

VELOCITY SEGREGATION IN GALAXY CLUSTERS

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ABSTRACT

We have investigated the velocity field of galaxies in clusters. Our sample consists of the 68 clusters with at least 30 galaxies for which redshifts are available in the literature; for 61 of these clusters we were also able to collect most of the galaxy magnitudes. We have unambiguously found that galaxies brighter than the magnitude of third-ranked object, m_3 , have velocities lower than average. These galaxies are preferentially located in the central regions. The effect is not induced by morphological segregation, it is not restricted to cD clusters, and it does not depend on the presence of substructures. The energy equipartition status seems to be achieved by these low-velocity galaxies. This evidence of segregation stresses the importance of m_3 as a boundary between different dynamical statuses, and constrains the theory of structure and evolution of clusters of galaxies. A possible interpretation is given in terms of dynamical friction and merging.

Subject headings: galaxies: clustering — galaxies: distances and redshifts — galaxies: kinematics and dynamics

1. INTRODUCTION

The dynamical and evolutionary status of clusters of galaxies can be described by the galaxy velocity field. In this paper we want to examine whether the velocity distribution of cluster galaxies depends on their luminosities and/or distances from the cluster center. In the following, we will refer to velocity segregation of galaxies with respect to their luminosities as *VLS*, meaning that more luminous galaxies move more slowly than fainter ones.

Claims have been made in the literature that more luminous and/or more centrally located cluster galaxies have a lower velocity (with respect to the cluster mean) than other cluster galaxies (Rood 1965; Rood et al. 1972; Chincarini & Rood 1977, hereafter CR; Struble 1979; Cowie & Hu 1986; Mellier et al. 1988; Bothun & Schombert 1990), yet little (Merrifield & Kent 1991) or no evidence of segregation (Kent & Gunn 1982; Kent & Sargent 1983) has been found in other works.

The presence of velocity segregation in galaxy clusters is often taken as evidence of advanced dynamical evolution. While the process of violent relaxation is thought to produce a velocity distribution independent of the galaxy mass, the dynamical friction process transfers kinetic energy from more massive galaxies to less massive ones (see, e.g., Sarazin 1986). The more massive galaxies are slowed down and spiral in toward the cluster center. In this framework, a lower velocity dispersion should be seen in the central galaxies and in the more luminous ones as well, if a correlation exists between galaxy masses and luminosities.

Related topics are galaxy merging and accretion phenomena in the cluster cores (see, e.g., Tonry 1987; Schombert 1987; Bothun & Schombert 1990, and references therein). The low-velocity galaxies are natural candidates for merging and accretion, which lead to galaxies of higher luminosities and even lower velocities (and eventually to the formation of a single dominant galaxy).

In § 2 we describe our data sample; in § 3 we describe our analysis and results; in § 4 we provide the relevant discussion.

2. THE DATA SAMPLE

We have collected data for 68 clusters from the literature, selecting only those clusters with measured radial velocities (and positions) for at least 30 galaxies (for a total of ≈ 6500 galaxies). Clusters with mean redshift > 0.15 have not been taken into account, in order to avoid dealing with evolutionary effects (see, e.g., Newberry, Kirshner, & Boroson 1988). Our clusters span a wide range of properties, from richness class 0 to 4, and are quite representative of the average cluster population.

Magnitude data are available in the literature for most galaxies in 61 clusters of our sample (i.e. ≈ 5500 galaxies). When necessary, we corrected these magnitudes for *K*-dimming effect and absorption by our Galaxy. We have taken the *K*-dimming corrections for the *R*, *V*, and *B* bands from Sandage (1973), for the *r*, *g* magnitude system from Schneider, Gunn, & Hoessel (1983), and for the *J* system from Phillips, Fong, & Shanks (1981). The absorptions by our Galaxy have been obtained by using the maps and prescriptions of Burstein & Heiles (1982), and the relations between absorption in different bands as given by Sandage (1973) and Schneider et al. (1983). We transformed these magnitudes into the same photometric band, i.e. the visual one using the formulas given by Oemler (1974), Schweizer (1976), Thuan & Gunn (1976), de Vaucouleurs (1977), Kirshner, Oemler, & Schechter (1978), Geller et al. (1984), Shanks et al. (1984), Postman, Huchra, & Geller (1986), and Colless (1989). The galaxy morphological types are known for most galaxies only in 38 clusters of our sample.

In order to reject the noncluster members from our samples, we first adopted a limiting radius of $3 h_{100}^{-1}$ Mpc from the cluster center (we assume $q_0 = \frac{1}{2}$ throughout this paper; however, this choice is not very important); we used the biweight location estimator for α , δ (see e.g., Beers, Flynn, & Gebhardt 1990), to define the center of each cluster. Then we eliminated the “obvious interlopers” in the redshift space following the procedure developed by Zabludoff, Huchra, & Geller (1990). For the remaining galaxies, we computed the “biweight center” again in order to select galaxies inside different limiting radii, from 0.25 to $3 h_{100}^{-1}$ Mpc. To identify the “true” cluster members, we applied the classical 3σ clipping procedure (Yahil & Vidal 1977, hereafter YV). The “gapping”

