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THE ROLE OF THE DWARF NOVA PERIOD DISTRIBUTION IN UNDERSTANDING THE EVOLUTION OF CATACLYSMIC VARIABLES

ALLEN W. SHAFTER
Department of Astronomy and Mount Laguna Observatory, San Diego State University, San Diego, CA 92182
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ABSTRACT

Within the context of disk instability theory, cataclysmic variables possessing accretion disks (i.e., nonmagnetic systems) are expected to exhibit disk instabilities that result in dwarf nova eruptions if the mass transfer rates fall below a critical level, $\dot{M}_{\rm crit}$. It is argued that the eruptive characteristics of cataclysmic variables can therefore be used to infer relative mass transfer rates among nonmagnetic cataclysmic variables. Here the dwarf nova period distribution is used to constrain the variation of mass transfer with orbital period. For orbital periods above the gap, agreement between the observed dwarf nova period distribution with those constructed from various magnetic braking models is generally disappointing. The differences arise both because the braking laws often result in $\dot{M}(P)$ relations that are unacceptably steep, and, more specifically, because the braking laws offer no explanation for the observed dearth of dwarf novae with orbital periods between 3 and $\sim$4 hr. The most promising braking law is that of Mestel and Spruit. Under certain conditions this braking law is able to produce a relatively flat $\dot{M}(P)$ relation, but the braking law is not entirely satisfactory because it offers no explanation for the complete dominance of stable over unstable accretors immediately above the period gap. Speculative ideas are presented that may eventually provide a complete and satisfactory explanation for the lack of dwarf novae with periods between 3 and $\sim$4 hr. As a general point, it is suggested that the dwarf nova period distribution, and not only the overall period distribution, should be considered when applying observational constraints to theories of mass transfer in cataclysmic variables.

Subject headings: accretion, accretion disks — binaries: close — novae, cataclysmic variables

1. INTRODUCTION

Considerable progress has been made during the past decade in understanding the secular evolution of cataclysmic variable stars. It is generally accepted that most, and possibly all, cataclysmic variables are the descendents of long-period, relatively high-mass binaries that have gone through a phase of common envelope evolution, losing much mass and angular momentum (Paczyński 1976; Webbink 1979). Because the white dwarf is usually the more massive component, cataclysmic variables are stable with respect to mass transfer. In a hypothetical system with total mass and angular momentum conserved, any mass transfer from the secondary star to the white dwarf primary will cause the stellar separation to increase, and the mass transfer to cease. Mass transfer can only be sustained by an expansion of the secondary star (due to nuclear evolution), or by a loss of orbital angular momentum from the binary. Since the low-mass secondary stars cannot have evolved significantly during the age of the Galactic disk, it must be the case that cataclysmic variables sustain mass transfer as a result of orbital angular momentum loss. In the relatively long-period systems ($P \geq 3$ hr), magnetic braking of the secondary star's rotation by its own stellar wind appears to drain enough angular momentum from the binary to account for the relatively high mass transfer rates observed (Verbunt & Zwaan 1981; Rappaport, Verbunt, & Joss 1983; Taam 1983; Patterson 1984). For the shorter period systems, angular momentum loss via gravitational radiation is believed to play a major role (Paczyński & Sienkewicz 1981; Rappaport, Joss, & Webbink 1982).

The evolution of cataclysmic variables produces a bimodal orbital period distribution consisting typically of periods between 80 minutes and approximately half a day, with a statistically significant gap between periods of $\sim$2 and $\sim$3 hr (Robinson 1983). The currently favored explanation for the period gap derives from the so-called disrupted magnetic braking model (Spruit & Ritter 1983; Rappaport et al. 1983; Hameury et al. 1988; McDermott & Taam 1989; Taam & Spruit 1989). The rapid mass loss associated with magnetic braking drives the secondary star out of thermal equilibrium causing it to bloat beyond its main-sequence radius. After the system has evolved to a period of about 3 hr, it is believed that the angular momentum loss rate is reduced significantly. That the secondary star becomes fully convective near this period ($M_2/M_\odot \sim 0.3$) appears to be responsible for the reduction (Robinson et al. 1981; Spruit & Ritter 1983; Hameury et al. 1987). The lower angular momentum loss rate that results slows the evolution, allowing the secondary star to reestablish thermal equilibrium and detach from its Roche lobe. The system then is believed to evolve in a detached state until angular momentum losses due to gravitational radiation brings the secondary star back into contact at a period of about 2 hr.

Despite these successes, the details of how magnetic breaking drives mass transfer in cataclysmic variables is not fully understood. For example, although some form of magnetic braking appears to be capable of driving the relatively high mass transfer rates thought to be present in the longer period cataclysmic variables, there is no one, generally favored, braking theory applicable to cataclysmic variables. In their pioneering paper, Verbunt & Zwaan (1981), were the first to show that magnetic braking could in principle account for the mass transfer in cataclysmic variables and low-mass X-ray binary systems. Their model, which is based on an extrapolation of the Skumanich law for slowly rotating G stars, has been criticized as not being applicable to the rapidly rotating secondary stars in close binary systems. Since the work of Verbunt & Zwaan (1981), several other magnetic braking prescriptions have been put forward, but deciding between these different models is difficult because there are many free parameters and few observational constraints.

From an observational standpoint, much of the problem concerns the difficulty in comparing the mass transfer rates predicted by models for magnetic breaking with observations. As with most other fundamental parameters of cataclysmic variables, it has proven difficult to determine accurate mass transfer rates for individual systems. To determine the absolute level of mass transfer, the distance to the system must be known. If one then attempts to determine the mass accretion rate from an estimate of the bolometric luminosity, then corrections need to be made for orbital inclination, for contamination of the disk light by the secondary star, and for flux outside the observed wave band (i.e., the bolometric correction). In most cases these corrections are poorly known, making the mass transfer rates uncertain. In the majority of cases where the distances are unknown, the situation is considerably worse. Mass transfer rates must be estimated from comparisons of the observed colors with those from available disk models. Work by Wade (1984, 1988), in particular, has shown that the disk models are incomplete and that mass accretion rates obtained in this way are unreliable, especially for dwarf novae whose disks are not in a steady state.

As an illustration of the difficulties involved in estimating reliable mass transfer rates, Patterson (1984), using variations of the above methods, was able to infer a power-law relationship between the average mass transfer rate and orbital period [$d \log \dot{M}(P)/d \log P \simeq 3.3$], while using similar data, but differing assumptions, Verbunt & Wade (1984) found no obvious correlation of $\dot{M}$ with orbital period. More recently Warner (1987) has taken a more conservative approach and used the absolute magnitudes of systems with reasonably reliable distance determinations to make statistical inferences about $\dot{M}$. This approach avoids many model-dependent assumptions necessary to convert the observations to derived quantities. Warner finds that there is a significant dispersion in the absolute magnitude, and presumably $\dot{M}$, at a given orbital period.

Although determining absolute mass transfer rates for specific systems at specific epochs has proven to be problematic, Shafter, Wheeler, & Cannizzo (1986) have suggested that the eruptive characteristics of cataclysmic variables can be used to provide meaningful constraints on the rate of mass transfer as a function of orbital period. Specifically these authors have argued that the occurrence of dwarf nova eruptions can be used to discriminate between high and low mass transfer rates in systems containing accretion disks (i.e., *non*magnetic systems). Their argument rests on one crucial assumption: Dwarf nova eruptions are triggered by thermal instabilities in the accretion disks. There are several variations of the disk instability model (Meyer & Meyer-Hofmeister 1981; Faulkner, Lin, & Papaloizou 1983; Cannizzo & Wheeler 1984; Mineshige & Osaki 1983), but they all share a common prediction: If the mass transfer rate lies above a critical value, $\dot{M}_{\rm crit}$, the instability is suppressed, the accretion rate through the disk is constant, and dwarf nova eruptions do not occur. In general, dwarf novae are expected to have mass accretion rates less than $\sim 10^{-9}\ M_\odot\ {\rm yr}^{-1}$, while other cataclysmic variables possessing accretion disks, but not exhibiting dwarf nova eruptions, have mass transfer rates that generally exceed $\sim 10^{-9}\ M_\odot\ {\rm yr}^{-1}$. Shafter et al. (1986) were the first to exploit this prediction, using the observed dwarf nova period distribution to argue against a steeply increasing $\dot{M}(P)$ relation such as the one found by Patterson (1984).

During the five years since the work of Shafter et al. (1986), there has been a significant increase in the number of cataclysmic variables with known orbital periods, making the dwarf nova period distribution better defined. In addition, progress has been made in our theoretical understanding of the processes that drive mass transfer in systems above the period gap. One example is a new magnetic braking model, proposed by Mestel & Spruit (1987), that has been claimed to drive mass transfer with an orbital period dependence more in accord with observations. In view of developments such as these, I have decided to reexamine the consistency between the observed dwarf nova period distribution and the predictions of current magnetic braking theory. My purpose is not to illuminate or focus on small deficiencies in the current state of magnetic braking models, as these models are admittedly still in an early stage of development, but rather to emphasize the point that the dwarf nova period distribution, not just the overall period distribution of cataclysmic variables, is an important observational constraint that should be used to constrain models of mass transfer above the period gap. For example, the observed width of the period gap is often used to constrain magnetic braking models, but the fact that the period gap for dwarf novae (presumably low-$\dot{M}$ systems) is almost *twice* as wide as the gap in the overall distribution is not usually considered.

The paper is organized as follows. In § 2 I discuss the observed dwarf nova period distribution. In § 3 I summarize the predictions of disk instability and magnetic braking theory. These predictions are then used to produce theoretical period distributions that can be compared with the observations. Finally, in § 4 I discuss possible avenues to explore in bringing the theory into better accord with observations.

