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HYDRODYNAMICS OF THE HOT COMPONENT OF THE GALACTIC HALO. II. RADIATIVE AND DYNAMICAL INSTABILITIES

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ABSTRACT

The role that hydrodynamical instabilities may play in the formation mechanism of H I high-velocity clouds (HVCs) in the Galactic halo is studied taking into account the effects of both convection and thermal conduction. The linearized MHD equations are solved numerically in order to analyze the behavior of small perturbations applied to different equilibrium configurations whose relevance in a Galactic halo context has been discussed in a previous paper (Einaudi & Ferrara). The conditions for the growth of a condensation mode, often invoked to explain the origin of HVCs, and its spatial profile are derived. It is shown that, for the range of parameters appropriate to the hot disk gas generated by multisupernova explosions, thermal instability is strongly inhibited by spatial gradients in the background flow. It is argued that HVCs can hardly form as a result of a thermal instability in a hot Galactic flow. A nonradiative but rather "dynamical" instability may develop whose growth rate is maximum when parameters close to those determining a transonic solution for the flow are adopted; transonic solutions themselves are found to be stable. Some discussion is devoted to the possibility that this "dynamical" instability may be connected with the onset of a thermal supersonic Galactic wind.

Subject headings: Galaxy: halo — hydrodynamics — instabilities — MHD

1. INTRODUCTION

This is the second of a series of papers whose aim is to investigate the properties of the hot component, if present, of the Galactic halo. Here we study the behavior of small perturbations which can arise in this component, whereas the first paper (Einaudi & Ferrara 1991, hereafter Paper I) was devoted to the analysis of the stationary solutions modeling hot gas outflows from the disk into the halo. The presence of such a gas at high latitudes in the Milky Way and in external galaxies has been suggested, but not definitely proved, by a growing number of evidences. The most obvious diagnostic technique for hot gas ($T \simeq 10^6$ K) is represented by X-ray observations. Bregman (1980b) developed a theoretical model of the emission properties of X-ray coronae around galaxies. He concluded that the bulk of the emission should occur below 1 keV and that for edge-on galaxies the corona may be more luminous than the disk which suffers the X-ray absorption by neutral gas. Bregman & Glassgold (1982) have searched for X-ray emission in two edge-on galaxies (NGC 3628 and NGC 4244) in the *Einstein* IPC 0.9–2.9 keV band, but they detected no diffuse emission from the coronae of those galaxies. On this basis they suggested that either the gas must be at a lower temperature ($< 6 \times 10^5$ K) or a very hot wind may be present. McCammon & Sanders (1984) also derived an upper limit ($T < 5 \times 10^5$ K) for the temperature of the hot gas in the halo of M101 from X-ray observations. The implications of these observations for the general structure and properties of the ISM in M101 are explored by Cox & McCammon (1986). Recently Wang (1991) has reported the detection of diffuse X-ray emission from the halo of the Small Magellanic Cloud using the *Einstein* IPC. He claims that the emission spectrum is consistent with a hot cooling flow from the galaxy's body. Extensive searches for hot gas in the halo of the Galaxy have been carried out by Nousek et al. (1982) and Marshall & Clark (1984), their work being based on different surveys of the soft X-ray sky. The signatures of the hot coronal component may be found in the emission detected by Nousek et al. (1982) in the MI band (0.5–1.0 keV) and by Marshall & Clark (1984) in the *SAS 3* carbon band (0.1–0.28 keV). These data are consistent with hot halo gas having a temperature of 2–3 $\times 10^6$ K, but unfortunately, because of many difficulties in the interpretation of the measurements, a local origin cannot be excluded. Indirect indications of hot gas in the halo come from *IUE* absorption-line measurements. The most relevant feature in this context is the N V absorption line, which has a scale height of about 2 kpc (Savage & Massa 1987). Because of its high ionization potential (77.5 eV), this ion is very likely to be produced collisionally and therefore has been taken as a good indicator of hot gas. In fact the abundance of N V, under a collisional equilibrium hypothesis, peaks at about $T_c = 2 \times 10^5$ K; however, Edgar & Chevalier (1986) have demonstrated convincingly that absorption lines are most likely to be formed under nonequilibrium ionization conditions in a gas cooling from temperatures higher than T_c. Theoretical foundations for the origin and existence of high-latitude hot gas are provided by several models. That which has attracted more attention in the last decade is the so-called galactic fountain (hereafter GF) model introduced by Shapiro & Field (1976) and subsequently developed by Bregman (1980a). In the GF model, the hot gas ($T \simeq 10^6$ K) present in the Galactic disk, which has a thermal scale height of 6 kpc and therefore is buoyant, tends naturally to flow into the halo where it cools through the combined effects of convection and radiation. Furthermore, Shapiro & Field (1976) suggest that some condensations (clouds) may form as a result of the cooling process and fall back to the plane at high velocities, thus indicating a connection with the observed high-latitude, H I high-velocity clouds (HVCs). Bregman (1980a) argued that thermal instabilities like those studied by Field (1965) may represent a suitable formation mechanism for the HVCs in the framework of the GF model. However, it has become increasingly clear that the filling factor of the hot component of the ISM in the disk is not as high as previously thought (Cox 1989, 1990; Cox & Slavin 1991). In fact the neutral layer, as revealed by H I 21 cm observations, has a thick exponential distribution superposed to the Gaussian one and reaches heights of about 500 pc (Lockman 1984; Dickey & Lockman 1990). With this recognition the circulation of gas proposed by the GF model should be considered as a localized phenomenon where most of the hot gas is created, i.e., regions of multiple supernovae explosions. This point is implemented correctly in the chimney model (Norman & Ikeuchi 1989) which predicts that the hot gas can flow freely into the halo only through the open top of a very large superbubble created by multiple supernovae explosions and subsequently fragmented by a Rayleigh-Taylor instability, thus forming what the authors define as a "chimney." Although this model is quite appealing, some important aspects are far from accepted. In particular, numerical simulations and analytical work seem to indicate that, if the exponential tail of the H I distribution is really present, the blowout does not occur unless the center of the superbubble is located about 100 pc or more from the Galactic plane (McCray & Kafatos 1987; Tenorio-Tagle & Bodenheimer 1989; Spitzer 1990). In addition, the situation is even worse if a magnetic field of a few μG parallel to the plane is present (Cox 1989; Tomisaka 1991).