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TABLE 1
THE DATA SAMPLE

NAME (1)	N_{tot}^a (2)	N_{true}^b (3)	N_{mag}^c (4)	REFERENCES ^d		
				Velocity (5)	Magnitude (6)	T (7)
ACO 194	268	65	65	1	1	2
ACO 262	88	47	47	3, 4, 5	6	3, 4, 5
ACO 426 (Perseus)	200	151	151	7	6, 7	7
ACO 458	45	31	31	8	9	
ACO 496	34	34	32	10	11	11
ACO 539	289	50	49	12	6, 13	12
ACO 548	134	117	117	11	11	11
ACO 576	51	47	30	14	15	15
ACO 754	89	66	66	11	11	11
ACO 999	45	23	23	16	17	17
ACO 1016	44	22	22	16	17	17
ACO 1060 (Hydra)	177	133	133	18, 19	18, 19	18, 19
ACO 1142	66	39	37	20	11	11
ACO 1146	84	54	...	21		
ACO 1367	94	72	72	22, 23, 24, 25	6	22, 23, 24, 25
ACO 1631	90	58	58	11	11	11
ACO 1644	102	77	77	11	11	11
ACO 1651	31	29	...	26		
ACO 1656 (Coma)	414	221	220	27	27	27
ACO 1736a	40	34	34	11	11	11
ACO 1736b	64	54	54	11	11	11
ACO 1795	45	40	...	28		
ACO 1983	100	64	64	11	11	11
ACO 2052	43	37	12	29	6	
ACO 2065	31	24	24	30	30	
ACO 2147/2152	45	38	34	31	6	31
ACO 2151 (Hercules)	105	94	94	11	11	11
ACO 2197	45	41	30	32	6	
ACO 2199	71	60	54	32	6	33, 34
ACO 2256	89	86	86	35	35	
ACO 2538	45	42	42	8	9	
ACO 2554	41	27	27	8	9	
ACO 2670	303	192	192	36	36	36
ACO 2717	45	33	33	8	9	
ACO 2721	88	57	22	21	9	
ACO 3126	45	38	38	8	9	
ACO 3128	45	44	44	8	9	
ACO 3158	37	34	30	37, 38	37	38
ACO 3225	44	41	41	8	9	
ACO 3266	172	129	...	21		
ACO 3334	41	32	32	8	9	
ACO 3360	40	36	36	8	9	
ACO 3376	84	67	67	11	11	11
ACO 3381	64	42	42	11	11	11
ACO 3389	39	38	37	21	11	11
ACO 3391	81	65	...	21		
ACO 3395	203	143	...	21		
ACO 3526 (Centaurus)	301	163	162	39, 40	39, 40	39, 40
ACO 3532 (Klemola 22)	44	42	42	41	41	
ACO 3558 (Shapley 8)	123	113	40	21, 42	42	
ACO 3574	42	35	35	43	43	43
ACO 3667	48	45	45	44	44	
ACO 3705	45	40	40	8	9	
ACO 3716	106	73	73	11	11	11
ACO 4067	41	29	...	21		
ACO S301	30	26	26	11	11	11
ACO S373 (Fornax)	57	56	56	40	40	40
ACO S463	100	79	79	11	11	11
ACO S753	43	31	31	43	43	43
ACO S805	50	42	42	45	45	45
AWM 1	56	24	23	46	46	46
AWM 7	33	33	25	46	46	46
Colless 67	45	37	36	8	9	
Dressler 0003-50	55	34	34	11	11	11
MKW 4	86	43	41	46, 47	46, 47	46, 47
MKW 12	75	25	25	46	46	46
Pegasus I	78	36	36	48, 49, 50	48, 49, 50	49, 50, 51
Virgo	572	410	410	51	51	15

procedure developed by Zabludoff et al. (1990) generally identified fewer galaxies as interlopers in our samples. In this way we have defined a sample of ≈ 3700 galaxies in 61 clusters, with both redshifts and magnitudes known.

In Table 1 we list the names of the clusters in our sample (col. [1]), and the number of collected galaxies with available redshift in each cluster region (col. [2]); column (3) lists the number of "true" members inside $1.5 h_{100}^{-1}$ Mpc (one Abell radius); column (4) lists the number of galaxies in column (3) with available magnitude; the relevant references to the galaxy redshifts, magnitudes, and morphological types are listed in columns (5), (6), and (7) respectively. A full description of our catalog will be given in Girardi et al. (1992).

3. ANALYSIS

3.1. Luminosity Segregation in Velocity Space—VLS

We wish to examine the velocity distributions of galaxies with different luminosities. So we compare, in each cluster, the unweighted velocity dispersion:

$$\sigma_u = \left[\frac{\sum_{i=1}^n (v_i - \bar{v})^2}{n-1} - \Delta_u \right]^{1/2} \quad (1)$$

with the luminosity-weighted velocity dispersion:

$$\sigma_{lw} = \left[\frac{\sum_{i=1}^n (v_i - \bar{v})^2 l_i}{\sum_{i=1}^n l_i} \frac{n}{n-1} - \Delta_{lw} \right]^{1/2}, \quad (2)$$

where the mean velocity is

$$\bar{v} = \frac{\sum_{i=1}^n v_i}{n}, \quad (3)$$

where v_i is the velocity of the i th galaxy corrected for cosmological effects (see, e.g., Danese, De Zotti, & di Tullio 1980), l_i is the apparent galaxy visual luminosity, and n is the number of "true" cluster members in our sample; Δ_u and Δ_{lw} are the correction terms which account for the measurement errors, in both the velocities and the weights (i.e., the luminosities). As concerns Δ_u , see Danese et al. (1980), while the expression we derived for Δ_{lw} is given in the Appendix; these correction terms are always very small, on the order of a few km s^{-1} . The choice of the unweighted mean \bar{v} is natural, because of the lack of extended evidence for energy equipartition. Nevertheless, we checked that our results do not change when using a luminosity-weighted mean.

The *Sign* and *Wilcoxon Signed-ranks* tests (hereafter referred to as S and W tests; e.g. Siegel 1956) were used to compare σ_u and σ_{lw} in our sample of 61 clusters with available magnitudes. We found that σ_u is larger than σ_{lw} at a $>99.99\%$ significance

level (hereafter s.l.) for both tests, for the $1.5 h_{100}^{-1}$ Mpc subsample. In the following we will always refer to this subsample, except where otherwise explicitly specified; other limiting radii always gave similar results. The cumulative distributions for σ_u and σ_{lw} are shown in Figure 1: the more luminous galaxies have indeed a lower velocity dispersion than the fainter ones, yet the difference is small.

We note here that the errors involved in the transformation of magnitudes into a common system are not very important in this analysis, since only nine clusters have their galaxy magnitudes originally given in different bands. We repeated the previous tests on the 52 remaining clusters, and we found similar results (99.94% and 99.97% s.l. for the S and W test, respectively). Another problem may be induced by the large error associated with the magnitudes given by Dressler & Shectman (1988a); we eliminated their 17 clusters from our analysis, and yet we again found similar results (99.98% and 99.99% s.l.). Having shown that magnitude uncertainties are not biasing our result, we feel confident in considering the whole sample of clusters hereafter in our analysis.

