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ON THE STUDY OF THE MASS RATIO OF SPECTROSCOPIC BINARIES

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ABSTRACT

We present a new algorithm to derive the true mass-ratio distribution of an observed sample of spectroscopic binaries. The algorithm includes an iterative procedure in which each binary of unknown inclination is replaced by an ensemble of virtual systems with a distribution of inclinations. This work shows that contrary to a widely held assumption, the inclination angles of each virtual ensemble should *not* be distributed randomly in space. We also suggest a way to account for the observational selection effect associated with the inability to detect single-line spectroscopic binaries with low-amplitude radial-velocity modulation. We present several numerical simulations which demonstrate the advantage of the new algorithm compared with the classical method to determine binary mass-ratio distribution.

Subject headings: binaries: spectroscopic

1. INTRODUCTION

In recent years the study of mass-ratio distribution of stellar binary systems (Trimble 1990; Duquennoy & Mayor 1990, 1991; Tout 1991) has been stimulated by the advent of new stellar speedometers (Griffin 1985; Mayor 1985; Latham 1985) which have yielded many new spectroscopic orbits. In particular, the results of systematic radial-velocity searches for spectroscopic binaries are now available for large preselected samples of stars (Latham et al. 1988; Duquennoy, Mayor, & Halbwachs 1991). Using the new data, the long controversy over the mass-ratio distribution of short-period binaries (e.g., Abt & Levy 1976; Halbwachs 1987; Trimble 1990) can now be resolved.

The mass-ratio distribution of short-period binaries provides important information about the formation of these systems (e.g., Bodenheimer, Ruzmaikina, & Mathieu 1992). The frequency of binaries with extremely low mass-ratios (Duquennoy & Mayor 1990, 1991) is another aspect of particular interest. It is related to the search for brown dwarfs as low-mass companions to nearby stars (Latham et al. 1989; Marcy & Benitz 1989; Mazeh et al. 1990), a subject into which a great deal of effort has been endowed recently.

The derivation of the mass-ratio distribution of a given sample of spectroscopic binaries is hampered by two well-known problems (Aitken 1935). First, the mass ratio of a specific system can be deduced only for double-line spectroscopic binaries. For the single-line binaries the mass ratio is unknown; only the mass function can be derived directly from the observations. For a binary system which consists of a primary with mass M_1 and a secondary with mass M_2, the mass function is

$$f(M_2) = M_1 \frac{q^3}{(1+q)^2} \sin^3 i \,, \qquad (1)$$

where q is the mass ratio ($=M_2/M_1$) and i is the inclination of the binary orbit relative to our line of sight. Since the inclination is not known, the mass ratio *cannot* be derived, even when the primary mass can be estimated from its spectral type.

Second, observational limits affect the detected binary sample. For example, systems with small radial-velocity amplitude are not easily detected, so that binaries with a small mass ratio tend to be excluded from the observed sample.

The problem of unknown inclinations was already addressed back in the 1920s (e.g., Aitken 1935), and since the 1970s by many workers (see Halbwachs 1987, Tout 1989, and Trimble 1990 for reviews). They have used statistical approaches to derive the true mass-ratio distribution. This work suggests a new iterative statistical algorithm, which replaces each binary with an ensemble of virtual systems with nonrandom orientations. A preliminary version of this work was presented at the workshop of Binaries as Tracers of Stellar Formation held in Bettmeralp, Switzerland, in 1991 September (Mazeh & Goldberg 1992).

In order to present the new algorithm, we first ignore the observational selection effects and use the new method to analyze idealized samples of binaries. Section 2 reviews previous methods used to derive the mass-ratio distribution for such ideal samples and points out their disadvantages. Section 3 discusses in detail the classical method of Campbell and Schlesinger (Aitken 1935). We present two examples, typical of the numerous simulations we have run, for which the old method fails to reconstruct the correct distribution. This work then points out the reasons for the failure of the original method. Sections 4 and 5 present the modified algorithm which successfully reveals the true mass-ratio distribution for any ideal sample. Section 6 presents a new procedure to correct for the observational selection effect associated with the inability to detect spectroscopic binaries with low-amplitude modulation. We present one typical simulation to show that the proposed procedure reproduces the correct mass-ratio distributions. The paper concludes with a short discussion of some additional observational effects which should be accounted for in order to derive the true distribution of the mass ratio.

2. THE TWO DIFFERENT APPROACHES

Two basically different statistical approaches have been used to address the problem of unknown orbital inclination. In the

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direct approach, which goes back to the 1920s (see, e.g., Aitken 1935), the expected value of $\sin^3 i$ is assigned to *all* single-line spectroscopic binaries of the sample. Aitken quoted Campbell and Schlesinger, who had suggested using an averaged value of 0.589 for $\sin^3 i$ in an ideal unbiased sample of binaries. They derived this value by averaging $\sin^3 i$ over all possible angles. (For real samples, Campbell and Schlesinger preferred a value of 0.679, obtained by assuming a detection probability which depends on the binary inclination and is proportional to $\sin i$; see § 6 below.) The philosophy behind this method is clear: For a large enough sample, the differences between the assigned values of $\sin i$ and the true values are averaged to zero.