2. THE OBSERVATIONAL DATA

Figure 1 shows the orbital period distribution for non-magnetic cataclysmic variables with periods less than 10 hr (only ∼5% of known systems have longer periods). I have excluded the few systems with periods longer than 10 hr because the period distribution is ill defined and the evolved secondary stars in such systems make their evolution more complicated. The data used in the period distribution have been culled from the compilations of Ritter (1990a) and Webbink (1990) with a small number of modifications to be discussed below. Because I propose to use the eruptive characteristics of cataclysmic variables to discriminate between high- and low-$\dot{M}$ systems, cataclysmic variables that may not have the *potential* to undergo a dwarf nova outburst regardless of their mass transfer rates must be excluded from the sample. Specifically, magnetic systems, the DQ Her (intermediate polars), and the AM Her stars (polars) have been strictly excluded because the accretion disks in the former systems are thought to be disrupted to varying degrees by the white dwarf's magnetic field, while the latter systems have no disks at all. Within the context of disk instability theory it is impossible for the diskless AM Her systems to exhibit dwarf nova eruptions; it is unclear whether the DQ Her systems, with partially (or completely) disrupted disks, can undergo a disk instability. The suspicion that DQ Her systems are incapable of undergoing disk instabilities is borne out empirically as essentially no bona

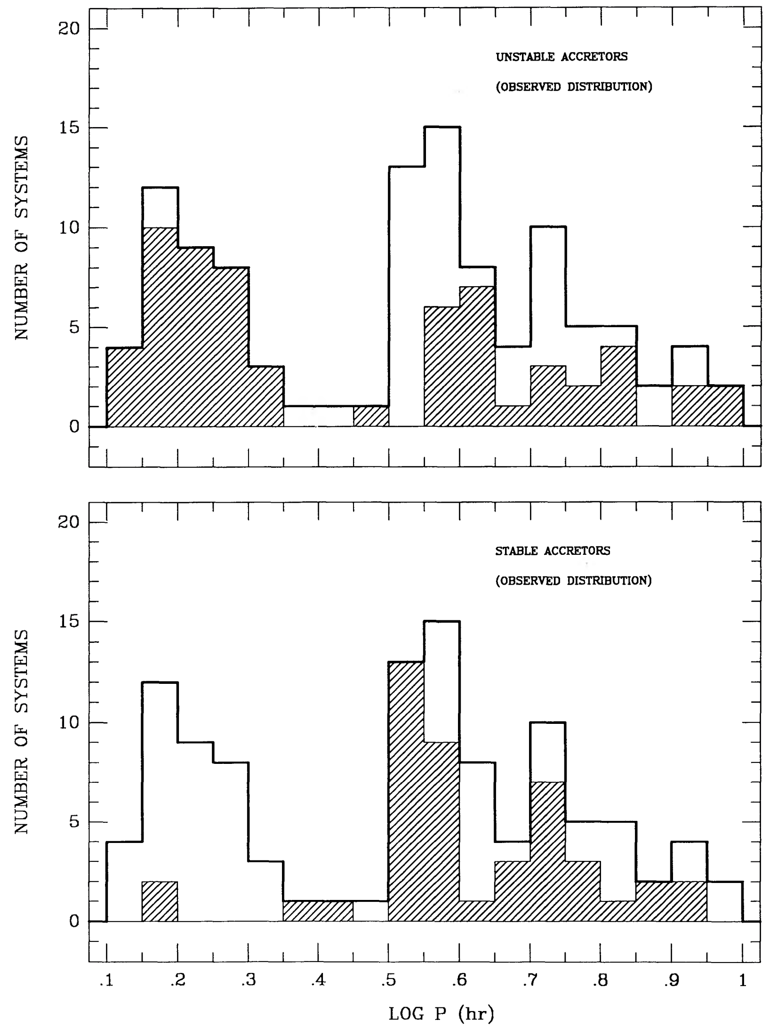

FIG. 1.—The observed orbital period distributions for unstable accretors (dwarf novae) and for stable accretors. The heavy solid histogram represents all cataclysmic variables that are thought to have accretion disks (i.e., the nonmagnetic systems). The shaded region of the upper panel represents the dwarf nova period distribution. For comparison, the complement of the dwarf nova period distribution, the period distribution for stable accretors, is shown in the lower panel. Note the dearth of dwarf novae immediately above the period gap.

fide member of this class has been observed to exhibit dwarf nova eruptions.[1] After the exclusion of magnetic systems, the remaining cataclysmic variables can be divided into two classes: those that exhibit dwarf nova eruptions, and those that do not. We will sometimes refer to these two classes as unstable and stable accretors, respectively.

Before proceeding further, it is necessary to discuss briefly the potential sources of error in the observational data, specifically the dwarf nova period distribution. There are two sources of uncertainty: the reliability of the orbital period and the classification of the system as either a stable or unstable accretor. Generally, the orbital period is one of the more reliably determined parameters of any cataclysmic variable. The principal source of error is orbital period aliasing. Occasionally nonorbital photometric periodicities (or quasi-periodicities) are incorrectly identified with the orbital period of the system.

[1] An exception is the very long orbital period system GK Per ($P \sim 2$ day). The accretion disk in GK Per is likely to be sufficiently large that disk instabilities can occur even if the inner portion of the disk is disrupted by the white dwarf's magnetic field.

In order to minimize the possibility of including systems with spurious periods, I have excluded systems whose periods are explicitly reported as uncertain. The orbital periods for the remaining systems should result in a reliable estimate of the orbital period distribution; the possibility that a few spurious periods are included in the final sample should not have a significant impact on the analysis presented here.

The situation with regard to identification of eruptive behavior is more problematic. It is possible, even likely, that the mass transfer rate of a given system fluctuates around some secular mean value. Although the direct estimates of $\dot{M}$ are wrought with uncertainty, the available observational data suggest that there may be a considerable spread in $\dot{M}$ at a given period above the gap (Hameury, King, & Lasota 1989; Warner 1987; Patterson 1984). Since, in the disrupted magnetic braking model, the width of the period gap depends on the mass transfer rate above the gap, these fluctuations cannot be too large or the gap would become ill defined, or would disappear altogether. Thus in a *statistical* sense, the ratio of stable to unstable accretors within a given orbital period bin should not be affected significantly by these short-term fluctuations, and the relative numbers of stable and unstable systems should provide a reliable tracer of the secular mean mass transfer rate.

A more immediate difficulty is simply that the long-term photometric monitoring of many systems is sporadic, leaving open the possibility that eruptive events may have been missed. Fortunately, for most systems the eruptive behavior is not in serious doubt. In fact, a principal tenet of this study is that the eruptive behavior of a cataclysmic variable, while not known with absolute certainty in all cases, is known more reliably than is the mass transfer rate for that system. Once again to minimize the effect of misclassifications, I have excluded from the sample systems whose eruptive characteristics are not clearly established. Because the process of selecting the most "reliable" systems is necessarily subjective at some level, a few systems that have been omitted deserve special comment. These systems are listed in Table 1, along with a brief explanation of why they were omitted from the sample.

A comparison of the dwarf nova period distribution in Figure 1 with that of the stable accretors reveals an obvious distinction. Almost all of the systems below the period gap are dwarf novae, while the majority of systems just above the period gap ($3 < P[\mathrm{hr}] < 4$) are stable accretors (i.e., they do *not* exhibit dwarf nova eruptions). The extremely high incidence of dwarf novae below the gap is easily understood if we make the usual assumption that gravitational radiation is the dominant angular momentum loss mechanism acting at these very short periods. Henceforth, I will comment little on the eruptive behavior of systems below the period gap. It is the eruptive behavior of systems above the gap, although not well understood at present, that may provide useful constraints on models for magnetic braking.

Above the gap there is a general admixture of stable and unstable accretors. The distributions for the two types of systems, however, are not similar. A Kolomogorov-Smirnov test yields a two-sample statistic of 1.46. A value this high or higher is expected to occur with a probability of 2.8×10^{-2} if the distributions are drawn from the same parent distribution. The primary reason for the dissimilarity is that, arranged in order of period, the first 15 systems in a row out of the 68 systems above the gap are all of one eruptive type—they are all stable accretors. Note that if the stable and unstable accretors above the gap were assumed to be distributed randomly with orbital period, the probability of a single "run" of 15 or more systems of one type out of a sample of 68 systems would be $\sim 2 \times 10^{-3}$ (e.g., von Mises 1964).

That the observed distributions of stable and unstable accretors are dissimilar is not necessarily surprising. In general the two distributions should be determined by the relationship between the mass transfer rates driven by magnetic braking and the critical mass accretion rate given by disk instability theory. Thus although there is no reason to expect the distributions to be similar, the functional form of $\dot{M}(P)$ given by the magnetic braking laws provides explicit predictions for the ratios of stable to unstable accretors as a function of period. In the next section, the observed dwarf nova period distribution is compared with those predicted by the currently popular magnetic braking prescriptions.

TABLE 1

STARS OMITTED FROM PERIOD DISTRIBUTION

Object	Class	Period	Reason for Rejection
RZ Leo	DN	0.071:	Period uncertain
TT Boo	DN	0.077:	Period uncertain
RW UMi	N	0.081:	Period uncertain[a]
V795 Her	DQ?	0.108	Class uncertain
BZ Cam	N?, VY?	0.139:	Period and class uncertain
WY Sge	N, DN?	0.154	Class uncertain
1329-294	UG?	0.159	Class uncertain
CM Del	DN?, VY?	0.162	Class uncertain[b]
UU Aql	DN	0.164	Period alias[c]
EY Cyg	DN	0.181:	Period uncertain
AR Cnc	UG?	0.215	Class uncertain
BV Pup	UG	0.225:	Period uncertain
V794 Aql	NL	0.23:	Period uncertain
EI Uma	UG?	0.268	Class uncertain

[a] The 0.081 d photometric modulation may not be orbital in nature.