From the above considerations it is possible to realize that our present comprehension of the general scenario of the disk-halo connection and of the coronal environment is somewhat uncertain. We believe that a crucial point, a potentially rich source of information in order to achieve a deeper understanding of the processes which govern the thermal and dynamical properties of the Galactic halo, is represented by the study of the origin and properties of the HVCs. In this paper we would like to clarify whether the existence of HVCs may be generally explained with thermal instabilities occurring in a hot coronal flow originating in the Galactic disk.

The properties of thermal instabilities were explored by Field (1965) who derived the dispersion relation of the modes existing in a homogeneous static fluid where radiative losses, mechanical heating, and thermal conduction are taken into account. His study was based on two essential assumptions: the first is that the fluid is homogeneous and the second is that the perturbations are periodic in space along with their derivatives. These assumptions allow solutions of the form $e^{i(\boldsymbol{kr}+\omega t)}$ and yield the dispersion relation between ω and k. It is evident that under these conditions the presence of a homogeneous velocity field produces only a Doppler shift of the frequencies without altering the physics of the modes, because the static situation can be recovered with a simple change of the reference frame. The above assumptions appear to be clearly unrealistic in a Galactic application. In fact, on the one hand, the outflow from the disk can be homogeneous only when the radiative cooling is exactly balanced by some form of mechanical heating everywhere. On the other, even if the actual boundary conditions which must be satisfied by the perturbations are unknown, the periodic ones appear to be highly unrealistic. It follows that a Fourier expansion is not allowed and Field's results are not applicable, even in a homogeneous configuration.

The choice of the boundary conditions in a stability analysis is crucial because it may change the nature of the possible perturbations which arise in the system (provided that perturbations scale lengths are not small compared to the equilibrium ones) and therefore may alter sensitively the stability properties of the same equilibrium structure. In the case of interest, there are two possible choices which in principle can lead to completely different results. One possibility is to believe that the perturbation is unable to change the conditions of the flow at the disk level and therefore perturbations of the governing parameters must vanish there. This possibility applies to a physical situation in which the flow propagates in the vacuum and does not have to coexist with an external medium. When an external medium is present, a stationary solution is achieved when the influence of the external medium propagates back to the origin of the flow, producing a "feedback" which changes the physical conditions there. We have seen in Paper I that once the above-mentioned values of the governing parameters are given, the properties of the stationary solutions are completely determined. There are solutions which develop singularities at the sonic point, thus unphysical; there are solutions which correspond to either subsonic or supersonic flows depending on whether the velocity at the disk is subsonic or supersonic; finally, we have critical solutions in which an initially subsonic flow becomes supersonic passing the sonic point or vice versa. All these solutions have been found by changing the values of velocity, density, temperature, and heat flux at the disk level. Following the above discussion, some of these values depend on the feedback of the external medium in which the flow is propagating and therefore the nature of the stationary solution, which is actually achieved by the system, can be found only by an inspection of the time-dependent equations governing the system.

The simplest approach to this problem is represented by a linear stability analysis of the stationary solutions discussed in Paper I considering boundary conditions for the perturbations such that the effects of an external medium can be taken into account. If the perturbation grows in time, the corresponding stationary solution is not physically feasible, since the conditions at the disk level producing such a solution vary in time as a result of the simulated feedback. The linear analysis can provide information only on the nature of the stable, physically realistic equilibria and on the time scales of the departure of the system from the initial configuration in the first phase. No information can be deduced on the nature of the final state or on the total transient time necessary to achieve the equilibrium.

In this paper we perform this analysis adopting a normal modes approach, i.e., looking for solutions of the linearized relevant equations whose time dependence is of the form $e^{i\omega t}$. The frequency ω, which is the eigenvalue of the problem, is in general a complex number. In the homogeneous case, with periodic boundary conditions, there are several eigenmodes which can be easily identified as sound waves, gravity waves, and one condensation mode. We are interested in the possible formation of HVCs in the galactic halo and therefore in the condensation mode, described by Field (1965) in the static and homogeneous case. This mode is practically isobaric; it is driven by a favorable energy balance and is a purely growing mode in the unstable static configurations; thus, the real part of the frequency ω_r is zero. We will study the properties of this thermal mode by adopting those boundary conditions capable of providing the halo conditions discussed above, and by including the effects of the dynamical terms not considered in the previous works. We will consider one-dimensional perturbations which depend only on the coordinate along the equilibrium streamlines, solving numerically the resulting eigenvalue problem and deriving the conditions for stability, the growth rate of the perturbations in the unstable configurations, and their spatial profiles whose inspection allows their identification. The plan of the paper is the following: in § 2 we describe the model and the fundamental equations, and in § 3 we present the results of the stability analysis. Section 4 gives a brief summary of the results along with the conclusions.

2. DEFINITIONS AND EQUATIONS

In Paper I we solved the equations governing the steady state of a hot outflow propagating from the disk of a spiral galaxy; here we study the stability properties of those structures. We just mention the approximations and assumptions made in Paper I, which are the same adopted here.

We assume that the flow is one-dimensional in the direction perpendicular to the Galactic disk along the field lines of the magnetic field. The magnetic flux tube defining the topology of the flow has a cross section $A(z)$ which enlarges with height. The only magnetic effect considered is the collimation both of the flow and of the heat flux in the direction of the field with the subsequent reduction of the dimensionality of the problem to one. The plasma is assumed to be optically thin and the energy equation adopted expresses the balance resulting from the combined effects of thermal conduction, radiative losses, convection and mechanical heating. The gas is supposed to the perfect, fully ionized with cosmic abundances (Allen 1973). As for the gravitational potential Φ, we use the model by Innanen (1973) for the Galaxy, modeled as a three-component (bulge, disk, and a dark massive halo) system. The adopted cooling function $\Lambda_{\rm rd}$ is the standard one for collisional equilibrium given by Raymond, Cox, & Smith (1976); the thermal conduction coefficient $\kappa_{\parallel}$ parallel to the magnetic field lines is the classical one given by Spitzer (1962). In the coronal environment, this is a fairly good approximation and saturation effects considered by Cowie & McKee (1977) may be neglected in the linear regime of perturbations, if it is negligible in the stationary flow.

