The Astrophysical Journal, 393:3–21, 1992 July 1

STATISTICAL PROPERTIES OF GRAVITATIONAL LENSES WITH A NONZERO COSMOLOGICAL CONSTANT

M. Fukugita,[1,2] T. Futamase,[3] M. Kasai,[3] and E. L. Turner[4,5]

Received 1991 July 29; accepted 1992 January 6

ABSTRACT

We investigate statistical properties of gravitational lensing in the presence of a cosmological constant, with emphasis given to the study of uncertainties in cosmological factors arising from different possible physical and statistical formulations of the problem. We find that a substantial uncertainty associated with the distance formulation for the prediction of the lensing optical-depth makes the discrimination between low- and high-density universe models difficult for high-redshift lens-quasar systems. We find, however, that the cosmological constant, if it dominates over the mass density, increases the optical depth greatly, and its effect is much larger than the uncertainty arising from details of the problem's formulation. Therefore, the lensing frequency can provide a simple and very useful test for the cosmological constant. For a low-redshift system ($z_S \lesssim 2$) the formulation uncertainties are rather moderate, though the difference among different cosmology models is not large. The optical-depth redshift distribution is also very sensitive to cosmological models, but it is less sensitive to statistical formulations. A realistic prediction is also made for the lensing frequency taking account of various selection effects for some particular samples. We also study the gravitational lens effect with a cosmological constant on the quasar-galaxy correlation and on fluctuations in the cosmic microwave background radiation. This paper thus attempts to present a systematic and reasonably complete discussion of statistical problems in gravitational lensing for $\Lambda \neq 0$ cosmological models.

Subject headings: cosmology: theory — gravitational lensing

1. INTRODUCTION

The determination of the world model is one of the main goals of cosmology. If we accept the view that the universe is homogeneous and isotropic on large scales, the model is described by Friedmann-Lemaitre-Robertson-Walker (FLRW) geometry which is characterized by a few parameters. One of them is the cosmological constant which has been regarded in general as anathema and usually set equal to zero without any compelling reasons. Various aspects of recent observations, however, suggest reconsideration of a nonvanishing cosmological constant. One example is the problem of the age of the universe. While the distance to the Virgo cluster center has been regarded as a matter of debate, recent work with the new technique of the planetary nebula luminosity function and surface brightness fluctuations applied to the Virgo cluster indicated that the distance to the center of the cluster is $\sim$15 Mpc rather than 20 Mpc (Jacoby, Ciardullo, & Ford 1990; Tonry, Ajhar, & Luppino 1990; Tonry 1991), in agreement with the earlier results of Aaronson et al. (1986) and Pierce & Tully (1988). This distance is translated into a Hubble constant of 75–100 km s^{-1} Mpc^{-1}. This value also receives support from the study of galaxies and clusters beyond the Virgo cluster (Aaronson et al. 1986; Fukugita et al. 1991; Fukugita & Hogan 1991). This larger value of H_0 can be reconciled with the cosmic age indicated by globular cluster evolution ($t_0 > 14$ Gyr) (e.g., VandenBerg 1983; Iben & Renzini 1984; Alcaino, Liller, & Alvarado 1988) only with a positive finite cosmological constant. Another example is given by the recent number counts of faint galaxies (Tyson 1988; Lilly, Cowie, & Gardner 1991). The number of galaxies at faint magnitudes is even more than expected in a open (low-density) universe, and the observation is reproduced best with a sizable cosmological constant in a low-density universe (Fukugita et al. 1990b). Typical cosmological parameters favored by this analysis are $\Omega_0 \sim 0.1$ for the mean mass density and $\lambda_0 \sim 0.5$–1.2, where $\lambda_0 \equiv \Lambda/(3H_0^2)$ is the normalized cosmological constant. While a nonvanishing cosmological constant, if confirmed, would carry a significant implication for our understanding of cosmology, the ways in which one can clearly test for its existence are limited: The cosmological constant has little effect on the local dynamics (Peebles 1984). The effect on dynamics can hardly be observed also with distant clusters because it appears always to be canceled in observable quantities (Lahav et al. 1991). The effect should not be important in the far past since its strength diminishes as $(1 + z)^{-3}$. Most promising are tests of geometry for an intermediate redshift $z_S \simeq 1$–3.

Bearing this in mind, we extensively examine the effect which the cosmological constant causes on gravitational lensing. (For general reviews on lensing, we refer to Canizares 1987; Blandford & Kochanek 1987; Turner 1989.) It is not a new idea that gravitational lenses be used as a tool to explore the cosmological parameters (e.g., Refsdal 1964; Press & Gunn 1973). Earlier studies, however, have shown that the lensing properties are rather insensitive to the mean mass density of the Universe (e.g., Blandford & Kochanek 1987), although the possibility of distinguishing the case of $\Omega_0 = 0$ from the flat ($\Omega_0 = 1$) universe using the

[1] Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606, Japan.
[2] Institute of Astronomy, Madingley Road, Cambridge CB3 0HA, England, UK.
[3] Department of Physics, Hirosaki University, Hirosaki 036, Japan.
[4] Princeton University Observatory, Princeton, NJ 08544.
[5] National Astronomical Observatory, Mitaka, Tokyo 181, Japan.

statistical property of lenses has been suggested by Turner, Ostriker, & Gott (1984; hereafter TOG). Fukugita, Futamase, & Kasai (1990a) and Turner (1990) have independently pointed out that the cosmological constant affects significantly the lensing probability, and that statistical properties, which are drastically affected by its presence, can be used to test for its existence.

There also already have been a few sporadic studies which investigated the effect of the cosmological constant on some specific aspects of lensing, especially in high-density universes (Paczyński & Gorski 1981; Alcock & Anderson 1986; Gott 1987; Gott, Park & Lee 1989). In this paper we focus upon the low-density universe that is favored by many present observations. For convenience of comparison we consider, for most cases, four typical choices of the cosmological parameters:

$$\text{case A: } \Omega_0 = 1 \text{ and } \lambda_0 = 0\,; \qquad \text{case B: } \Omega_0 = 0.1 \text{ and } \lambda_0 = 0\,;$$
$$\text{case C: } \Omega_0 = 0.1 \text{ and } \lambda_0 = 0.9\,; \qquad \text{case D: } \Omega_0 = 0 \text{ and } \lambda_0 = 1\,. \tag{1}$$

Case A is included as a reference, since most of the studies for gravitational lensing in the past have assumed this case. Case D is the opposite extreme $k = 0$ limiting case.