[b] AAVSO records do not support the DN classification.

[c] Periods of 0.140 and 0.164 d are possible; the latter is slightly preferred (J. A. Thorstensen 1991, private communication).

3. PREDICTIONS FROM THEORY

3.1. *Disk Instability Models*

It is beyond the scope of this paper to discuss disk instability theory in detail. The basic idea is that material initially transferred from the secondary star is temporarily stored in a low-viscosity disk surrounding the white dwarf primary. When the material reaches a critical temperature and surface density, an instability ensues, heating the material, raising its viscosity, and causing it to accrete onto the white dwarf. Here we only make use of the general result that disk instabilities (dwarf nova eruptions) do not occur if the mass transfer rate from the secondary star is above a critical rate, $\dot{M}_{\rm crit}$. As pointed out by Shafter et al. (1986), the various models for disk instabilities give remarkably similar expressions for this critical mass transfer rate. Specifically, the models of Meyer & Meyer-Hofmeister (1983), Faulkner et al. (1983), Cannizzo & Wheeler (1984), Mineshige & Osaki (1983), and Smak (1984) yield expressions for $\dot{M}_{\rm crit}$ that can be approximated well by

$$\dot{M}_{\rm crit} \simeq 10^{16} r_{10}^{2.6} M_1^{-0.87} \ {\rm g\ s^{-1}}\ , \quad (1)$$

where r_{10} is the radius of the accretion disk in units of 10^{10} cm and M_1 is the mass of the white dwarf. If we adopt Eggleton's (1983) expression relating the dimensionless radius of the Roche lobe to the mass ratio of the binary, and assume that the accretion disk fills a fraction ζ of the primary's Roche lobe, we can cast the expression for $\dot{M}_{\rm crit}$ in terms of the orbital period, P. Specifically,

$$\dot{M}_{\rm crit}(P) = 6.54 \times 10^{-10} [\zeta F(q)]^{2.6} P({\rm hr})^{1.73}\ M_\odot\ {\rm yr^{-1}}\ , \quad (2)$$

where $F(q)$ is a weakly varying function of $q\ (= M_2/M_1)$ and is given by

$$F(q) = \frac{(1+q)^{1/3}}{0.6 + q^{2/3} \ln(1 + q^{-1/3})}\ . \quad (3)$$

A comparison of $\dot{M}_{\rm crit}(P)$ with expressions for $\dot{M}(P)$ given by evolutionary models for cataclysmic variables will enable us to compute theoretical dwarf nova period distributions, which will then be compared with the observed distribution.

3.2. *Angular Momentum Loss Mechanisms*

To determine how the mass transfer rate varies as a function of orbital period I have followed the general procedure outlined in Taam (1983). I assume $\dot{M}$ is driven by angular momentum losses resulting from the combined effects of gravitational radiation ($\dot{J}_{\rm gr}$) and magnetic braking ($\dot{J}_{\rm mb}$). As an option I include the angular momentum loss that results from material expelled from the white dwarf during nova eruptions ($\dot{J}_{\rm nova}$). I assume that gravitational radiation acts for all periods, while the effects of magnetic braking are included only for periods above the gap. It may be the case that magnetic braking contributes to angular momentum losses below the gap, but with much reduced efficiency. However, since we are primarily concerned with systems above the period gap, a precise treatment of possible angular momentum loss mechanisms below the gap is not necessary in the analysis. Following Taam (1983), the angular momentum loss rate can be written

$$\dot{J}_{\rm orb} = \dot{J}_{\rm gr} + \eta_1 \dot{J}_{\rm mb} + \eta_2 \dot{J}_{\rm nova} , \quad (4)$$

where $\eta_1 = 0(1)$ for periods below (above) the gap and $\eta_2 = 0(1)$ for conservative (nonconservative) mass transfer.

Expressions for each of the angular momentum loss mechanisms require that the mass-radius relation for the secondary star be specified. In order to provide a consistent comparison between the different magnetic braking laws, I assume a general mass-radius relation of the form

$$\frac{R_2}{R_\odot} = \alpha\left(\frac{M_2}{M_\odot}\right)^\xi . \quad (5)$$

The precise form of $\dot{J}_{\rm orb}$ depends on whether we have conservative mass exchange. However, a general expression for the orbital angular momentum, $J_{\rm orb}$, can be written as

$$J_{\rm orb} = 9.0 \times 10^{51} \alpha^{1/2} M^{-1/3} M_1 M_2^{(5+3\xi)/6} \text{ g cm}^2 \text{ s}^{-1} , \quad (6)$$

where, following Patterson (1984), I have made use of the convenient Roche lobe approximation for small q given by Paczyński (1971).

Similarly the angular momentum loss rate due to gravitational radiation is given by the well-known dipole formula, modified by the general mass-radius relation for the secondary star,

$$\dot{J}_{\rm gr} = -1.1 \times 10^{34} \alpha^{-7/2} M^{-2/3} M_1^2 M_2^{(19-21\xi)/6} \text{ ergs} . \quad (7)$$

In the case of nonconservative mass transfer, the angular momentum loss due to mass expelled from the white dwarf can be written (Taam 1983) as

$$\dot{J}_{\rm nova} = \frac{J_{\rm orb} M_2}{M_1 M} \dot{M}_2 \text{ ergs} , \quad (8)$$

where $J_{\rm orb}$ is given in equation (6).

I consider three proposed magnetic braking laws: (1) the Verbunt & Zwaan (1981, hereafter VZ81) formulation; (2) the Patterson (1984, hereafter P84) law; and (3) the Mestel & Spruit (1987, hereafter MS87) law. Using equation (5) the angular momentum loss rate due to magnetic braking, $\dot{J}_{\rm mb}$, for the three magnetic braking laws can be cast in general form. I discuss each in turn below.

3.2.1. *The Verbunt and Zwaan Formulation*

The VZ81 formulation assumes that magnetic braking of the secondary stars in cataclysmic variables can be obtained by an extrapolation of the Skumanich law (Skumanich 1972), which describes how the equatorial rotation rate of main-sequence G stars depends on age. The VZ81 law can be expressed as

$$\dot{J}_{\rm VZ81} = -1.8 \times 10^{37} \alpha^{-1/2} f^{-2} k^2 M_2^{(5-\xi)/2} \text{ ergs} , \quad (9)$$

where f is an empirically determined parameter obtained from the calibration of the Skumanich law and k is the radius of gyration of the secondary star. The value of f has been estimated to lie between 0.73 (Skumanich 1972) and 1.78 (Smith 1979); here I adopt $f = 1$. The value of k is dependent on the mass of the secondary star's convective envelope. For fully convective stars (i.e., near the upper edge of the period gap) $k^2 \simeq 0.2$, while for a 0.7 $M_\odot$ secondary star that would be typical of a system at a period of $\sim$8 hr, $k^2 \simeq 0.03$ (e.g., Hameury et al. 1988). For simplicity, I follow VZ81 and adopt $k^2 = 0.1$ independent of orbital period. The effect on the analysis of including the period dependence of k^2 will be discussed further below.

3.2.2. *The Patterson Formulation*

The P84 formulation makes use of observed period changes in close binaries to estimate the angular momentum loss rate directly. The expression is given simply as

$$\dot{J}_{\rm P84} = -10^{37} M_2^4 \text{ ergs} . \quad (10)$$

Because this is an empirical relation, there are no free parameters as are found in the theoretical magnetic braking prescriptions.

3.2.3. *The Mestel and Spruit Formulation*

The MS87 formulation is the most recent of the magnetic braking prescriptions. The region surrounding the secondary star is divided into two regions, a so-called dead zone of closed magnetic field lines, and a wind zone of open field lines where mass, and hence angular momentum, can be drained from the secondary star. Because mass and angular momentum cannot be lost from the dead zone, the net effect of including this region is to reduce the effectiveness of the magnetic braking. The strength of the braking can be characterized by a parameter, n, which relates the coronal X-ray luminosity to the magnetic field strength: $L_x \propto B^n$ (cf. Hameury 1991). The MS87 relation can be written as

$$\dot{J}_{\rm MS87} = -9.1 \times 10^{35} \alpha^{(4-3n)/6} M_2^{[6+n+\xi(4-3n)]/6} \text{ ergs} . \quad (11)$$

3.3. *Mass Transfer Rates*

The orbital-period-dependent mass transfer rates that we seek can be obtained by equating the angular momentum loss rates described above with the appropriate expression for $\dot{J}_{\rm orb}$. In the case of conservative mass transfer,

$$\dot{J}_{\rm orb} = \left.\frac{\partial J_{\rm orb}}{\partial t}\right]_{M={\rm const}} , \quad (12)$$

while if the mass accreted by the white dwarf is expelled from the system during nova eruptions, it seems reasonable to expect that M_1 would remain approximately constant during the secular evolution of the binary. If so, then

$$\dot{J}_{\rm orb} = \left.\frac{\partial J_{\rm orb}}{\partial t}\right]_{M_1={\rm const}} . \quad (13)$$

Figure 2 shows the variation of $\dot{M}(P)$ for an example of conservative mass transfer with a total system mass, $M = 1.6$ $M_\odot$. Below the period gap the mass transfer rate driven by gravitational radiation alone is comfortably below the critical mass transfer rate. It may be possible for magnetic braking to be acting at a diminished level below the period gap and still not produce a significant number of stable accretors. Above the gap both gravitational radiation and magnetic braking are assumed to drive mass transfer. The dependence on orbital period is shown for three representative expressions for the mass-radius relation of the secondary star. The simplest form, used by VZ81 is given by $\xi = \alpha = 1$. A more realistic form, based on observations of low-mass main-sequence stars, was