The other statistical approach starts with an assumed quantitative model for the mass-ratio distribution of the sample. The model prediction for a distribution of some observable quantity, such as the mass function divided by the primary mass, is then compared with the actually observed one. This model-fitting approach was suggested for the mass-ratio problem by Jaschek & Ferrer (1972) and was adopted by Halbwachs (1987) and Trimble (1990).

Trimble (1990), in her fundamental review of the problem, strongly advocated the model-fitting approach. She stated, truly, that "one of the important lessons that astronomers have been slow to learn (e.g., when analyzing HR diagrams of clusters) is that one should always transform the predictions of a model into the coordinates of the observations, rather than the other way around." However, the analysis of binary mass-ratio distribution is slightly different from the analysis of HR diagrams of clusters. In the latter, the astrophysics behind the models is clear, albeit complicated, and the evolutionary models have as free parameters only the cluster age and the chemical composition. Here, on the other hand, the astrophysics of binary formation is still unclear, and there is no unique model to predict the mass-ratio distribution. In previous studies, the observed mass-ratio distributions have been fitted only *phenomenologically* to somewhat arbitrary analytical functions. The model-fitting approach enables us only to find the best parameters of an *assumed* function to represent the mass-ratio distribution, but cannot be used to find new, possibly better, functions to describe the distribution.

The preferred approach to analyze the mass-ratio distribution might be, therefore, to use a direct method, which operates without assuming any preselected function. However, the direct method of Campbell and Schlesinger (hereafter CS) is inappropriate (Halbwachs 1987), because of a few subtle features which will be discussed in the next section. Another direct method, suggested by Abt & Levy (1976), introduces high noise into the derived distribution (Halbwachs 1987; Trimble 1990), due to its recursive nature (see Halbwachs 1987 for a clear critical description of this method). A novel approach suggested very recently by Tokovinin (1992), which uses the maximum likelihood technique, still reveals some problems when applied to relatively small samples and might need some modifications. Thus, despite all the effort devoted to the analysis of the problem, there is still a need for a better algorithm to analyze the observational results.

3. THE FAILURE OF THE CAMPBELL AND SCHLESINGER DIRECT METHOD

Our algorithm is a modification of the classical method of Campbell and Schlesinger. As this method has been used widely, we discuss first this method in detail, demonstrating its failure in a few examples.

To do that, we applied the CS direct method to a *simulated* sample of 2000 binaries, with a uniform mass-ratio distribution and with random orientations. We set the period distribution of the sample to be uniform in log P, between 1 and 1000 days, and primary mass was chosen to be 1 $M_\odot$. We then calculated the mass function for each binary by using equation (1). The simulated sample was then analyzed, using the only information available for each binary in real samples—the primary mass and the mass function. In this analysis we ignored the information about the mass ratio of each binary.

Testing any method over a simulated sample, as Trimble (1974) did in one of her early studies of the problem, enabled us to use a sample as large as needed. Furthermore, we could consider an idealized sample in which all binaries are detected and therefore ignore any observational selection effects in the present discussion.

The results of the simulation testing the CS model are presented in Figure 1*a*. Clearly this method is drastically biased toward the low end of the mass-ratio spectrum. In Figure 1*b*

