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MAGNETIC FIELD DECAY IN ISOLATED NEUTRON STARS

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ABSTRACT

We investigate three mechanisms that promote the loss of magnetic flux from an isolated neutron star.

Ohmic decay produces a diffusion of the magnetic field with respect to the charged particles. It proceeds at a rate that is inversely proportional to the electric conductivity and independent of the magnetic field strength. Ohmic decay occurs in both the fluid core and solid crust of a neutron star, but it is too slow to directly affect magnetic fields of stellar scale.

Ambipolar diffusion involves a drift of the magnetic field and charged particles relative to the neutrons. The drift speed is proportional to the second power of the magnetic field strength if the protons form a normal fluid. Variants of ambipolar diffusion include both the buoyant rise and the dragging by superfluid neutron vortices of magnetic flux tubes. Ambipolar diffusion operates in the outer part of the fluid core where the charged particle composition is homogeneous, protons and electrons being the only species. The charged particle flux associated with ambipolar diffusion decomposes into a solenoidal and an irrotational component. Both components are opposed by frictional drag. The irrotational component perturbs the chemical equilibrium between neutrons, protons, and electrons, thus generating pressure gradients that effectively choke it. The solenoidal component is capable of transporting magnetic flux from the outer core to the crust on a short time scale. Magnetic flux that threads the inner core, where the charged particle composition is inhomogeneous, would be permanently trapped unless particle interactions could rapidly smooth departures from chemical equilibrium.

Magnetic fields undergo a *Hall drift* related to the Hall component of the electric field. The drift speed is proportional to the magnetic field strength. Hall drift occurs throughout a neutron star. Unlike ohmic decay and ambipolar diffusion which are dissipative, Hall drift conserves magnetic energy. Thus, it cannot by itself be responsible for magnetic field decay. However, it can enhance the rate of ohmic dissipation. In the solid crust, only the electrons are mobile and the tangent of the Hall angle is large. There, the evolution of the magnetic field resembles that of vorticity in an incompressible fluid at large Reynolds number. This leads us to speculate that the magnetic field undergoes a turbulent cascade terminated by ohmic dissipation at small scales. The small-scale components of the magnetic field are also transported by Hall drift waves from the inner crust where ohmic dissipation is slow to the outer crust where it is rapid. The diffusion of magnetic flux through the crust takes $\sim 5 \times 10^8/B_{12}$ yr, where B_{12} is the crustal magnetic field strength measured in units of 10^{12} G.

Subject headings: stars: magnetic — stars: neutron

1. INTRODUCTION

Young neutron stars are seen as ordinary radio pulsars and X-ray pulsars. Their surface magnetic field strengths are deduced to be of order 10^{12}–10^{13} G. Older neutron stars are observed as recycled pulsars and low mass X-ray binaries. Their surface fields are weaker, $\lesssim 10^{10}$ G. The association of weaker fields with older objects suggests that the magnetic fields of neutron stars are subject to decay. Since the neutron stars found in recycled pulsars and low-mass X-ray binaries have accreted substantial amounts of matter, it is difficult to resolve whether the decay results from age or accretion (Bisnovatyi-Kogan & Komberg 1975). Evidence favoring age comes from some statistical studies of ordinary, single, radio pulsars which conclude that the magnetic fields of these objects decay on time scales of order 10^7 yr (Lyne, Manchester, & Taylor 1985; Narayan & Ostriker 1990). However, other studies reach the opposite conclusion (Bhattacharya et al. 1992). The detection in γ-ray burst spectra of what appear to be cyclotron lines formed in 10^{12}–10^{13} G fields (Murakami et al. 1988) would provide evidence in favor of accretion should the bursts emanate from old neutron stars (Shibazaki et al. 1989).

The purpose of this paper is to identify decay mechanisms for the magnetic field of an isolated neutron star and to estimate their time scales. We do not address questions related to the origin of the field. We merely assume that the initial field threads the interior of the star and inquire as to how it would evolve. To do so, we solve the equations of motion for charged particles in the presence of a magnetic field and a fixed background of neutrons while allowing for the creation and destruction of particles by weak interactions. Strictly speaking, these equations apply to normal neutrons and protons. However, we extend our interpretations of their solutions to cover cases of neutron superfluidity and proton superconductivity.

The organization of the paper is set out below. We present continuity equations and equations of motion for the protons and electrons in § 2. These equations are manipulated to prove that, in the presence of a magnetic force, the charged particles cannot be simultaneously in magnetostatic equilibrium and in chemical equilibrium with the neutrons. In § 3, the equations are solved and two mechanisms for the decay of the magnetic energy are identified, *Ohmic dissipation* and *ambipolar diffusion*. Speculations concerning turbulent field evolution by *Hall drift* are offered in § 4. Finally, § 5 contains a discussion of the application of our results to real neutron stars.

Each of the three mechanisms we investigate, ohmic decay, ambipolar diffusion, and Hall drift, has already received attention in relation to neutron star magnetic fields. Baym, Pethick, & Pines (1969b) were the first to properly calculate the ohmic decay time in the fluid core under the assumption that the neutrons and protons were normal (not superfluid and superconducting). Ewart, Guyer, & Greenstein (1975) and Sang & Chanmugam (1987) estimated the ohmic decay of fields supported by currents in the solid crust. The ambipolar diffusion time scale for normal neutrons and protons was evaluated by Haensel, Urpin, & Yakovlev (1990), although these authors mistakenly attributed it to enhanced ohmic decay (Pethick 1991). Harrison (1991) properly appreciated the connection between ambipolar diffusion and the buoyant rise in flux tubes. Hall drift was part of the picture of the thermoelectric generation of magnetic fields detailed by Blandford, Applegate, & Hernquist (1983). Jones (1988) proposed that Hall drift could transport magnetic flux across neutron star crusts. Relations between our results and those obtained in earlier papers are mentioned in § 5.

2. EQUATIONS OF MOTION FOR THE CHARGED PARTICLES

We model the interior of a neutron star as a lightly ionized plasma consisting of neutrons, protons, and electrons labeled by the indices n, p, e. The equation of state for each particle species is taken to be that of an ideal, completely degenerate, gas. Modifications associated with the presence of other particle species and the strong interactions are discussed in §§ 3.5 and 5.2. We neglect thermal contributions to the Brunt-Väisälä frequency on the grounds that the thermal conductivity of neutron star interiors is so high that they are unimportant for the slow motions of interest here.

We specify the local state of each species by its internal chemical potential, μ_i, which is equal to the Fermi energy including rest mass. The protons and electrons are described as two separate fluids coupled by electromagnetic forces. Drag forces due to elastic binary collisions impede the relative motions of the different particle species. Weak interactions tend to erase pertubations away from chemical equilibrium among the neutrons, protons, and electrons.

