropriate color versus temperature calibrations and Kurucz (1979) model atmospheres. Then for each value of S_V and V_0, a value of the angular diameter θ is calculated, and the θ curve thus derived is matched with the linear displacement ΔR curve, obtained by integrating the radial velocity curve, to find the distance.

The first procedure has been applied only with the optical colors, since the V and K photometries are not simultaneous, and allows a direct comparison with the previous results by Cacciari et al. (1989a and CCB). The second procedure has been applied with the infrared colors in order to have a direct comparison with the IF method, and with the optical colors in order to test the sensitivity of both procedures to the various assumptions and approximations. A detailed discussion is given in § 4.1; here we present the results of both SB procedures applied to the program stars.

TABLE 9

SUMMARY OF RESULTS FROM THE INFRARED FLUX METHOD

Parameter	UU Cet	RV Phe	W Tuc
$\langle R/R_\odot \rangle$	5.75	5.11	5.88
d(pc)..............	1887	1585	1555
$\langle M_V \rangle$	0.626	0.872	0.485
$\langle M_K \rangle$	−0.537	−0.284	−0.606
$\langle \log L/L_\odot \rangle$	1.678	1.583	1.738
$\langle M/M_\odot \rangle$	0.70	0.53	0.68

3.2.1. *RV Phoenicis*

The SB method using the first procedure and the optical colors was applied to RV Phe by CCB. We repeat this analysis

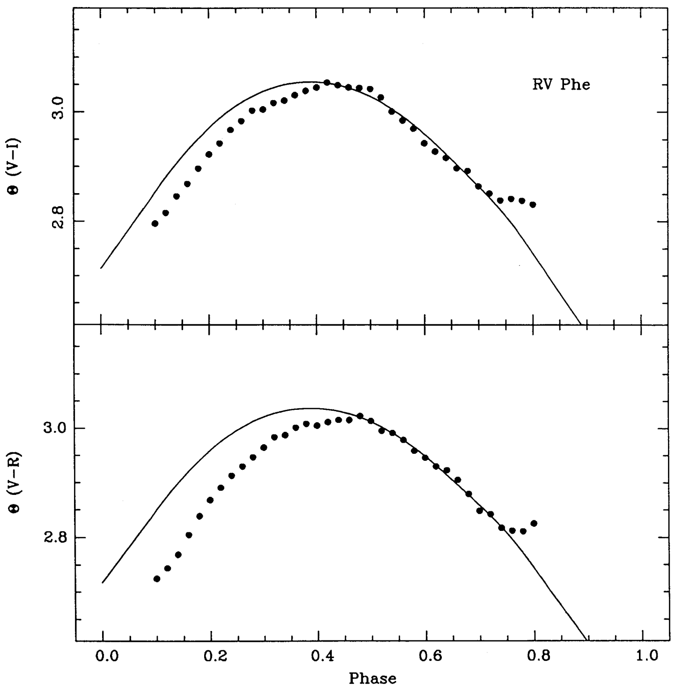

FIG. 5.—The angular diameter θ vs. phase for RV Phe, as derived using the first procedure and $V-R$ (*bottom*) and $V-I$ (*top*) colors. The solid lines represent the radial displacements ΔR which best fit the θ curves over the phase intervals 0.48–0.74 and 0.42–0.74, respectively, and correspond to $M_V = 0.65$ and 0.69.

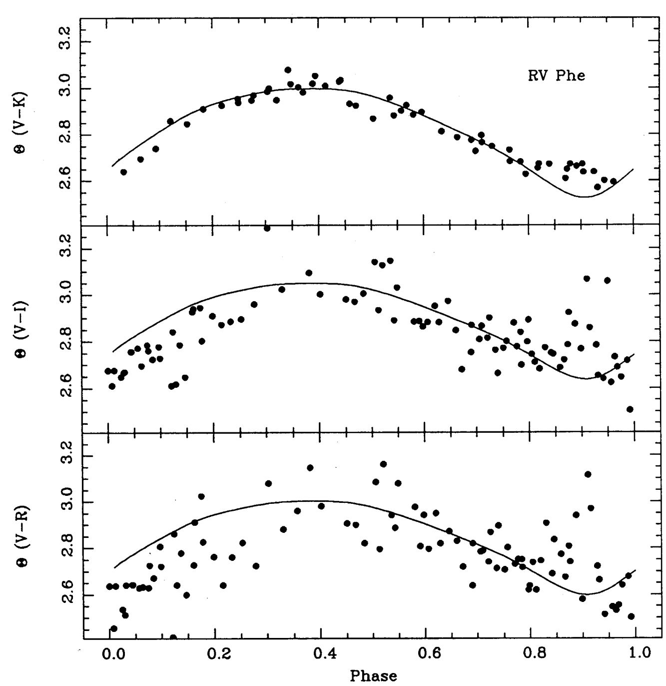

FIG. 6.—The same as in Fig. 5, as derived using the second procedure and $V-R$ (*bottom*), $V-I$ (*middle*), and $V-K$ (*top*) colors. The solid line in the $V-K$ panel corresponds to the best fit over the phase interval 0.35–0.85 and $M_V = 0.97$, while the lines in the $V-R$ and $V-I$ panels are the same as in Fig. 5 (see text).

here using the presently adopted values for the reddening and γ-velocity (see §§ 2.3 and 2.4), and we obtain the values of angular diameter θ shown in Figure 5. The solid lines represent the linear radius variations ΔR which best fit (i.e., linear least-squares regression) the θ curves over the phase intervals 0.48–0.74 (using $V-R$ colors) and 0.42–0.74 (using $V-I$ colors), and correspond respectively to distances $D = 1759$ and 1727 pc, and absolute magnitudes $M_V = 0.65$ and 0.69 using $V_0 = 11.872$. As discussed in § 3, item [4], the phase intervals are defined by the problems affecting the optical energy distribution, i.e., flux excess at $\phi \leq 0.40$ and a bump around phase 0.75, which are somewhat emphasized by the first procedure (see § 4.1). The present estimate of absolute magnitude from visual color is $\sim$0.1 mag brighter than the value found by CCB, the difference being mostly due to the different γ-velocity used in the analysis. The effect of the γ-velocity on the derived magnitude, however, cannot be simply extrapolated from the result on RV Phe. We have performed simulations on W Tuc and UU Cet repeating the analysis with slightly different radial velocity curves and γ-velocities, and we have found magnitude differences $\leq$0.03 and 0.05 mag for differences in γ-velocity $\sim$0.2 and 0.07 km s^{-1} in W Tuc and UU Cet, respectively. Therefore, the effect of the γ-velocity combines somehow with the characteristics of the angular diameter curves to provide the final accuracy of the fitting.