All the quantities entering the equations must be understood as nondimensional, each having been normalized to the appropriate scale. We use as basic dimensional quantities time, density, and temperature, whose scales are indicated by τ, ρ^*, T^*; τ will be identified with the cooling time corresponding to ρ^* and T^*, which are the values of density and temperature at the disk level. In terms of these basic scales, we derive the scales of all the remaining physical quantities. In particular, the velocity scale is given by the adiabatic sound speed c_s^* at the base, the length scale by τc_s^*. In the absence of self-gravity, the relevant equations can be written as follows:

$$\frac{d}{dt}(\rho A) + \rho A \frac{\partial v}{\partial z} = 0 \,, \tag{2.1}$$

$$\rho \frac{dv}{dt} = -\frac{\partial p}{\partial z} - \frac{1}{L_g} \rho g_z \,, \tag{2.2}$$

$$\rho^\gamma \frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right) = \frac{1}{\tau_{\rm cd}} \frac{\partial}{\partial z}\left(T^{5/2} \frac{\partial T}{\partial z}\right) - \rho^2 T^f + \frac{1}{\tau_{\rm he}} H \,, \tag{2.3}$$

$$p = \rho T \,, \tag{2.4}$$

where ρ is the density of the fluid, v is the velocity, p is the thermal pressure, and T is the temperature. We have also introduced the conductive and heating time scales $\tau_{\rm cd}$ and $\tau_{\rm he}$, both normalized to the radiative one (for their explicit expression, the reader is referred to Paper I). A misprint has introduced a gravitational term in the energy equation of Paper I which does not appear in the above equation (2.3), and which was not considered in the calculations. The term f denotes the appropriate exponent of the cooling function in the range of temperatures considered, γ is the specific heats ratio, g_z is the z-component of the gravitational field, and L_g is the nondimensional gravitational scale length (which is equal to 1 if the scale length is normalized to the sonic radius). The exponent f is temperature-independent, which seems to be a good assumption in a linear regime. In general, as Gaetz, Edgar, & Chevallier (1988) have demonstrated, in a fountain flow, the cooling function may be different. In fact, it would be necessary to perform the ionization and cooling function calculations along with the hydrodynamics; we will address this issue in a forthcoming paper. For comparison with Field's original analysis, we consider a constant heat source term H in the energy equation (2.4), which will be dropped subsequently. A homogeneous steady solution is attained when H is equal to the radiative losses computed at ρ^* and T^*. This term has been introduced in the present analysis for the following reason. The properties of thermal instabilities were studied by Field (1965) adopting a homogeneous equilibrium, resulting from a balance between radiative losses and mechanical heating and neglecting gravity. Although we are not interested in homogeneous configurations, we found it instructive to start our analysis by deriving the thermal stability properties of a static homogeneous equilibrium and then modifying the boundary conditions with respect to Field's analysis, in order to identify the condensation mode. This mode will then be studied in more realistic configurations obtained by decreasing to zero the source term H, and by increasing the disk velocity v_0^*.

We perform a linear stability analysis; any quantity ζ may be expressed as $\zeta = \zeta_0 + \delta\zeta$, where the perturbation $\delta\zeta$ is such that $|\delta\zeta|/|\zeta| \ll 1$ and has the form

$$\delta\zeta(z, t) = \delta\zeta(z) e^{i\omega t} \,. \tag{2.5}$$

By linearizing equations (2.1)–(2.4), we obtain the following set of equations for the perturbed quantities ρ, v, T:

$$v_0 \rho' + \rho_0 v' + (i\omega + \Gamma v_0 + v_0')\rho + (\Gamma \rho_0 + \rho_0')v = 0 \,, \tag{2.6}$$

$$T_0 \rho' + \rho_0 v_0 v' + \rho_0 T' + \left(v_0 v_0' + \frac{1}{L_g} g_z + T_0'\right)\rho + (i\omega\rho_0 + \rho_0 v_0')v + \rho_0' T = 0 \,, \tag{2.7}$$

$$\begin{aligned} -\frac{1}{\tau_{\rm cd}} T_0^{5/2} T'' &+ (v_0 T_0 - \gamma v_0 T_0)\rho' + \left(v_0 \rho_0 - 5 \frac{1}{\tau_{\rm cd}} T_0^{3/2} T_0'\right) T' \\ &+ (i\omega T_0 - i\gamma\omega T_0 + v_0 T_0' + 2\rho_0 T_0^f)\rho \\ &+ (\rho_0 T_0' + T_0 \rho_0' - \gamma T_0 \rho_0')v \\ &+ \left\{ i\omega\rho_0 + v_0 \rho_0' - \gamma v_0 \rho_0' + f\rho_0^2 T_0^{f-1} \right. \\ &\left. - \frac{1}{\tau_{\rm cd}}\left[\frac{15}{4} T_0^{1/2} T_0'^2 + \frac{5}{2} T_0^{3/2} T_0''\right]\right\} T = 0 \,, \end{aligned} \tag{2.8}$$

where the subscript 0 refers to the unperturbed quantities, the prime indicates the z-derivative, and $\Gamma = 1/A(dA/dz)$. In order to solve the problem numerically, the system (2.6)–(2.8) has been put in a different form, involving two coupled equations for T and ρ, i.e., eliminating v, whose explicit expressions are given in the Appendix. Equations (2.6)–(2.8) form a homogeneous fourth-order ordinary differential system which implies that four boundary conditions (BCs) are required. When they are homogeneous, we have to deal with an eigenvalue problem where the eigenvalue is represented by the normalized growth rate ω. Following the discussion given in the Introduction, we want to simulate the effect of the feedback of the external medium through a proper choice of the BCs. Therefore we choose to impose two BCs at the disk level and two at the upper boundary located at z_l, where we assume that the external pressure is comparable to the internal one. The actual value of z_l in this framework is evidently a free parameter of the problem and has been varied in the range 3–10 kpc.