In order to understand whether a limiting magnitude exists for velocity segregation, we excluded the most luminous galaxies from our samples and repeated the previous analysis. The difference between σ_u and σ_{lw} loses significance (i.e., disappears) when we exclude those galaxies with magnitude $m \leq m_3$, m_3 being the magnitude of the third-ranked galaxy in each cluster; m_3 corresponds to a visual absolute magnitude $M_3 = -21.38 + 5 \log h_{100}$, averaged over all our 61 clusters, with a rms error of 0.08 mag. From this result, we see that VLS applies only to the brightest galaxies of the clusters.

However, VLS is not caused by the presence of a single dominant galaxy (e.g., a cD). In fact, we considered only the 33 clusters in our sample with known Rood-Sastry type (taken from Struble & Rood 1987), and we selected the 25 non-cD clusters: the difference between the σ_u and σ_{lw} was confirmed (S: 99.95% and W: 99.93% s.l.). Moreover, even by excluding Bautz-Morgan type I (hereafter BM I) clusters, as identified by Abell, Corwin, & Olowin (1989), the difference was confirmed (S: 99.99% and W: 99.99% s.l.).

We verified that the presence of substructure in our clusters (estimated as in Dressler & Shectman 1988b) does not induce the effect we found: the 41 clusters without substructures (at a 90% s.l.) do show evidence of VLS (S: 99.98% and W: 99.85% s.l.).

Moss & Dickens (1977) found a trend for spiral and irregular galaxies to have greater velocity dispersion than ellipticals and lensicals, a result later confirmed by other authors (e.g., Sodré et al. 1989). This holds for our larger cluster sample too: the velocity dispersion of S and Irr is larger than that of E and

NOTES TO TABLE 1

^a Total number of galaxies with available redshift in the original sample.

^b Number of "true" member galaxies (see text), within $1.5 h_{100}^{-1}$ Mpc from the cluster center.

^c As in col. (3), but only galaxies with available magnitudes.

^d References to velocities, magnitudes, and morphological types, respectively.

REFERENCES.—(1) Chapman, Geller, & Huchra 1988; (2) Chincarini & Rood 1977; (3) Giovanelli et al. 1982; (4) Gregory, Thompson, & Tift 1981; (5) Moss & Dickens 1977; (6) Zwicky et al. 1961–1968; (7) Kent & Sargent 1983; (8) Colless & Hewett 1987; (9) Colless 1989; (10) Mazure et al. 1986; (11) Dressler & Shectman 1988b; (12) Ostriker et al. 1988; (13) Nilson 1973; (14) Hintzen et al. 1982; (15) Fanti et al. 1982; (16) Chapman, Geller, & Huchra 1987; (17) Adams, Strom, & Strom 1980; (18) Richter 1987; (19) Richter 1989; (20) Geller et al. 1984; (21) Teague, Carter, & Gray 1990; (22) Gavazzi 1987; (23) Gregory & Thompson 1978; (24) Tift 1978; (25) Dickens & Moss 1976; (26) Zabludoff, Huchra, & Geller 1990; (27) Kent & Gunn 1982; (28) Hill et al. 1988; (29) Quintana et al. 1985; (30) Postman, Geller, & Huchra 1988; (31) Tarenghi et al. 1979; (32) Gregory & Thompson 1984; (33) Tift 1974; (34) Butcher & Oemler 1985; (35) Fabricant, Kent, & Kurtz 1989; (36) Sharples, Ellis, & Gray 1988; (37) Chincarini, Tarenghi, & Bettis 1981; (38) Lucey et al. 1983; (39) Dickens, Currie, & Lucey 1986; (40) Lauberts & Valentijn 1989; (41) Cristiani et al. 1987; (42) Metcalfe, Godwin, & Spenser 1987; (43) Willmer et al. 1991; (44) Proust et al. 1988; (45) Bell & Whitmore 1989; (46) Beers et al. 1984; (47) Malumuth & Kriss 1986; (48) Richter & Huchtmeier 1982; (49) Bothun et al. 1985; (50) Chincarini & Rood 1976; (51) Binggeli, Sandage, & Tammann 1985.

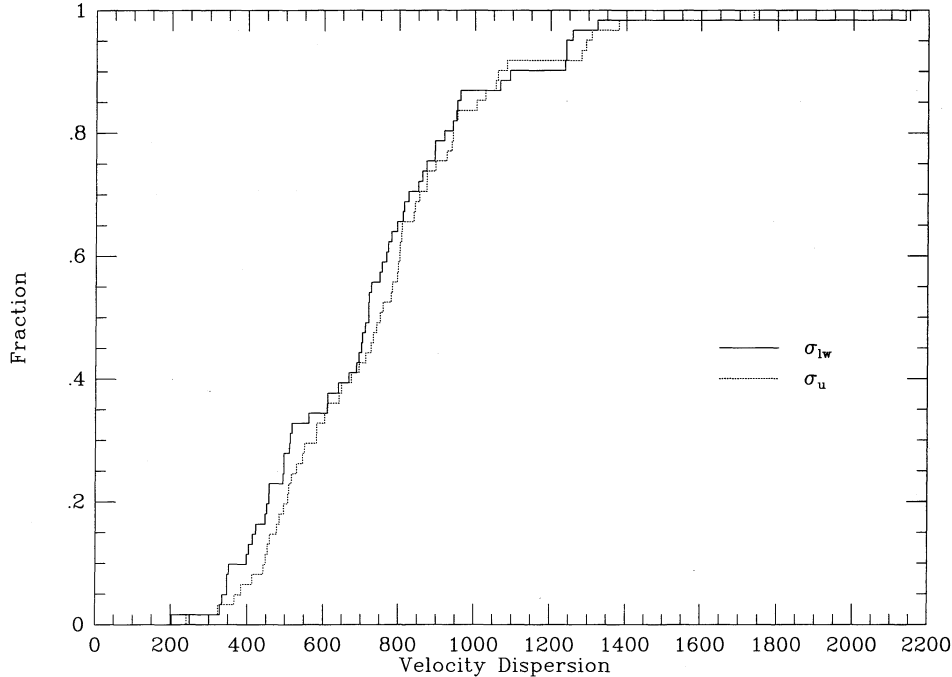


FIG. 1.—Cumulative distributions for the luminosity-weighted (full line) and unweighted (dotted line) velocity dispersions

S0 at the 99.62% and 99.86% s.l. according to the S and W tests (37 clusters with known galaxy morphological types were considered). Moreover, galaxies of different morphological types have different luminosity distributions (e.g., Binggeli, Sandage, & Tammann 1988). The coupling of the above mentioned effects might mimic the *VLS* we have found. We showed this was not the case with our sample by considering separately galaxies of different morphological types. The difference between σ_u 's and σ_{lw} 's, as computed on the E galaxies only, was still found to be significant (S: 99.98% and W: >99.99% s.l.). The difference is less significant for S0 galaxies (S: 96.74% and W: 97.66% s.l.), and only marginally significant for S plus Irr galaxies (S: 93.38% and W: 92.02% s.l.). This is consistent with the fact that the most luminous galaxies in our clusters are early types (see below).