The general problem in using gravitational lensing to study the world model is that there are a number of uncertainties in the lens model. It is a rare case that we know the property of the lensing object in detail, and many observational quantities may depend rather strongly on the details of the lensing object's mass distribution. Most of the known lenses require a study case by case. More important for cosmology studies is that there is an ambiguity as to the choice of the redshift-distance formula. This is associated with the fact that the light propagates through the inhomogeneous spacetime rather than the averaged smooth spacetime; the light ray feels the local metric which deviates from the smoothed Robertson-Walker metric. Therefore, even if the global parameters such as the density parameter and the cosmological constant are fixed, the propagation of light rays, and hence the distance formula, is not uniquely determined (Zeldovich 1964; Dashevskii & Slysh 1966; Dyer & Roeder 1972; 1973 hereafter DR; Futamase & Sasaki 1989). This causes a significant uncertainty, especially for the case of $\Omega_0 + \lambda_0 = 1 (\Omega_0 \neq 0)$, as large as a factor 1.5–2 in the distance. The lens equation is formulated in terms of the distance formula and thus the observables for lenses necessarily depend on its choice. At present there is no general agreement as to which distance formula represents the most realistic case. In order to circumvent this problem, therefore, it is crucial to know that these uncertainties in the cosmological formalism do not preclude discriminating among cosmological models. To examine this point, we have adopted two different distance formulae and two different formulations of the lens statistics and then look for quantities that do not depend much on their choice. We calculate various quantities with two extreme lens models, the point particle and the singular isothermal sphere, the latter of which is more suitable to describe a galaxy lens. We find that some of the effects of the cosmological constant on the statistical properties of lenses do not depend much on the choice of the model or on the statistical formalism. The most important is the optical depth of the lens distribution. Furthermore, the effect of the cosmological constant on it is very large, especially in the low-density universe, and gives a promising possibility to test for its value. In order to make our calculation useful for a realistic comparison with the observations, we include the various selection effects according to Fukugita & Turner (1991, hereafter FT; see also Kochanek 1991a) to predict lensing frequencies.

We also discuss the effect of the cosmological constant on some other aspects of statistical lensing, (1) the quasar-galaxy association and (2) anisotropy enhancement of the cosmic background radiation (CBR). Webster et al. (1988; hereafter WHHW; see also Fugmann 1988) showed that the apparent number density of galaxies is enhanced in the vicinity of a quasar, and interpreted this as an effect of gravitational lensing. It has been claimed, however, that the enhancement factor that Webster et al. found was too large to be accounted for (Narayan 1989; Kovner 1989) by lensing. We consider the effect of the cosmological constant with the hope that it might boost this effect. Another aspect concerns the lensing effect in the CBR anisotropy. Kashlinsky (1988) and Tomita (1988) suggested that the small-scale anisotropy might be decreased by randomly distibuted lenses. It was recognized soon, however, that lensing actually increases the rms anisotropy, but the effect is too small to be detected observationally (Cole & Efstathiou 1989). The presence of a large cosmological constant might naively be expected to enhance the effect substantially. We also include a brief discussion, although it is not on a statistical property, on the effect of cosmological constant on the differential time delay, which may be used to derive the Hubble constant.

In summary, this paper has two major goals. First, it is intended to catalog a more comprehensive and systematic set of gravitational lens calculations in nonzero Λ cosmologies than has previously been presented in various papers (FFK; Turner 1990; FT; Gott et al. 1989; Alcock & Anderson 1986) which have been devoted to particularly chosen aspects of the problem. Thus, we attempt to cover a wide variety of topics in lensing theory, not all closely related, which have or might be thought to have an interesting dependence on the cosmological constant. Second, it is to compare the uncertainties resulting from current ambiguities in the formulation of problems in lens statistics with the size of the various cosmological effects, particularly the Λ dependent effects. Barring qualitative improvements in our understanding of these fundamental theoretical problems, these uncertainties limit what can be learned from lens statistics, however, accurate the input data and no matter how numerous and reliable the observations become.

The plan of this paper is as follows. In § 2, we write down the basic equations for describing statistical properties of lenses for each of the lens models. We discuss the choice of the distance formula in § 3. In § 4 the results of calculations are summarized. In § 5, the quasar-galaxy association and the effect on CBR anisotropies are discussed. Our conclusions are summarized in § 6. In an Appendix useful cosmological factors are tabulated for known candidate lens systems and a brief discussion of the differential time delay is given.

2. BASIC EQUATIONS FOR STATISTICAL LENSING

To discuss statistical properties of gravitational lenses we assume that the universe is well approximated by the FLRW geometry on large scales. As mentioned in § 1, this does not determine uniquely the distance formula. We may assume, however, that the relation between the affine distance of the null geodesic and the redshift of the source object in a clumpy universe is the same as that in the FLRW geometry. This may be regarded as a mathematical expression of the assumption that the universe is described by FLRW geometry on large scales. This is supported by theoretical as well as numerical arguments (Futamase & Sasaki 1989; Watanabe & Tomita 1990). We write the basic equations for the statistical lensing for two models of the lensing object; i.e., (1) point masses, which are an appropriate model for stellar mini-lensing or concentrated sources such as black holes, and (2) singular isothermal spheres ($\rho \propto r^{-2}$), which would model the matter distribution of an isolated galaxy. We assume that the comoving number density of the lensing object is conserved in cosmic time. In the following we use the notation

$$D_{OL} = d(0, z_L)\,, \quad D_{LS} = d(z_L, z_S)\,, \quad D_{OS} = d(0, z_S)\,, \tag{2}$$

where $d(z_1, z_2)$ is the angular diameter distance between the redshift z_1 and z_2, and the arguments z_L and z_S are the redshift of the lens and the source, respectively. The formulation and notation of TOG is basically followed in this paper.