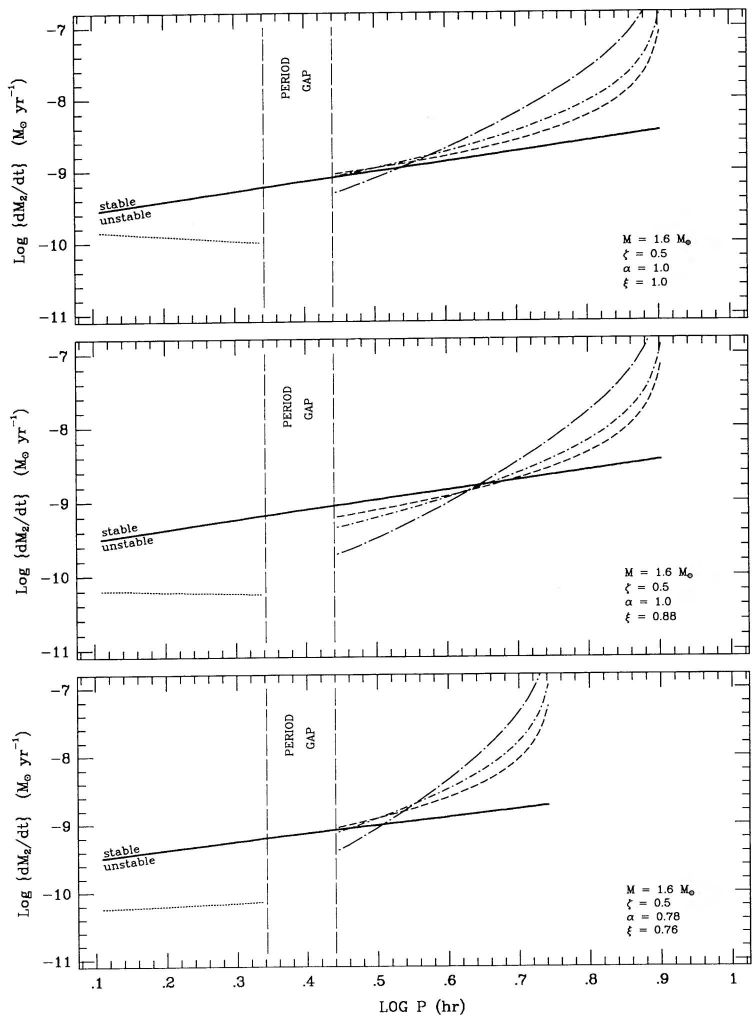

FIG. 2.—The comparison between the critical mass transfer rate dividing stable from unstable accretion, $\dot{M}_{crit}$, and the mass transfer rates expected from evolutionary theory in the case of *conservative* mass transfer. The total system mass is 1.6 $M_{\odot}$, and the disk is assumed to fill 50% of the primary's Roche lobe. The heavy solid line represents $\dot{M}_{crit}$. The dotted line represents the mass transfer driven by angular momentum loss due to gravitational radiation. The other curves represent the mass transfer rates driven by the sum of angular momentum loss due to magnetic braking and gravitational radiation. The short-dash-dot curve represents the VZ81 braking law, the log-dash-dot curve represents the P84 braking law, and the short-dashed curve represents the MS87 braking law. The three panels show the effect on $\dot{M}(P)$ of varying the mass-radius relation of the secondary star.

adopted by P84, who found $\xi = 0.88$ and $\alpha = 1$. Also shown are curves based on the mass-radius parameters adopted by McDermott & Taam (1989) in their magnetic braking models ($\xi = 0.76$; $\alpha = 0.78$). In general, the smaller the value of ξ, the steeper the dependence of $\dot{M}$ on orbital period. The value of α essentially acts like a scale factor. The slope of the $\dot{M}(P)$ relation, $(d \log \dot{M}/d \log P)$, can be approximately by a power law of index γ. For conservative mass transfer,

$$\dot{M} \propto \frac{P^{\gamma}}{\alpha[(5/3 + \xi)M_1 - 2M_2]}, \quad (14)$$

while in the case of nonconservative mass transfer,

$$\dot{M} \propto \frac{P^{\gamma}}{\alpha[(5/3 + \xi)M_1^2 + (1 + \xi)M_1 M_2 - 2M_2^2]}, \quad (15)$$

where the value of γ depends on the particular mass-radius relation and braking law considered. Specifically,

$$\gamma = \frac{25/3 - \xi}{3\xi - 1}, \quad (\text{P84}), \quad (16)$$

$$\gamma = \frac{16/3 - 2\xi}{3\xi - 1}, \quad (\text{VZ81}), \quad (17)$$

and, for $n = 1$, the MS87 braking law gives

$$\gamma = \frac{8/3 - 2\xi/3}{3\xi - 1}, \quad (\text{MS87}). \quad (18)$$

The denominators in equations (14) and (15) are slowly varying functions of the mass ratio, q, for small q. For mass ratios aproaching unity, the slope $(d \log \dot{M}/d \log P)$ becomes larger than that given by γ alone, particularly in the case of conservative mass transfer. This effect would be partially offset if the period dependence of the radius of gyration of the secondary star, k, were taken into account. Since the radius of gyration is actually a slowly decreasing function of the orbital period, not a constant as I have assumed, inclusion of the period dependence would result in a slight *decrease* in the slope of $\dot{M}(P)$. Thus γ is a good estimate of the slope of the $\dot{M}(P)$ relation. Table 2 gives a few representative values of γ for each of the three braking laws and mass-radius parameter ξ.

TABLE 2

THE SLOPE OF THE $\dot{M}(P)$ RELATION

	$\gamma = (d \log \dot{M}/d \log P)$		
BRAKING LAW	($\xi = 1.0$)	($\xi = 0.88$)	($\xi = 0.76$)
P84	3.67	4.54	5.92
VZ81	1.67	2.18	2.98
MS87 ($n = 0.5$)	1.17	1.43	1.85
MS87 ($n = 1.0$)	1.00	1.27	1.69
MS87 ($n = 2.0$)	0.67	0.94	1.35

Observations of nova shells suggest that the mass ejected in nova eruptions is at last as great as the amount of material accreted from the secondary star. Thus the inclusion of angular momentum losses from mass lost from the white dwarf during nova eruptions and the assumption that M_1 remains constant during the secular evolution of the system would appear to be a more realistic approach than simply adopting the idealized case of conservative mass transfer. Figure 3 shows the dependence of $\dot{M}(P)$ for an example of nonconservative mass transfer with $M_1 = 1\ M_{\odot}$. Once again I show the variation of $\dot{M}(P)$ for three representative mass-radius relations for the secondary star. A comparison with Figure 2 reveals that the nonconservative mass transfer case yields a somewhat flatter dependence of $\dot{M}$ on orbital period. Despite this difference, the conservative and nonconservative cases are similar in one important respect: *the slope of $\dot{M}(P)$ is significantly steeper than $\dot{M}_{\rm crit}(P)$.*

Although the mono-mass examples shown in Figures 2 and 3 are useful for didactic purposes, they are not necessarily satisfactory for predicting the dwarf nova period distribution. It is almost certainly the case that the observed cataclysmic variable period distribution is populated by an ensemble of systems having a range of masses. In addition to the expected variation of the secondary star's mass with orbital period, there is likely to be significant dispersion in the white dwarf mass at a given orbital period. The variation in white dwarf mass will affect both the rate of angular momentum loss, and hence $\dot{M}$, as well as the radius of the accretion disk, and hence $\dot{M}_{\rm crit}$.

Because we want to compare the predictions of magnetic braking theory with the *observed* dwarf nova period distribution, we must consider the selection effect favoring the discovery of systems containing massive white dwarfs (Ritter & Burkert 1986; Ritter & Özkan 1986; Ritter 1986). As discussed

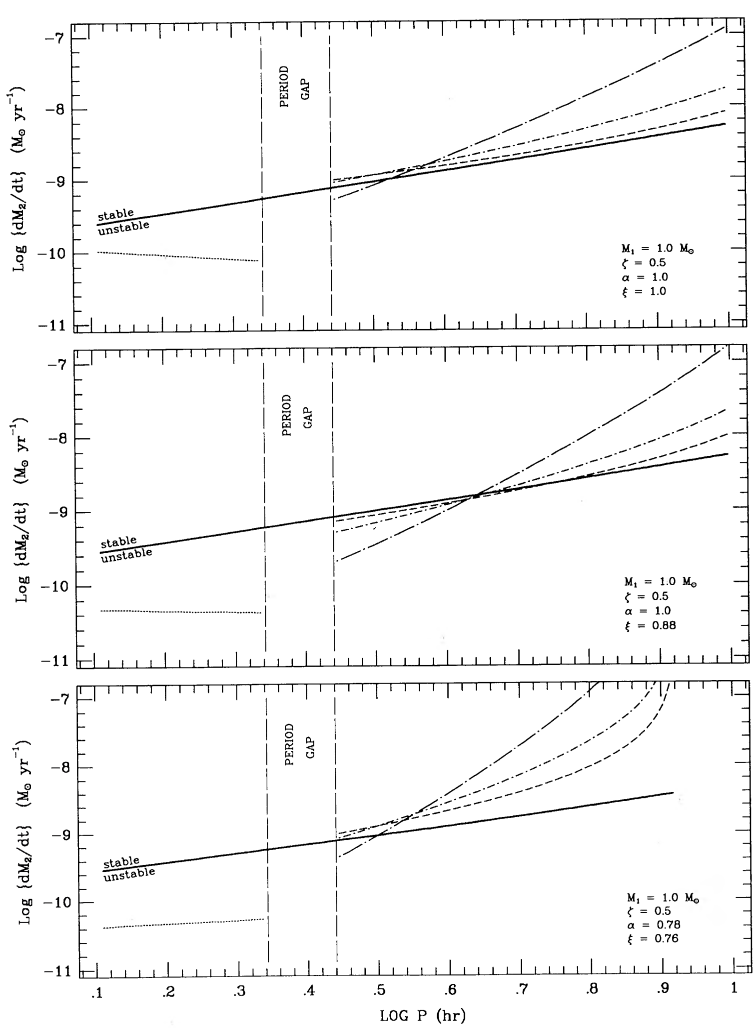

FIG. 3.—Same as Fig. 2, except for *nonconservative* mass transfer. The mass accreted by the white dwarf is assumed to be expelled from the system during a succession of nova eruptions occurring during the secular evolution of the binary. The mass of the white dwarf is assumed constant at 1 $M_{\odot}$. Note that the mass transfer rates do not increase quite as steeply with orbital period as in conservative mass exchange.