The neutrons are assumed to form a fixed background in diffusive equilibrium. This assumption, while not entirely realistic, simplifies the algebra and does not lead us astray. Its justification is that the combined fluid of neutrons, protons and electrons is stably stratified (Reisenegger & Goldreich 1992). The stratification is associated with the chemical composition gradient; the equilibrium ratio of the number densities of charged particles to neutrons increases with depth. The ratio of the magnetic field stress to the pressure of the charged particles is small. Thus, the magnetic field cannot force significant displacements of the combined fluid, at least not ones in which the composition is frozen. We show in § 5 that the interactions which smooth perturbations of chemical equilibrium are so slow that these are the only displacements of practical interest. The density profile of the neutrons, as determined by

$$\mu_n + m_n\psi = \text{constant} , \quad (1)$$

gives rise to a Newtonian gravitational potential, ψ; contributions to ψ by protons and electrons are neglected, as are corrections due to general relativity.

The charged particles satisfy the equations of motion:

$$m_p \frac{\partial \boldsymbol{v}_p}{\partial t} + m_p(\boldsymbol{v}_p \cdot \nabla)\boldsymbol{v}_p = -\nabla\mu_p - m_p\nabla\psi + e\left(\boldsymbol{E} + \frac{\boldsymbol{v}_p}{c} \times \boldsymbol{B}\right) - \frac{m_p \boldsymbol{v}_p}{\tau_{pn}} - \frac{m_p(\boldsymbol{v}_p - \boldsymbol{v}_e)}{\tau_{pe}} , \quad (2)$$

$$m_e^* \frac{\partial \boldsymbol{v}_e}{\partial t} + m_e^*(\boldsymbol{v}_e \cdot \nabla)\boldsymbol{v}_e = -\nabla\mu_e - e\left(\boldsymbol{E} + \frac{\boldsymbol{v}_e}{c} \times \boldsymbol{B}\right) - \frac{m_e^* \boldsymbol{v}_e}{\tau_{en}} - \frac{m_e^*(\boldsymbol{v}_e - \boldsymbol{v}_p)}{\tau_{ep}} . \quad (3)$$

Here, $m_e^* = \mu_e/c^2$ is the effective inertia of the electrons, $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric and magnetic fields, $\boldsymbol{v}_i$ is the mean velocity of the particles of species i, and τ_{ij} is the relaxation time for collisions of particles of species i against particles of species j. The average velocity of the neutrons is assumed to vanish, $\boldsymbol{v}_n = 0$. Conservation of momentum implies that $m_p/\tau_{pe} = m_e^*/\tau_{ep}$. We ignore relativistic corrections to both the inertia of the neutrons and protons and to the gravitational forces acting upon them. To be consistent, we also drop the gravitational force acting on the electrons and take the neutron and proton masses to be equal. Without the essential additions of the forces due to pressure and gravity, our equations of motion would yield an electrical conductivity tensor similar to that applied by Haensel, Urpin, & Yakovlev (1990).

The processes under consideration involve small velocities that change over time scales much longer than any of the relaxation times. Thus, we neglect the acceleration terms on the left-hand sides of equations (2) and (3). Then, combining equations (1), (2), and (3), we arrive at

$$\frac{\boldsymbol{f}_B}{n_c} - \nabla(\Delta\mu) = \frac{m_p \boldsymbol{v}_p}{\tau_{pn}} + \frac{m_e^* \boldsymbol{v}_e}{\tau_{en}} \equiv \left(\frac{m_p}{\tau_{pn}} + \frac{m_e^*}{\tau_{en}}\right)\boldsymbol{v} , \quad (4)$$

where $\Delta\mu \equiv \mu_p + \mu_e - \mu_n$ is the departure from chemical equilibrium, $n_c \approx n_p \approx n_e$ is the number density of charged particles, $\boldsymbol{f}_B$ is the magnetic force density,

$$\boldsymbol{f}_B = \frac{\boldsymbol{j} \times \boldsymbol{B}}{c} , \quad (5)$$

with the electric current, $\boldsymbol{j}$, given by

$$\boldsymbol{j} \equiv en_c(\boldsymbol{v}_p - \boldsymbol{v}_e) . \quad (6)$$

Each of the terms in equation (4) admits a simple interpretation. Clearly, $\boldsymbol{f}_B/n_c$ is the magnetic force per proton-electron pair. From the thermodynamic identity $(\partial\mu/\partial p)_T = 1/n$, it follows that $-\nabla(\Delta\mu)$ is the net of the forces due to particle pressure plus gravity acting on a proton-electron pair. Equation (4) shows that magnetostatic equilibrium requires $\boldsymbol{f}_B/n_c$ to be the gradient of a potential. Only in this special circumstance can the gradient of the perturbed chemical potential balance the magnetic force density. If magnetostatic equilibrium does not apply, the forces drive the charged particles through the fixed background of neutrons at the *ambipolar diffusion velocity*, $\boldsymbol{v}$, defined by the second equality in equation (4).

Weak interactions tend to erase chemical potential differences between the charged particles and neutrons. The difference between the rates, per unit volume, at which the reactions $p + e^- \rightarrow n + \nu_e$ and $n \rightarrow p + e^- + \bar{\nu}_e$ occur is

$$\Delta\Gamma \equiv \Gamma(p + e^- \rightarrow n + \nu_e) - \Gamma(n \rightarrow p + e^- + \bar{\nu}_e) = \lambda\Delta\mu , \quad (7)$$

where the coefficient λ is a temperature-dependent proportionality constant in the limit $\Delta\mu \ll k_B T$.

The protons and electrons each satisfy a continuity equation

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{v}_i) = -\lambda \Delta\mu \, . \tag{8}$$

Approximate charge neutrality implies $n_p \approx n_e \equiv n_c$ from which it follows that

$$\frac{\partial n_c}{\partial t} + \nabla \cdot (n_c \boldsymbol{w}) = -\lambda \Delta\mu \, , \tag{9}$$

where

$$\boldsymbol{w} \equiv \frac{\boldsymbol{v}_p + \boldsymbol{v}_e}{2} = \boldsymbol{v} - \left(\frac{m_p/\tau_{pn} - m_e^*/\tau_{en}}{m_p/\tau_{pn} + m_e^*/\tau_{en}}\right) \frac{\boldsymbol{j}}{2n_c e} \, . \tag{10}$$

Since the Eulerian variations of n_c are of order $n_c B^2/p_e \ll 1$, where p_e is the electron pressure, equation (9) simplifies to

$$\nabla \cdot (n_c \boldsymbol{w}) \approx -\lambda \Delta\mu \, . \tag{11}$$

3. OHMIC DISSIPATION AND AMBIPOLAR DIFFUSION

In this section we study the dissipation of magnetic energy in a fluid mixture of neutrons, protons, and electrons that is close to both magnetostatic and chemical equilibrium. To avoid the proliferation of inessential terms, we neglect gravity and treat m_e^*, τ_{pn}, τ_{en}, and λ as constants throughout most of the section. Moreover, we assume that the magnetic field is spatially bounded and that the fluid medium is of infinite extent. In the final subsection, § 3.5, we consider extensions and refinements of our results to inhomogeneous, gravitating media.