The second procedure was applied with the $V-K$ colors, and the results are shown in Figure 6, where the fit has been performed over the phase interval 0.35–0.85 and corresponds to $D = 1515$ pc and $M_V = 0.97$ mag. This result is consistent with the results of the IF method but is significantly fainter than the absolute magnitudes derived from optical colors using the first procedure. We have therefore applied the second procedure to the optical colors as well; however, the large scatter of the θ data points makes the numerical fitting quite inaccurate and unstable. A visual inspection of the ΔR curves corresponding to the previous solutions superposed on the "optical" θ points shows that the bright magnitude solutions seem to match the data slightly better than the faint magnitude one, although the scatter in the data does not exclude any of them. Therefore, we do not consider the results of the second procedure with optical colors as conclusive for RV Phe, and we shall not use them in the following considerations.

From the $V-K$ colors we have determined the mean temperature $\langle T \rangle = 6290$ K by averaging over the entire pulsation cycle, with the only purpose of deriving an estimate for the bolometric correction (i.e., $BC_V = -0.09$ mag). Then, from the results of the first procedure with the optical colors and the second procedure with the infrared colors, one derives the stellar luminosities $\log L/L_\odot = 1.668$ and 1.548 respectively, assuming $M_{bol}(\odot) = 4.75$ (Allen 1976). By direct integration of the radial velocity curve, one obtains the mean radius of the star $\langle R \rangle = 5.42\ R_\odot$ and $\langle R \rangle = 4.59\ R_\odot$, respectively, with a radius variation $\Delta R/R = 14\%$ and 17%. The method suggested by Lub (1977) to calculate the average radius, i.e., $\langle R \rangle = \frac{2}{3}R_{max} + \frac{1}{3}R_{min}$ leads to a value that agrees to better than 1% with the values derived from direct integration of the radial velocity curve. From the Stefan-Boltzmann law the equilibrium temperatures can be estimated, i.e., $T_e = 6486$ and 6578 K from optical and infrared colors, respectively, which in turn lead to values of mass and gravity $M = 0.62\ M_\odot$ and log $g = 2.76$, and $M = 0.41\ M_\odot$ and log $g = 2.73$ from the stellar pulsation theory (van Albada & Baker 1971). These results are also summarized in Table 10.

As we have noticed above, the results of the analysis using

TABLE 10

SUMMARY OF RESULTS FROM THE SURFACE BRIGHTNESS METHOD

Parameter	UU Cet	RV Phe	W Tuc
$\langle R/R_\odot \rangle$	5.36, 5.50	5.42, 4.59	5.94, 5.85
d(pc)	1825, 1982	1743, 1515	1667, 1601
$\langle M_V \rangle$	0.70, 0.52	0.67, 0.97	0.33, 0.42
$\langle M_K \rangle$	−0.46, −0.64	−0.49, −0.19	−0.76, −0.67
$\langle \log L/L_\odot \rangle$	1.648, 1.720	1.668, 1.548	1.794, 1.758
$\langle M/M_\odot \rangle$	0.59, 0.62	0.62, 0.41	0.70, 0.67

NOTE.—Optical colors (*left*) and $(V-K)$ colors (*right*).

infrared colors, although internally more accurate, are rather discrepant from those that make use of the optical colors and lead to physical parameters of the star which are hardly consistent with the values predicted by the stellar evolution theory for an RR Lyrae star. Since, as we shall see later, the results on the two other program stars are much more consistent, it may well be that RV Phe has some problem, e.g., Blazhko effect and/or varying period, that was not taken properly into account in the present analysis. Although no clear evidence of the Blazhko effect can be seen, the data do show some larger scatter. On the other hand, there is a clear indication that the period is varying, but the observed epochs are too few to allow an accurate determination of the ephemerides. In order to take these possible effects into account, one would need strictly simultaneous spectroscopy and photometry. Since the *JHK* photometry has been taken a few years later, the match is probably not totally correct and more trust should be given to the results obtained from optical data which were taken simultaneously with the radial velocity data. An extensive error analysis is given in Cacciari et al. (1989b) and is applicable to the present work as well. We adopt in general the error $\Delta M_V = 0.20$, which is "intrinsic" to the method and does not take into account possible systematic effects. For RV Phe, however, the accuracy may be worse for the reasons discussed above, and we have adopted an error 0.25 mag.

3.2.2. *UU Ceti*

The same procedures of the SB method described above have been applied to UU Cet using the optical data published by CCL and the infrared data listed in Table 4. Figure 7 shows the values of ΔR and θ derived from the first procedure. The match was performed over the phase interval $0.48 \leq \phi \leq 0.74$ in order to avoid the very weak bump around phase 0.75. The result of this fit yields distances $D = 1718$ and 1932 pc from $V-R$ and $V-I$, respectively. The average distance is then $D = 1825$ pc, and from the mean dereddened magnitude $V_0 = 12.005$ mag we derive the absolute visual magnitude $M_V = 0.70$.

In Figure 8 we show the results of the second procedure applied to the optical and infrared colors, where the fit was performed over the phase intervals 0.45–0.85 for the optical colors and 0.35–0.85 for the infrared colors. The fit with the optical colors can be performed as far as $\theta = 0.85$ since there is no evidence of bumps in the θ curve, and provides distances $D = 1724$ and 1981 pc from $V-R$ and $V-I$, respectively, the average value being 1853 pc and $M_V = 0.67$, in very good agreement with the results of the first procedure. The fit with the infrared colors leads to $D = 1982$ pc and $M_V = 0.52$ mag, and all these values compare quite well among themselves and

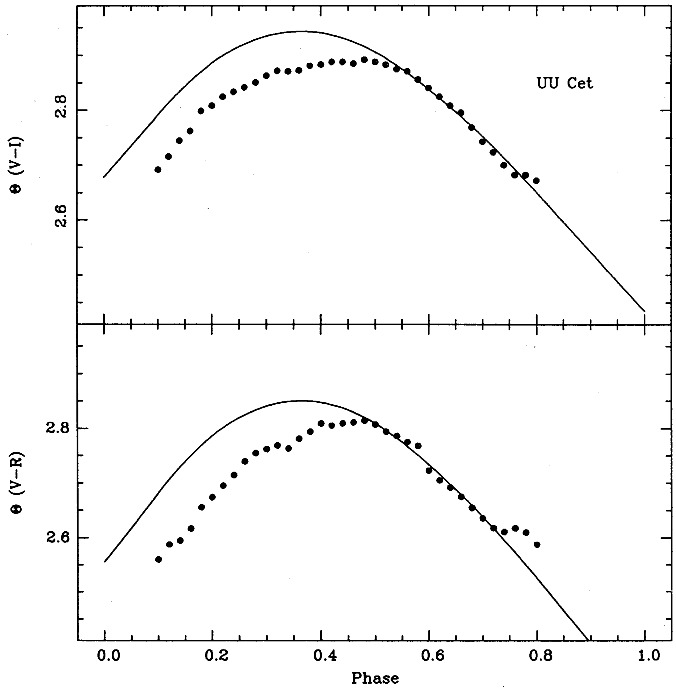

FIG. 7.—The angular diameter θ vs. phase for UU Cet, as derived using the first procedure and $V-R$ (*bottom*) and $V-I$ (*top*) colors. The solid lines represent the radial displacements ΔR which best fit the θ curves over the phase interval 0.48–0.74 and correspond to $M_V = 0.83$ and 0.57, respectively.