by Ritter and collaborators, in systems where the accretion disk dominates the luminosity of the system (dwarf novae in eruption and nova-like variables), the strength of the selection effect is determined primarily by the depth of the white dwarf's gravitational potential well. To first order, the strength of the selection is proportional to $(M_1/R_1)^{\nu/2}$, where M_1 and R_1 are the mass and radius of the white dwarf and ν is a parameter that depends on both the nature of interstellar absorption and the spatial distribution of the systems. The selection effect is weakest for a disklike distribution where the effects of interstellar extinction are included. In this case $\nu = 1$. The selection is strongest in the case where the systems are assumed to be spherically distributed in an absorption-free environment; here $\nu = 3$. In a magnitude-limited sample with a faint limiting magnitude (so that most systems are discovered) a value of $\nu = 1$ probably best describes the selection effect. For rather bright limiting magnitudes on the other hand, where only nearby systems are sampled, the stronger selection effect given by $\nu = 3$ may be more appropriate.

To model the intrinsic dispersion in M_1, I have adopted the theoretical mass distribution computed by Politano (1988; Politano & Webbink 1990). The mass distribution, $f(M_1)$, is reproduced in Figure 4 for three degrees of observational selection: $\nu = 0$ (volume-limited sample; in our approximation, no selection effect), $\nu = 1$, and $\nu = 3$. It is clear that selection effects can have a significant effect on the observed white dwarf mass spectrum. In particular, strong observational selection dramatically reduces the fraction of known systems containing He white dwarfs.

The full range of white dwarf masses shown in Figure 4 is not always appropriate for systems at a given orbital period. Specifically, for a given orbital period (secondary star mass), there is a *minimum* white dwarf mass below which mass transfer will be thermally and dynamically unstable. The minimum value, $M_{1,\mathrm{min}}(P)$, is indicated in Figure 4 and is given by $M_2(P)/q_{\mathrm{crit}}$, where q_{crit} depends on the structure of the secondary star. The value of q_{crit} ranges from $\sim 2/3$ for low-mass secondary stars with deep convective envelopes to $\sim 5/4$ for higher mass secondary stars. Following Politano (1991), I adopt

$$q_{\mathrm{crit}} = \begin{cases} \frac{2}{3} & (M_2/M_\odot) < 0.4\ , \\ \frac{2}{3} + 2.24(M_2/M_\odot - 0.4)^{1.36} & 0.4 \le (M_2/M_\odot) \le 0.8\ , \\ \frac{5}{4} & (M_2/M_\odot) > 0.8\ . \end{cases} \quad (19)$$

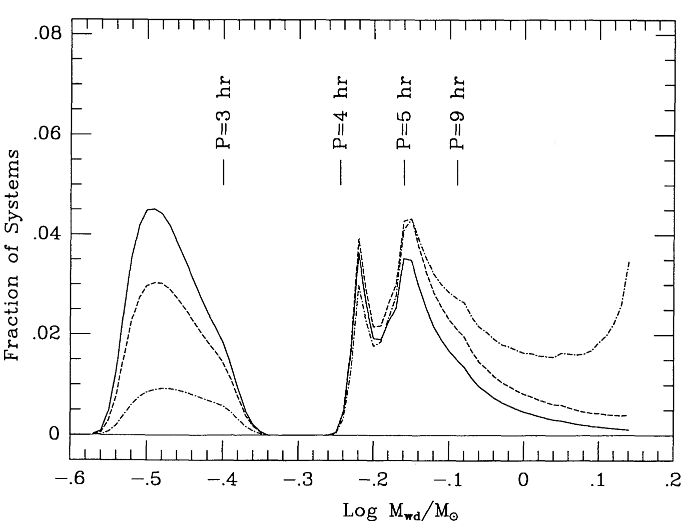

FIG. 4.—The theoretical white dwarf mass distribution taken from calculations of Politano (1988, 1991). The solid line represents the theoretical distribution, uncorrected for observational selection characterized by $\nu = 1$ and $\nu = 3$, respectively (see text for details). The minimum white dwarf mass, $M_{1,\mathrm{min}}(P)$, is shown for selected orbital periods.

For the sample of cataclysmic variables adopted in this study, it is likely that the selection effect is adequately described by a value of ν between the extremes of $\nu = 1$ and $\nu = 3$. We begin by considering the case having the weakest selection. Figure 5 shows the $\dot{M}(P)$ relations computed using the theoretical white dwarf mass distribution characterized by $\nu = 1$. The $\dot{M}(P)$ and $\dot{M}_{\mathrm{crit}}(P)$ relations have been computed using the *median* white dwarf mass appropriate for a given orbital period (the median rather than the mean was used because the truncated mass distributions can be highly skewed). In this way the approximate ratio of stable to unstable accretors at a given orbital period can be estimated by visually comparing the $\dot{M}(P)$ and $\dot{M}_{\mathrm{crit}}(P)$ relations. The requirement for thermally and dynamically stable mass transfer prohibits low-mass white dwarfs at long orbital periods and results in a median white dwarf mass that increases with orbital period. The net effect is to decrease the slope of the $\dot{M}(P)$ relation, and to increase the slope of the $\dot{M}_{\mathrm{crit}}(P)$ relation, thus bringing the two into better agreement.

To explore the effect of varying ν, Figure 6 compares $\dot{M}(P)$ relations computed using the three theoretical white dwarf mass distributions shown in Figure 4. For consistency all three cases assume a secondary star mass-radius relation characterized by $\alpha = 1.0$ and $\xi = 0.88$. It is obvious that the character of the $\dot{M}(P)$ curves is not significantly affected by the precise form of the mass distribution. Although at first this result may seem surprising, it is not unexpected; the reason is simple. The selection effect raises the average white dwarf mass for all periods roughly equally (the effect is somewhat stronger at shortest orbital periods where the largest range of masses are allowed). As the selection effect becomes increasingly stronger, thereby favoring systems with increasingly more massive white dwarfs, the results approach those where the white dwarf masses are high and independent of obital period, such as the mono-mass examples explored previously (see Fig. 3).[2]

Since the slopes of the $\dot{M}(P)$ and $\dot{M}_{\mathrm{crit}}(P)$ relations are quite insensitive to the poorly determined value of ν, I will retain the white dwarf mass distribution characterized by $\nu = 1$ in the analysis to follow. It is worth keeping in mind that since stronger observational selection can only *steepen* the $\dot{M}(P)$ relations relative to the $\dot{M}_{\mathrm{crit}}(P)$ curve and thus *increase* the fraction of dwarf novae at shorter orbital periods, a choice of $\nu = 3$ would only worsen the agreement with the observed period distribution above the period gap.

3.4. *The Theoretical Dwarf Nova Period Distribution*

In order to make a quantitative comparison of theory with observation, I have used the results of the previous section to compute model dwarf nova period distributions. Specifically, a comparison of $\dot{M}(P, M_1)$ with $\dot{M}_{\mathrm{crit}}(P, M_1)$, will establish whether a system with a given white dwarf mass and orbital period can be expected to undergo disk instabilities. By con-

[2] Classical nova systems are subject to even stronger selection for systems containing massive white dwarfs (e.g. see Ritter et al. 1991). In this case, mono-mass calculations similar to those shown in Fig. 3 may be more appropriate.

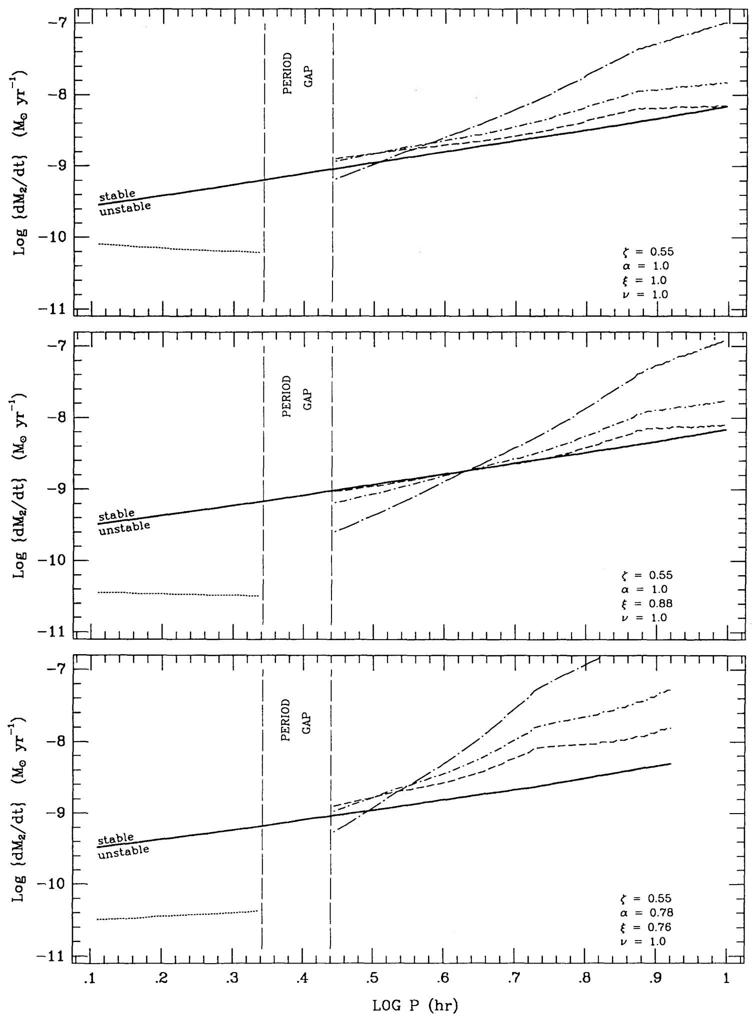

FIG. 5.—Same as Fig. 3, except that the curves are computed using the median white dwarf mass computed from the $\nu = 1$ mass distribution shown in Fig. 4. The median white dwarf mass is a function of the orbital period because the minimum white dwarf mass, $M_{1,\min}$ is constrained by the requirement of thermally and dynamically stable mass transfer. The major effect of the variation of M_1 with period is to flatten the period dependence of the mass transfer rates. The small irregularities are the result of the coarse binning of the white dwarf mass distribution.