Which is the more realistic choice of the BCs depends on the response of the external fluid at $z = 0$ and z_l to the flow perturbation, which is observationally unclear. However, arguments can be found in favor of several different possibilities. The external fluid can be considered either a mass reservoir, such that the perturbed density must vanish, or a perfect absorber, such that the perturbed pressure must vanish. It may act as a thermostat, implying that the perturbed temperature vanishes, or alternatively as an insulator, maintaining the heat flux unaltered. In order to check the possible difference in the stability properties of the same equilibrium configuration induced by a different choice of the BCs, we parameterize the external fluid response in the following way:

$$K^{(1)}_{(0,l),1}\,\rho' + K^{(2)}_{(0,l),1}\,\rho + K^{(3)}_{(0,l),1}\,T' + K^{(4)}_{(0,l),1}\,T = 0\ ; \quad (2.9a)$$

$$K^{(1)}_{(0,l),2}\,\rho' + K^{(2)}_{(0,l),2}\,\rho + K^{(3)}_{(0,l),2}\,T' + K^{(4)}_{(0,l),2}\,T = 0\ ; \quad (2.9b)$$

the coefficients $K^{(j)}$ are considered as free parameters to be varied and subscripts 0, l refer to the boundaries $z = 0$ and $z = z_l$, respectively. Some particularly meaningful limiting cases can be obtained by an appropriate choice of the $K^{(j)}$ coefficients at each boundary. All the detailed results presented in this paper refer to a choice of $K^{(j)}$ modeling the boundaries as a thermostat and mass reservoir. We have found the solutions to be insensitive to variations of the type of BCs used in the calculation. The numerical procedure adopted is a finite difference scheme with variable grid. The grid size is a function of ω, determined by the local scale of the equilibrium quantities. We make an initial guess for the perturbations T and ρ using a number of grid points N equal to the number of grid points necessary to compute the equilibrium. Then we increase N in order to attain a satisfactory accuracy on T and ρ by using the routine D02RAG of the NAG numerical library. The convergence is quite fast, and we are able to compute the eigenvalue ω and the profiles of the corresponding eigenfunctions with an error of a few percent.

3. RESULTS AND DISCUSSION

As we have shown in Paper I, it is possible to obtain various equilibrium configurations depending on the actual thermodynamic conditions at the disk-halo interface. In this section, we discuss their stability properties which are deduced from the behavior of the eigenvalues and the eigenfunctions of the system (2.6)–(2.8) as a function of the background flow variables (density, temperature, velocity, heat flux, heating, etc.). Given the form of the elementary perturbation (2.5), an instability will occur if the imaginary part of the eigenvalue Im (ω) is negative.

We first study the static homogeneous solutions of the steady state equations, i.e., constant temperature and density and zero velocity. This solution exists when the equations are simplified by considering a constant cross section for the flow, by neglecting the gravity and by assuming the following mechanical heating:

$$H - \Lambda_{\rm rd}(\rho^*, T^*) = 0\ . \quad (3.1)$$

If this condition is not fullfilled, gradients in the profiles of the various equilibrium quantities are always present. As far as the equilibrium configuration is concerned, this is the solution studied by Field (1965), but the use of the BCs (2.9) modifies the properties of the various modes present in the system. A spatial Fourier expansion of the perturbations is not consistent with such a choice of the BCs and therefore it is impossible to determine a simple dispersion relation $\omega(k) = 0$ as in the case of the periodic BCs adopted by Field. We find an infinite spectrum of modes, and we identify those corresponding to pure imaginary values of ω and to negligible pressure variations with the condensation ones. Among them we look for the eigenvalue with the minimum imaginary part of the frequency, $\hat{\omega}_i$: its sign determines whether or not the system is stable and, when the sign is negative, its modulus represents the growth rate of the instability. The sign of $\hat{\omega}_i$, given the density and the temperature of the system, depends only on z_l, and it becomes positive for sufficiently short structures. We can then define the critical length z_{l_c} as the minimum length for a given temperature and density in order to have instability. Using a set of values of ρ_0^*, v_0^*, T_0^*, and z_l which lead to unstable solutions, we start to decrease z_l. The spectrum of the eigenvalues is then shifted toward more stable values, and we take as z_{l_c} the largest z_l for which all the eigenvalues have a positive imaginary part. In order to quantify the differences introduced by our BCs with respect to the periodic ones, in Table 1 we compare the values of z_{l_c} computed when $K^{(2)}_{(0,l),1} = K^{(4)}_{(0,l),2} = 1$, $K^{(j)}_{(0,l),i} = 0$; $i = 1, 2$; $j \neq 2, 4$, with the corresponding critical lengths $z^{\rm F}_{l_c}$ obtained from Field's dispersion relation using our cooling function.

We find values of z_{l_c} systematically larger than those of Field, which means that some configurations which are unstable against periodic perturbations are stabilized when our BCs are introduced. This result is not surprising because BCs (2.9) limit the class of perturbations which can arise in the system with respect to the periodic case. When z_l is slightly bigger than z_{l_c}, we find only one mode with negative $\hat{\omega}_i$. The corresponding eigenfunctions ρ and T are such that the pressure perturbation for the adopted BCs is

$$P = \rho_0\,T' + T_0\,\rho' \simeq {\rm const} = 0\ , \quad (3.2)$$

as is evident from the example shown in Figure 1. These results are in perfect agreement with the well-known fact that in the

TABLE 1

CRITICAL LENGTHS

n^* (cm^{-3})	T^* (K)	z_{l_c} (kpc)	$z^{\rm F}_{l_c}$ (kpc)
10^{-3}	10^6	0.88	0.7
10^{-3}	$3. \times 10^6$	8.4	6.5
$5. \times 10^{-3}$	10^6	0.19	0.14

NOTE.—Comparison of the critical length z_l (this paper) with the one obtained by Field 1965 for different base parameters of the flow.