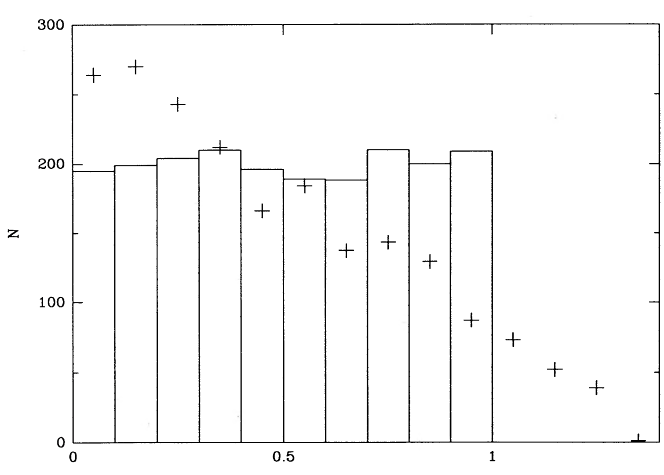

FIG. 1*a*

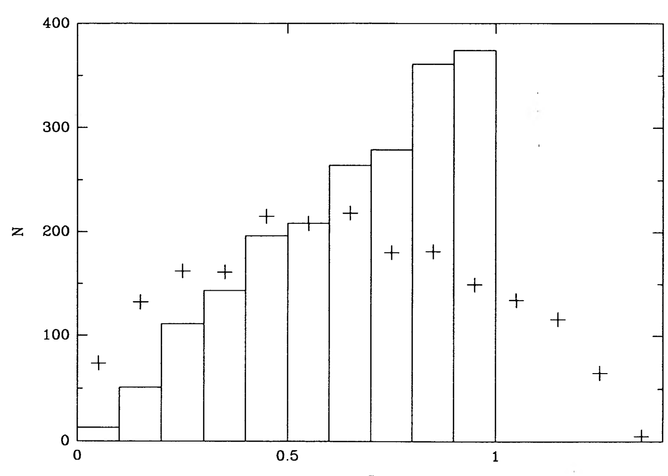

FIG. 1*b*

FIG. 1.—Numerical simulations to test the classical Campbell and Schlesinger method for deriving the mass-ratio distribution. The histogram shows the true distribution of the simulated sample. The pluses represent the results of the method. (*a*) A simulation with a uniform distribution. (*b*) A simulation with a monotonic increasing distribution of $N(q) = 2q$.

we present a similar simulation with a mass-ratio distribution which rises toward $q = 1$. The intrinsic bias of the method is very prominent again. Many of our simulations yielded similar results.

To understand the reasons for the failure of the CS method, we must first clarify its basic assumptions. Explicitly the method assumes that the orbits of the sample are randomly oriented. However, this assumption is not enough. Actually the CS method implicitly assumes that the orientations of binaries within any subset of the sample, for which the mass function is equal to some particular value, are also completely random. Otherwise it would not be possible to assign the same value of 0.589 to every binary of the sample.

This further assumption is not correct, because the mass function is not an independent variable of the sample. Our basic assumption is that the independent variables are the mass ratio, the inclination, and possibly the periods of the binaries. The mass function depends on these variables through equation (1) and therefore is a dependent variable. To demonstrate this point we plot in Figure 2 some constant mass-function contours on the q–sin i plane, for primary mass of unity. The two axis represent independent variables, while clearly the contours do not.

Figure 2 shows in three ways that the assumption of totally random distribution of inclinations of any subset of the sample is incorrect.

1. The figure shows that if the sample includes only binaries with mass ratio smaller than unity, then for any given mass-function value, some of the inclination angles must be excluded. The CS direct method, by averaging over *all* angles, incorrectly ignores this constraint and thus underestimates $\sin^3 i$. As a result, the mass ratio is overestimated for most binaries. Moreover, for quite a few systems, the assigned value of $\sin^3 i$ is actually smaller than its lower limit, which results in q larger than unity. The classical direct method, by assigning all binaries the same value for $\sin^3 i$, ignores some specific additional information available for the individual systems.

2. Any subset of binaries, with some mass function value $-f$, must have some finite width df, because the probability of finding a binary with $df = 0$ is zero. Therefore any subsample with nonvanishing probability will spread over a stripe in the q–sin i plane, with the mass function between f and $f + df$. Three such stripes are plotted in Figure 2. The figure shows that the width of these stripes, when measured parallel to the q-axis, varies as a function of sin i. This results from the fact that the partial derivative of the mass function with regard to q varies as a function of sin i. The variation of the finite width introduces another deviation from the random distribution.

3. Finally, Figure 2 shows that for a given subset of binaries with a constant mass function, the inclination distribution depends on the mass-ratio distribution. Therefore, in order to assign an averaged value of $\sin^3 i$ to any binary, we need to know the mass-ratio distribution. This is, however, precisely the distribution that we try to extract from the sample! The CS method assumes a uniform mass-ratio distribution when averaging over the inclination angles. However, this is at best only a zero-order approximation, which might induce some bias. Clearly, to overcome this problem we need some iterative approach.

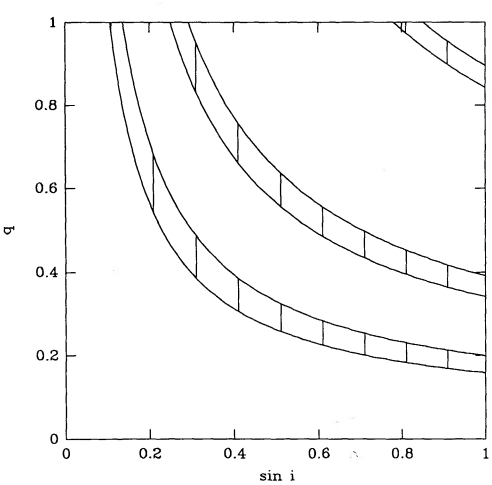

FIG. 2.—"Stripes" of constant mass function in the mass-ratio–sin i plane. The three "stripes" are confined by the values (0.001, 0.002), (0.01, 0.015), and (0.15, 0.18), respectively. Dashed line parallel to the q-axis cover the stripes. Their varying length reflects the fact that the partial derivative of f with respect to the mass ratio varies as a function of sin i.