3.1. *Magnetic Field Evolution*

The evolution of the magnetic field is related to the electric field, $\boldsymbol{E}$, by Faraday's induction law,

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\nabla \times \boldsymbol{E} \, . \tag{12}$$

The electric field, obtained from a suitable combination of equations (2) and (3) without the inertial terms, reads:

$$\boldsymbol{E} = \frac{\boldsymbol{j}}{\sigma_0} - \frac{\boldsymbol{v}}{c} \times \boldsymbol{B} + \left(\frac{m_p/\tau_{pn} - m_e^*/\tau_{en}}{m_p/\tau_{pn} + m_e^*/\tau_{en}}\right) \frac{\boldsymbol{j} \times \boldsymbol{B}}{n_c e c} + \frac{(\tau_{pn}/m_p)\nabla\mu_p - (\tau_{en}/m_e^*)\nabla\mu_e}{e(\tau_{pn}/m_p + \tau_{en}/m_e^*)} \, , \tag{13}$$

where

$$\sigma_0 = n_c e^2 \left(\frac{1}{\tau_{ep}/m_e^*} + \frac{1}{\tau_{pn}/m_p + \tau_{en}/m_e^*}\right)^{-1} \tag{14}$$

is the electrical conductivity in the absence of a magnetic field.

Substituting equation (13) into equation (12), we obtain the governing equation for the magnetic field,

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\nabla \times \left(\frac{\boldsymbol{j}}{\sigma_0}\right) + \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \left(\frac{m_p/\tau_{pn} - m_e^*/\tau_{en}}{m_p/\tau_{pn} + m_e^*/\tau_{en}}\right) \nabla \times \left(\frac{\boldsymbol{j} \times \boldsymbol{B}}{n_c e}\right) . \tag{15}$$

where $\boldsymbol{j}$ is related to $\boldsymbol{B}$ by Ampère's law,

$$\boldsymbol{j} = \frac{c\nabla \times \boldsymbol{B}}{4\pi} \, . \tag{16}$$

The terms on the right-hand side of equation (15) describe, in order, the effects of ohmic decay, ambipolar diffusion, and Hall drift. Since $\boldsymbol{j}$ and $\boldsymbol{v}$ are linear and quadratic functionals of $\boldsymbol{B}$, these terms scale as B, B^3, and B^2, respectively.

3.2. *Dissipation of Magnetic Energy*

The total magnetic energy is given by

$$E_B = \frac{1}{8\pi} \int d^3x \, |\boldsymbol{B}|^2 \, . \tag{17}$$

We write its time derivative, with the aid of equation (12) and after an integration by parts, in the form

$$\frac{dE_B}{dt} = -\frac{1}{4\pi} \int d^3x \boldsymbol{j} \cdot \boldsymbol{E} \, . \tag{18}$$

Neither the Hall term nor the potential term in the electric field contribute to dE_B/dt. The former is orthogonal to $\boldsymbol{j}$ and the latter is eliminated by the use of Ampère's law in the derivation of equation (18). Thus,

$$\frac{dE_B}{dt} = \left(\frac{dE_B}{dt}\right)_{\text{ohmic}} + \left(\frac{dE_B}{dt}\right)_{\text{ambip}} \, . \tag{19}$$

The contribution from ohmic dissipation reads

$$\left(\frac{dE_B}{dt}\right)_{\text{ohmic}} = -\frac{1}{4\pi} \int d^3x \frac{|\boldsymbol{j}|^2}{\sigma_0} \, . \tag{20}$$

The ambipolar term is given by

$$\left(\frac{dE_B}{dt}\right)_{\text{ambip}} = -\frac{1}{4\pi} \int d^3x \boldsymbol{v} \cdot \boldsymbol{f}_B = -\int d^3x n_c \left(\frac{m_p}{\tau_{pn}} + \frac{m_e^*}{\tau_{en}}\right) |\boldsymbol{v}|^2 - \int d^3x n_c \boldsymbol{v} \cdot \nabla(\Delta\mu) \, , \tag{21}$$

where we arrive at the second expression by using equation (4) to eliminate $\boldsymbol{f}_B$ in favor of $\boldsymbol{v}$ and $\Delta\mu$. Another integration by parts, together with equation (11), yields

$$\left(\frac{dE_B}{dt}\right)_{\text{ambip}} = -\int d^3x \left[n_c \left(\frac{m_p}{\tau_{pn}} + \frac{m_e^*}{\tau_{en}}\right) |\boldsymbol{v}|^2 + \lambda(\Delta\mu)^2\right] . \tag{22}$$

The first piece in the integrand arises from energy lost to frictional drag. The second piece accounts for the energy carried away by the neutrinos and anti-neutrinos that are emitted during the inverse and direct beta decays that smooth departures from chemical equilibrium.

As is evident from equations (20) and (22), ohmic dissipation and ambipolar diffusion always act to decrease the magnetic energy.

3.3. *Ambipolar Drift Velocity*

To relate the chemical potential imbalance, $\Delta\mu$, and the drift velocity, $\boldsymbol{v}$, to the magnetic force, $\boldsymbol{f}_B$, we start from equations (4) and (11). It is convenient to resolve $\boldsymbol{v}$ and $\boldsymbol{f}_B$ into solenoidal (divergence-free) and irrotational (curl-free) components, $\boldsymbol{v}^s$

and f_B^s, and v^{ir} and f_B^{ir}.[1] Because $\nabla(\Delta\mu)$ is irrotational, the solenoidal and irrotational components of equation (4) can be written as

$$v^s = \frac{f_B^s}{n_c(m_p/\tau_{pn} + m_e^*/\tau_{en})}, \quad (23)$$

$$v^{ir} = \frac{f_B^{ir} - n_c\nabla(\Delta\mu)}{n_c(m_p/\tau_{pn} + m_e^*/\tau_{en})}. \quad (24)$$

Note that v^s is directly proportional to the local value of f_B^s with a coefficient that is inversely proportional to the frictional coupling between the charged particles and neutrons. Because v^{ir} perturbs the chemical equilibrium between the neutrons and charged particles, its response to f_B^{ir} is more complicated. The details are worked out below.

Since the fractional variations of n_c are of order $B^2/p_e \ll 1$, equation (11) simplifies further to

$$\nabla \cdot v^{ir} \approx -\frac{\lambda\Delta\mu}{n_c}. \quad (25)$$

Taking the divergence of equation (24) and using equation (25) to eliminate $\nabla \cdot v^{ir}$, we obtain

$$\nabla^2(\Delta\mu) - \frac{\Delta\mu}{a^2} = \frac{\nabla \cdot f_B^{ir}}{n_c}, \quad (26)$$

where the length scale a satisfies

$$a \equiv \left[\frac{\lambda}{n_c}\left(\frac{m_p}{\tau_{pn}} + \frac{m_e^*}{\tau_{en}}\right)\right]^{-1/2}. \quad (27)$$

The solution of equation (26) is conveniently expressed in terms of the Green's function

$$G(x - x') = -\frac{\exp(-|x - x'|/a)}{4\pi|x - x'|} \quad (28)$$

as

$$\Delta\mu(x) = \frac{1}{n_c}\int d^3x' G(x - x')\nabla' \cdot f_B^{ir}(x'). \quad (29)$$

Next, we relate v^{ir} to f_B^{ir} by substituting equation (29) into equation (24) and performing an integration by parts:

$$v^{ir}(x) = \frac{\lambda a^2}{n_c^2}\left[f_B^{ir}(x) - \int d^3x' G(x - x')\nabla[\nabla' \cdot f_B^{ir}(x')]\right]. \quad (30)$$

Let us denote by L the characteristic length scale over which f_B^{ir} varies. The response of v^{ir} to f_B^{ir} depends upon the relative sizes of L and a.