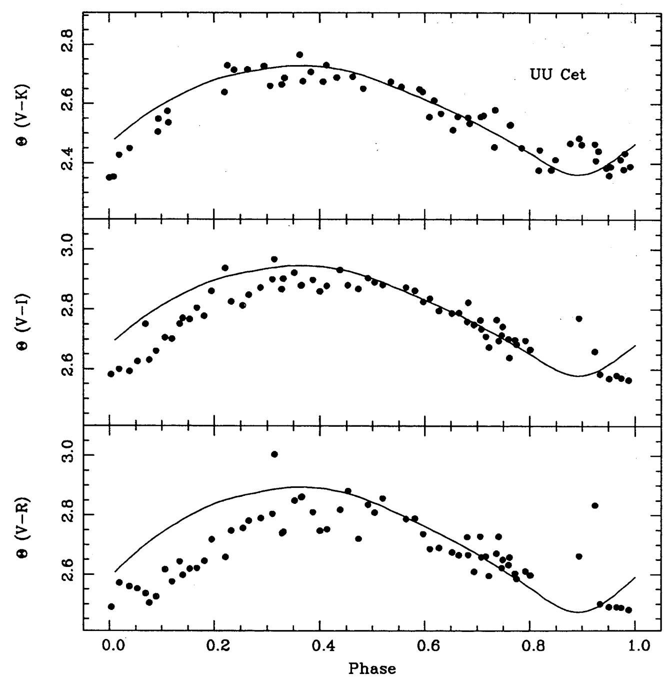

FIG. 8.—The same as in Fig. 7, as derived using the second procedure and $V-R$ (*bottom*), $V-I$ (*middle*), and $V-K$ (*top*) colors. The solid lines correspond to the best fit over the phase intervals 0.45–0.85 for the optical colors and 0.35–0.85 for $V-K$. The corresponding magnitudes are $M_V = 0.82$, 0.52, and 0.52.

with the results of the IF method. The estimated magnitude appears slightly brighter than one would expect for a variable of this metallicity (see Fig. 12a, b). However, this discrepancy would be reduced if the reddening were lower as suggested by Blanco (1992), or the metallicity were lower as suggested by Jones's & Lub's photometric indices, or both. Independently, the inspection of Figure 11 shows no evidence that this star is intrinsically overluminous or evolved.

The physical parameters of the star have been calculated as before, adopting $BC_V = -0.07$ from $\langle T \rangle \sim 6300$ K and are summarized in Table 10.

3.2.3. *W Tucanae*

Figure 9 shows the values of the radial displacement ΔR and angular diameters θ determined from the V magnitudes and $V-R$, $V-I$ colors using the first procedure. The $\theta(V-I)$ curve shows a strong bump at $\phi \geq 0.60$ and deficiency at $\phi \leq 0.40$, which restrict the possibility of matching the ΔR and θ curves over the phase interval $0.40 \leq \phi \leq 0.60$ only, leading to a distance $D = 1497$ pc and $M_V = 0.57$ (from $V_0 = 11.444$). The $\theta(V-R)$ curve is very "bumpy," and the fitting could be performed over an even smaller portion of the pulsation cycle, i.e., $0.40 \leq \phi \leq 0.55$, leading to a distance $D = 1836$ pc and $M_V = 0.12$. In this case, however, the phase interval is extremely small and the result is not reliable.

The second procedure was then applied using the infrared colors, and the results are shown in Figure 10, where the solid line corresponds to the best fit over the phase interval 0.35–0.85 and leads to the distance $D = 1601$ pc and magnitude $M_V = 0.42$, in very good agreement with the results of the IF method. The use of the optical colors over the phase interval 0.35–0.85 would lead to a distance $D = 1535$ pc and magnitude $M_V = 0.51$ from $V-R$, and 1741 and 0.24 from $V-I$. In the case of W Tuc, however, the bump around phase 0.6 shows up quite clearly and the θ points beyond this phase lie mostly above the ΔR curve, suggesting that the fits extended until phase 0.85 and the corresponding results may not be very reliable. This may be due to the fact that W Tuc has the largest light and temperature variation, which affect much more significantly the surface brightness and angular diameter values derived from optical colors than those derived from infrared colors. As a test, we have also performed the ΔR-θ match over the same phase interval 0.40–0.55 that was used in the first procedure, but the combination of short phase interval and scatter in the data leads to very inaccurate results. In Figure 10 we show therefore the ΔR curves from the first procedure superposed on the "optical" θ points calculated with the second procedure, for comparison.

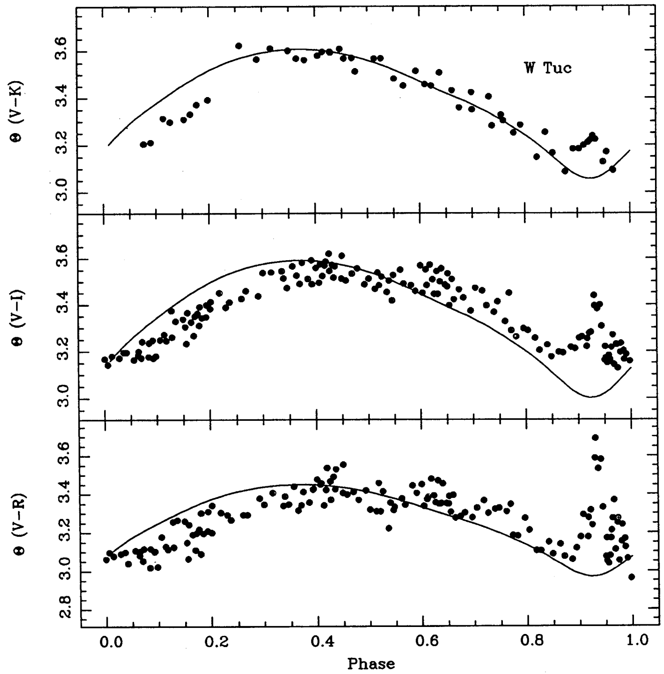

FIG. 10.—The same as in Fig. 9, as derived using the second procedure and $V-K$ colors (*top*). The solid line corresponds to the best fit over the phase interval 0.35–0.85 for $V-K$ and leads to $M_V = 0.42$. The lines in the $V-R$ and $V-I$ panels are the results of the first procedure as in Fig. 9, here shown for comparison.