2.1. *Point Masses*

We first define the length $a_{\rm cr}$ which characterizes the effective radius of the lens,

$$a_{\rm cr}^2 = \frac{4GM}{c^2}\frac{D_{OL}D_{LS}}{D_{OS}}\,, \tag{3}$$

where M is the mass of the lensing object (Press & Gunn 1973; TOG). Then the cross section σ for "strong" lensing events as defined by TOG is given by

$$\sigma = \pi a_{\rm cr}^2\,. \tag{4}$$

The differential probability $d\tau$ of a beam encountering a lens in traversing the path of dz_L is given by

$$\begin{aligned} d\tau &= n_L(0)(1+z_L)^3\sigma\frac{c\,dt}{dz_L}\,dz_L \\ &= \frac{3}{2}\,\Omega_L(0)(1+z_L)^3\frac{D_{OL}D_{LS}}{R_0 D_{OS}}\frac{1}{R_0}\frac{c\,dt}{dz_L}\,dz_L\,, \end{aligned} \tag{5}$$

where $\Omega_L(0) = 8\pi G M n_L(0)/3H_0^2$ is the lens density parameter which is the ratio of the local lens density to the critical density, R_0 is the Hubble distance ($R_0 = c/H_0$ with H_0 the Hubble constant), and t stands for the lookback time. This yields the correct probability for the standard FLRW distance. The expression, however, might have to be modified when the Dyer-Roeder distance is employed (Ehlers & Schneider 1986) as will be discussed in § 3 below. The quantity $c\,dt/dz_L$ is calculated in the FLRW geometry to be

$$\frac{c\,dt}{dz_L} = \frac{R_0}{1+z_L}\frac{1}{\sqrt{\Omega_0(1+z_L)^3 + (1-\Omega_0-\lambda_0)(1+z_L)^2 + \lambda_0}}\,, \tag{6}$$

where Ω_0 is the total mass density of the universe. By integrating the differential probability along the line of sight to the source, we obtain the total probability

$$\tau(z_S) = \int_0^{z_S}\frac{d\tau}{dz_L}\,dz_L\,. \tag{7}$$

Another interesting quantity is the mean image separation at a given z_L, and its average over the lens redshift distribution. The former is obtained by averaging the image separation with respect to the impact parameter at z_L.

$$\overline{\Delta\theta}(z_L) = \frac{10\sqrt{5}-16}{3}\frac{a_{\rm cr}}{D_{OL}} = \frac{10\sqrt{10}-16\sqrt{2}}{3}\left(\frac{R_L}{R_0}\right)^{1/2}\frac{R_0}{D_{OL}}\left(\frac{D_{OL}D_{LS}}{R_0 D_{OS}}\right)^{1/2}\,, \tag{8}$$

where $R_L = 2GM/c^2$ is the Schwarzschild radius of the lensing object. If we normalize the angle in terms of $\Delta\theta_0 \equiv (R_L/R_0)^{1/2}$, we have

$$\frac{\overline{\Delta\theta}}{\Delta\theta_0} = 2.998\,\frac{R_0}{D_{OL}}\left(\frac{D_{OL}D_{LS}}{R_0 D_{OS}}\right)^{1/2}\,. \tag{9}$$

The average image separation is obtained by

$$\langle\overline{\Delta\theta}\rangle = \frac{1}{\tau}\int_0^{z_S}\overline{\Delta\theta}\,\frac{d\tau}{dz_L}\,dz_L\,. \tag{10}$$

2.2. *Singular Isothermal Spheres*

Let us give the corresponding quantities in the case of the singular isothermal sphere. The lens model is characterized by the one-dimensional velocity dispersion v. The deflection angle is given for all impact parameters to be $\alpha = 4\pi v^2/c^2$. The lens produces two images if the angular position of the source is less than the critical angle $\beta_{\rm cr} \equiv \alpha D_{LS}/D_{OS}$. Then the critical impact parameter is defined by $a_{\rm cr} \equiv D_{OL}\beta_{\rm cr}$ and the cross section is given by

$$\sigma = \pi a_{\rm cr}^2 = 16\pi^3\left(\frac{v}{c}\right)^4\left(\frac{D_{OL}D_{LS}}{D_{OS}}\right)^2 . \tag{11}$$

The differential probability of a lensing event is

$$\begin{aligned} d\tau &= n_0(1+z_L)^3\sigma\frac{c\,dt}{dz_L}\,dz_L \\ &= F(1+z_L)^3\left(\frac{D_{OL}D_{LS}}{R_0 D_{OS}}\right)^2\frac{1}{R_0}\frac{c\,dt}{dz_L}\,dz_L , \end{aligned} \tag{12}$$

where F measures the effectiveness of matter in producing double images (TOG)

$$F = 16\pi^3 n_0\left(\frac{v}{c}\right)^4 R_0^3 . \tag{13}$$

The total probability is obtained by integrating the differential probability along the line of sight to the source as in equation (7).

The mean image separation for the lens at z_L takes a simple form

$$\overline{\Delta\theta} = 2\alpha\frac{D_{LS}}{D_{OS}} , \tag{14}$$

and the average over the lens redshift is obtained by an integration similar to equation (10).

3. AMBIGUITIES IN THE DISTANCE FORMULA

There are ambiguities in the distance formula and hence in the predicted lensing properties. The earliest studies of gravitational lenses have paid little attention to this problem and used the standard Friedmann-Lemaitre distance formula which we call the standard distance (e.g., Refsdal 1964). It was soon realized, however, that gravitational lensing takes place only in a clumpy universe and light rays from distant galaxies have propagated through intergalactic space in which the density is much lower than the averaged density. Thus the distance formula derived by solving the null geodesic in such a realistic clumpy universe should be different from that derived in the homogeneous universe. The distance formula which takes this effect into account was proposed by Zeldovich (1964), Dashevskii & Slysh (1966) and later by DR in more general form, which is now known as the Dyer-Roeder distance. They assumed that a certain mass fraction $\tilde{\alpha}$ of all matter is distributed uniformly, whereas the rest is clumped into galaxies, and that light rays travel well away from all clumps of matter, feeling only the effect of the fraction $\tilde{\alpha}$ of all matter. It has been shown that their formula gives a good approximation to the one more generally derived, when their assumptions are satisfied (Futamase & Sasaki 1989). Many of the previous studies of gravitational lenses have used this distance (Vietri & Ostriker 1983; Press & Gunn 1973; Canizares 1982; TOG).