For $L/a \gg 1$ the second term in equation (30) is smaller than the first by a factor of order $(a/L)^2 \ll 1$, and

$$v^{ir} \approx \frac{\lambda a^2}{n_c^2} f_B^{ir} = \frac{f_B^{ir}}{n_c(m_p/\tau_{pn} + m_e^*/\tau_{en})}. \quad (31)$$

In this limit chemical equilibrium is achieved so rapidly that only the frictional drag exerted by the neutrons on the charged particles is available to balance the magnetic force.

In the opposite limit, $L/a \ll 1$, the relation between v^{ir} and f_B^{ir} is nonlocal, and therefore more complicated. It is best revealed in Fourier space, since the Fourier components of the irrotational parts of vector fields are parallel to k. Taking the Fourier transforms of equations (25) and (26) yields

$$k \cdot v^{ir}(k) = \frac{\lambda a^2}{n_c^2(1 + k^2a^2)} k \cdot f_B^{ir}(k) \approx \frac{\lambda L^2}{n_c^2} k \cdot f_B^{ir}(k), \quad (32)$$

for $L = k^{-1} \ll a$. For $L/a \ll 1$, f_B^{ir} is balanced by the pressure gradient, leaving only f_B^s to be balanced by frictional drag.

3.4. *Decay Time Scales*

Here, we collect formulae giving the characteristic time scales over which ohmic decay and ambipolar diffusion dissipate magnetic energy. We reserve until § 5 the numerical evaluation of these time scales under different hypotheses concerning the state of matter in neutron star interiors.

The time scale for ohmic decay, which follows immediately from equations (15) and (16), has the familiar form

$$t_{ohmic} \sim \frac{4\pi\sigma_0 L^2}{c^2}. \quad (33)$$

Ohmic decay involves a diffusion of the magnetic field lines with respect to the charged particles. Note that t_{ohmic} is proportional to L^2 and independent of the field strength.

There are two time scales for ambipolar diffusion, one for the solenoidal component of the charged particle flux and the other for the irrotational component. Following equations (23) and (32), we find

$$t_{ambip}^s \sim \frac{L}{v^s} \sim \frac{4\pi n_c L^2}{B^2}\left(\frac{m_p}{\tau_{pn}} + \frac{m_e^*}{\tau_{en}}\right), \quad (34)$$

$$t_{ambip}^{ir} \sim \frac{L}{v^{ir}} \sim \frac{4\pi n_c(L^2 + a^2)}{B^2}\left(\frac{m_p}{\tau_{pn}} + \frac{m_e^*}{\tau_{en}}\right). \quad (35)$$

Ambipolar diffusion involves the motion of the magnetic field lines together with the charged particles relative to the neutrons. Note that both expressions for t_{ambip} are inversely proportional to B^2. Also, for $L/a \ll 1$, $t_{ambip}^s \approx (L/a)^2 t_{ambip}^{ir}$.

We show in § 5.2 that t_{ambip}^{ir} is larger than the Hubble time. However, if it were not, we would be compelled to consider displacements of the combined fluid of neutrons and charged particles. This is because magnetic forces would drive a solenoidal flux of baryons (neutrons plus protons) if particle interactions could maintain chemical equilibrium. This solenoidal motion of the combined fluid would not suffer the frictional retardation that the solenoidal component of the charged particle fluid does. It would only have the milder effects of viscosity to contend with.

3.5. *Extensions and Refinements*

It is easy to extend most of the results obtained in this section so that they apply to inhomogeneous media in gravitational fields.

The expressions for the dissipation of magnetic energy by ohmic decay and ambipolar diffusion given by equations (20) and (22) are unchanged in an inhomogeneous medium. However, the derivation of $(dE_B/dt)_{ambip}$ is complicated by the spatial variations of m_e^*, τ_{pn}, τ_{en}, and λ. We leave the proofs to the reader.

The flow of charged particles in a homogeneous medium tends to upset chemical equilibrium if $\nabla \cdot (n_c v) \neq 0$. This generalizes in an inhomogeneous medium to $\nabla \cdot (n_c w) \neq 0$ (see eq. [9]). It is useful to resolve the charged particle flux $n_c w$ into its solenoidal and irrotational components. If beta reactions do

[1] This decomposition is unique since the fields are spatially bounded.

not erase perturbations from chemical equilibrium, the irrotational component is choked by pressure gradients. We note that $\boldsymbol{w}$ differs from the ambipolar diffusion velocity $\boldsymbol{v}$ by a term proportional to the current density $\boldsymbol{j}$. Since $\nabla \cdot \boldsymbol{j} = 0$ as a consequence of charge neutrality,

$$\nabla \cdot (n_c \boldsymbol{w}) = \nabla \cdot (n_c \boldsymbol{v}) - \frac{\boldsymbol{j}}{2e} \cdot \nabla \left(\frac{m_p/\tau_{pn} - m_e^*/\tau_{en}}{m_p/\tau_{pn} + m_e^*/\tau_{en}} \right) . \quad (36)$$

The difference between $\nabla \cdot (n_c \boldsymbol{w})$ and $\nabla \cdot (n_c \boldsymbol{v})$ vanishes in either the limit $m_p \tau_{en} \gg m_e^* \tau_{pn}$ or the limit $m_e^* \tau_{pn} \gg m_p \tau_{en}$. The first limit would be relevant if the protons were normal since that would imply $\tau_{en}/\tau_{pn} \gg 1$ because neutron-proton scatterings are mediated by the strong force, whereas neutron-electron scatterings are due to electromagnetic interactions involving the neutron's magnetic moment. The consequences of proton superconductivity are less clear. However, we shall assume that $\nabla \cdot (n_c \boldsymbol{w}) \approx \nabla \cdot (n_c \boldsymbol{v})$ wherever ambipolar diffusion might be important inside neutron stars. Thus, we write

$$\nabla \cdot (n_c \boldsymbol{v}) \approx -\lambda \Delta\mu , \quad (37)$$

from here on.