sidering an ensemble of such systems, mass-specific dwarf nova period distributions, $N(P, M_1)$, can be constructed. An overall theoretical dwarf nova period distribution, $N(P)$, can then be obtained by integrating the mass-specific distributions over the range of white dwarf masses appropriate for each period and weighting by the white dwarf mass frequency distribution, $f(M_1)$. Specifically,

$$N(P) = \frac{\int_{M_{1,\min}(P)}^{M_{\rm ch}} f(M_1)N(P, M_1)dM_1}{\int_{M_{1,\min}(P)}^{M_{\rm ch}} f(M_1)dM_1}, \quad (20)$$

where $M_{\rm ch}$ is the Chandrasekhar mass.

The final parameters that must be specified before we can compute the period distributions are those describing the mass-radius relation for the secondary star and the size of the accretion disk relative to the mean Roche lobe radius, ζ. Although a mass-radius relation characterized by $\alpha = \xi = 1$ yields the flattest $\dot{M}(P)$ dependence, these values have been chosen historically, not because they are the most accurate, but because they provide a particularly simple mass-radius relation. Observations of low-mass main-sequence stars suggest a rather weaker dependence of the radius on mass. A good example is the relation adopted by P84, where $\alpha = 1$, and $\xi = 0.88$. The models computed by McDermott & Taam (1989) suggest an even weaker dependence. As a compromise, I have adopted the mass-radius relation advocated by P84 in the simulations.

The value of ζ affects $\dot{M}_{\rm crit}$ and hence whether a given system will undergo disk instabilities; it is therefore constrained by the fraction of all systems observed to be dwarf novae. In computing model period distributions, I have chosen the value of ζ to normalize the model distribution so that it yields the same number of dwarf novae as in the observed distribution. In all cases of interest, the required value of ζ is found to lie between 0.5 and 0.6. Although the data are limited, values in this range are consistent with many observational estimates.

It is possible that ζ may have a weak orbital period dependence. The maximum size that an accretion disk may attain relative to the primary's Roche lobe, $\zeta_{\max}$, is believed to be a very weakly varying function of the mass ratio, and hence possibly the orbital period of the system (Paczyński 1977). Specifically, the value of $\zeta_{\max}$ increases by roughly 10% from $\zeta_{\max} \simeq 0.85$ to $\zeta_{\max} \simeq 0.95$, when the mass ratio decreases from unity to $M_2/M_1 = 0.1$. For simplicity, and because observa-

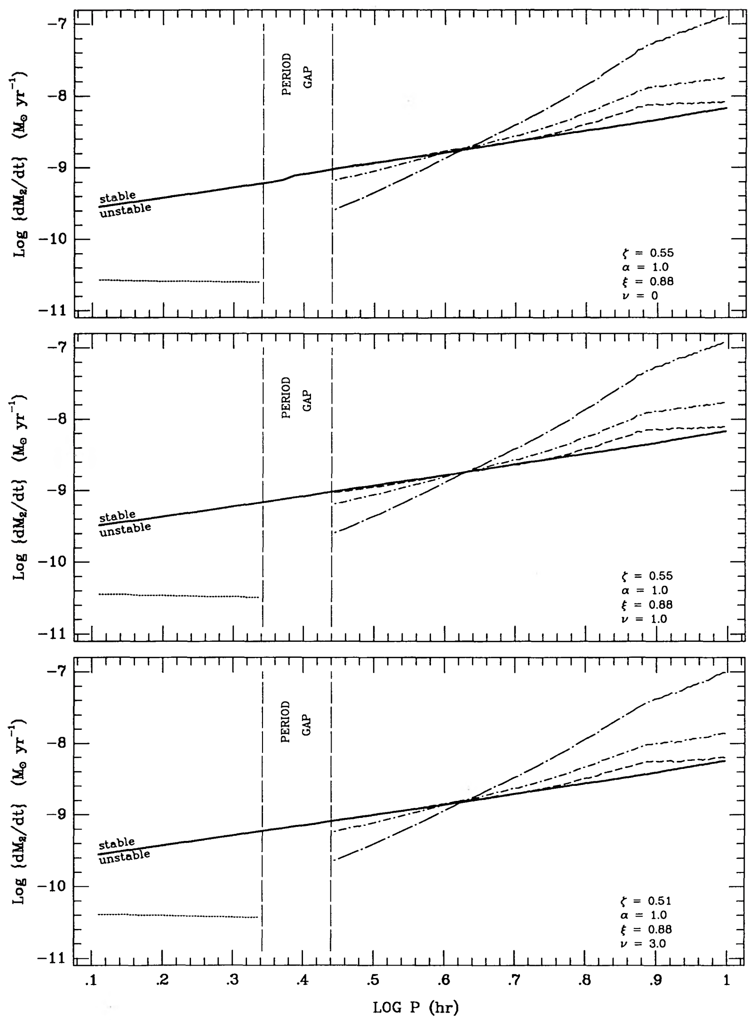

FIG. 6.—A comparison between the $\dot{M}_{\rm crit}(P)$ and $\dot{M}(P)$ curves for differing values of the parameter, ν, controlling the strength of the observational selection for systems containing massive white dwarfs. All panels represent calculations performed with the same secondary star mass-radius parameters ($\alpha = 1$; $\xi = 0.88$). The top panel represents $\nu = 0$ (no selection), the middle panel, $\nu = 1$, and the bottom panel, $\nu = 3$ (*see text for details*). It is clear that the relationship between the $\dot{M}_{\rm crit}(P)$ and $\dot{M}(P)$ curves is insensitive to the strength of observational selection.

tional data do not suggest that real accretion disks necessarily grow to the maximum allowable size, I make the assumption that ζ is independent of orbital period. If ζ does indeed have a weak orbital period dependence, the effect would be similar to that of increasing the strength of the observational selection in favor of massive white dwarfs: it would increase the discrepancy between the slopes of the $\dot{M}(P)$ and $\dot{M}_{\rm crit}(P)$ relations, resulting in an unacceptably large fraction of dwarf novae at short orbital periods.

Figure 7 shows three representative model dwarf nova period distributions that result from each of the three magnetic braking laws. Unlike the observed dwarf nova period distribution, in all cases the ratio of stable to unstable accretors increases as a function of orbital period. This is especially evident in the case of the P84 braking law, where a particularly steep $\dot{M}(P)$ relation results in an abrupt transition from dwarf novae to nova-like systems. A χ^2 testing procedure confirms that none of the model distributions satisfactorily reproduces the observed dwarf nova period distribution. The χ^2 probability functions are 1×10^{-4}, 6×10^{-3}, and 2×10^{-1}, for the P84, VZ81, and MS87 braking laws, respectively. It is worth noting that since an increase in the strength of the observational selection for systems containing massive white dwarfs has the effect of reducing the dispersion in the white dwarf masses for a given orbital period, a more abrupt transition from unstable to stable accretors in the theoretical period distributions would be expected. Thus, the inclusion of stronger selection in the model calculations would result in even poorer agreement between the observed and theoretical period distributions.

The relatively flat braking law of MS87 comes the closest to reproducing the observed dwarf nova period distribution. The MS87 law with $n = 1$ and mass-radius parameters $\alpha = \xi = 0.88$ gives an orbital period dependence that has essentially the same slope as does the critical mass transfer rate. Unlike the braking laws of VZ81 and P84, the parameter n in the MS87 law can affect not only the absolute level of mass transfer at a given period, but also the slope of the $\dot{M}(P)$ relation. Following MS87 and McDermott & Taam (1989), I adopted $n = 1$ for the initial analysis. However, this parameter is poorly constrained, and n may plausibly exceed unity. As pointed out by Hameury et al. (1988), the slope of the $\dot{M}(P)$ relation is inversely proportional to n. For example, if we assume $\xi = 0.88$ and take $n = 1.45$ as did Hameury (1991), we