On the other hand, a possible inconsistency in the use of the DR distance in a calculation of the statistical property of lensing is pointed out by Ehlers & Schneider (1986, hereafter ES). In the derivation of the probability of a source being multiply imaged by a galaxy along the line of sight, one integrates along a random line of sight to the source. ES argued that the direction to the source cannot be a random variable in the clumpy universe, since the assumption is crucial in the DR distance that the light rays are well away from all clumps of matter. By choosing the position of the source on a sphere of $z = z_S$ as a random variable, they proposed a new derivation of the probability which we call the ES probability.

A recent numerical study (Kasai, Futamase, & Takahara 1990) shows that which angular diameter distance is appropriate depends upon the angular scale that one considers. Whereas the standard distance gives the correct description for the angular scale comparable to or larger than the mean separation of clumps, a substantial deviation from the standard distance is observed for smaller angular scales. Monte Carlo simulations for small angular scales showed that the distance is statistically distributed; i.e., it is not uniquely described by the standard or the DR distance, although the latter is probabilistically slightly more favored (Kasai, Futamase, & Takahara 1990). In this sense neither the standard nor the DR distance formula may be regarded as quite appropriate for a realistic case, and the ES prescription also lacks compelling justification.

Under this circumstance we calculate quantities of interest using the various formulations, and look for those quantities which are insensitive to different distance measures and statistical formalisms. We shall employ in this paper both the standard distance and the DR distance with the two alternative derivations of the lensing probability. For the DR distance we only consider the extreme empty beam (namely, $\tilde{\alpha} = 0$) case.

For convenience we summarize the distance formulae for each case of the cosmological parameters and the ES expression for the lensing probability.

Case 1: *Standard Distance.*—The angular diameter distance is given by

$$d_S(z_1, z_2) = \begin{cases} \dfrac{R_0}{(1+z_2)\sqrt{\Omega_0+\lambda_0-1}}\sin(\chi_2-\chi_1) & \text{for } k=+1 , \\ \dfrac{R_0}{(1+z_2)}(\chi_2-\chi_1) & \text{for } k=0 , \\ \dfrac{R_0}{(1+z_2)\sqrt{1-\Omega_0-\lambda_0}}\sinh(\chi_2-\chi_1) & \text{for } k=-1 , \end{cases} \tag{15}$$

where

$$\chi_2 - \chi_1 = \sqrt{|\Omega_0 + \lambda_0 - 1|} \int_{z_1}^{z_2} \frac{dz}{\sqrt{\Omega_0(1+z)^3 + (1 - \Omega_0 - \lambda_0)(1+z)^2 + \lambda_0}} \tag{16}$$

for $k \neq 0$. (For $k = 0$ universes, the square root in front of the integral should be omitted.) For $\lambda_0 = 0$ it simplifies to

$$d_S(z_1, z_2) = \frac{2R_0}{\Omega_0^2(1+z_1)(1+z_2)^2} \{(2 - \Omega_0 + \Omega_0 z_2)\sqrt{1 + \Omega_0 z_1} - (2 - \Omega_0 + \Omega_0 z_1)\sqrt{1 + \Omega_0 z_2}\} \tag{17}$$

and for $\Omega_0 + \lambda_0 = 1$,

$$d_S(z_1, z_2) = \frac{R_0}{1 + z_2} \int_{z_1}^{z_2} \frac{dz}{\sqrt{\Omega_0(1+z)^3 + (1 - \Omega_0)}} . \tag{18}$$

This case is often referred to as the filled beam.

Case 2: *Dyer-Roeder Distance* (empty beam).—It is given by

$$d_{\rm DR}(z_1, z_2) = R_0(1 + z_1) \int_{z_1}^{z_2} \frac{dz}{(1+z)^2\sqrt{\Omega_0(1+z)^3 + (1 - \Omega_0 - \lambda_0)(1+z)^2 + \lambda_0}} . \tag{19}$$

In Figure 1 we present the angular distance $d(0, z)/R_0$ for the cases 1 and 2 for the four sets of the cosmological parameters A–D.

Case 3: *Lensing Probability of Ehlers-Schneider.*—We use the DR distance in this case. The probability of lensing as given in equations (5) and (12) is modified into

$$d\tau_{\rm ES} = \left[\frac{d_{\rm DR}(0, z_S)}{d_S(0, z_S)}\right]^2 \left[\frac{d_S(0, z_L)}{d_{\rm DR}(0, z_L)}\right]^2 d\tau . \tag{20}$$

4. STATISTICAL PROPERTIES OF LENSES

The formulae given in § 2 show that the distances appear in two typical combinations D_{LS}/D_{OS} and $D_{OL}D_{LS}/D_{OS}$ in various expressions for the observables. Before giving the results for statistical properties, we first show these combinations of the distances as a function of the lens redshift for the two choices of distances. D_{LS}/D_{OS} is given in Figures 2*a* and 2*b* and $D_{OL}D_{LS}/D_{OS}$ in Figures 3*a* and 3*b*, where the source redshift is assumed to be $z_S = 3$. In Figure 2*a* we see almost no difference in the ratio D_{LS}/D_{OS} between the flat (case A) and the open (case B) universes; a rather large difference in the standard distances themselves (see Fig. 1*a*) cancel in this combination. This is, however, not the case for the DR distance; an appreciable difference is visible between cases A and B. In both cases the ratio D_{LS}/D_{OS} becomes appreciably larger with a cosmological constant. Figures 3*a* and 3*b* show that the ratio $D_{OL}D_{LS}/D_{OS}$ is substantially larger in the presence of the cosmological constant compared to the change due to the increase of the mass density parameter ($\Omega_0 = 0.1 \rightarrow 1$) in the $\lambda_0 = 0$ models for both choices of the distance. In the Appendix we present D's and their combinations for many known candidate lenses.