Ambipolar diffusion in a homogeneous medium is driven by unbalanced magnetic stresses. In an inhomogeneous medium subject to a gravitational field buoyancy forces also play a role (Parker 1979). To estimate the buoyancy forces, consider a thin, circular, magnetic flux tube of outer radius r that surrounds the center of a spherical star. The pressure of the charged particles, p_c, mostly due to electrons, is lower inside the tube than outside by $\delta p_c \approx -B^2/(8\pi)$. The density deficit inside the tube is $\delta\rho/\rho \approx -3B^2/(32\pi p_c)$. Thus, the buoyancy force density is given by

$$\boldsymbol{f}_{\rm buoy} \approx -\frac{3B^2\rho}{32\pi p_c} \boldsymbol{g} \approx \frac{3B^2}{32\pi H} \hat{\boldsymbol{r}} , \quad (38)$$

where $\hat{\boldsymbol{r}}$ is the radial unit vector, and H is the pressure scale height of the charged particle fluid. It is easy to show that the magnitude of $\boldsymbol{f}_{\rm buoy}$ exceeds that of the inward directed force density due to magnetic tension provided $H < 3r/4$. The buoyancy force density is to be compared to $B^2/(8\pi L)$, the characteristic magnitude of the force density associated with a magnetic field of scale L. Since $L \lesssim H$ in the fluid core of a neutron star, the addition of buoyancy forces does not alter the time scales for ambipolar diffusion given by equations (34) and (35).

Our treatment of ambipolar diffusion is predicated on the assumption that the charged particle fluid is homogeneous; more specifically, that it is composed of equal number densities of protons and electrons. This crucial assumption insures that the charged particle fluid is neutrally stratified. The solenoidal component of the charged particle flux does not perturb the density and pressure of a homogeneous fluid. However, it is likely that additional species of charged particles appear in the equilibrium composition at pressures below the central pressure of a neutron star. We refer to this region, where there is a gradient in the charged particle composition, as the inner core. Unfortunately, the size and composition of the inner core are uncertain. However, it is clear that the charged particle fluid in the inner core is stably stratified. This has serious implications for ambipolar diffusion. Displacements of the charged particle fluid at frozen composition would raise the potential energy. Unless particle interactions could rapidly erase perturbations from chemical equilibrium, ambipolar diffusion could not occur in the inner core.

4. HALL DRIFT AND MAGNETIC TURBULENCE

In this section we examine the third term in equation (15), the one that describes advection of the field by Hall drift. This term does not change the total magnetic energy. However, it cannot be ignored in neutron star interiors because, in places, its magnitude exceeds that of the terms which account for ambipolar diffusion and ohmic decay. We begin by describing Hall drift waves. Then, we go on to consider the possibility that the magnetic field in the crust evolves through a turbulent cascade.

We simplify the induction equation (15) by taking the limit $\tau_{pn} \to 0$ and $\tau_{en} \to \infty$. With the protons immobilized, the electrons carry all the current and ambipolar diffusion is eliminated. The medium resembles a metallic solid. Then, the reduced version of equation (15) reads

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\frac{c}{4\pi n_c e} \nabla \times [(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}] + \frac{c^2}{4\pi\sigma_0} \nabla^2 \boldsymbol{B} . \quad (39)$$

Application of dimensional analysis to equation (39) yields a relation between the linear size, L, and characteristic evolution time scale, $t_{\rm Hall}$, of field structures:

$$t_{\rm Hall} = \frac{4\pi n_c e L^2}{cB} . \quad (40)$$

Jones (1988) proposed that Hall drift could transport magnetic field from the inner crust where ohmic decay is slow to the outer crust where it proceeds rapidly. Here we show that there is a class of Hall drift waves that carry magnetic energy and whose dispersion relation is closely related to equation (40). To obtain the dispersion relation for linear waves in a uniform magnetic field $\boldsymbol{B}_0$, we substitute the elementary disturbance $\boldsymbol{B}_1 = \tilde{\boldsymbol{B}}_1 \exp i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t)$ into equation (39). After a little algebra, we obtain

$$\omega = \frac{ck|\boldsymbol{k} \cdot \boldsymbol{B}_0|}{4\pi n_c e} , \quad (41)$$

where $k \equiv |\boldsymbol{k}|$. For $\boldsymbol{k} \cdot \boldsymbol{B}_0 \gtrless 0$, the corresponding group velocity is

$$\boldsymbol{v}_{\rm gp} = \pm \frac{ck[\boldsymbol{B}_0 + (\hat{\boldsymbol{k}} \cdot \boldsymbol{B}_0)\hat{\boldsymbol{k}}]}{4\pi n_c e} , \quad (42)$$

where $\hat{\boldsymbol{k}} \equiv \boldsymbol{k}/k$.

There is reason to doubt whether these waves could transport magnetic energy from the inner to the outer crust. In particular, they might be reflected as they propagate upward toward lower density. To expose the problem, we interpret equation (41) as a WKBJ dispersion relation. Consider a plane-parallel model for the crust with n_c decreasing monotonically in the z-direction. The validity of the WKBJ approximation requires $k_z H \gg 1$, where H is the local scale height. Let us assume that a wave packet which satisfies this inequality is launched upward from the lower crust. For the moment, we focus on the special case with $\boldsymbol{B}_0$ constant and aligned along the x-axis. As the wave packet propagates toward lower density, k must decrease in direct proportion to n_c, since ω remains constant. Because of the symmetry of the problem, the decrease of k comes entirely at the expense of k_z. Since H also decreases with height, the inequality $k_z H \gg 1$ must eventually be violated. It is plausible that the wave packet would be reflected downward at about the level where $k_z H \sim 1$. Although the details differ when $\boldsymbol{B}_0$ is aligned along the z-axis, the reflection of upward propagating wave packets still seems likely.

The above considerations suggest that only disturbances whose wavelengths in the inner crust are very much shorter than the local scale height could propagate to the outer crust. Below, we argue that Hall drift tends to produce short wavelength magnetic structures. This enhances the local rate of ohmic dissipation as well as the ability of Hall waves to transport magnetic energy upward.

We proceed by rewriting equation (39) in dimensionless form as

$$\frac{\partial \boldsymbol{b}}{\partial \tau} = -\nabla_\xi \times [(\nabla_\xi \times \boldsymbol{b}) \times \boldsymbol{b}] + \frac{1}{\mathscr{R}_B} \nabla_\xi^2 \boldsymbol{b} . \tag{43}$$

Here, $\boldsymbol{\xi} \equiv \boldsymbol{x}/L$, $\boldsymbol{b} \equiv \boldsymbol{B}/B_0$, and $\tau \equiv t/t_{\rm Hall}$, with L and B_0 scale factors appropriate to the largest magnetic structures. The parameter

$$\mathscr{R}_B = \frac{\sigma_0 B_0}{n_c ec} = \frac{eB_0 \tau_{ep}}{m_e^* c} \tag{44}$$

is the tangent of the Hall angle; $\mathscr{R}_B$ may be large inside neutron stars. Note that $\mathscr{R}_B$ has a couple of interpretations. It is equal to 2π times the ratio of the electron relaxation time to the electron cyclotron period, and it is also equal to $t_{\rm ohmic}/t_{\rm Hall}$.