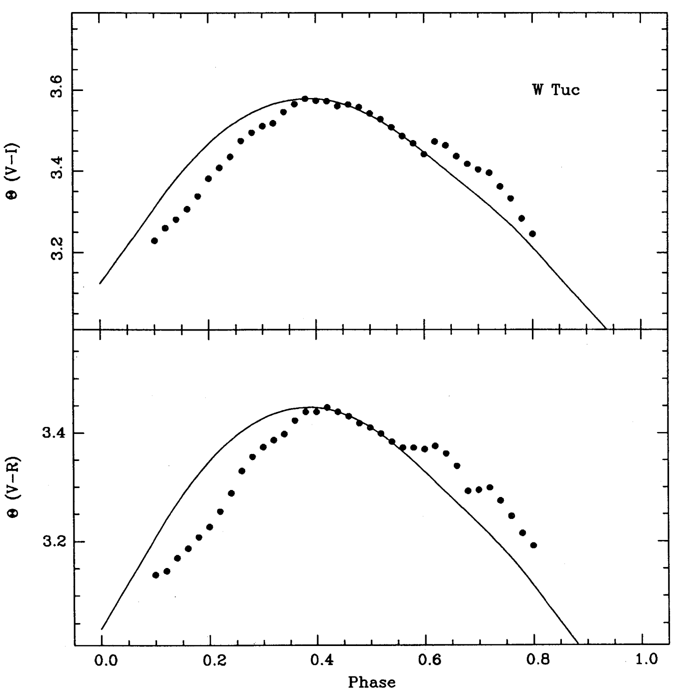

FIG. 9.—The angular diameter θ vs. phase for W Tuc, as derived using the first procedure and $V-R$ (*bottom*) and $V-I$ (*top*) colors. The solid lines represent the radial displacements ΔR which best fit the θ curves over the phase intervals 0.40–0.55 and 0.40–0.60, respectively, and correspond to $M_V = 0.12$ and 0.57.

The physical parameters of W Tuc, calculated as before and adopting $BC_V = -0.065$ from $\langle T \rangle = 6440$ K, are listed in Table 10.

All values of absolute magnitude determined above, including those with low weight, consistently indicate that this star is rather bright. Its position in the log P–A_B diagram (i.e., −0.192d, 1.51) is very close to that of SS Leo (see Fig. 11 in J92), thus suggesting that W Tuc too may be overluminous because evolved off the zero-age horizontal branch (ZAHB), as J92 indicated for SS Leo. We refer to § 4.2 for a more detailed discussion on the problem of evolved stars.

4. DISCUSSION AND COMPARISON WITH PREVIOUS WORK

4.1. *Confrontation between the IF and SB Methods*

The two procedures we have used in the application of the SB method differ in the following aspects:

1. In the first procedure, the colors are estimated at evenly spaced phase intervals from the smoothed color curves. Combined with the V magnitudes estimated from the smoothed light curves, this leads to angular diameter θ curves where the observational scatter has been eliminated by the smoothing, and all θ points have the same weight in the fitting procedure. The subsequent least-squares fitting with the linear radius values ΔR therefore is tightly defined by and very sensitive to the individual values of θ. On the other hand, the smoothing procedure, as careful as it may be, tends to emphasize features or introduce "wiggles" which may be due to statistical or sampling effects, thus introducing spurious data points with the same weight as the good ones. This "magnification" effect is particularly strong with the optical colors because of the steep dependence of the color–surface brightness relation on temperature. In this procedure, therefore, it is extremely important to use as accurate light and color curves as possible, and to avoid those parts of the pulsation cycle which are affected by some of the problems discussed above. If the usable phase interval is not too small, then the θ-ΔR fitting may be sufficiently accurate and provide a reliable estimate of distance. As discussed by Fernley et al. (1990a) in connection with the RR Lyrae star VY Ser, the use of very small phase intervals can give spurious results.

2. In the second procedure, which follows more closely the IF method, only the V magnitudes (as the integrated flux in the IF method) are estimated from the smoothed curves at evenly spaced phase intervals. The observed R, I, and K values are used in combination with V to obtain the colors, which retain therefore the corresponding observational scatter and transfer it down the line to the final θ values. In this case no smoothing is performed (except for the V light curve which is usually extremely well defined) and spurious errors are less likely to occur, while the observational scatter is naturally and properly taken into account by the least-squares fitting procedure to the ΔR curve. On the other hand, different parts of the pulsation cycle have different weights in the final θ-ΔR match depending on the distribution of the observations with phase, and in the case of the optical colors the scatter on the θ values may be so large as to hamper the possibility of a meaningful fit if the phase interval is also too small.

In conclusion, it appears that the virtues are optimized with respect to the shortcomings if the optical colors are analyzed with the first procedure, and the infrared colors are analyzed with the second procedure, considering that the use of infrared data provides in any case more accurate and reliable results in this type of analysis.

As far as the IF method is concerned, it is in principle superior to the SB method since it derives the integrated flux directly from the observations rather than from model atmospheres via the BC_V, and the temperatures are derived with an iterative procedure where the IR flux also plays a role. In any other respect the two methods seem rather equivalent (provided the SB method with $V-K$ colors is used), and the results are compatible within the quoted uncertainties, as shown by the results listed in Tables 9 and 10.

4.2. *Comparison with Previous Work*

It is clear that the use of both optical and infrared photometry leads to more reliable results from B-W analysis than the use of purely optical photometry. The physical basis for this has been discussed elsewhere (e.g., Fernley et al. 1989) and there are now numerous examples in the literature (e.g., Jones et al. 1987 and this paper). In recent years three groups have employed infrared photometry in their B-W analyses, Liu & Janes (1990) studied 13 stars, Jones et al. (1988 and references therein) plus J92 have studied nine stars, and Fernley et al. (1990a and references therein) plus this paper have also studied nine stars. The first two groups have used the SB method with K and $V-K$ and the latter group has used the IF method, while in this paper we have used both the IF method and the SB method with V and $V-K$. In Table 11 we list the periods, metallicities, and absolute V and K magnitudes for the RR Lyrae stars studied with infrared data.