4.1. *Lensing Probabilities*

4.1.1. *Singular Isothermal Sphere Case*

We present in Figures 4*a*, 4*b*, and 4*c* the total probabilities τ for singular isothermal sphere lenses with the use of the standard distance, the DR distance and the DR distance with the ES probability. Figures 5*a*, 5*b*, and 5*c* show the corresponding differential

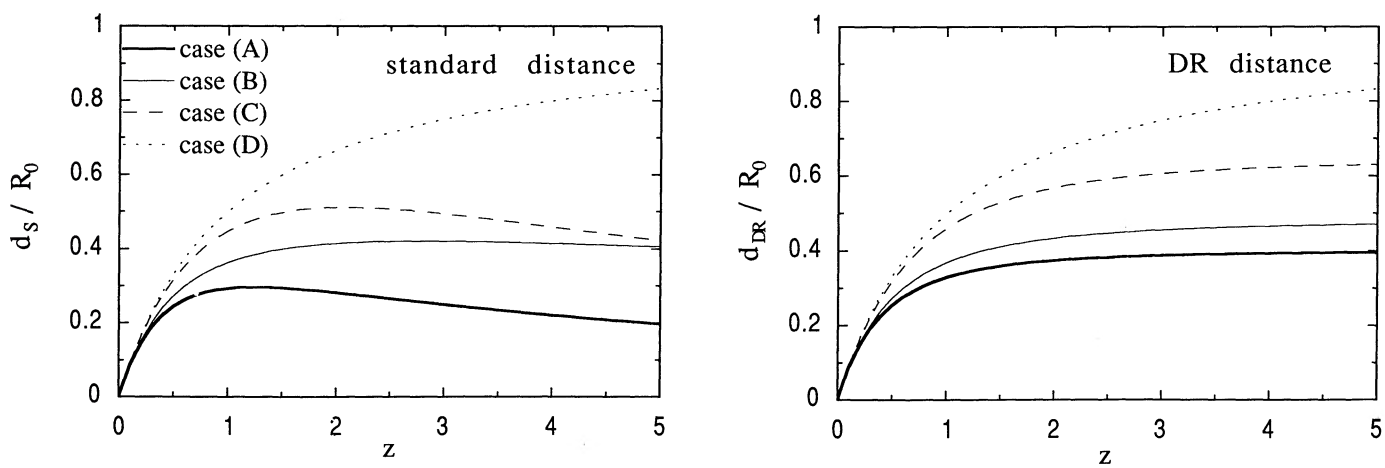

FIG. 1*a* FIG. 1*b*

FIG. 1.—Angular diameter distances in the (*a*) standard and (*b*) Dyer-Roeder (DR) formalisms as a function of redshift z. The distances are shown for the four cases: (A) $\Omega_0 = 1, \lambda_0 = 0$; (B) $\Omega_0 = 0.1, \lambda_0 = 0$; (C) $\Omega_0 = 0.1, \lambda_0 = 0.9$; (D) $\Omega_0 = 0, \lambda_0 = 1$, and are normalized by the Hubble distance $R_0 = c/H_0$.

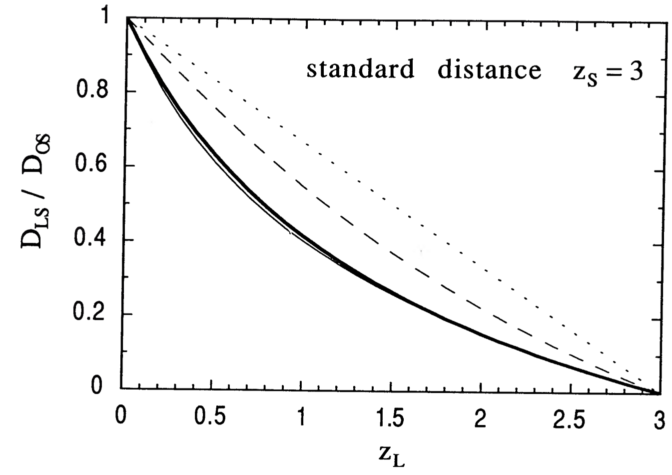

FIG. 2a

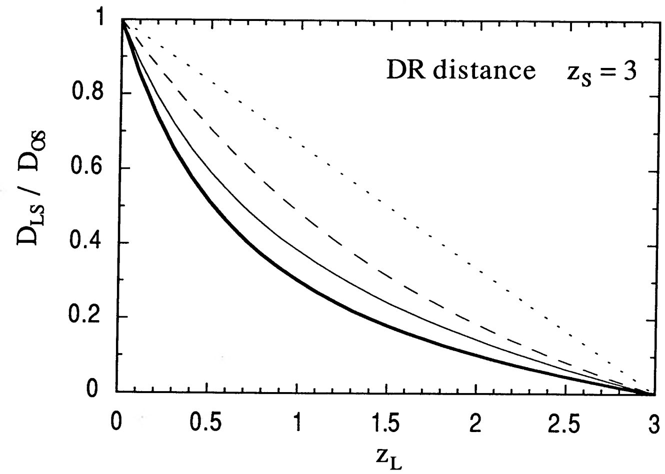

FIG. 2b

FIG. 2.—Ratio D_{LS}/D_{OS} as a function of lens redshift z_L for source redshift fixed at $z_S = 3$; (*a*) the standard distance, (*b*) the DR distance. The meaning of curves are the same as Fig. 1.

probability $d\tau/dz_L$ for $z_S = 1$ and 3. The total probability is normalized by the effectiveness parameter of lenses F, and the differential probability is normalized to total probability of unity. Let us note that for case D ($\Omega_0 = 0$, $\lambda_0 = 1$) the total probability is $(1/30)Fz_S^3$, and the normalized differential probability is $30z_L^2(z_S - z_L)^2/z_S^5$ independent of the formalism.