We then examined the presence of *VLS* in each single cluster. Following Struble (1979) we adopted the *F*-test for equality of variances, which uses the statistics $F = \sigma_{\text{faintest}}^2 / \sigma_{\text{brightest}}^2$. We took the galaxies with magnitude $m \leq m_3 + 0.5$, and $m > m_3 + 0.5$ to compute $\sigma_{\text{brightest}}$ and, respectively, σ_{faintest} . We did not find clear evidence of *VLS* in many clusters. Following CR, we correlated $\log |v_i - \bar{v}|$ with the visual magnitude m_i for the “true” members (see above) of each cluster. As before, we did not detect *VLS* in many clusters. The results of the CR and *F*-tests are in agreement, since their significance levels are correlated at a 99.79% s.l. Moreover, these significance levels both correlate (at >99.99% s.l.) with the normalized difference between unweighted and weighted dispersions, $(\sigma_u - \sigma_{lw}) / \sigma_u$. Finally, we divided our cluster sample into two subsamples with a *VLS* probability larger and, respectively, lower than the median value, according to the *F*-test: the difference between σ_{lw} and σ_u still holds for the subsample with high *VLS* probability (S: 99.99%, W: 99.99% s.l.), but not for the other one. The same holds for the CR test probabilities, too (S: >99.99%, W: >99.99% s.l. for the high-probability

sample). In Table 2 we list the probabilities, of *VLS* in each single cluster, according to the *F*- and CR tests, and $(\sigma_u - \sigma_{lw}) / \sigma_u$.

The failure to detect *VLS* in some of our clusters is possibly due to the low number of galaxy redshifts and luminosities, and/or to the different degree of segregation present in different clusters (e.g., Yepes, Dominguez-Tenreiro, & del Pozo-Sanz 1991). In order to overcome these problems, we joined all cluster galaxies together, by using the normalized velocities $|v_i - \bar{v}| / \sigma_u$ and normalized magnitudes $m_i - m_3$. These two quantities are correlated at a 98.64% s.l., more luminous galaxies having lower velocities. In Figure 2, we show the average normalized velocities binned in 0.5 mag intervals versus normalized magnitudes. The averages and the corresponding errors have been computed by using a bootstrap technique (with 1000 resamplings). In Figure 2, galaxies brighter than m_3 have significantly lower velocities than others: the mean values of the normalized velocities are 0.55 ± 0.05 and 0.81 ± 0.01 , respectively. We therefore again found that m_3 can be regarded as an approximate limit for velocity segregation.

We considered the morphological types and clustercentric distances of bright galaxies in our sample. We found the galaxies more luminous than the third-ranked one have an average morphological type $\bar{T} = -3$ (T is coded as in de Vaucouleurs, de Vaucouleurs, & Corwin 1976), while the others have $\bar{T} = -1$. The more luminous galaxies are also more centrally located than the others (the average distances from the centre are $\bar{d} = 0.38 \pm 0.04 h_{100}^{-1}$ Mpc, and $0.54 \pm 0.01 h_{100}^{-1}$ Mpc, respectively). This evidence is in agreement with previous findings that the more luminous galaxies are more centrally located (e.g., Bahcall 1973; Oemler 1974; Godwin & Peach 1977; Quintana 1979; Mazure, Capelato, & des Forêts 1984; Oegerle, Hoessel, & Ernst 1986; Yepes et al. 1991).

Finally, we gave a quantitative estimate of the difference among the velocity dispersions of galaxies in different magni-

TABLE 2
LUMINOSITY SEGREGATION

NAME (1)	SIGNIFICANCE LEVEL ^a		$(\sigma_u - \sigma_{lw})/\sigma_u^b$ (4)
	F-Test (2)	CR Test (3)	
ACO 194	96%	95%	0.09
ACO 262	98	98	0.12
ACO 426 (Perseus)	83	77	0.02
ACO 458	...	95	0.16
ACO 496	54	93	0.06
ACO 539	6	11	-0.08
ACO 548	52	10	0.05
ACO 576	71	93	0.06
ACO 754	...	13	0.00
ACO 999	...	92	0.19
ACO 1016	...	90	0.16
ACO 1060 (Hydra)	80	8	0.08
ACO 1142	100	97	0.10
ACO 1367	80	96	0.10
ACO 1631	25	0	-0.04
ACO 1644	99	18	0.07
ACO 1656 (Coma)	93	83	0.05
ACO 1736a	18	12	0.05
ACO 1736b	72	78	0.07
ACO 1983	100	100	0.17
ACO 2052	...	75	0.06
ACO 2065	69	86	0.02
ACO 2147/2152	96	94	0.05
ACO 2151 (Hercules)	78	76	-0.05
ACO 2197	...	18	0.11
ACO 2199	89	87	0.04
ACO 2256	95	17	0.04
ACO 2538	26	1	-0.07
ACO 2554	40	94	0.04
ACO 2670	90	93	0.06
ACO 2717	51	10	0.06
ACO 2721	...	94	0.14
ACO 3126	88	88	0.11
ACO 3128	42	76	0.07
ACO 3158	...	21	0.05
ACO 3225	...	78	-0.04
ACO 3334	...	24	0.05
ACO 3360	...	9	0.04
ACO 3376	...	9	0.05
ACO 3381	18	7	-0.24
ACO 3389	...	76	0.23
ACO 3526 (Centaurus)	40	96	0.06
ACO 3532 (Klemola 22)	50	88	0.04
ACO 3558 (Shapley 8)	6	12	0.05
ACO 3574	35	24	-0.04
ACO 3667	100	87	0.10
ACO 3705	27	82	-0.04
ACO 3716	78	88	0.07
ACO S301	...	18	-0.10
ACO S373 (Fornax)	...	10	-0.01
ACO S463	...	4	-0.06
ACO S753	11	87	0.10
ACO S805	98	92	0.28
AWM 1	86	12	0.00
AWM 7	...	9	-0.17
Colless 67	...	7	-0.23
Dressler 0003-50	78	99	0.14
MKW 4	61	83	0.10
MKW 12	33	23	-0.01
Pegasus I	99	75	0.12
Virgo	...	92	0.04

^a Significance level (%) of the F-test or CR test for luminosity segregation.