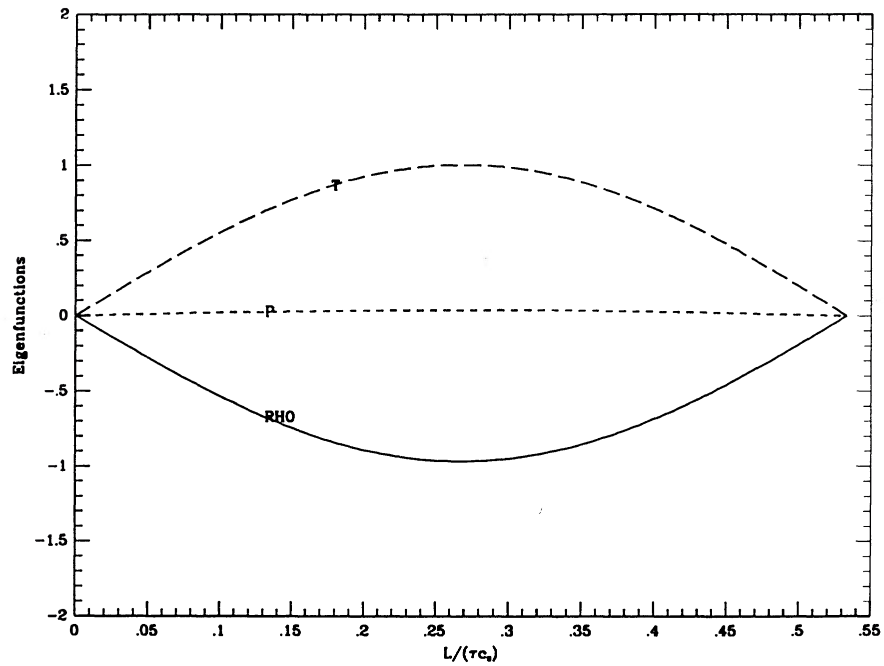

FIG. 1.—Eigenfunctions for the homogeneous case with $n^* = 10^{-3}$ cm^{-3}, $T^* = 10^6$ K, $z_l = 1.61$ kpc, and $\hat{\omega}_i = -1.17$. *Solid line*, density; *dashed line*, pressure; *long-dashed line*, temperature.

homogeneous case, the condensation mode is almost perfectly isobaric, that, if $P = 0$, the problem can be reduced to a Sturm-Liouville one (McClymont & Craig 1985) with the consequence that the most unstable condensation mode, identified as the only one with negative $\hat{\omega}_i$ in this case, is the fundamental harmonic with no nodes in the eigenfunctions.

We want to determine the properties of such a mode in more realistic nonstatic and nonhomogeneous configurations. It is interesting to note that, also in these cases, the system of equations (2.6)–(2.8) admits nonoscillatory solutions, which we find throughout the paper to correspond to the condensation mode, whose $\hat{\omega}_i$ is modified by the presence of velocity fields and of the gradients of the equilibrium quantities.

The presence of a homogeneous equilibrium velocity v_0^* induces a shift toward more stable values of the spectrum of the eigenvalues with respect to the static case. Eventually, for any given value of z_l such that the static configuration is unstable, $\hat{\omega}_i$ changes sign and stability is achieved for $v_0^* > v_c$. Some computed values of v_c corresponding to different choices of ρ_0^* and T_0^* are given in Table 2, with $K^{(2)}_{(0,l),1} = K^{(4)}_{(0,l),2} = 1$, $K^{(j)}_{(0,l),i} = 0; i = 1, 2; j \neq 2, 4$.

A decrease of H from the value necessary to have homogeneity has the effect of creating gradients in the steady state

TABLE 2

CRITICAL VELOCITIES

n^* (cm^{-3})	T^*(K)	z_l (kpc)	v_c (km s^{-1})
10^{-3}	10^6	1.6	25.0
10^{-3}	$3. \times 10^6$	9.0	33.0
5×10^{-3}	10^6	0.3	27.0

NOTE.—Values of the velocities of which a stability is achieved for different base parameters of the flow (homogeneous equilibrium case).

solutions, with a consequent increase of the importance of thermal conduction. In general, the presence of gradients influences the growth rates of the condensation mode. In order to show this effect, we choose two representative disk values of the density and temperature, namely $n^* = 10^{-3}$ cm^{-3}, $T^* = 10^6$ K, and we also fix $z_l = 10$ kpc and $K^{(2)}_{(0,l),1} = K^{(4)}_{(0,l),2} = 1$, $K^{(j)}_{(0,l),i} = 0$; $i = 1, 2$; $j \neq 2, 4$. For the adopted values of n^* and T^*, the cooling time is equal to $\tau = 2.6 \times 10^7$ yr. We also use a very low value of the Mach number of the flow at the disk level, namely $M^* = 8.8 \times 10^{-4}$, which is unlikely to be realistic, but, as it turns out from the energy terms analysis performed in Paper I, ensures that radiation strongly dominates the energetics of the flow. When H is given by equation (3.1), the corresponding equilibrium is unstable, as shown in Table 1, and the unstable mode is driven by radiation losses and corresponds to a thermal instability. In the next table (Table 3), we show the variations of $\hat{\omega}_i$ as a function of the heating. For values of H below $\simeq 10^{-28}$ ergs cm^{-3} s^{-1}, the equilibrium solutions do not differ appreciably from the one without heating. When the gradients in the steady state solution are increased through a progressive decrease of the heating term, the growth rate becomes slower until complete stability is reached once the heating is suppressed. The numbers in Table 3 have a variation of a few percent for a different choice of the set $K^{(j)}_{(0,l)}$.

Up to this point we have shown that thermal instabilities are suppressed when (1) flows are present such that the crossing time is so short that the cooling does not have the time to be effective (Table 1) and (2) temperature gradients are present such that the corresponding heat flux is enhanced enough to stabilize the system. With these results in mind, we turn to the stability analysis of some of the structures described in Paper I in which convective and conductive effects are included. Therefore, from now on, we consider $H = 0$, we turn on the gravity, and we allow the cross section $A(z)$ to vary as described in

TABLE 3
HEATING EFFECTS

$H/10^{-28}$ (ergs cm^{-3} s^{-1})	$\hat{\omega}_i$
3.74	−1.348
2.	−0.231
1.8	−0.134
1.	−0.064
<1.	Stable

NOTE.—Effects of the variation of the mechanical heating on the growth rate.