J92 made a comprehensive review of recent B-W results and attempted to "homogenize" them. Clearly this is worthwhile to do; however, it does involve making decisions, specifically what are the correct values for the p factor in converting radial velocities, what is the most appropriate wavelength to use in the IF method (H or K), what is the least problematic phase interval over which to run the B-W solutions and what is the correct zero-point (S_0) in the SB method. While we are in agreement with some of the decisions made by J92, we are not in agreement with others, and in the remainder of this section we will discuss these problems.

1. Fernley et al. have used the same value for the p factor ($=1.33$) for all stars. As pointed out by J92 this is probably incorrect since the value of p depends on the dispersion of the spectra used to measure the radial velocities (Parsons 1972). Thus for DX Del, where the radial velocity curve is from CORAVEL, a more appropriate value is $p = 1.36$ and for X Ari, DH Peg and VY Ser, where the velocities are from the CfA Spectrometer, $p = 1.30$. For the stars SS Leo and V445 Oph, $p = 1.33$ is probably reasonable given the dispersion of the spectra that were used to measure the radial velocities. Tests with our data show that the radius varies linearly with p and thus DX Del should be 0.05 mag brighter and X Ari, DH Peg, and VY Ser 0.05 mag fainter.

2. Jones et al. and Liu & Janes use slightly different values for the constant S_0 in the SB method. We know of no physical reason for preferring one value to another, and simply because Liu & Janes have more stars, we make the adjustment to Jones et al. Thus all their magnitudes should be 0.03 mag fainter. The theoretical models used by Liu & Janes are normalized to Procyon, while the models we have used in § 3.2 are normalized to Vega. Normalizing to Procyon would make our magnitudes from the SB method $\sim$0.02 mag fainter, and this adjustment has been made to our SB results in Table 11.

3. Fernley et al. use both H and K in the IF method, whereas both Jones et al. and Liu & Janes use only K in the SB method. To try and ensure consistency between the SB and IF methods, J92 decided to ignore the H results from the IF method. We do not agree with this since it is throwing away good data and an independent solution. On the other hand, it is clear from Figure 3 in this paper and the plots of T_e versus phase in previous IF papers that there is a difference of $\sim$3% between the model atmosphere H and K fluxes in the sense that the models have an excess at H relative to K. We know of no physical reason for preferring one wavelength to the other but to ensure conformity with the SB method, which uses K, we make the adjustment to H. Tests show that reducing the H model flux by 3% leaves radii unchanged but reduces temperatures by $\sim$50 K and distance moduli by 0.05 mag. Since the H results are half the final solutions (the other half being the K results), this means all IF magnitudes should be fainter by 0.025 mag.

TABLE 11

SUMMARY OF ABSOLUTE MAGNITUDES FROM BAADE-WESSELINK ANALYSES AND INFRARED DATA

Star	Log P	[Fe/H]	<M_V>	<M_K>	σ[Fe/H]	σM_V	Comments
Stars from Liu and Janes (1990)							
UU Vir	−0.322	−0.50	0.98	−0.03	0.15	0.15	
RR Cet	−0.257	−1.25	0.78	−0.32	0.10	0.15	
SU Dra	−0.181	−1.60	0.73	−0.38	0.20	0.15	Evolved
RX Eri	−0.231	−1.40	0.76	−0.41	0.20	0.25	Data
RR Gem	−0.401	−0.30	0.99	0.08	0.25	0.25	Data
RR Leo	−0.345	−1.15	0.86	−0.06	0.20	0.15	
TT Lyn	−0.224	−1.35	0.75	−0.45	0.20	0.15	
AV Peg	−0.409	0.00	1.20	0.24	0.10	0.15	
AR Per	−0.371	−0.30	0.97	0.04	0.20	0.25	Reddening
TU UMa	−0.253	−1.25	0.80	−0.31	0.20	0.15	
SW And	−0.354	−0.15	0.97	−0.05	0.15	0.15	
TV Boo	−0.380	−2.30	0.68	−0.10	0.15	0.25	c-type
T Sex	−0.364	−1.20	0.76	0.03	0.15	0.25	c-type
Stars from Jones *et al* (1988, 1992)							
X Ari	−0.186	−2.20	0.77	−0.42	0.10	0.15	
SW Dra	−0.244	−1.14	0.81	−0.25	0.20	0.15	
TW Her	−0.398	−0.50	0.93	0.06	0.15	0.15	
UU Vir	−0.322	−0.50	0.85	−0.14	0.15	0.15	
SW And	−0.354	−0.15	1.13	0.13	0.15	0.15	
DX Del	−0.325	−0.20	0.71	−0.32	0.15	0.15	
VY Ser	−0.146	−1.80	0.93	−0.36	0.15	0.25	Blazhko
RS Boo	−0.423	−0.40	0.98	0.13	0.25	0.25	Blazhko
DH Peg	−0.468	−0.80	0.92	0.22	0.15	0.25	c-type
Stars from Fernley *et al* (1990 and refs therein) and This Paper							
X Ari	−0.186	−2.20	0.59	−0.51	0.10	0.15	
DX Del	−0.325	−0.20	0.86	−0.16	0.15	0.15	
V445 Oph	−0.401	−0.30	1.17	0.38	0.25	0.25	Reddening
SS Leo	−0.203	−1.50	0.31	−0.77	0.15	0.15	Evolved
VY Ser	−0.146	−1.80	0.51	−0.75	0.15	0.25	Blazhko
DH Peg	−0.468	−0.80	0.69	−0.01	0.15	0.25	c-type
UU Cet (1)	−0.218	−1.00	0.65	−0.52	0.20	0.15	
W Tuc (1)	−0.192	−1.50	0.51	−0.59	0.25	0.15	Evolved
RV Phe (1)	−0.224	−1.35	0.89	−0.26	0.25	0.25	Blazhko?
UU Cet (2)	−0.218	−1.00	0.54	−0.62	0.20	0.15	
W Tuc (2)	−0.192	−1.50	0.44	−0.65	0.25	0.15	Evolved
RV Phe (2)	−0.224	−1.35	0.99	−0.17	0.25	0.25	Blazhko?

NOTE.—Absolute magnitudes have been adjusted in order to be homogeneous with Liu & Janes 1990. See § 4.2 for details. (1) Magnitudes from IF method. (2) Magnitudes from SB method.