The dimensionless induction equation (43) resembles the vorticity equation for an incompressible fluid. In dimensionless form, the latter equation reads

$$\frac{\partial \boldsymbol{\omega}}{\partial \tau} = \nabla_\xi \times (\boldsymbol{v} \times \boldsymbol{\omega}) + \frac{1}{\mathscr{R}} \nabla_\xi^2 \boldsymbol{\omega} , \tag{45}$$

where $\boldsymbol{v}$ and $\boldsymbol{\omega} = \nabla_\xi \times \boldsymbol{v}$ are the dimensionless velocity and vorticity, and $\mathscr{R}$ is the Reynolds number. The analogy between equations (43) and (45) would be complete if $\boldsymbol{v}$ were the curl, rather than the inverse curl, of $\boldsymbol{\omega}$.[2]

Turbulence is a generic property of homogeneous, incompressible flows under circumstances where the Reynolds number is large. It is easy to rationalize this fact from equation (45) by noting that the nonlinear advection term is much larger than the linear diffusion term for $\mathscr{R} \gg 1$. We speculate that, where $\mathscr{R}_B \gg 1$ in the solid crust, the generic magnetic field evolves through a turbulent cascade. In other words, nonlinear couplings transfer magnetic energy from larger to smaller scales where it is ultimately dissipated by ohmic decay. The similarity between equations (43) and (45) leads us to speculate that the generic magnetic field is turbulent for $\mathscr{R}_B \gg 1$. The material in the remainder of this section is based on that speculation. It is so intriguing that we present it in advance of serious investigation.

Having guessed that magnetic fields are turbulent for $\mathscr{R}_B \gg 1$, it is natural to inquire about their spectra. We take a first cut at this problem by adapting a method devised by Kolmogoroff (1941) for fluid turbulence. We assume that the nonlinear interactions transfer magnetic energy from large to small scales where it is ultimately dissipated by ohmic diffusion. The outer, or energy bearing, scale has linear size L, magnetic field strength B_0, and lifetime $t_{\rm Hall}$. Smaller structures of size λ have magnetic field strengths B_λ and lifetimes t_λ. The inner scale, at which ohmic decay becomes important, is denoted by λ_*. We assume that magnetic turbulence is space filling and that the nonlinear transfer of magnetic energy is local in wave number space. Then, the steady flow of energy toward smaller scales implies

$$\frac{B_\lambda^2}{t_\lambda} \sim \frac{B_0^2}{t_{\rm Hall}} . \tag{46}$$

We determine t_λ from the form of the nonlinear term in equation (43). A simple scaling argument suggests that

$$\frac{t_\lambda}{t_{\rm Hall}} \sim \left(\frac{\lambda}{L}\right)^2 \frac{B_0}{B_\lambda} . \tag{47}$$

This is the choice made by Vainshtein (1973) and amounts to assuming that the turbulence is strong. However, the period of Hall waves of wavelength λ is shorter than t_λ by a factor $\sim B_\lambda/B_0$. Thus, Hall turbulence consists of weakly interacting waves (Kingsep, Chukbar, & Yan'kov 1990). The lowest order nonlinear interactions are those that couple three resonant waves which satisfy, $\omega = \omega_1 + \omega_2$ and $\boldsymbol{k} = \boldsymbol{k}_1 + \boldsymbol{k}_2$. It is easy to verify that the dispersion relation (eq. [41]) permits these conservation laws to be satisfied simultaneously. The characteristic time scale for the transfer of energy among resonant triplets is

$$\frac{t_\lambda}{t_{\rm Hall}} \sim \left(\frac{\lambda}{L}\right)^2 \left(\frac{B_0}{B_\lambda}\right)^2 , \tag{48}$$

which is longer by the factor B_0/B_λ than t_λ given in equation (47). The larger value for t_λ arises because the transfer of energy and momentum among wave packets of unit fractional bandwidths in ω and $\boldsymbol{k}$ takes place in steps of dimensionless size B_λ/B_0, each of duration $\sim 1/\omega_k$ with $\lambda k \sim 1$. Our derivation of t_λ is a heuristic one. However, the same result may also be derived by the rigorous methods described by Zakharov (1971, 1983).

Together, equations (46) and (48) yield

$$\frac{B_\lambda}{B_0} \sim \left(\frac{\lambda}{L}\right)^{1/2} , \tag{49}$$

$$\frac{t_\lambda}{t_{\rm Hall}} \sim \frac{\lambda}{L} . \tag{50}$$

The inner scale is set by $t_{\rm ohmic} \sim t_\lambda$. From equations (33) and (50), we arrive at

$$\frac{\lambda_*}{L} \sim \frac{1}{\mathscr{R}_B} . \tag{51}$$

The one-dimensional power spectrum of the magnetic field is determined by $kB^2(k) \sim B_\lambda^2$. Thus,

$$B^2(k) \sim \frac{B_0^2}{Lk^2} . \tag{52}$$

By way of comparison, the Kolmogoroff power spectrum of a turbulent velocity field $v^2(k) \propto k^{-5/3}$. Just as most of the energy in a turbulent flow is contained in the largest eddies, most of the energy in a (Hall) turbulent magnetic field is contained in the largest magnetic structures. However, the small scales dominate the vorticity density in fluid turbulence and the current density in (Hall) magnetic turbulence.

The turbulent cascade of magnetic energy leads to an enhanced ohmic decay of the magnetic field. The large-scale components of the field weaken as magnetic energy is conservatively transported to smaller scales.

Hall drift occurs in electrically conducting fluids as well as solids. However, its implications in fluid media are less clear. The reason is that Hall drift changes the magnetic force

[2] The minus sign in front of the nonlinear term in eq. (43) is not crucial. It arises because the current carriers have negative charge.

density, $\boldsymbol{j} \times \boldsymbol{B}/c$. In a fluid, the magnetic force density drives motions at the Alfvén speed, $v_A = B/(4\pi\rho)^{1/2}$, which in cases of interest here is much greater than the speed of the Hall drift. The situation in a solid is simpler, because the magnetic force density is ultimately balanced by the divergence of the lattice stress tensor.

5. APPLICATION TO NEUTRON STARS

Our goal is to determine how magnetic fields in neutron stars decay. We discuss the possible roles played by ohmic dissipation, ambipolar diffusion, and Hall drift. Lack of knowledge concerning the states of matter inside neutron stars is a great hindrance. We adopt the following approach for dealing with this problem.

We assess each decay mechanism as it would apply if the modified URCA reactions were the principal means for smoothing departures from chemical equilibrium, if the neutrons and protons were normal, and if neutrons, protons and electrons were the only particles present in the fluid core. Then, we relax various combinations of these assumptions and consider how our assessments must be modified.

Many of the uncertainties regarding the properties of matter in neutron star interiors stem from our inadequate knowledge of particle interactions at above nuclear density. This impedes prediction of the equilibrium number densities of different species of particles. It also limits our ability to determine whether and where the neutrons form a superfluid and the protons form a superconductor. These unresolved issues impact the discussion of the decay of the magnetic field in many ways, a few of which are mentioned below.