Paper I. The equations which govern the equilibrium state of the flow admit solutions which present critical points (critical solutions), where a transition between the subsonic and supersonic regime occurs. We discuss in detail the stability analysis of configurations with the values of $n^* = 0.6 \times 10^{-3}$ cm^{-3}, $T^* = 4.1 \times 10^6$ K, and of the heat flux $(dT/dz)^* = 4.52 \times 10^{-17}$ K cm^{-1}, which, for $v^* = 157$ km s^{-1}, gave rise to the critical solution presented in Figure 1 of Paper I. We can then derive the stability properties of a whole set of equilibria, including the critical one, with Mach numbers at the base ranging from 8.8×10^{-4} to 10. We choose $z_l = 10$ kpc and $K^{(2)}_{(0,l),1} = K^{(4)}_{(0,l),2} = 1, K^{(j)}_{(0,l),i} = 0; i = 1, 2; j \neq 2, 4$.

For $M^* = 8.8 \times 10^4$, the system is stable, as expected from the results presented in Table 3. It becomes unstable for very low $M^* > M^*_{\rm cr} \simeq 0.004$. Figure 2 illustrates the behavior of the imaginary growth rate $\hat{\omega}_i$ as a function of the Mach number M^* for the unstable configurations. It is seen that the system becomes more and more unstable as v^* approaches the value $v^* = 157$ km s^{-1} (at which $M^* = 0.68$) corresponding to the critical solution. We find that the critical solution is instead fully stable, a fact that has important consequences which we will discuss in § 4. As explained in Paper I, the value of v^* cannot be increased beyond 157 km s^{-1} in this case because the corresponding solutions present an O-type singularity. Physically meaningful solutions are found again when $v^* > 278$ km s^{-1} ($M^* = 1.2$). Increasing v^* to supersonic values, $|\hat{\omega}_i|$ becomes smaller, and eventually the system becomes stable again for high M^*, for the order of 10. In Figure 3 we present the profiles of the temperature, density and pressure perturbations for a typical solution ($v^* = 70$ km s^{-1}). There are two important points, resulting from an inspection of Figures 2–3, which we want to outline. The first is that the values of $|\hat{\omega}_i|$ are significantly greater than 1 for a large range of Mach numbers. Thus, the corresponding modes grow much faster than the thermal modes, whose typical growth time is comparable to the cooling time; the second is that the perturbation is not isobaric. We have also evaluated for the solution shown in Figure 3 the relevance of the various contributions to the perturbation energy, namely the dynamical, conductive, gravitational, and radiative terms, and we have found that the energetics of the perturbed flow and dominated by the action of convection and thermal conduction. It follows that this instability is favored by the presence of perturbed pressure gradients, and it is energetically driven by mechanisms different from radiative losses. Therefore, it seems more suitable to refer to it as a dynamical instability rather than a thermal one.

The value of the heat flux at the actual Galactic disk cannot be deduced at any reasonable confidence level from the observations. Taking this point into account, we have studied its influence on the stability properties of the flow. A decrease of $(dT/dz)^*$ reduces the thermal conduction, and the system tends to be more unstable for supersonic flows. As an example, when $(dT/dz)^* = 0$ K cm^{-1}, $n^* = 10^{-3}$ cm^{-3}, and $T^* = 10^6$ K, the behavior of $\hat{\omega}_i$ with v^* is similar to that presented in Figure 2, as regards the subsonic part of the curve; in the supersonic part, $|\hat{\omega}_i|$ is not a decreasing function of v^* but rather tends to saturate. The variation of the coefficients $K^{(j)}_{(0,l)}$ does introduce differences in the results, but they are not appreciable.

All the above presented results referring to nonhomogeneous, nonstatic configurations correspond to $z_l = 10$ kpc. We have investigated the effects of varying z_l, finding that $|\hat{\omega}_i|$ is a decreasing function of z_l. If the influence of the external medium becomes important at smaller distances from the disk,

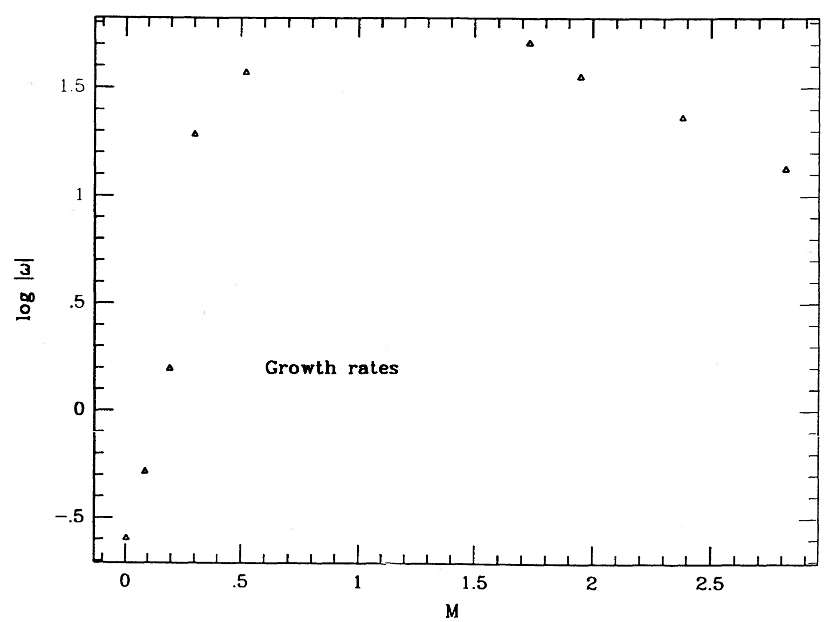

FIG. 2.—Growth rate of the dynamical instability as a function of a Mach number of the flow