Replacing $\sin^3 i$ by its expectation value introduces two additional error sources to the derivation of the correct mass-ratio distribution. First, as pointed out by Halbwachs (1987) in his seminal work, the specific shape of the distribution of $\sin^3 i$ is heavily weighted toward the two extreme points, where $\sin^3 i$ equals zero or unity (see Fig. 2 of Halbwachs 1987 and the discussion there). The average value of $\sin^3 i$, 0.589, used by the CS method, is, therefore, far from being the most probable value. This feature led Halbwachs to reject the CS method as an appropriate approach to the problem.

Second, to obtain the correct average mass-ratio for each binary, one should not average $\sin^3 i$ over the inclinations and then solve for the mass ratio, but rather average the mass ratio directly. This means that for each possible inclination, one should solve for the mass ratio, given the mass function and the primary mass, and then average the mass ratio over the range of inclinations, taking into account the correct distribution of inclinations (Torres 1991).

When evaluating the CS method and its importance to the advance of the study of binary characteristics, we should recall that, despite all its drawbacks, the method was conceived and first used when computing machines were not available. The beauty of this zero-order approximation relies on the fact that no computing is needed, and the averaging can be done analytically. The next section outlines our modified approach which takes into account all the points mentioned above. The new algorithm takes some amount of numerical computing, a feature which should not be considered disadvantageous in these days, contrary to the situation in the good old days of Aitken.

4. THE MODIFIED ALGORITHM TO DERIVE THE MASS-RATIO DISTRIBUTION

The proposed algorithm considers each observed binary as drawn from a subset of binaries with orientations that are *not* randomly distributed, but depends on the mass function of each binary. Three main features characterize each subset:

1. For each binary we find a lower limit for the possible inclinations, i_{min}. Inserting $q = 1$ into equation (1) yields

$$\sin^3 i_{min} = \frac{4f(M_2)}{M_1}. \qquad (2)$$

The upper limit of the inclinations is, of course, $\sin i = 1$.

2. The inclination distribution is corrected for the variation of the partial derivative of the mass function with regard to q. We multiply any distribution of angles by a correction factor

$$A(q) = \frac{(1 + q)q}{3 + q} , \qquad (3)$$

where q, the mass ratio, depends on the inclination and the mass function. To do that, we have to find q, using equation (1), for each possible inclination.

3. The inclination distribution of each subset with fixed mass function depends on an assumed mass-ratio distribution. Therefore, the algorithm is of an *iterative* nature. To begin the iteration we first assume a uniform distribution of q, assign each binary the resulting inclination distribution, and calculate q for this binary. We then derive the q distribution of the whole sample by summing over all binaries. This new distribution is the first-order approximation derived by the algorithm. It is then used as the input for calculating the second-order iteration, and so on. This process is continued till the nth order approximation is statistically indistinguishable from the $(n$-1)-th one.

To take into account the special shape of the inclination distribution, we replace each binary by an ensemble of N virtual systems with the same period, primary mass, and mass function. The inclination angles of the ensemble are distributed according to features 1–3 discussed above. Obviously, the generated ensembles of inclinations will necessarily be different for different binaries. For normalization purposes, each of the virtual systems represents $1/N$ binaries. For each virtual system we know the mass function, the primary mass, and the inclination, and therefore its mass ratio. The mass-ratio histogram of the ensembles of all binaries included in the sample represents our best estimation of the mass-ratio distribution of the observed sample.

To test the proposed new algorithm we applied it to the same simulated samples which were used to demonstrate the drawbacks of the CS method. The results are presented in Figure 3. A comparison of Figures 1 and 3 establishes the advantage of the proposed algorithm over the CS method. Numerous tests with different samples yielded the same conclusion. The simulations indicate that the difference between our algorithm and the CS method is small for distributions that peak toward smaller q, and is most pronounced for mass-ratio distributions that rise toward unity.

5. SOME CAVEATS

One of the basic *assumptions* of the proposed algorithm is that the mass ratio is smaller than unity. This assumption is true for most of the spectroscopic samples considered recently (e.g., Duquennoy & Mayor 1991; Latham et al. 1988). These samples include only main-sequence primaries, and therefore the secondary (=less luminous) component can be more massive only if it is a compact object. For example, in binaries with a G-dwarf primary (Duquennoy & Mayor 1991), the secondary (fainter component) can be more massive only if it is either a very massive white dwarf or a neutron star. These two possibilities are of negligible probability. For a sample with M-dwarf primaries, the possibility that the mass ratio is larger than unity cannot be excluded for all binaries, as the secondary (fainter) component could be a white dwarf with mass of, say, 0.6 $M_{\odot}$. However, even in such a sample, the frequency of the binaries with massive secondaries is very small.