It is apparent that the cosmological constant strongly affects both quantities. The probability of lensing increases much faster as the source redshift increases for cases C and D than for cases A and B, irrespective of the choice of the formalism. This may be understood by noting that the combination $D_{OL} D_{LS}/D_{OS}$ that is affected strongly by a cosmological constant appears in the expression of $d\tau$. In addition, the fact that the number of galaxies per redshift (dt/dz_L) increases with Λ and that the average redshift of relevant galaxies deepens (see discussion in the next paragraph) with Λ are all partly responsible for the accumulated large increase of the optical depth in cases C and D.

At a more quantitative level there is a sizable difference among the behaviors of curves with three different formulations (1), (2), and (3) especially at a redshift $z_S > 2$. The ES probability gives values appreciably larger than others for all cosmological models. For the $\Omega_0 = 1$ (case A), the value of τ_{ES} at $z_S = 3$ is larger by almost a factor of 2 than the corresponding τ. For the open universe, on the other hand, τ_{ES} is larger than τ only slightly, and the two curves for cases A and B agree closely with formulation (3). The fact that the difference from the formulation dependence is larger than that among cosmological models A and B diminishes the hope, proposed by TOG, of using the total optical depth for high-redshift ($z > 3$) quasars in distinguishing the $\Omega_0 = 0.1$ model from the $\Omega_0 = 1$ model, unless the correct formulation can be determined. On the other hand, the effect of the cosmological constant, if it is as large as in case C, is enough to distinguish the case from that for $\lambda = 0$ even in the presence of the ambiguity associated with the formulation. For a small source redshift ($z_S < 2$) the formulation dependence is moderate, and one can discriminate between case A and B using the total optical depth; however, τ is quite small, so a large and well-understood lens sample would be required.

The differential probability is also affected significantly by the cosmological constant; it increases the average lens redshift substantially. We plot in Figure 6 the position at which $d\tau/dz_L$ reaches a maximum as a function of the source redshift. Figure 5 and

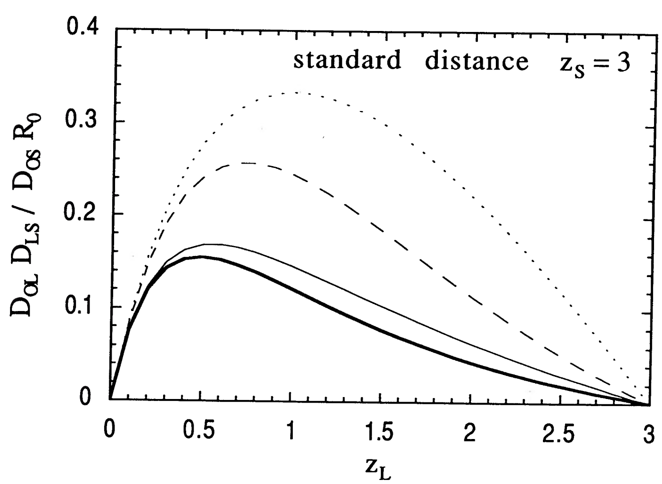

FIG. 3a

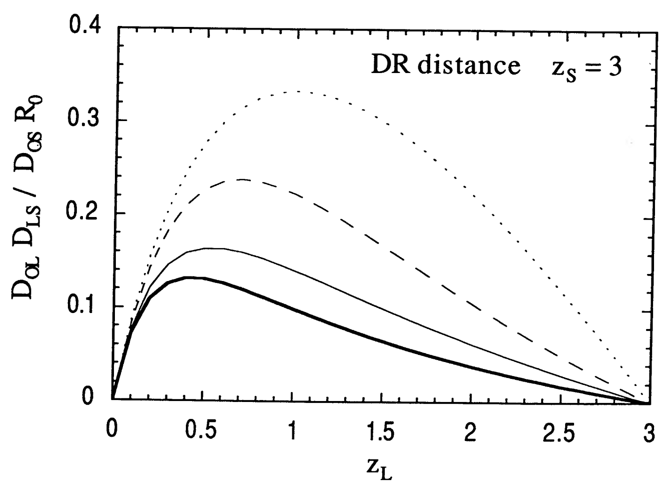

FIG. 3b

FIG. 3.—Combination $D_{OL} D_{LS}/D_{OS} R_0$ as a function of z_L for $z_S = 3$; (*a*) the standard distance, (*b*) the DR distance. The meaning of curves are the same as Fig. 1.