^b Normalized difference between unweighted and weighted velocity dispersions.

tude ranges. Beers et al. (1990) showed that there are better estimators of the velocity dispersion than YV's. In particular, they suggested using the *gapper* scale estimator when the number of galaxies in the sample is ≤ 10 , and the *biweight* estimator otherwise. We will refer to these scale estimators as σ_{rob} . Note that σ_{rob} has always been computed without applying YV's clipping technique; therefore, the galaxy samples on which σ_{rob} has been computed are larger. We computed σ_{rob} for three magnitude intervals, $m \leq m_3 + 0.5$, $m_3 + 0.5 < m \leq m_3 + 1.5$, and $m > m_3 + 1.5$. In this way we succeeded in having enough galaxies for computing the velocity dispersion in each magnitude bin, for 34 clusters (the choice of $m \leq m_3$ as the first bin would not have allowed it). We then considered the average of these 34 σ_{rob} values, weighting each value according to its bootstrap error (see Beers et al. 1990). The mean values and rms errors for the three magnitude intervals are 634 ± 49 , 723 ± 49 , and 786 ± 44 , respectively (note that use of YV's dispersion estimate does not affect this result significantly). Once again, the existence of some *VLS* is suggested by the data.

Another quantitative estimate of *VLS* can be obtained by joining all cluster samples together. Specifically, we joined all the first-ranked galaxies, m_1 , in a first sample, all galaxies with $m_1 \leq m \leq m_3$ in a second sample, and all galaxies with $m > m_3$ in a third one. In Figure 3 we show the fractional histogram of the galaxies binned in intervals of $|v - \bar{v}|$ (i.e., the velocity of each galaxy with respect to its cluster mean velocity). The velocity dispersion increases from $\sigma_{rob} = 475_{-79}^{+114}$, in the first-ranked galaxy sample, $\sigma_{rob} = 665_{-35}^{+57}$ in the second sample, and $\sigma_{rob} = 876_{-10}^{+13}$ in the low-luminosity sample. The errors quoted represent the 68% probability intervals as derived via the bootstrap resampling technique (1000 resamplings). The value of σ_{rob} for the first-ranked galaxies is comparable to the velocity dispersion of dumbbell components as quoted by Valentijn & Casertano (1988).

3.2. Segregation in Distance

As shown in the previous paragraph, the more luminous galaxies move more slowly and are preferentially located in the central regions. We then considered how the velocity dispersion depends on the distance from each cluster center. Therefore, we used the distance-weighted velocity dispersion, σ_{dw} :

$$\sigma_{dw} = \left[\frac{\sum_{i=1}^n (v_i - \bar{v})^2 d_i^{-1}}{\sum_{i=1}^n d_i^{-1}} \frac{n}{n-1} - \Delta_{dw} \right]^{1/2} \quad (4)$$

where d_i is the galaxy radial distance from its cluster center (see, for comparison, eq. [2]).

We found that σ_u is larger than σ_{dw} only in the central region of the cluster ($d \leq 0.25 h_{100}^{-1}$ Mpc), where this result is only partially significant (S test: 94.44% s.l.; W test: 97.56% s.l.) We did not find any significant difference between σ_u and σ_{dw} at larger radii. This lack of evidence does not necessarily imply that segregation in distance does not exist: the effect may be far too small to be seen by using projected distances, or may be weakened by the presence of subclustering.

However, the evidence for velocity segregation increases when we take into account not only luminosities but distances as well; this can be seen by using the double-weighted velocity dispersion, σ_{ldw} :

$$\sigma_{ldw} = \left[\frac{\sum_{i=1}^n (v_i - \bar{v})^2 l_i d_i^{-1}}{\sum_{i=1}^n l_i d_i^{-1}} \frac{n}{n-1} - \Delta_{ldw} \right]^{1/2} \quad (5)$$

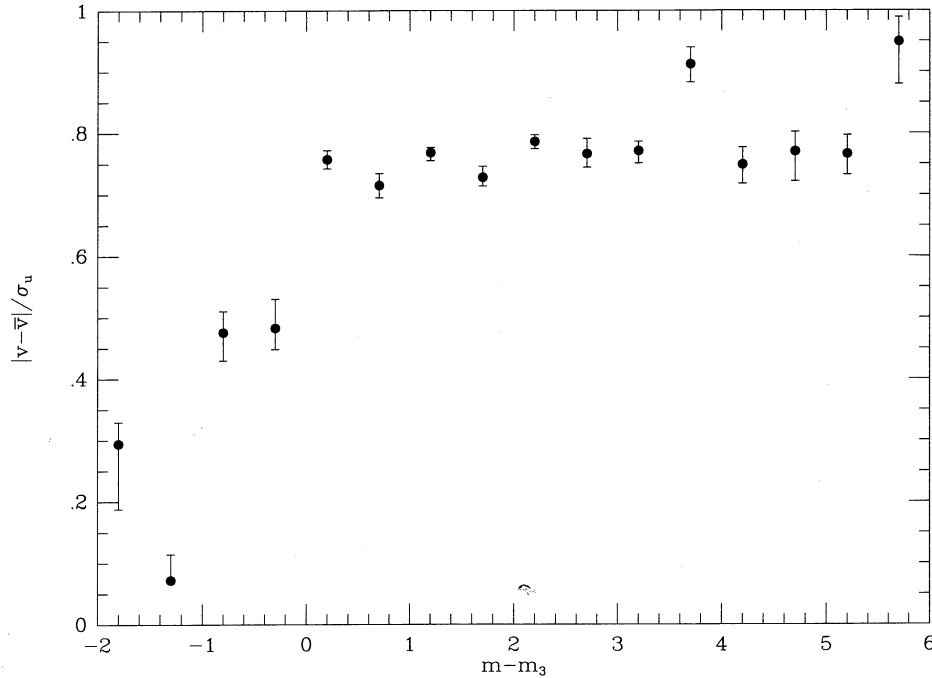


FIG. 2.—Normalized velocities $|v - \bar{v}|/\sigma_u$ binned in intervals of 0.5 mag vs. $m - m_3$; the error bars (at the 68% confidence level) are obtained via a bootstrap resampling procedure.