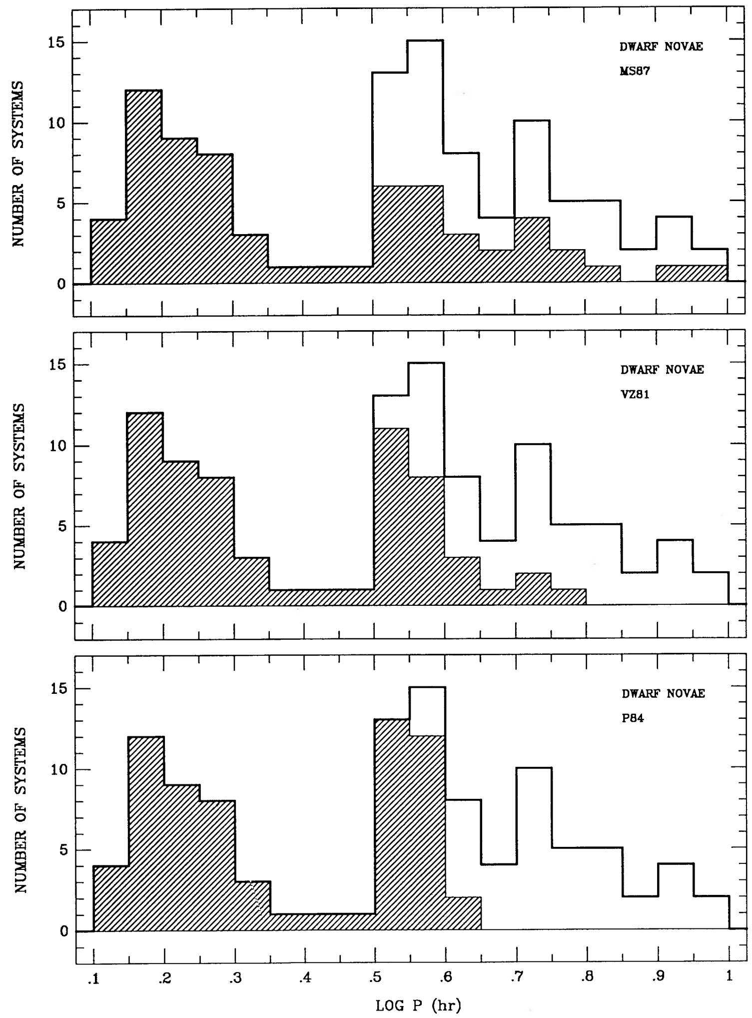

FIG. 7.—Theoretical orbital period distributions based on a comparison of the mass transfer rates with $\dot{M}_{crit}$. The number of dwarf novae (unstable accretors) in each period bin has been computed using the $\nu = 1$ white dwarf mass distribution from Fig. 4. Note the tendency for dwarf novae to dominate the distribution at short orbital period, particularly in the case of the P84 braking law.

find that $\gamma = 1.12$, which is significantly smaller than the value, $\gamma = 1.27$, obtained assuming $n = 1$ (see Table 2).

Panel one of Figure 8 shows $\dot{M}(P)$ for the extreme, but plausible values: $n = 0.5$ and $n = 2.0$. Panels two and three show theoretical period distributions based on these braking laws. It is clear that higher values of the index n reduce the slope of the $\dot{M}(P)$ relation, and increase the ratio of stable to unstable accretors at orbital periods just above the gap. A χ^2 test shows that the MS87 braking law with $n = 2$ produces a model dwarf nova period distribution that is not inconsistent with the observed distribution.

Although it is difficult to draw firm conclusions from the simulations performed here, one general point seems clear: It is unlikely that the mass transfer rate can increase as steeply with orbital period as is suggested by most currently popular braking laws. Including a weak period dependence for the fractional disk radius ζ, or increasing the strength of the observational selection, will only exacerbate the problem by increasing the discrepancy between the slopes of the $\dot{M}(P)$ and $\dot{M}_{crit}(P)$ relations. Steep braking laws such as the one advocated by P84 are particularly difficult to reconcile with the observed dwarf nova period distribution. The relatively flat braking law of MS87, on the other hand, offers a significant quantitative improvement when the model period distribution is compared with observations.

4. DISCUSSION

Despite the fact that the MS87 braking law comes close to reproducing the observed dwarf nova period distribution, it still cannot be considered to be entirely satisfactory. In particular, it is difficult to imagine how any simple magnetic braking prescription can account for the complete dominance of stable over unstable accretors just above the period gap. It seems likely that another explanation must be found to account for this aspect of the dwarf nova period distribution. Below I describe a few speculative ideas that may provide better agreement with the observed period distribution.

4.1. *Additional Angular Momentum Loss*

One obvious way to account for the abundance of stable accretors with periods between 3 and ~4 hr is to suppose that there is an additional angular momentum loss mechanism acting at periods just above the gap, resulting in mass transfer rates that are significantly above $\dot{M}_{crit}$. The major objection to this idea is that the disrupted magnetic braking model for the period gap places constraints on the level of mass transfer at

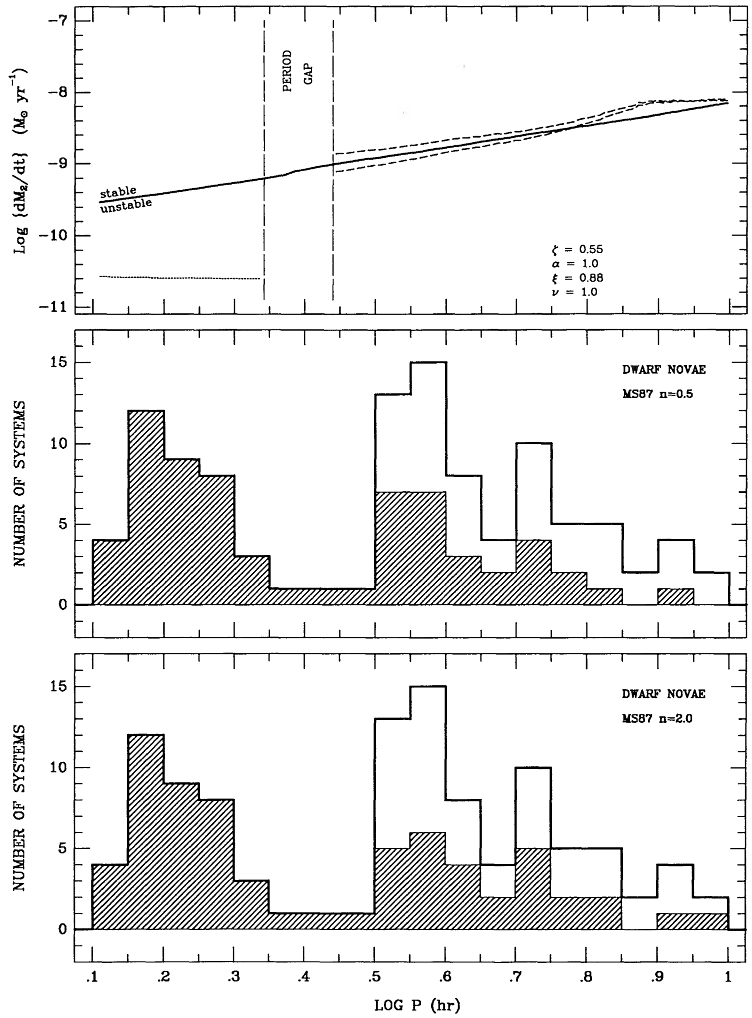

FIG. 8.—Variations of the MS87 braking law. The top panel shows the effect of varying the parameter n on the expected mass transfer rates. The lower dashed curve represents $n = 0.5$, and the upper curve $n = 2.0$. The lower two panels show the theoretical period distributions that result from the two cases. The rather flat orbital period dependence given by the MS87 law with $n = 2.0$ comes the closest to reproducing the observed dwarf nova period distribution.

the upper edge of the gap. Specifically, the width of the gap depends on how far out of thermal equilibrium the secondary star is when magnetic braking is diminished, which in turn depends on the secondary star's mass-loss rate. The lower the mass transfer rate, the narrower the resulting period gap. The magnetic braking prescriptions considered here can all be adjusted to give $\dot{M}(P = 3 \text{ hr}) \sim 10^{-9}\ M_\odot \text{ yr}^{-1}$, a value that results in the observed width of the gap. If an additional angular momentum loss mechanism is operating that significantly raises the mass transfer rate for a period of ~ 3 hr, then the ability of the models to reproduce the observed width of the gap will be destroyed. It is worth emphasizing here that the period gap is actually *wider* for dwarf novae (presumably the low-$\dot{M}$ systems) than it is for cataclysmic variables in general. If dwarf novae and stable accreting systems retain their distinction during the secular evolution of the system, then this behavior is exactly the opposite of what one would expect from the disrupted magnetic braking model: the low $\dot{M}$ systems should have the narrowest gap. It therefore appears that the dearth of dwarf novae with periods between 3 and ~ 4 hr is unlikely to be the result of an unusually high mass transfer rate in this period range.

4.2. *Weakly Magnetic Systems*

Besides having a high mass accretion rate, another characteristic that would suppress dwarf nova eruptions would be for the system to have its inner disk disrupted by a magnetic white dwarf. It may be possible for the field to be strong enough to disrupt the accretion flow in the inner disk, but yet sufficiently weak to avoid transforming the system into a conspicuous magnetic accretor (displaying strong X-ray emission and polarization). It is not obvious why such systems would be particularly common between 3 and 4 hr. However, the relatively small orbital dimensions in these short period systems would result in a greater fraction of the accretion disk being disrupted by the white dwarf's weak magnetic field. Thus weak field primaries in long orbital period systems may not suppress dwarf nova eruptions.

The possibility that a class of weakly magnetic systems may exist has gained some support by the eclipse analyses of Williams (1989), who pointed out that the emission-line profiles in many eclipsing nova-like systems do not display the classic rotational disturbance during eclipse ingress and egress that one expects for systems with accretion disks. The spectra of many of these systems display prominent He II λ4686 emission, characteristic of emission from a high-temperature region in the system. The strength of He II emission is often used as an indicator of either magnetic accretion or of a high mass transfer rate. Analysis of eclipsing systems suggest that the He II emission is produced near the white dwarf. If the mass accretion rate is high, the temperature of the inner disk may be sufficiently high to produce significant He II emission. Alternatively, if the accretion is channeled onto a restricted area of the white dwarf's surface, as expected for a magnetic white dwarf, a high-temperature region can also be created. As we have just seen, it appears unlikely that the mass transfer rates are unusually high for systems just above the gap; therefore the possibility that magnetic accretion is playing a role in these systems cannot be easily dismissed. It is perhaps not surprising that many of the stable accretors displaying strong He II emission and ill-defined rotational disturbances have periods between 3 and 4 hr. The newly defined class of novalike variables displaying strong He II emission and large emission-line phase offsets, the SW Sex stars (see Thorstensen et al. 1991), are good examples.