The relative number densities of protons and electrons to neutrons determines whether the regular URCA process can occur in neutron stars. Until recently, it was thought that only the much slower modified URCA reactions could operate (Chiu & Salpeter 1964). However, this issue seems less settled now (Lattimer et al. 1991). If the regular URCA reactions function, both neutron star cooling and the smoothing of perturbations away from chemical equilibrium would proceed much faster than previously estimated.

Neutron superfluidity would greatly reduce the collision rates between neutrons and charged particles. The energy gap would impede the reactions that restore chemical equilibrium. The effects of proton superconductivity would depend upon whether the superconductor was type I or II. The prevailing view is that the protons form a type II superconductor (Baym, Pethick, & Pines 1969a). If so, the arrangement of the magnetic field in quantized flux tubes would modify the magnetic stress (Easson & Pethick 1977). In particular, the components of the stress tensor would be proportional to the first power of the mean magnetic field strength. Thus, the time scales for ambipolar diffusion would be inversely proportional to B instead of B^2.

The presence of exotic species of particles would affect the static stability of neutron star interiors as measured by the Brunt-Väisälä frequency. The dynamics of ambipolar diffusion would be complicated by the presence of additional species of charged particles.

5.1. *Ohmic Decay*

Shortly after the discovery of pulsars, Baym, Pethick & Pines (1969b) calculated the electrical conductivity, σ_0, of neutron star interiors under the assumption that the neutrons, protons, and electrons are degenerate but normal (not superfluid), and that the magnetic field is weak. They found that σ_0 is so high that the time scale for ohmic dissipation of neutron star magnetic fields exceeds the age of the universe. We take the electrical conductivity of the core fluid, as given by equation (14), to be $\sigma_0 = 4.2 \times 10^{28} T_8^{-2} (\rho/\rho_{\rm nuc})^3\ {\rm s}^{-1}$, where T_8 denotes the temperature in units of 10^8 K, and $\rho_{\rm nuc} \equiv 2.8 \times 10^{14}{\rm g\ cm}^{-3}$ (Haensel, Urpin, & Yakovlev 1990).[3] This corresponds to an ohmic decay time scale (cf. eq. [33])

$$t_{\rm ohmic} \sim 2 \times 10^{11} \frac{L_5^2}{T_8^2} \left(\frac{\rho}{\rho_{\rm nuc}}\right)^3 {\rm yr}\,, \tag{53}$$

where $L_5 \equiv L/(10^5\ {\rm cm})$.

We can draw a rigorous, although qualified, conclusion from equation (53). It is that magnetic fields of stellar scale supported by currents in the fluid core of a neutron star would not suffer significant ohmic decay if the core matter were normal. This conclusion can be extended in several directions. Superconductivity of either type would certainly decrease the rate of ohmic decay, but might lead to the expulsion of magnetic fields by other means. If crustal currents support neutron star magnetic fields, ohmic decay would be faster. However, unless the currents are confined to the outer crust, ohmic decay would fall short of accounting for the magnitude of the decline in field strength estimated from observations of neutron stars (Ewart, Guyer, & Greenstein 1975; Sang & Chanmugam 1987).

Haensel, Urpin, & Yakovlev (1990) reopened the issue of the ohmic decay with the claim that the resistivity is enhanced in directions perpendicular to strong magnetic fields.[4] They proposed that ohmic decay could reduce arbitrary initial fields to strengths below $B \sim 10^{12} G$ in $\sim 10^7$ yr. However, as we show below, and as has also been recognized by Pethick (1991), the decay mode identified by Haensel, Urpin, & Yakovlev is *ambipolar diffusion* rather than *ohmic dissipation.*

We conclude that large-scale magnetic structures in neutron stars do not suffer significant ohmic decay.

5.2. *Ambipolar Diffusion*

Ambipolar diffusion involves a coupled motion of the magnetic field lines and the charged particles (protons and electrons) relative to the neutrons. The flux of charged particles associated with ambipolar diffusion, $n_c \boldsymbol{v}$, resolves into a solenoidal and an irrotational component. The solenoidal component does not disturb the chemical equilibrium between neutrons, protons, and electrons. Therefore, it is only opposed by friction between the charged particles and the neutrons. However, the irrotational part of $n_c \boldsymbol{v}$ is also retarded by pressure gradients that build up in response to the departures from chemical equilibrium that it causes. Since the weak interactions that restore chemical equilibrium are very sluggish at low temperatures,[5] the pressure gradients effectively choke $n_c \boldsymbol{v}^{\rm ir}$.

The square of the length scale ratio L/a provides a quantitative measure of the relative importance of frictional drag and pressure gradients in limiting the irrotational component of the charged particle flux. We find

$$\frac{L}{a} \approx \left(\frac{\lambda m_p}{n_c \tau_{pn}}\right)^{1/2} L \sim 7 \times 10^{-4} T_8^4 L_5 \left(\frac{\rho_{\rm nuc}}{\rho}\right)^{1/3}, \tag{54}$$

[3] Electrons are the main current carriers and their important collisions are with protons.

[4] Their work is based on the assumption that the neutrons and protons are normal.

[5] We are assuming that only the modified URCA reactions can operate inside neutron stars.

where we use

$$n_c \sim 5 \times 10^{-2} \frac{\rho}{m_n} \approx 8 \times 10^{36} \frac{\rho}{\rho_{\rm nuc}} \text{ cm}^{-3} , \quad (55)$$

$$\frac{1}{\tau_{pn}} = 4.7 \times 10^{16} T_8^2 \left(\frac{\rho_{\rm nuc}}{\rho}\right)^{1/3} \text{ s}^{-1} \gg \frac{m_e^*}{m_p} \frac{1}{\tau_{en}} , \quad (56)$$

from Yakovlev & Shalybkov (1990), and

$$\lambda = 5 \times 10^{27} T_8^6 \left(\frac{\rho}{\rho_{\rm nuc}}\right)^{2/3} \text{ ergs}^{-1} \text{ cm}^{-3} \text{ s}^{-1} , \quad (57)$$

due to the modified URCA reactions from Sawyer (1989).

Next, we evaluate the time scales for ambipolar diffusion at $\rho = \rho_{\rm nuc}$ from equations (34), (35), (54), (55), and (57), and arrive at

$$t^s_{\rm ambip} \sim 3 \times 10^9 \frac{T_8^2 L_5^2}{B_{12}^2} \text{ yr} , \quad (58)$$

$$t^{\rm ir}_{\rm ambip} \sim \frac{5 \times 10^{15}}{T_8^6 B_{12}^2} (1 + 5 \times 10^{-7} T_8^8 L_5^2) \text{ yr} , \quad (59)$$

where $B_{12} \equiv B/(10^{12} G)$. The expression for $t^s_{\rm ambip}$ is equal to the second term in $t^{\rm ir}_{\rm ambip}$. They account for the retardation of the charged particle flux by frictional drag and approximately reproduce the time scale that Haensel, Urpin, & Yakovlev (1990) attribute to enhanced ohmic decay. The first term in $t^{\rm ir}_{\rm ambip}$ expresses the choking of the irrotational part of the charged particle flux by pressure gradients. It dominates under conditions expected to hold inside neutron stars. The minimum value of $t^{\rm ir}_{\rm ambip}$ as a function of T is of order $10^{11} L_5^{3/2} B_{12}^{-2}$ yr and occurs for $T_8 \approx 7 L_5^{-1/4}$.