4. Finally we must seek uniformity in phase interval. Following the extensive discussion of § 3.1 we believe that the phase interval ~0.35 to ~0.85, when the star is contracting near isothermally, is least problematic. This was approximately the phase interval used by Liu & Janes (i.e., 0.30–0.80) so no adjustments are necessary to their work. Fernley et al. in their analyses took an average of several different phase intervals; however, in all their papers solutions are given for the 0.35–0.85 phase interval. Their published results have been corrected to these values. The situation with regard to Jones et al. is more difficult. While they used slightly different phase intervals for different stars, in general they used the phase interval ~0.1 to ~0.6. In our opinion, this is a more problematic phase region than ~0.35 to ~0.85 since first, it contains a

larger temperature variation, from near maximum temperature at ~ 0.1 to minimum temperature at ~ 0.35 and beyond, and second, a lower radius variation, from mid-radius at ~ 0.1 to maximum radius at ~ 0.35 and back to mid-radius at ~ 0.6. As regards the first point, it was argued in § 3.1 that there is a temperature-dependent error in the Kurucz models; thus using a phase interval that covers a large temperature variation is clearly a potential source of error. As regards the second point, it is obvious that greater numerical stability will be present in the solutions that use the largest possible radius variation. We have made extensive tests on our data to see if any simple relation exists for transforming B-W solutions run over $0.1 \leq \phi \leq 0.6$ to solutions run over $0.35 \leq \phi \leq 0.85$ but the results are inconclusive: in some cases the radii are larger and in other cases smaller. Thus we cannot make any correction to the published results of Jones et al., and we turn to this point later in this section.

The data in Table 11 are as published except that the absolute magnitudes have been corrected as per the foregoing discussion. The periods of the c-types have been transformed to the fundamental mode by using the relation (van Albada & Baker 1971) $\log P_0 = \log P_1 + 0.125$ where we assume typical values for an RR Lyrae of $M = 0.6\ M_{\odot}$, $M_{\rm bol} = 0.75$, and $\langle T_e \rangle = 6500$. Several stars have been studied by two groups, and in the case of SW And and UU Vir the metallicities adopted by the two groups differed slightly. We have taken the average values. In addition, SW Dra, RR Gem, and RS Boo have recent metallicity estimates by Clementini et al. (1991b), and their [Fe/H] values have been adjusted to take account of this work. Also we have increased the metallicity of V445 Oph to $[{\rm Fe/H}] = -0.3$. In Fernley et al. (1990a) the metallicity was taken as $\Delta S = 1$ from the Preston (1959) work. Due to an oversight, no account was taken of the Lub (1977) metallicity determination which gives $\Delta S = 0$. Finally the $\langle M_V \rangle$ and $\langle M_K \rangle$ magnitudes of V445 Oph have been made brighter by 0.22 and 0.03 mag, and the reasons for these adjustments are discussed later in this section.

Because there are errors in both the absolute magnitudes and the [Fe/H] values, then formally a least-squares fit in the $\langle M_V \rangle$-[Fe/H] plane is not correct, and instead we have used a maximum likelihood method. This requires error estimates for both absolute magnitude and [Fe/H], and these are also listed in Table 11. The metallicity errors are based on both the number of independent determinations and the type of determination, higher weight being given to curve-of-growth analyses and high-resolution spectroscopy (e.g., Butler 1975; Butler & Deming 1979; Carney & Jones 1983) than to ΔS and Ca II estimates (e.g., Preston 1959; Clementini et al. 1991b), which in turn are given higher weight than estimates based on photometric indices (e.g., Lub 1979). The M_V errors are by default set to ± 0.15 mag but with higher values for the following stars:

1. V445 Oph and AR Per which have large and less certain reddenings. We note here that J92 omitted these stars from their summary of B-W work because of their large reddenings. This cannot be correct since formally it is giving the stars zero-weight or infinite error which clearly is not the case.

2. RS Boo, VY Ser, and RV Phe which are afflicted by the Blazhko effect or possible phasing problems.

3. TB Boo, T Sex, and DH Peg which are c-types. As discussed in Fernley et al. (1990b), the lower amplitude of the radius variation $\Delta R/R$ in the c-types makes the B-W analysis intrinsically more difficult and a higher error therefore seems appropriate.

4. RX Eri and RR Gem for which the data are of lower quality, i.e., large gaps in the phase coverage of the radial velocity curve (RX Eri) or the infrared light curve (RR Gem). We note that J92 included stars with the Blazhko effect, c-type RR Lyraes, and RX Eri and RR Gem with the same weight as singly periodic ab-types with good data.

5. Stars which may be more evolved. According to stellar evolution theory, as the mass of a horizontal branch (HB) star decreases, so its initial location on the ZAHB moves to the blue, and so for a star below a certain mass (depending on Y and Z), its ZAHB position will be blueward of the instability strip. At the end of their HB lifetime, such stars then evolve rapidly to the AGB and thus cross the instability strip at higher luminosities than the corresponding ZAHB models. It has been suggested that this may be the case for most RR Lyrae stars in metal-poor globular clusters (Lee, Demarque, & Zinn 1990), thus offering an explanation for the strong dependence of period and luminosity on metallicity observed by Sandage (1982, 1990) in both cluster and field variables. If the ideas of Lee et al. are correct and if we are concerned with the mean level of the horizontal branch, then evolved stars have to be included in any $\langle M_V \rangle$-[Fe/H] or $\langle M_K \rangle$–log P relations; however, if the ideas of Lee et al. are not correct or if we are concerned with the zero-age horizontal branch, then such stars must not be included. We will discuss this point further at the end of this section, and for now we will consider how such stars may be identified. Evolved stars may be detected from a star's position in the period-luminosity or period-temperature plots, since at a given period an evolved star will be at higher luminosity and higher temperature. Assuming the blue amplitude of an RR Lyrae is a function of temperature and metallicity, then a series of period-amplitude plots at a range of metallicities provides a direct test for whether or not a star is significantly evolved. J92 constructed such plots for two metallicities and concluded that SS Leo was evolved and that SU Dra, UU Vir, and DX Del may also be evolved. Plotting the stars from this paper in these diagrams also suggests that W Tuc is evolved (B. W. Carney 1991, private communication). To examine this further we show in Figures 11*a* and 11*b* plots of $\log P$ versus $\langle M_V \rangle - \langle M_K \rangle$ for both the metal-rich ($[{\rm Fe/H}] > -1.0$) and metal-poor ($[{\rm Fe/H}] \leq -1.0$) groups in Table 11. The solid line is the theoretical relation derived from combining $\log P \propto -3.48 \log T_e$ (from the pulsation equation of van Albada & Baker 1971 with mass and luminosity held constant) with $\log T_e \propto -0.12(V-K)$ (from the Fernley 1989 calibration of $V-K$ versus T_e with $\log g = 2.75$ and $-2.0 \leq [{\rm Fe/H}] \leq 0.0$ and $5750 \leq T_e \leq 7250$). The zero point of the $\log T_e$–$(V-K)$ line has been determined by fitting by eye. For the range in [Fe/H] of ± 0.25 the relative insensitivity of $V-K$ to metallicity means the change in zero point on account of the metallicity is ± 0.01 in $V-K$. It can be seen in Figure 11 that among the metal-poor group, SS Leo, W Tuc, and SU Dra all lie well above the other stars, while among the metal-rich group, V445 Oph and AV Peg lie below the other stars, UU Vir slightly above, and DX Del insignificantly above. The position of UU Vir, AV Peg, and DX Del is reasonably consistent with their metallicity and the Sandage period shift relation discussed below, the suggestion that UU Vir and DX Del are evolved being not strongly supported by Figure 12 in J92 either; however, V445 Oph is clearly discrepant. First we consider whether the positions of SU Dra, W Tuc, SS Leo, and V445 Oph can be explained by observational error. An error of ± 0.1 in $V-K$ would be sufficient to bring the stars into agreement. This is equivalent to an error of ± 0.03 in $E(B-V)$.