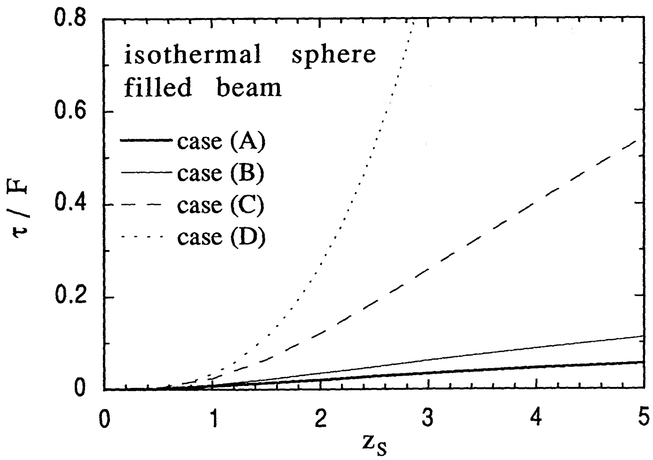

FIG. 4a

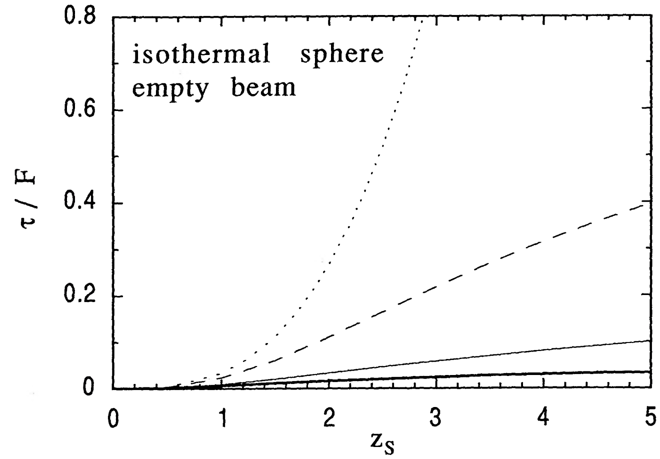

FIG. 4b

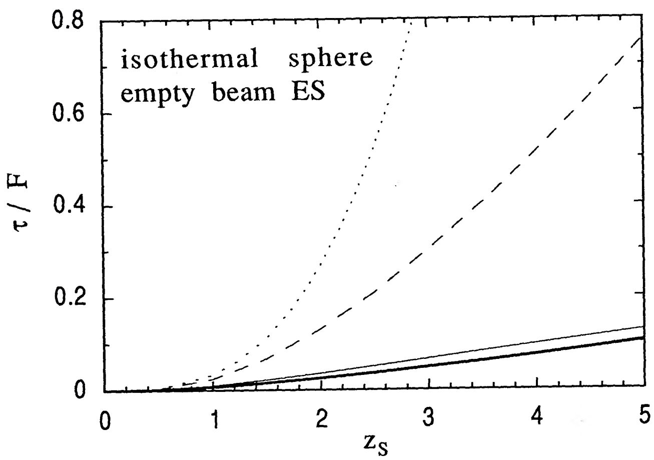

FIG. 4c

FIG. 4.—Probability of a beam encountering with a lens event (lens optical depth) for a source at z_S normalized by the lens efficiency parameter F defined in eq. (13). The lens is modeled by a singular isothermal sphere. (*a*) is for the filled beam (standard distance) and (*b*) for the empty beam with the DR distance. In (*c*) the Ehlers-Schneider prescription is adopted for the probability (with the empty beam).

more clearly, Figure 6 show that the formulation dependence is less in this quantity. The position of peak redshifts resolves not only between cases B and C, but also cases A and B. Fukugita, Futamase, & Kasai (1990) and Kochanek (1991b) have proposed Λ tests based on this effect.

From the fact that the Λ dependence of the lensing probabilities is much larger than the uncertainty caused by the choice of the lens model and of the distance and probability formulae, we conclude that both the total and the differential probabilities are the quantities suitable to test the existence of the cosmological constant. The differential optical depth is less sensitive to the ambiguities in the formulation, and it may clearly test even the density parameter of the universe.

4.1.2. *Point Mass Case*

We show in Figure 7 the total probabilities, which are normalized by the lens density $\Omega_L(0)$. As seen in these figures, the point mass lens model yields results very similar to the singular isothermal sphere case, though the ratio of the optical depth of C to A or B is smaller in the point mass case (compare eq. [12] with eq. [5]). The distribution of $\tau^{-1}d\tau/dz_L$ and its dependence on cosmological parameters and formalism are very similar to those for the isothermal sphere case, except for the behavior close to $z_L \sim z_S$, where the distribution falls as $\propto |z_S - z_L|$ rather than $\propto |z_S - z_L|^2$ in the case for the singular isothermal sphere. All discussion given for the singular isothermal sphere case also applies to the point mass case. In general, this means that much larger microlensing effects are expected in Λ-dominated models.

4.2. *Image Splittings*

4.2.1. *Singular Isothermal Sphere Case*

We now present calculations of the statistics of angular separations between the two outer images. The image separation for the isothermal sphere model is given by

$$\Delta\theta = 2\alpha \frac{D_{LS}}{D_{OS}} = 8\pi\left(\frac{v}{c}\right)^2 \frac{D_{LS}}{D_{OS}} . \tag{21}$$

We note that the expected value $\langle \Delta\theta \rangle = \alpha$ for the standard distance when $\Omega_0 + \lambda_0 = 1$, as also pointed out by Gott, Park, & Lee

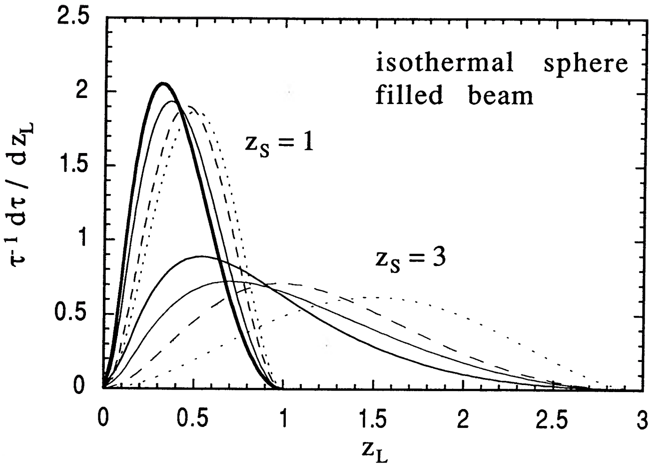

FIG. 5a

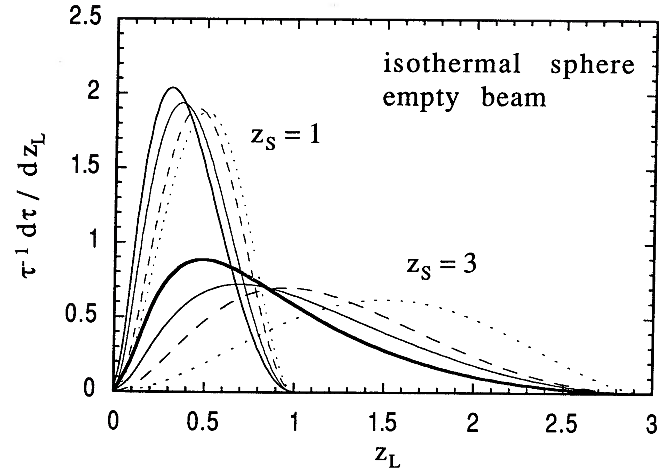

FIG. 5b

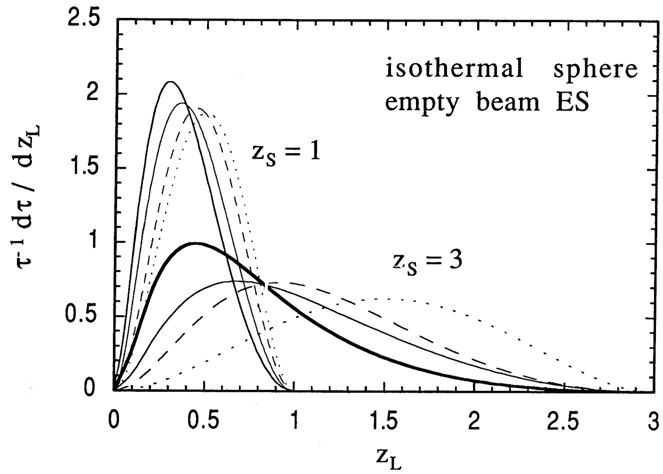

FIG. 5c

FIG. 5.—Differential probability of a lens event for singular isothermal spheres normalized to a total probability of unity. Curves are shown for $z_S = 1$ and $z_S = 3$. (*a*) is for the filled beam, (*b*) for the empty beam, and (*c*) with the ES formalism. The meaning of curves is the same as Fig. 4.