4.3. *Correlation of White Dwarf Mass With Orbital Period*

If the white dwarf masses are positively correlated with orbital period, the systematic increase in accretion disk size will modify the ratio of stable to unstable accretors as a function of orbital period. Unfortunately the masses of white dwarfs in cataclysmic variables are poorly known, so it is not possible to test the above idea observationally. Nevertheless, the scanty data that are available do suggest that the mean white dwarf mass may be weakly correlated with orbital period (Shafter 1983). As we have seen, from a theoretical perspective this is not surprising since long-period systems containing low-mass white dwarfs are unstable with respect to mass transfer on a dynamical time scale. Such systems, once formed, would probably experience a common envelope stage of evolution where they would quickly evolve to shorter orbital period. A further possibility that massive white dwarfs may be rare at short orbital periods is raised by observations of nova shells.

Spectroscopy of nova shells shows an overabundance (relative to solar) of CNO elements, and in some cases (the "Neon" novae) an overabundance of neon. As it is difficult to produce these elements in the required abundance during the nova eruptions, it is thought that some fraction of the ejecta comes directly from the underlying white dwarf (e.g., Truran & Livio 1986). For these systems, not only will the mass of the white dwarf fail to increase during the secular evolution of the binary, as assumed in the case of conservative mass transfer, it may actually decrease as successive nova eruptions gradually whittle away the white dwarf. Thus as systems evolve toward shorter orbital period, the mean white dwarf mass should decrease. The smaller the mass of the white dwarf for a given orbital period, the smaller will be the white dwarf's Roche lobe, and hence the smaller will be the accretion disk. Since the value of $\dot{M}_{\rm crit}$ is strongly dependent on the size of the disk (see eq. [1]), the frequency of systems displaying dwarf nova eruptions will be lower in short-period systems containing low-mass white dwarfs than it would be if the masses were higher.

Finally it may be possible that some of the systems with periods just above the period gap may contain low-mass helium white dwarfs. In the model simulations considered in the previous section, the P84 mass-radius relation gave $M_2 \simeq 0.27\ M_\odot$ for a period of 3 hr. Thus, the minimum allowed white dwarf mass, $M_{1,\rm min} \simeq 0.4\ M_\odot$, and helium white dwarfs were excluded. However, if the secondary stars in these systems are severely out of thermal equilibrium, the mass of the secondary stars may be significantly lower than we have assumed. In this case, $M_{1,\rm min}$ may become sufficiently low to allow otherwise unstable systems to exist.

4.4. *The Role of Hibernation*

The idea that dwarf novae, novae, and nova-like behavior represent different stages in the evolution of the same system has been used to explain various problems in understanding the physics of nova eruptions (Shara et al. 1986). The outbursts of classical (and recurrent) novae are caused by a thermonuclear runaway in the material accreted onto the surface of the white dwarf (Starrfield et al. 1972; Starrfield, Sparks, & Truran 1974a, b). Models show that the nature of the eruption is critically dependent on parameters such as the mass and luminosity of the white dwarf, the accretion rate, and the chemical composition of the accreted material, specifically the CNO abundance (Shara, Prialnik, & Shaviv 1980; MacDonald 1983). Of particular importance is the mass-transfer rate. If the rate is too low, it will take a prohibitively long time to build up a critical envelope mass, and nova eruptions will be rare. On the other hand, if the mass-transfer rate is too high, the accreted material will not become sufficiently degenerate prior to ignition, the envelope will burn nonexplosively, and a nova eruption is unlikely to occur. The critical mass-accretion rate depends on the properties of the white dwarf, but in general accretion rates above $10^{-9}\ M_\odot\ {\rm yr}^{-1}$ are not expected to produce strong thermonuclear runaways (Kutter & Sparks 1980; Prialnik et al. 1982). If boundary layer heating is taken into account, the mass-transfer rates must be even lower (Shaviv & Starrfield 1987).

Although, at present, relatively few orbital periods are known for novae, the available data seem to suggest that novae occur primarily in systems with periods above the period gap. As pointed out by Ritter (1990b), the requirement that novae have mean mass-transfer rates below $\sim 10^{-9}\ M_\odot\ {\rm yr}^{-1}$ is not easy to reconcile with the disrupted magnetic braking period gap models, which require that $\dot{M}(P = 3\ {\rm hr}) \sim 2 \times 10^{-9}\ M_\odot\ {\rm yr}^{-1}$. Thus, on immediate inspection, it would appear that models of thermonuclear runaways on white dwarfs and the magnetic breaking models are incompatible.

Ritter (1990b) has reviewed possible resolutions of the $\dot{M}$ conflict of novae. Perhaps the most promising resolution lies in the "hibernation" scenario of Shara and collaborators (Shara et al. 1986). In this picture, the mass that was transferred with a high accretion rate to the white dwarf immediately following a nova eruption has time to cool and become degenerate during the subsequent period of hibernation (Prialnik & Shara 1986). After the system emerges from hibernation, mass transfer resumes and a nova eruption soon follows. As long as the *mean* mass-transfer rate throughout the inter-eruption cycle is greater than $\sim 2 \times 10^{-9}\ M_\odot\ {\rm yr}^{-1}$, the period gap model is not affected. Thus the conflict between models for the period gap and those for thermonuclear runaways on white dwarfs disappears.

An implication of the hibernation scenario is that novae, dwarf novae, and nova-like variables represent cyclical stages in the evolution of the same systems. In particular, a nova-like stage is expected to occur immediately prior to and following a nova eruption if the mass-transfer rate becomes high enough to suppress disk instabilities. On the other hand, during the transitions into and out of hibernation, the mass-transfer rates should be such as to permit dwarf nova eruptions. The observed frequency of each type of system within a given period range will depend on the fraction of time that the system spends in each cyclical stage. If the hibernation is sufficiently deep (i.e., if the mass transfer is greatly diminished or if it ceases altogether) and the transition into and out of hibernation is sufficiently rapid, the observed frequency of dwarf novae may be quite small. Calculations by Livio & Shara (1987) suggest that hibernation will be deepest, at a given period, for systems with mass ratios near unity, or, in the case of a given mass ratio, for systems with the shortest orbital period. Note that these are the conditions that are expected to prevail at periods between 3 and 4 hr if the white dwarf mass is correlated with the orbital period as discussed in the previous section. If systems with orbital periods between 3 and 4 hr spend a large fraction of their time in deep hibernation, the observed ratio of stable to unstable accretors may therefore not reflect the ratio expected based on a simple comparison of $\dot{M}_{\rm mb}$ with $\dot{M}_{\rm crit}$.

5. CONCLUSIONS

I have argued that the eruptive characteristics of cataclysmic variables can be used to infer mass-transfer rates for nonmagnetic cataclysmic variables. Theoretical $\dot{M}(P)$ relations that result from models of angular momentum loss due to magnetic braking were used to construct theoretical dwarf nova period distributions. Agreement between the observed dwarf nova period distribution with those constructed from various magnetic braking models was generally disappointing. The differences arise both because the braking laws often resulted in $\dot{M}(P)$ relations that were unacceptably steep, and, more specifically, because the braking laws offer no explanation for the observed dearth of dwarf novae with orbital periods between 3 and 3.8 hr. These results are robust in that increasing the strength of observational selection for systems containing massive white dwarfs, or allowing for a possible orbital period dependence in the fractional size of the accretion disk relative to the primary's Roche lobe, ζ, would only exacerbate the discrepancy.

The most promising braking law was that of Mestel & Spruit (1987). Under certain conditions this braking law was able to produce a relatively flat $\dot{M}(P)$ relation that resulted in a model dwarf nova period distribution formally consistent with the observed distribution. Despite this result, the MS87 braking law cannot be considered entirely satisfactory because it was unable to account for the dominance of stable over unstable accretors immediately above the period gap. Magnetic braking models, even the MS 87 models, generally predict that dwarf novae should be relatively common in this period regime. It is unlikely that this discrepancy can be removed by hypothesizing that the mass-transfer rates are anomalously high in the 3–3.8 hr regime because the observed width of the period gap places an upper limit to $\dot{M}(P = 3 \text{ hr})$ of $\sim 2 \times 10^{-9}$ $M_\odot$ yr^{-1}, even if there were a known physical process available to drive the additional mass transfer. Alternative explanations include suppression of disk instabilities in these short-period systems by weakly magnetic white dwarf primaries, deep hibernation for short period, near-unity mass ratio systems, or suppression of disk instabilities in systems having low-mass white dwarfs as the disks in such systems would be relatively small. Despite these rather speculative ideas, it is fair to say that no compelling explanation for the dominance of stable accretors between periods of 3 and 3.8 hr has yet been identified.

As a general point, I have argued that the dwarf nova period distribution, and not only the overall period distribution, should be considered when applying observational constraints to theories of mass transfer in cataclysmic variables. The lack of dwarf nova systems in the 3–3.8 hr period range causes the width of the period gap for dwarf novae to be essentially *twice* as wide as the traditional 2–3 hr period gap for cataclysmic variables in general. Furthermore, since essentially all nonmagnetic novalike variables have periods "longward" of the period gap and the vast majority of AM Her systems have periods below the gap, the traditional period *gap* is only clearly defined for dwarf novae in any case! Finally, the fact that the disrupted magnetic braking model for the period gap predicts a *narrower* gap for low-$\dot{M}$ systems than for high-$\dot{M}$ systems again seems to be in serious conflict with the observations.

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