Both the S and W tests showed that σ_u is larger than σ_{ldw} at a significance level $>99.99\%$. The cumulative distributions for σ_u and σ_{ldw} are plotted in Figure 4. The increased difference between the weighted and unweighted dispersions can be seen by comparing Figure 4 with Figure 1.

4. DISCUSSION

We detected significant evidence for *VLS* in a sample of 61 clusters. The velocity dispersions are significantly lower for galaxies with higher luminosities. There is no evidence for *VLS* of galaxies fainter than the third ranked. The segregated galaxies are those with $m \leq m_3$; these are preferentially early-type galaxies and are located in the central regions. The segregation was clearly detected by using the velocity dispersions weighted both on galaxy luminosities and on the inverse of their clustercentric distances.

Clear-cut evidence of segregation in each of our clusters was not detected; this may be due to the smallness of the effect and/or to the limited quantity of data available for each single cluster. The fact that previous analyses based on a few clusters have sometimes failed to detect a significant amount of segregation is, then, not surprising. Moreover, clusters are likely to be in different evolutionary stages, and, consequently, in different dynamical statuses (see, e.g., Yepes et al. 1991).

The segregation we detected could be a natural characteristic of galaxies formed at the highest density peaks. Being at the cluster center, these galaxies are free enough from size limitations imposed by the mean tidal field of the cluster (see, Dressler 1984; Merritt 1984; Richstone 1990). Another possible explanation is the process of dynamical friction. The relaxation time for this process depends on the mass of the galaxy considered, as well as on the cluster velocity dispersion and core radius. We took the core radii, r_c 's for 20 clusters in our sample from Sarazin (1986), and estimated their σ_{rob} 's, and

the luminosities, l_3 , of their third-ranked galaxies. From these values we computed the relaxation time, t_3 , for the third-ranked galaxy in each cluster—using equation (2.36) in Sarazin (1986):

$$t_3 \simeq 6 \times 10^9 \text{ yr} \left(\frac{\sigma_{rob}}{1000 \text{ km s}^{-1}} \right) \left(\frac{r_c}{0.125 h_{100}^{-1} \text{ Mpc}} \right)^2 \times \left(\frac{l_3}{l_*} \right)^{-1} \left[\frac{(\mathcal{M}/l)_{gal}}{10 \mathcal{M}_{\odot}/l_{\odot}} \right]^{-1}, \quad (6)$$

where l_* is the characteristic visual luminosity of galaxies in the Schechter luminosity function (Schechter 1976), and \mathcal{M}/l is the galaxy mass-to-luminosity ratio. The values of $t_3[(\mathcal{M}/l)_{gal}/(10 \mathcal{M}_{\odot}/l_{\odot})]$, for the clusters considered are listed in Table 3, in units of 10^9 yr. We denote as t_{eff} , the effective time during which the dynamical friction process operates, in 10^9 yr. We found that t_3 , averaged over these 20 clusters, is less than a cluster lifetime, if reasonable values for the galaxy mass-to-luminosity ratio are assumed, i.e. $\sim 50/t_{eff}$, in solar units. The extreme values found for t_3 allow us to constrain the average mass-to-luminosity ratio of cluster galaxies, in the visual band, from a minimum of $\simeq 10/t_{eff}$ to a maximum of $\simeq 150/t_{eff}$, in solar units. Moreover, we divided this sample of 20 clusters into two subsamples: clusters with t_3 higher and, respectively, lower than the median value. We noticed that the evidence for *VLS* is somewhat stronger for the low t_3 subsample (S: 99.90% and W: 99.90% s.l.) than for the high t_3 one (S: 94.53% and W: 95.80% s.l.). These results are compatible with the segregation being induced by the dynamical friction process.

As a further consideration, we have fitted the logarithm of the normalized velocities $\log(|v_i - \bar{v}|/\sigma_u)$ versus the normalized magnitudes $m_i - m_3$ (see Fig. 2), using a standard least-squares procedure. The straight line fitted on the data for galaxies brighter than m_3 was found to have a slope of 0.2, just

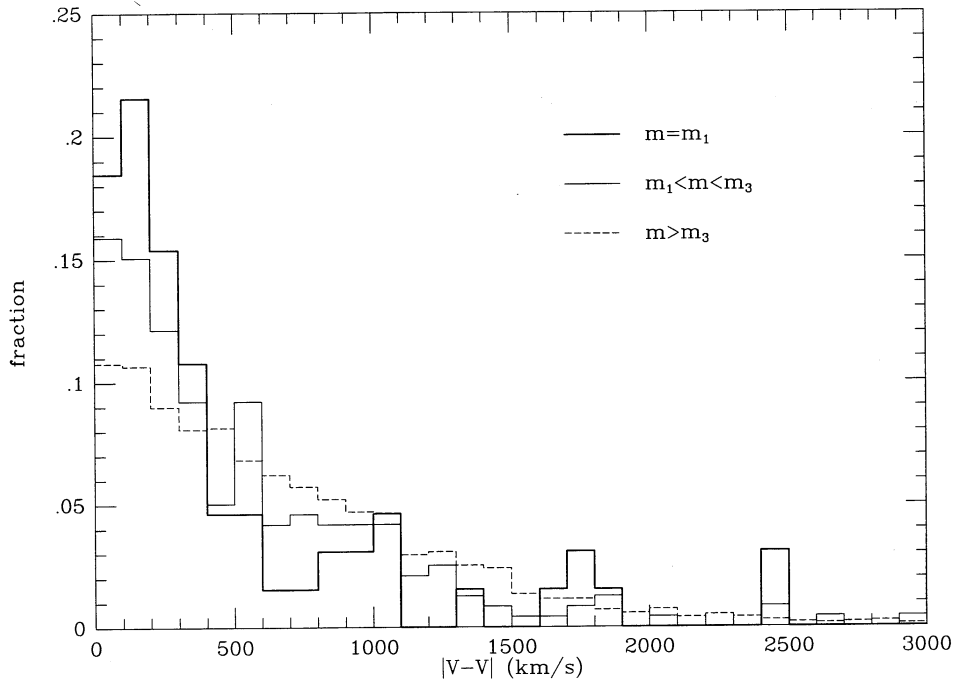


FIG. 3.—Fractional histogram of the galaxies binned in intervals of $|v - \bar{v}|$, for three cumulative cluster samples: *solid line*, first-ranked galaxies only; *continuous line*, galaxies with magnitude $m_1 \leq m \leq m_3$; *dashed line*, galaxies fainter than m_3 .