However, the assumption about the mass-ratio upper limit is not true for all samples; the sample of giants observed by Griffin (Trimble 1990) is one recent example. For such samples one can ignore this assumption and still use the algorithm. It is still very different from the CS method, because of the introduction of the $A(q)$ factor (eq. [3]), and the iterative nature of the algorithm.

The iterative nature of the proposed algorithm raises a few doubts about its effectiveness. To express these doubts more formally, let us consider any mass-ratio distribution as an histogram of, say, M bins; any distribution being a point in an M-dimensional phase space. Any realistic distribution must follow two constraints: Each bin cannot have negative numbers of binaries, and the total number of binaries in all the bins must equal the number of binaries in the sample. The iteration can be regarded as a mapping of part of the phase

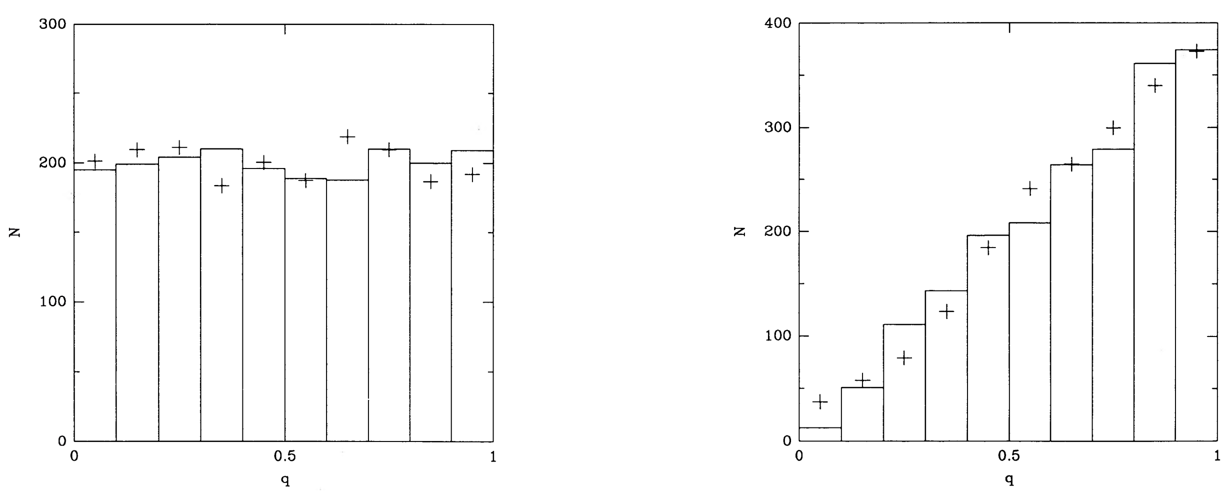

FIG. 3*a* FIG. 3*b*

FIG. 3.—Numerical simulations to test the proposed modified algorithm for deriving the mass-ratio distribution. The histogram shows the true distribution of the simulated sample. The pluses represent the results of the algorithm. The figure should be compared with Fig. 1. (*a*) A simulation with a uniform distribution. (*b*) A simulation with a monotonic increasing distribution of $N(q) = 2q$.

space into itself, the mapping being dependent on the sample. It was not clear a priori whether the iterations of a given sample converge at all. Even if they do, is the converging point independent of the starting point? And the most important doubt, is the converging point the true one?

Our numerous numerical simulations yielded positive answers to all these questions, provided the input mass-ratio distribution for each step of the iteration is continuous. The resulting histogram of any iteration is obviously not continuous. We therefore used the histogram values obtained at each iteration step for the input distribution of the next iteration only at the center of each bin. To get continuous distributions, we linearly interpolated between those points. We ran many tens of simulations and found that the iterations do converge, usually in a few steps, that the converging point is independent of the starting point, and, most important, that the result is the true distribution.

The requirement for continuous function as input distribution diminishes the ability of the algorithm to detect sharp features of the true distribution. We regard this feature of our algorithm as the "instrumental profile" of the method. The algorithm will convolve any real sharp feature of the distribution of the sample with the inherent profile. To demonstrate this profile we present in Figure 4 a simulation with a sample of binaries which all have exactly the same mass ratio $q = 0.85$, and the same primary mass. The figure presents the results of the original CS direct method and the outcome of the new proposed algorithm. Clearly, the response of the algorithm to the "delta function" input has a profile with a finite width. Nevertheless, the results indicate again that the proposed approach, even with its intrinsic profile, is advantageous over the original method, which is strongly biased.