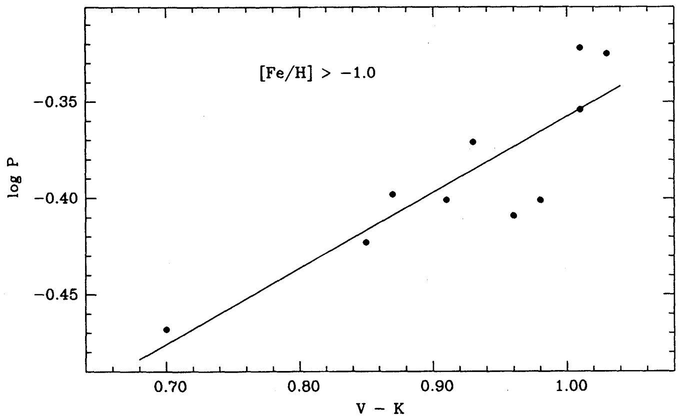

FIG. 11a

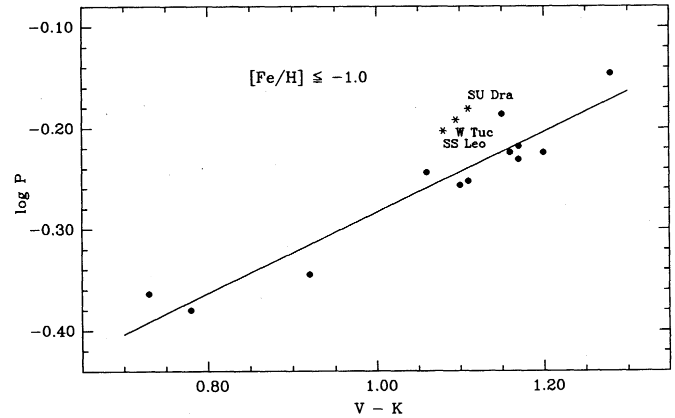

FIG. 11b

FIG. 11.—The log period–$(V-K)$ relations for (*a*) 10 metal-rich and (*b*) 15 metal-poor stars using the data from Table 11 but before making the corrections to V445 Oph discussed in § 4.2.

For the metal-poor stars, this is impossible since the sense of the error is that the present $E(B-V)$ is too large by ~ 0.03; however, the present $E(B-V)$ values for the three stars are all 0.01. For V445 Oph with $E(B-V) = 0.27$ a correction of $+0.03$ is not unrealistic (see Fernley et al. 1990a) and is actually consistent with Blanco's (1992) estimated reddening 0.29. Also it should be noted that there is considerable evidence that the reddening ratios $R_\lambda = A_\lambda/E(B-V)$ are higher in the region of the Ophiucus cloud (e.g., Martin & Whittet 1990 and references therein). If we adopt $R_V = 4.0$ instead of 3.2 and $R_K = 0.45$ instead of 0.35, then $V-K$ would be bluer by 0.19 for V445 Oph. Alternatively, the error could be in the metallicity. The Sandage period shift relation (Sandage 1990 and references therein) is still controversial (e.g., Carney et al. 1992 and references therein); however, for the present purpose we adopt the "standard" value $\Delta \log P = -0.10$[Fe/H]. Thus if the three metal-poor stars had [Fe/H] ~ -1.85 rather than -1.50 and V445 Oph had [Fe/H] ~ 0.00 rather than -0.03, then their position relative to the other stars would be consistent. Considering the metallicity estimates of the metal-poor stars, we note that Lub (1977) finds [Fe/H] $= -1.50$ from the Walraven photometric indices of both SS Leo and W Tuc. The value for SS Leo is confirmed by Preston's (1959) low-resolution spectroscopic survey that shows $\Delta = 8$ or [Fe/H] $= -1.51$, and by Suntzeff, Kinman, & Kraft (1991) who find $\Delta S = 8.4$ (i.e., [Fe/H] $= -1.57$). Also in the study of Clementini et al. (1991b), SS Leo appears as a standard star; however, one can derive its metallicity from the equivalent width of the Ca II K line W'(K) and the W'(K)-[Fe/H] relation defined by the standard stars excluding SS Leo itself and SU Dra obtaining [Fe/H] ~ -1.45. For SU Dra, the ΔS index is either 10 (Preston 1959) or 9.6 (Butler 1975) or 8.6 (Suntzeff et al. 1991) which are equivalent of [Fe/H] $= -1.83$ or -1.77 or -1.61. A high-dispersion abundance analysis by Butler & Deming (1979) gave [Fe/H] $= -1.39$ based on only three Fe II lines and a spectrum taken at maximum light (which, however, would underestimate the metal abundance because of temperature effects), while Clementini et al. (1991b) using the same procedure as for SS Leo would obtain [Fe/H] ~ -1.3. Based on the average of these estimates it may be that [Fe/H] $= -1.6$ is not unrealistic. The situation for V445 Oph has been discussed earlier in this section, and the star has [Fe/H] $= -0.4$ from Preston and -0.2 from Lub. A final point is that the metal-poor stars have a positive period shift which can be understood theoretically by their being more evolved, while V445 Oph has a negative period shift, and as far as we know, this is impossible to understand theoretically. For V445 Oph we are therefore inclined to believe the problem is with the reddening, in particular the reddening ratios. We adopt the reddening ratios discussed above, namely 4.0 and 0.45 for $A_V/E(B-V)$ and $A_K/E(B-V)$, respectively, and have corrected the original data by making the absolute magnitudes of this star brighter by 0.22 in $\langle M_V \rangle$ and 0.03 in $\langle M_K \rangle$. On the other hand, the metal-poor stars SU Dra, W Tuc, and SS Leo are likely to be evolved, and for the purposes of forming $\langle M_V \rangle$-[Fe/H] and $\langle M_K \rangle$–log P relations the only appropriate action is to omit them, if the use of such relations is intended for stars on or near the ZAHB. Also, it may be interesting to notice that while SU Dra appears to be evolved in Figure 11 but does not appear to be overluminous in Figure 12, UU Cet has the opposite behavior. Incidentally, we notice that in the sample of stars listed in Table 11, the evolved ones belong only to the metal-poor group, but none appears to be evolved among the metal-rich ones. While this is in general agreement with the predictions of the stellar evolution theory (Lee et al. 1990; Lee & Zinn 1991 and references therein), the statistics is too poor to say whether or not the relative numbers are consistent with such predictions. From the HB models presented by Lee & Zinn (1991) one can estimate that an evolved 0.64 $M_\odot$ RR Lyrae star of [Fe/H] $= -1.5$ is ~ 0.3–0.4 mag brighter than a variable near its ZAHB location, which is consistent with the values in Table 11.