If the regular URCA reactions operate, λ would be larger by a factor of order $5 \times 10^7 T_8^{-2}$ than the value given in equation (57) (Lattimer et al. 1991). This would not affect the value of $t^s_{\rm ambip}$, but the appropriate expression for $t^{\rm ir}_{\rm ambip}$ would become

$$t^{\rm ir}_{\rm ambip} \sim \frac{10^8}{T_8^4 B_{12}^2} (1 + 3 \times 10^1 T_8^6 L_5^2) \text{ yr} . \quad (60)$$

The minimum value of $t^{\rm ir}_{\rm ambip}$ would be reduced to $\sim 10^9 L_5^{4/3} B_{12}^{-2}$ yr and occur at $T_8 \approx 0.6 L_5^{-1/3}$. This great reduction of $t^{\rm ir}_{\rm ambip}$ at fixed T would be less significant than one might think because it would be accompanied by very rapid cooling. Thus, it is almost certain that the irrotational part of the charged particle flux would still be choked by pressure gradients.

If the neutrons form a superfluid, the drag associated with ambipolar diffusion would be greatly reduced. This would increase the magnitude of the solenoidal part of the charged particle flux. However, the superfluid energy gap would block the URCA reactions that are required to maintain the irrotational part of the flux. These considerations emphasize that the distinction between ohmic decay and ambipolar diffusion is more than semantic. For example, in their study of the electrical conductivity of magnetized neutron stars, Yakovlev & Shalybkov (1990) conclude that magnetically enhanced ohmic decay of cross field currents does not occur if the neutrons form a superfluid. However, realizing that ambipolar diffusion and not ohmic dissipation is under investigation makes it clear that neutron superfluidity speeds up the dissipation of magnetic energy.

There has been considerable discussion of the loss of magnetic flux from neutron star cores under the assumption that the neutrons are superfluid and the protons form a type II superconductor. The most popular ideas are that the quantized flux tubes rise due to magnetic buoyancy (Muslimov & Tsygan 1985; Jones 1987), or are pinned to and dragged by neutron vortices that migrate away from the rotation axis as the star is despun (Srinivasan et al. 1990). Although it was not recognized by the authors, these proposals are variants of ambipolar diffusion. Because the radii of curvature of the proton and electron orbits are much larger than the spacing between flux tubes, the charged particle fluids satisfy macroscopic equations of motion. Any drift of magnetic flux tubes faster than that permitted by ohmic decay must be accompanied by a flux of charged particles (Harrison 1991). Of course, the relation between the average magnetic flux density and the magnetic stress is modified by proton superconductivity (Easson & Pethick 1977). Harrison (1991) appreciated the relation between the buoyant rise of flux tubes and ambipolar diffusion. However, he incorrectly surmised that pressure gradients would block the ambipolar drift. In so doing he, like Pethick (1991), overlooked the distinction between the solenoidal and irrotational parts of the charged particle flux. It would be worth reexamining the motion of the flux tubes with the added restriction that the charged particle flux is purely solenoidal. This could spell trouble for the hypothesis that flux tubes are pulled along by neutron vortices.

We have been proceeding as though protons and electrons are the only species of charged particles in the fluid cores of neutron stars. Nevertheless, as discussed in § 3.5, it is plausible that other charged particle species make an appearance not far above nuclear density. A composition gradient in the charged particle fraction of the core fluid would impede the solenoidal component of the charged particle flux. The severity of this effect would depend upon the rate at which interactions could act to smooth departures from chemical equilibrium. These rates could be very slow if weak interactions among highly degenerate particles were involved or if superfluid energy gaps were present. A residual field would be trapped in the inner core if ambipolar diffusion were blocked there. The residual strength of the surface field would be related to that in the inner core by $(R_i/R)^3$, where R_i is the radius of the inner core.

We summarize our discussion of ambipolar diffusion as follows. Ambipolar diffusion is a viable mechanism for the dissipation of magnetic energy in regions where the charged particle fluid is chemically homogeneous. The charged particle flux associated with ambipolar diffusion is purely solenoidal, the irrotational part being choked by pressure gradients. These qualitative conclusions are independent of whether or not the direct URCA reactions occur, the neutrons form a superfluid, or the protons are superconducting. Charged particle composition gradients would inhibit the solenoidal component of the particle flux.

5.3. *Hall Drift*

The time scale for Hall drift is obtained from equation (40) using n_c from equation (55):

$$t_{\rm Hall} \approx 5 \times 10^8 \frac{L_5^2}{B_{12}} \left(\frac{\rho}{\rho_{\rm nuc}}\right) \text{ yr} . \quad (61)$$

Unlike ohmic decay or ambipolar diffusion, Hall drift is insensitive to the state of matter in the neutron stars. It occurs in both the fluid core and solid crust, although its implications are less obvious in the former than in the latter. Since Hall drift conserves magnetic energy, it cannot be a direct cause of magnetic field decay. However, if the speculative picture of magnetic turbulence advanced in § 4 is valid, it could tangle the field, thus enhancing ohmic dissipation. We evaluate the tangent of the Hall angle, $\mathscr{R}_B$, by forming the ratio of t_{ohmic} given in equation (53) to t_{Hall} from equation (61) above:

$$\mathscr{R}_B \sim 4 \times 10^2 \frac{B_{12}}{T_8^2} \left(\frac{\rho}{\rho_{\text{nuc}}}\right)^2 . \quad (62)$$

What we are interested in is the Hall drift in the crust. For $\rho = \rho_{\text{nuc}}$, the numerical expressions for t_{Hall} and $\mathscr{R}_B$ apply to the boundary between the core and crust. Higher in the crust, the low-temperature electrical conductivity depends on the abundance of lattice impurities. It is likely that these are so rare that $\mathscr{R}_B \gg 1$, at least in the inner crust. Should $\mathscr{R}_B \lesssim 1$, then ohmic dissipation would limit the lifetimes of crustal currents.

Our estimate for t_{Hall} is robust and suggests that Hall drift might be an important process in the decay of a neutron star's magnetic field if the currents that support the field are confined to the crust (Jones 1988). Should Hall drift be the limiting factor in the decay of a neutron star's magnetic field, the field strengths would decline approximately at t^{-1}, as least while $\mathscr{R}_B \gg 1$. Note that, if the magnetic field as well as the currents that support it is confined to the crust, the surface field strength would be about an order of magnitude smaller than the crustal field strength.

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