We have checked the new algorithm on simulated *small* samples of binaries. Again, when using the interpolation scheme we were able to reconstruct the true mass-ratio distribution in all the cases we have tried. We therefore conclude, albeit without proof, that the algorithm is robust and unbiased and can be used to derive the true mass-ratio distribution.

Another subtle question is how to handle the double-line spectroscopic binaries (SB2) in the sample. The SB2's provide direct information about the mass ratio and therefore do not require replacement by an ensemble of virtual systems. We therefore suggest that one find out first the mass ratio distribution of the single-line binries (SB1) with our algorithm, and then *add* the SB2's. Again, many numerical simulations in which we included some of the high mass-ratio binaries as SB2's proved this approach to be correct. Obviously one can always ignore the information about the mass ratio and handle the SB2's as SB1's (Halbwachs 1987). In principle, the results shoud not be substantially different, albeit more noisy. Indeed, many simulations we have tried proved this point.

When analyzing real samples, one has to account for the observational selection effects, in order to derive the true mass-ratio distribution. For many samples, the selection effects for the SB1 and the SB2 binaries are different. Therefore, the unification of the two populations of binaries, as suggested here, should be done with extreme care. The corrections for undetected binaries of the two groups should be applied separately, before combining one distribution with the other.

6. COMPENSATING FOR THE UNDETECTED BINARIES

We turn now to discuss one of the observational selection effects found in samples of spectroscopic binaries, the one associated with the inability to detect binaries with low-amplitude radial-velocity modulation. We suggest a new procedure to compensate for this effect.

Specifically, we consider a radial-velocity survey of a well-defined preselected sample of stars, of which any binary with an amplitude $K_1 \geq K_{\min}$ is detected. This selection effect applies, to first-order approximation, to the large surveys of Latham et al. with the CfA stellar speedometer (Latham et al. 1988) and those of Mayor et al. with the CORAVEL machine (Duquennoy & Mayor 1990, 1991; Duquennoy et al. 1991). Note that this observational selection effect is very different from the one assumed by Trimble (1974, 1990), Staniucha (1979), and Halbwachs (1987), because the recent samples are very different from samples included in any catalog of known spectroscopic binaries (e.g., Batten, Fletcher, & Mann 1978).

The detection threshold of the radial-velocity amplitude

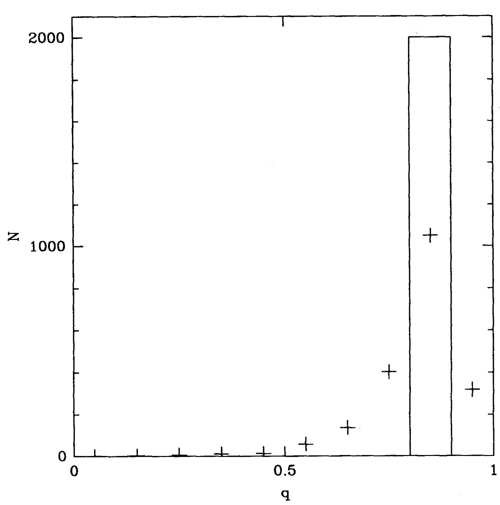

FIG. 4*a*

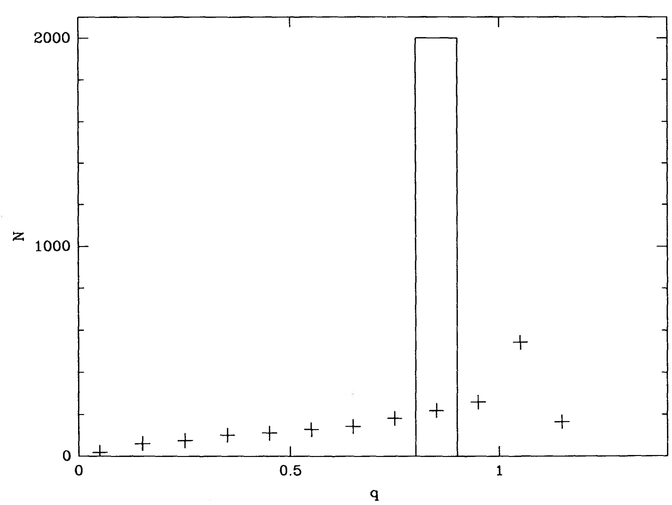

FIG. 4*b*

FIG. 4.—Numerical simulations to test the two methods for deriving the mass-ratio distribution. The histogram shows the true distribution of the simulated sample—all binaries have $q = 0.85$. The pluses represent the results of the algorithm. (*a*) The results of the modified algorithm. (*b*) The results of the old CS method.