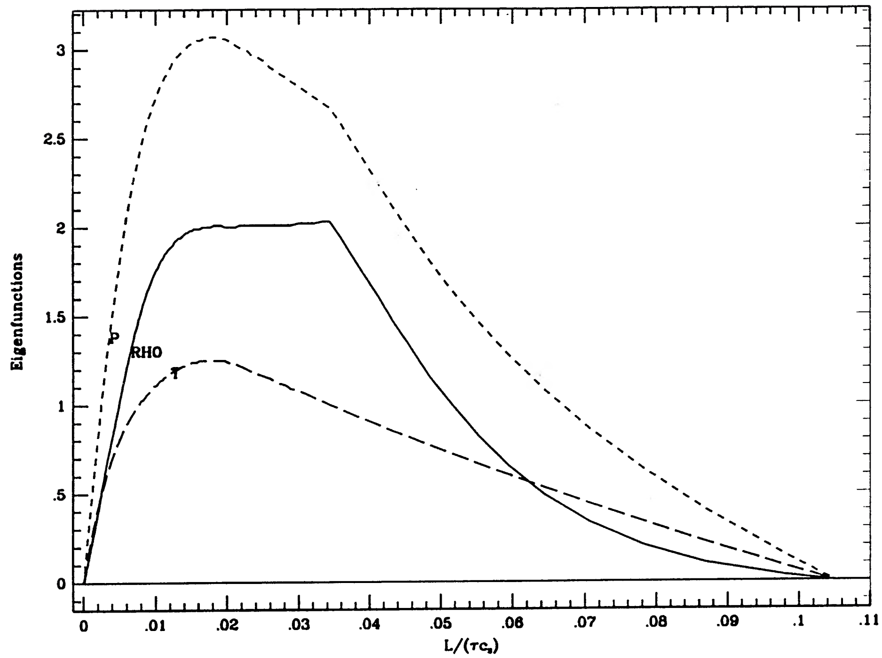

FIG. 3.—As Fig. 1, but case with $n^* = 0.6 \times 10^{-3}$ cm^{-3}, $T^* = 4.1 \times 10^6$ K, $\dot{M}^* = 0.3$, $z_l = 10.0$ kpc, and $\hat{\omega}_i = -19.22$

the nontransonic configurations are more unstable. As an example, for $z_l = 3$ kpc the growth time of the unstable modes is 30% shorter than the corresponding ones for $z_l = 10$ kpc. If z_l is so small that no transonic solutions can exist, then all equilibria are stable.

Finally, we conducted the stability analysis of a particularly interesting solution, namely one resulting from the choice of the parameters suggested by the chimney model (Norman & Ikeuchi 1989). The most suitable choice for the model parameters is $n^* = 10^{-3}$, $T^* = 3 \times 10^7$ K, and a velocity v^* close to the sound speed. The results of the numerical stability analysis show the "chimney" to be unconditionally stable for a large velocity range ($500 < v^* < 700$ km s^{-1}) for the background flow. A peculiar characteristic of the "chimney" solutions, as shown in Paper I, is the absence of critical solutions.

In order to investigate the role that the presence of critical solutions plays in determining the stability properties of hot radiative flows, we have studied a number of different cases with values of n^*, v^*, T^*, $(dT/dz)^*$ for which no critical solution is present. We always find that all the existing configurations (evidently, either completely subsonic or supersonic) are stable. We will discuss this point within a more general context in the following section.

4. SUMMARY AND CONCLUSIONS

In this paper we have investigated the linear stability properties of the hot component of the Galactic halo, studying the evolution of small perturbations which satisfy realistic boundary conditions. Using a normal modes technique, we have derived the growth time and the spatial profile of the most unstable perturbation as functions of certain parameters of the gas at the disk level, such as density, velocity, temperature, heat flux, and heating.

We have found that in a static homogeneous configuration, the critical length for the system to be thermally unstable is systematically larger than the one obtained by Field using periodic boundary conditions. The condensation mode can be stabilized, no matter what is the length of the system, by the presence of either sufficiently large homogeneous velocity fields or equilibrium gradients, whose effect is to enhance convection and conduction, respectively. Both the Mach numbers and the level of nonhomogeneity sufficient to eliminate thermal instabilities may reasonably exist under Galactic halo conditions.

In nonhomogeneous flows, a new instability appears whose growth rate $|\hat{\omega}_i|$ is strongly dependent on the Mach number of the flow. Referring to the steady state configurations presented in Paper I, we found that $|\hat{\omega}_i|$ is an increasing function of M^* until the value corresponding to the transonic solution is attained, while, in the supersonic regime, it decreases approaching, in some cases, an asymptotic value. We have shown that this instability has a typical growth time $\simeq 10^{-2}$ of the cooling time, that its energy balance is dominated by convection, and that the corresponding perturbations are not isobaric. Moreover, when a transonic solution does not exist for any value of the disk Mach number, all the configurations are stable, whereas, when the transonic solutions does exist, it is the only one stable.

It is impossible, with the analysis performed in the paper, to derive the subsequent evolution of the instability: to this aim, a full nonlinear treatment is required. However, we sketch here a tentative scenario, which, although not completely proved, represents a reasonable step toward the comprehension of the results concerning these instabilities. We have extensively discussed throughout the paper that a "feedback" effect due to an external medium pressure should be very important in the evolution of the system towards its final state. We have simulated this feedback with a particular choice of the boundary conditions that time-dependent perturbations must satisfy. It has been shown that the stability properties of a given configuration do not depend on this choice. What matters is the *presence* of some conditions at the external boundary producing the "feedback" and the consequent fact that the configuration is free to rearrange the values of some parameters at the disk level, during the time evolution. It is reasonable to argue that the existence of dynamical instabilities with the above described properties is the signature of the natural tendency of the system to relax on a transonic solution whenever it is allowed to rearrange dynamically the values of the parameters at the base. This conclusion is strengthened by the fact that critical solutions themselves are stable and the instability of the other solutions is faster when the external medium counter-pressure producing the feedback becomes relevant closer to the disk. This result indicates that when the gas present in the interior of superbubbles, if allowed to blow out of the disk, has the time to reach a steady state, then it is more likely to generate a supersonic Galactic wind rather than nongravitational condensations. The nature of such a wind and the time scales of its onset can be studied only through a nonlinear analysis, presently underway.