A least-squares fitting weighted by the σ values has been performed using the data in Table 11, taking mean values of magnitudes for the stars with two determinations. The following series of relations can be derived:

A.—Using all stars:

$$\langle M_V \rangle_{RR} = 0.22[\mathrm{Fe/H}] + 1.04 \text{ (25 stars, rms} = 0.122)$$

$$\langle M_K \rangle_{RR} = -3.05 \log P - 1.11 \text{ (25 stars, rms} = 0.096) .$$

B.—Same as (A) but excluding the possibly evolved stars SS Leo, SU Dra, and W Tuc:

$$\langle M_V \rangle_{RR} = 0.18[\mathrm{Fe/H}] + 1.03 \text{ (22 stars, rms} = 0.087)$$

$$\langle M_K \rangle_{RR} = -2.86 \log P - 1.04 \text{ (22 stars, rms} = 0.080) .$$

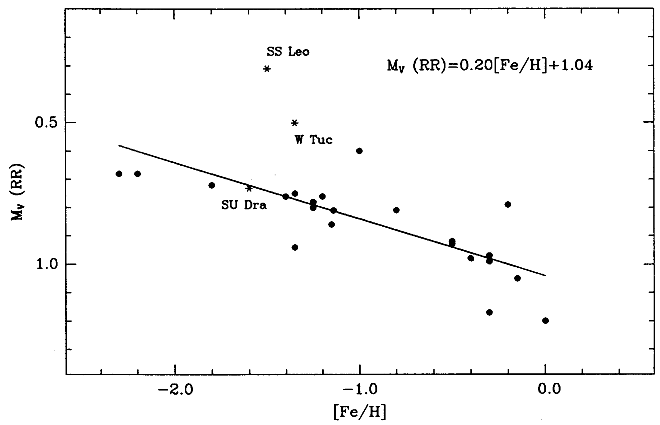

FIG. 12*a*

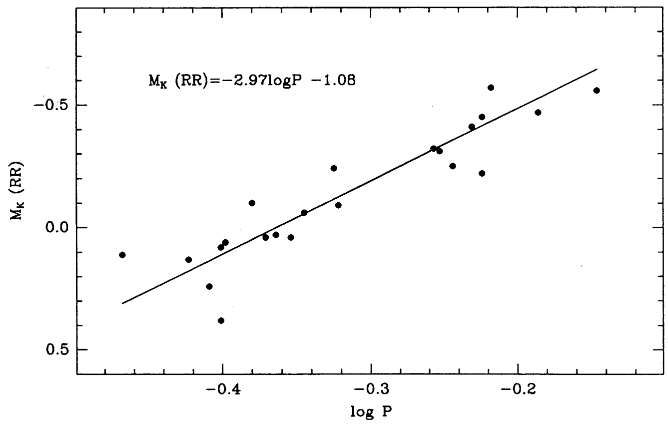

FIG. 12*b*

FIG. 12.—Summary of the RR Lyrae absolute magnitude determinations obtained from the data in Table 11. The solid lines represent (*a*) the $\langle M_V \rangle$-metallicity and (*b*) $\langle M_K \rangle$–log period relations defined in § 4.2. The stars appear only once, with their average magnitudes.

C.—Using only stars with solutions from the preferred phase interval of $0.35 < \phi < 0.85$:

$$\langle M_V \rangle_{RR} = 0.25[Fe/H] + 1.05 \text{ (22 stars, rms = 0.144)}$$

$$\langle M_K \rangle_{RR} = -3.19 \log P - 1.16 \text{ (22 stars, rms = 0.098)} .$$

D.—Same as (C) but excluding the possibly evolved stars SU Dra, SS Leo, and W Tuc:

$$\langle M_V \rangle_{RR} = 0.22[Fe/H] + 1.05 \text{ (19 stars, rms = 0.108)}$$

$$\langle M_K \rangle_{RR} = -3.09 \log P - 1.12 \text{ (19 stars, rms = 0.085)} .$$

It is clear that the slopes of the $\langle M_V \rangle$ and $\langle M_K \rangle$ relations vary according to the phase interval used in the solutions and the treatment of possibly evolved stars. Our preferred phase interval $0.35 < \phi < 0.85$ gives steeper slopes in both relations. Steeper slopes are also found including the possibly evolved stars, as obviously expected since these stars have unusually high luminosities and long periods. These latter values, however, are only indicative since they depend on the relative number of evolved and non-evolved stars used to derive these relations. Although it may be argued that some fraction of evolved HB stars is present in globular clusters, and therefore evolved stars should be considered in the above relations, the exact population ratio is still an open question, and for the time being we adopt a solution which is the average of the above cases (B) and (D), namely:

$$\langle M_V \rangle_{RR} = 0.20[Fe/H] + 1.04$$

$$\langle M_K \rangle_{RR} = -2.97 \log P - 1.08 .$$

However, if the evolved stars are a significant fraction of all HB stars in metal-poor globular clusters, as suggested by Lee et al. (1990), then the slopes of these relations are steeper by approximately 0.03 and 0.15, respectively, while the zero points remain essentially unchanged.

The program stars are plotted in Figures 12*a* and 12*b* in both the $\langle M_V \rangle$-[Fe/H] and $\langle M_K \rangle$–log *P* planes, using the mean estimated values of magnitude from Table 11. Our value for the slope of the $\langle M_V \rangle$-[Fe/H] relation is very similar to the value found by Sandage (1990) and Sandage & Cacciari (1990), and thus does not alter their discussion and conclusions, in particular about the ages of the Galactic globular clusters, to which we refer the interested reader.

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