introduces an observational selection effect on the binary inclinations, due to the dependence of K_1 on sin i:

$$K_1 = 212.9P^{-1/3}M_1^{1/3} \frac{q}{(1+q)^{2/3}} \sin i \ (\mathrm{km\ s^{-1}}) , \quad (4)$$

where P is the orbital period in days and M_1 is in solar mass units. We considered, for simplicity, only circular orbits. Consider now a subsample of binaries, all with the same binary period, mass ratio, and primary mass, and with a given distribution of inclinations. Out of this subsample, all systems with an inclination smaller than some minimal inclination, i_0, are *not* detected. One gets from equation (4) that

$$\sin i_0 = 4.697 \times 10^{-3} K_{\min} P^{1/3} M_1^{-1/3} \frac{(1+q)^{2/3}}{q} . \quad (5)$$

We therefore define a detection function, $D_{K_{\min}}(P, q, M_1)$, which is the fraction of detected binaries out of all binaries with the same P, q, and M_1:

$$D_{K_{\min}}(P, q, M_1) = \int_{\sin i_0}^{1} \Psi(i) d(i) = \sqrt{1 - \sin^2 i_0} , \quad (6)$$

where Ψ is the random distribution of the angles. Putting equation (5) into equation (6) gives for circular orbit

$$D_{K_{\min}}(P, q, M_1) = \sqrt{1 - \left[4.7 \times 10^{-3} P^{1/3} K_{\min} M_1^{-1/3} \frac{(1+q)^{2/3}}{q}\right]^2} . \quad (7)$$

The detection function was derived by assuming that the same value of $K_{\min}$ prevails for the whole sample. This is obviously only a zero-order approximation. In practice, $K_{\min}$ may depend on the binary period (e.g., Duquennoy & Mayor 1991), the primary spectral type, and possibly other parameters of the system. For eccentric orbits equation (4) should be modified, and some possible dependence of $K_{\min}$ on the eccentricity should also be taken into account. Moreover, $K_{\min}$ might be different for SB1 and SB2. We symbolically denote the $K_{\min}$ variability by

$$K_{\min} = K_{\min}(P, e, M_1, \ldots) , \quad (8)$$

to be inserted into equation (7).

The zero-order dependence of D on P and q, as expressed in equation (7), is plotted in Figure 5, for $K_{\min} = 2$ km s^{-1}, $M_1 = 1\ M_\odot$, and for circular orbits. The possible dependence of $K_{\min}$ on P was neglected. The function is close to unity for most values of P and q, except for low q, where the function is very steep. The low values of D for small q indicate that any sample of spectroscopic binaries is vulnerable to large statistical errors in this range of the mass ratio domain. We therefore regard the derived mass-ratio distribution for small q values as highly uncertain, unless $K_{\min}$ is very small.

The detection function enables us to assign each binary a correction factor,

$$C(K_{\min}, P, q, M_1) = \frac{1}{D} , \quad (9)$$

if we know its period, primary mass, and its mass ratio in particular. To compensate for the undetected binaries, we consider each binary as representing C number of systems with the same parameters. Unfortunately, the mass ratio of each binary is not known in real samples, and the correction factor cannot be applied. The correction factor procedure can work, however, within the proposed new algorithm. Each virtual system is assigned a mass ratio, and therefore its correction factor can be derived and used.

The advantage of the procedure outlined above is its capability to apply the correction for the undetected binaries to each virtual system independently. However, for small enough mass ratios, even a single virtual system is not assigned; the specific values depend on the range of observable periods of the survey and on $K_{\min}$. In such a case, the individual correction procedure cannot be applied, and the number of systems with small mass ratios has to be extrapolated. The extrapolation must assume some period distribution, the natural choice being the observed period distribution of the whole sample. This correction introduces again, a large uncertainty to the low end of the derived mass-ratio distribution. In the recent surveys of the CORAVEL and the CfA stellar speedometers this problem usually appears only for mass ratios smaller than 0.1.

To demonstrate the effectiveness of the new procedure we, again, used a simulated sample of binaries. We used the same sample with uniform mass-ratio distribution. Out of this sample we generated a subsample of the "detected" binaries, in which we included only binaries with an amplitude larger than a threshold detection of $K_{\min} = 2$ km s^{-1}. We first applied our modified iterative algorithm, to find the mass-ratio distribution of the *detected* sample. We then used the correction factor to find out the true distribution of the whole sample. The results are depicted in Figure 6, together with the CS method result, where we assigned to each binary a value of 0.679 for $\sin^3 i$. The figure suggests, again, that the combination of the iterative proposed algorithm with the individual correction factor procedure might be useful in some cases.