what is expected in the case of energy equipartition (assuming a constant mass-to-luminosity ratio for galaxies, which may be true for the narrow range of luminosities considered, from m_1 to m_3). The data for galaxies fainter than m_3 were best fitted by a line with a slope of 0.0, as expected in the case of velocity equipartition. Given the uncertainties in the normalizing procedure, it is difficult to assess the significance of this result, yet

it seems to be in agreement with a partial equipartition process having occurred (see, e.g., Capelato et al. 1981). All this evidence of segregation stresses the importance of m_3 as a boundary between different dynamical statuses.

Other evolutionary effects could be important in increasing the luminosities of slower galaxies (via merging, tidal accretion, etc.; see, e.g., Tonry 1987; Schombert 1987; Borthun & Schom-

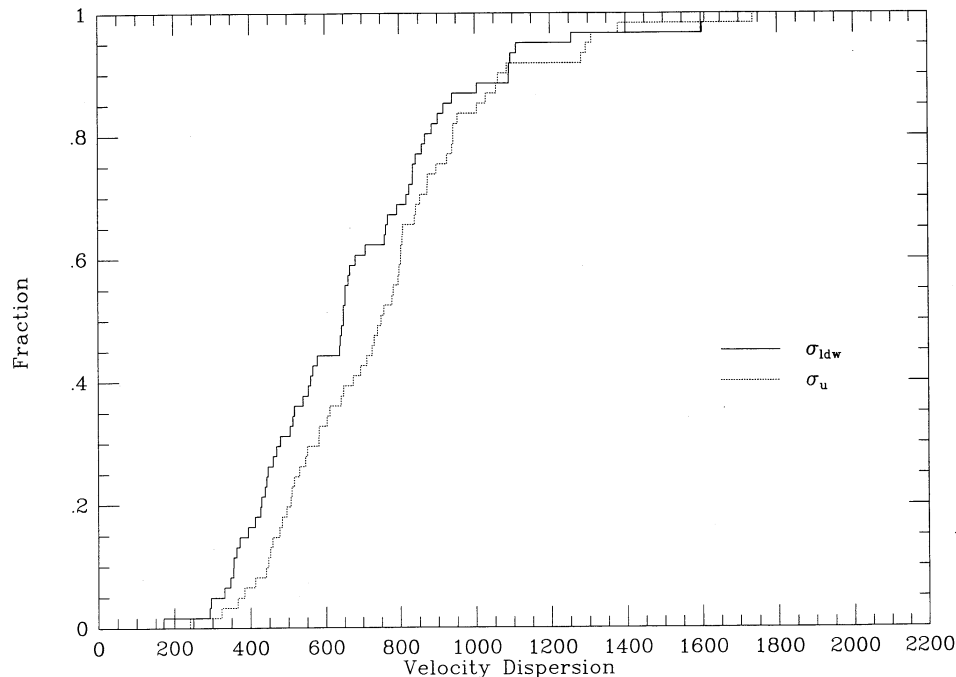


FIG. 4.—Cumulative distributions for the luminosity and distance double-weighted (*full line*) and unweighted (*dotted line*) velocity dispersions

TABLE 3
RELAXATION TIME

Name (1)	$t_3 \times \left[\frac{(\mathcal{M}/l)_{\text{gal}}}{10 \mathcal{M}_\odot/l_\odot} \right]^n$ (2)
ACO 194	1.3
ACO 426 (Perseus)	4.9
ACO 539	15.1
ACO 548	1.3
ACO 576	7.5
ACO 754	2.1
ACO 1060 (Hydra)	1.4
ACO 1367	3.6
ACO 1656 (Coma)	5.0
ACO 1983	5.0
ACO 2052	3.3
ACO 2065	5.2
ACO 2151 (Hercules)	3.0
ACO 2197	4.1
ACO 2199	2.0
ACO 2256	1.0
ACO 2670	6.6
ACO 3158	6.1
ACO 3558	5.2
Virgo	14.4

^a Relaxation time t_3 in 10^9 yr, for galaxies with $m = m_3$, multiplied by the galaxy mass-to-luminosity ratio, \mathcal{M}/l , in units of $10 \mathcal{M}_\odot/l_\odot$; see eq. (6) in the text.

bert 1990, and references therein). In order to understand the importance of merging and/or accretion phenomena, it would be interesting to look for segregation effect in the cD clusters alone. In fact, cD galaxies may have already accreted most of the slow core galaxies; if so, no segregation effect would be visible.

Therefore, we considered separately clusters of different

Bautz-Morgan type; we found evidence for *VLS* in any sample, except for the 13 clusters of BM I. Nevertheless, by considering the 33 clusters of our sample with known Rood-Sastry type, we still found significant evidence for *VLS* in the 8 cD clusters. These results seem contradictory. On the other hand, the average Bautz-Morgan type of our 8 cD clusters is I/II, i.e. the luminosity difference between the first and the second ranked galaxy is smaller than in BM I clusters. Therefore, the first-ranked galaxies in our sample of BM I clusters, may have already cannibalized most of the bright members slowed down by the cluster internal dynamics. In this way, the luminosity gap between the first and the second-ranked galaxies is increased. On the contrary, this accretion processes seems to be (or to have been) less effective in our Bautz-Morgan type I/II clusters. The analysis of the average absolute magnitude of the first-ranked galaxies, M_1 , supports this scenario, since $M_1 = -22.42 \pm 0.18 + 5 \log h_{100}$ mag, for the 8 cD clusters, and $M_1 = -22.78 \pm 0.17 + 5 \log h_{100}$ for the 13 Bautz-Morgan I clusters, i.e., a higher luminosity. However, the low number (13) of clusters considered does not allow us to draw a definite conclusion.