The presence of a supersonic wind has important implications for the Galaxy. If a systematic mass loss of presumably high metallicity material takes place, this should be properly taken into account in any realistic model for the chemical evolution of the Galaxy (Tosi 1991). Furthermore, shocks may form in the interaction between the wind and preexisting halo material, and they should become detectable with forthcoming observational facilities.

Following the above discussion, it seems clear that, if hot outflows from the Galactic plane exist, HVCs can hardly be produced by a thermal instability. Suitable conditions for the formation of overdense regions are achieved if mechanical heating is provided to the system, thus leading to an almost homogeneous and quasi-static configuration of the background flow. Our knowledge of the heating mechanism in the halo is rather poor; nevertheless Bregman (1988) pointed out that given the large scale height of Type I supernovae, they could represent an effective heating source in the halo. Unfortunately, it seems likely that their effect may disrupt the flow. On the other hand, the kinematic structure of the halo is far from simple and large systematic velocities are definitely present (Danly 1989, 1991) so that a quasi-static halo configuration does not seem to be an appropriate one.

Hence, one is led to consider different answers to the still unresolved question of the origin of the HVCs. Several interpretations of the available data are possible, mostly because the distance to the clouds cannot be derived easily from H I 21 cm observations. As an example, Bajaja et al. (1989) report evidence of small clouds with negative velocities about 160 km s^{-1} in the direction of the Galactic center, to which they refer to as very distant objects falling into the Galaxy. The above result is consistent with the explanation that at least some clouds are intergalactic material which is accreted by the Galaxy, or remnants of the protogalaxy collapse. Also, H I fragments with ballistic motions may be created at low latitudes by several processes. Franco et al. (1991) suggest that diffuse clouds may be raised at high Galactic latitudes by the "photolevitation" effect in which the clouds are driven into a soft Galactic fountain. Furthermore, the fragmentation of the walls of the chimney may produce dense blobs of neutral material which fall at high velocities on the disk (Lockman 1991). Given the many possible scenarios, a realistic picture of the nature of HVCs needs more theoretical work and observational input; however, we feel that a non-unique explanation may represent more suitably the complexity of the problem.

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APPENDIX

In order to solve the differential system equations (2.6)–(2.8) numerically with a finite difference scheme, it is necessary to put it in the form of two coupled second-order differential equations. Here we give the expressions of the reduced equations in terms of the following coefficients A_i, B_i, C_i:

$$A_1 = v_0 , \tag{A1}$$

$$A_2 = \rho_0 , \tag{A2}$$

$$A_3 = i\omega + \Gamma v_0 + v_0' , \tag{A3}$$

$$A_4 = \Gamma \rho_0 + \rho_0' ; \tag{A4}$$

$$B_1 = T_0 , \tag{A5}$$

$$B_2 = \rho_0 v_0 , \tag{A6}$$

$$B_3 = \rho_0 , \tag{A7}$$

$$B_4 = v_0 v_0' + \frac{1}{L_g} g_z + T_0' , \tag{A7}$$

$$B_5 = i\omega\rho_0 + \rho_0 v_0' , \tag{A8}$$

$$B_6 = \rho_0' ; \tag{A9}$$

$$C_1 = -\frac{1}{\tau_{\rm cd}} T_0^{5/2} , \tag{A10}$$

$$C_2 = v_0 \rho_0 - 5 \frac{1}{\tau_{cd}} T_0^{3/2} T'_0 , \tag{A11}$$

$$C_3 = i\omega\rho_0 + v_0 \rho'_0 - \gamma v_0 \rho'_0 + f\rho_0^2 T_0^{f-1} - \frac{1}{\tau_{cd}} \left[\frac{15}{4} T_0^{1/2} (T'_0)^2 + \frac{5}{2} T_0^{3/2} T''_0 \right] . \tag{A12}$$

$$C_4 = \rho_0 T'_0 + T_0 \rho'_0 - \gamma T_0 \rho'_0 , \tag{A13}$$

$$C_5 = v_0 T_0 - \gamma v_0 T_0 , \tag{A14}$$

$$C_6 = i\omega T_0 - i\gamma\omega T_0 + v_0 T'_0 + 2\rho_0 T_0^f . \tag{A15}$$

Furthermore we define the following quantities σ_i which are combinations of the previous coefficients:

$$\sigma = A_2 B_5 - B_2 A_4 , \tag{A16}$$

$$\sigma_1 = A'_2 B_5 + A_2 B'_5 - B'_2 A_4 - B_2 A'_4 / \sigma , \tag{A17}$$

where the prime has the usual meaning of a spatial derivative.

We finally obtain

$$(A_2 B_3) T''' - (\sigma_1 A_2 B_3 - A'_2 B_3 - A_2 B'_3 - A_2 B_6 - A_4 B_3) T' - (\sigma_1 A_2 B_6 - A'_2 B_6 - A_2 B'_6 - A_4 B_6) T + (A_2 B_1 - B_2 A_1) \rho''$$
$$- \left[\sigma_1 (A_2 B_1 - B_2 A_1) - (A'_2 B_1 + A_2 B'_1 - B'_2 A_1 - B_2 A'_2) - (A_2 B_4 - B_2 A_3) + \sigma \frac{A_1}{A_2} - \frac{A_4}{A_2} (A_2 B_1 - B_2 A_1) \right] \rho'$$
$$- \left[\sigma_1 (A_2 B_4 - B_2 A_3) - (A'_2 B_4 + A_2 B'_4 - B'_2 A_3 - B_2 A'_3) + \sigma \frac{A_3}{A_2} - \frac{A_4}{A_2} (A_2 B_4 - B_2 A_3) \right] \rho = 0 ; \tag{A18}$$

$$C_1 T'' + \left(C_2 - \frac{C_2}{\sigma} A_2 B_3 \right) T' + \left(C_3 - \frac{C_4}{\sigma} A_2 B_6 \right) T + \left[C_5 - \frac{C_4}{\sigma} (A_2 B_1 - B_2 A_1) \right] \rho' + \left[C_6 - \frac{C_4}{\sigma} (A_2 B_4 - B_2 A_3) \right] \rho = 0 . \tag{A19}$$

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