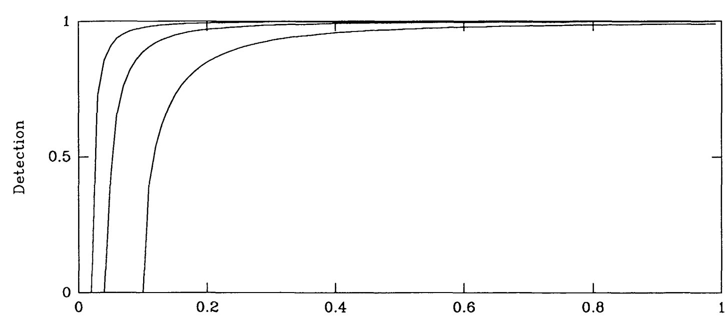

FIG. 5.—The detection function as a function of the binary mass ratio, for different values of the period P (10, 100, and 1000 d). The detection threshold was taken as $K_{\min} = 2$ km s^{-1}.

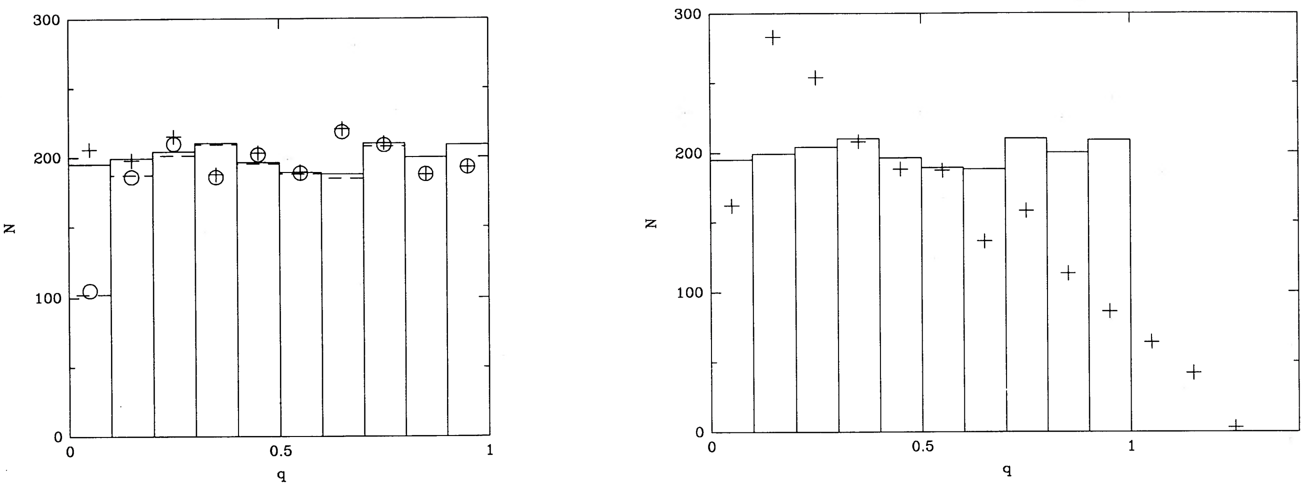

FIG. 6a

FIG. 6b

FIG. 6.—Numerical simulations to test the two methods for deriving the mass-ratio distribution. The continuous-line histogram represents the true distribution of the original simulated sample, including all binaries. The distribution used is a uniform one. The broken-line histogram shows the distribution of the "detected" binaries. (*a*) The results of the modified algorithm—the circles represent the derived distribution of the detected binaries, while the pluses show the obtained total distribution. (*b*) The results of the old CS method, assuming a detection probability which depends on the binary inclination and is proportional to sin *i*.

7. CONCLUSIONS

We have proposed a modified algorithm to derive the mass-ratio distribution of an observed sample of spectroscopic binaries. The effectiveness of the algorithm was demonstrated with several illustrative numerical simulations, in which we were able to reconstruct the initial distribution of idealized samples of binaries.

The derivation of the *observed* distribution is only the first step to obtain the true mass-ratio distribution of any real sample of binaries. As already pointed out by previous researchers (Halbwachs 1987; Trimble 1990; Duquennoy & Mayor 1991), the correction for the different selection effects of each survey might be more important than the specific algorithm to derive the mass-ratio distribution. We have outlined a new procedure to correct for one important observational effect associated with the detection limit of the radial-velocity amplitude. However, a few more effects should be accounted for when deriving the true distribution of any sample. Some famous examples are the bias introduced by binaries detected by visual means, and the distortion caused by a magnitude definition of the survey limits (Öpik 1924; Branch 1976; Halbwachs 1987; Trimble 1990). Another extremely important point is the relative weight of the double-line binaries (Trimble 1990), a feature that can distort completely the high end of the distribution. All these selection effects should be taken into account when analyzing real sample of spectroscopic binaries.

The analysis of the very recent samples of binaries (Latham et al. 1988; Duquennoy & Mayor 1991) is deferred to subsequent work, where our algorithm will be applied and the observational selection effects will be addresed thoroughly. Hopefully, the new data will enable us to determine soon the true shape of the mass-ratio distribution of stellar binaries.

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