In closing, we wish to stress that velocity segregation has been unambiguously detected for the brightest members of 61 clusters. This result may help to constrain the evolutionary history and the modeling of clusters of galaxies, as well as their present dynamical status.

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APPENDIX

ERROR ON WEIGHTED DISPERSIONS

Consider the square of the *observed* weighted velocity dispersion:

$$(\sigma_w^0)^2 = \frac{\sum_{i=1}^n (v_i - \bar{v})^2 w_i}{\sum_{i=1}^n w_i} \frac{n}{n-1}, \quad (\text{A1})$$

where v_i is the observed velocity of the i th galaxy, w_i is the weight associated with this galaxy (e.g., the observed luminosity, l_i , or the inverse of the galaxy distance from its cluster center, $1/d_i$), n is the number of galaxies considered, and \bar{v} is the unweighted average of the velocities:

$$\bar{v} = \frac{\sum_{i=1}^n v_i}{n}. \quad (\text{A2})$$

Similarly, we can define a *true* weighted velocity dispersion, σ_w^1 , via a substitution of the observed quantities of equations (A1) and (A2), with the true (unknown) quantities: V_i , W_i , \bar{V} . The number of galaxies, n , is fixed. The observed quantities are related to the true via their errors: $v_i = V_i + \phi_i$, $w_i = W_i + \psi_i$, ϕ_i and ψ_i being positive, negative, or null. We assume that the error on each velocity and weight is drawn from a Gaussian distribution with null mean and dispersion equal to the uncertainty quoted in the literature, Φ_i , for the velocity, Ψ_i , for the weight. Therefore, the expectation values of the errors are (see, e.g., Lèna 1988)

$$\begin{aligned} E(\phi_i^{2k}) &= (2k-1)!! \Phi_i^{2k}, \\ E(\phi_i^{2k-1}) &= 0, \\ E(\psi_i^{2k}) &= (2k-1)!! \Psi_i^{2k}, \\ E(\psi_i^{2k-1}) &= 0, \end{aligned} \quad (\text{A3})$$

with $k \in \mathcal{N}$, where E denotes the expectation value. In order to compute $\Delta_w = (\sigma_w^0)^2 - (\sigma_w^t)^2$, we first make a series expansion around $\sum_{i=1}^n \psi_i / \sum_{i=1}^n W_i \simeq 0$:

$$\frac{1}{\sum_{i=1}^n w_i} = \frac{1}{\sum_{i=1}^n W_i} \left[1 + \sum_{m=1}^{\infty} (-1)^m \left(\frac{\sum_{i=1}^n \psi_i}{\sum_{i=1}^n W_i} \right)^m \right]. \quad (\text{A4})$$

This series converges for $|\sum_{i=1}^n \psi_i / \sum_{i=1}^n W_i| < 1$, i.e., when the sum of errors on weights is smaller than the sum of weights (which is likely to be the case in our data sample). Equation (A4) is used in the expression for $(\sigma_w^0)^2$. We next take the expectation value of $(\sigma_w^0)^2$ and find

$$E[(\sigma_w^0)^2] = (\sigma_w^t)^2 + \frac{n}{n-1} \left[A_0^2 + \sum_{m=1}^{\infty} (-1)^m (A_m^2) \right] \quad (\text{A5})$$

with

$$\begin{aligned} A_0^2 &= \frac{n-2}{n} \left(\frac{\sum_{i=1}^n \Phi_i^2 W_i}{\sum_{i=1}^n W_i} \right) / \left(\frac{\sum_{i=1}^n \Phi_i^2}{\sum_{i=1}^n W_i} \right) / n^2 \\ A_m^2 &= m!! \left[\frac{n-2}{n} \left(\frac{\sum_{j=1}^n \Phi_j^2 \Psi_j^{m+1}}{\sum_{j=1}^n W_j} \right) + \sum_{j=1}^n (V_j - \bar{V})^2 \Psi_j^{m+1} \right] / \left(\frac{\sum_{k=1}^n W_k}{\sum_{k=1}^n W_k} \right)^{m+1} \quad \text{if } m \text{ odd} \\ A_m^2 &= \left(\frac{\sum_{i=1}^n \Psi_i^m}{\sum_{i=1}^n W_i} \right) \left[\frac{n-2}{n} \left(\frac{\sum_{j=1}^n \Phi_j^2 W_j}{\sum_{j=1}^n W_j} \right) + \sum_{j=1}^n (V_j - \bar{V})^2 W_j \right] / \left(\frac{\sum_{k=1}^n W_k}{\sum_{k=1}^n W_k} \right)^{m+1} \quad \text{if } m \text{ even.} \end{aligned}$$

In order to estimate σ_w^t we must subtract from σ_w^0 the second term on the right-hand side of equation (A5). This expression reduces to that given by Danese et al. (1980), when $w_i = W_i = 1$ for each i .

In order to apply equation (A5) to our data, we approximated the true quantities with the observed ones. We took the errors Φ_i on the velocities from the relevant references. When we weighted with luminosity, l_i , we considered an error of 0.5 mag (which is an overestimate of the true error in most cases), which translates into an error of $\Psi_i = \delta l_i = (1 - 10^{-0.2}) l_i$. When we weighted with the inverse of the radial distance, $1/d_i$, we took from the relevant sources the error on angular position, δd , for each cluster, yielding the error on $1/d_i$, $\Psi_i = \delta d / d_i^2$. Finally, the error on the weight $w_i = l_i / d_i$, was

$$\Psi_i = \left[\left(\frac{\delta l_i}{l_i} \right)^2 + \left(\frac{\delta d}{d_i} \right)^2 \right]^{1/2}.$$

Let us denote the approximation to σ_w^t obtained by stopping the series at order m , σ_m^t . We found that $|\sigma_3^t - \sigma_2^t|$ is always less than 6 km s^{-1} , whichever weight we used, and the median value of this difference is much smaller than 1 km s^{-1} . Therefore, in our computations, we stopped the series expansion at the third order.

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