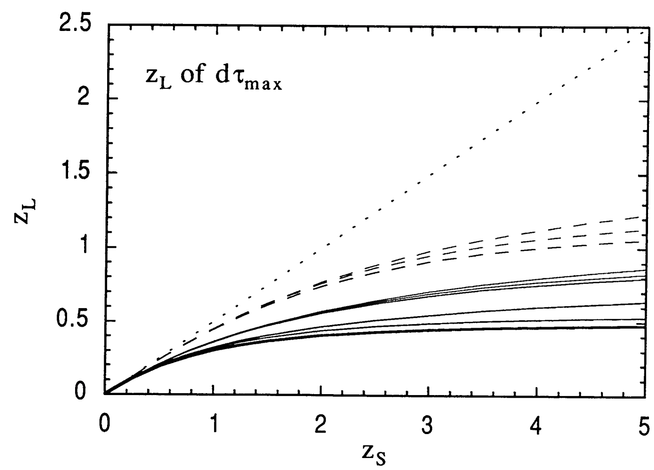

FIG. 6.—Positions at which the differential probability (Fig. 5) takes a maximum as a function of the source redshift. Clustered three curves correspond to the three formalisms, the filled beam, the empty beam, and the ES prescription in order from the top to the bottom. The four clusters correspond to the different cosmology models A–D.

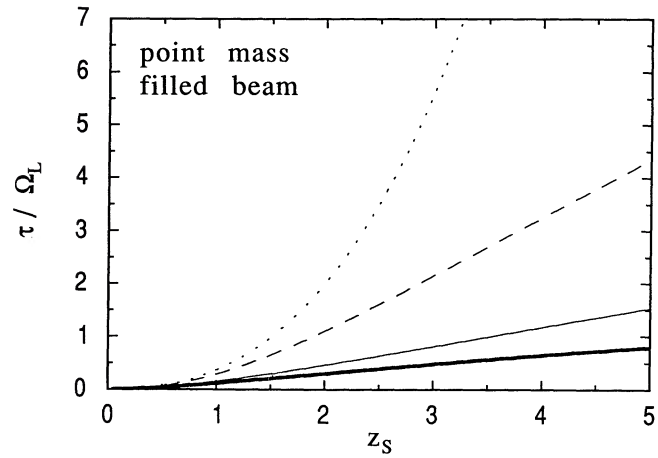

FIG. 7*a*

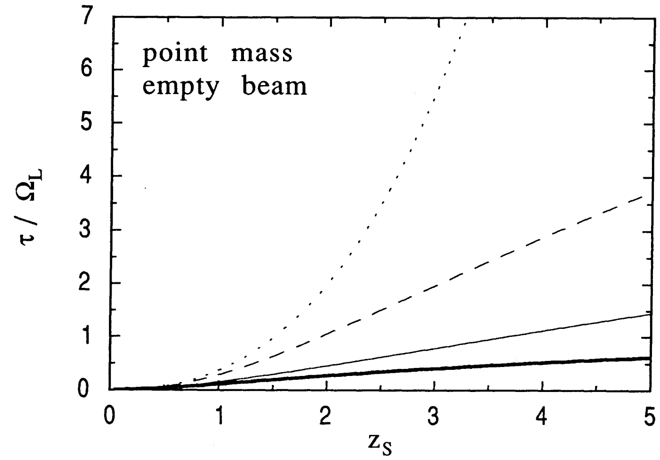

FIG. 7*b*

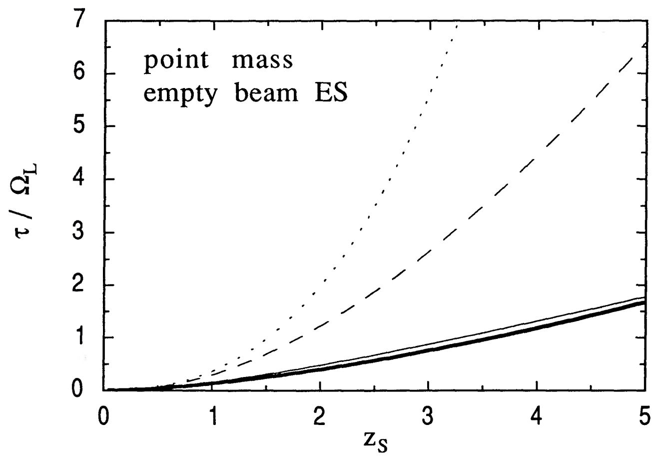

FIG. 7*c*

FIG. 7.—Lens optical depth for point mass lenses for a source at z_S, normalized by the lens density parameter $\Omega_L(0)$. The meaning of lines is the same as Fig. 4.

(1989). This relation does not hold for the DR distance, but the image separation is in any case insensitive to the cosmological parameters and to the choice of the distance formula (Fig. 8).

Let us now predict the distribution of image separations. In order to estimate the distribution of the velocity dispersion v, we assume the Schechter form for the luminosity function

$$\Phi(L)dL = \phi^*\left(\frac{L}{L^*}\right)^\alpha \exp\left(\frac{-L}{L^*}\right)\frac{dL}{L^*} \tag{22}$$

and use the empirical relationship between the luminosity and v, i.e., $L \propto v^4$ for ellipticals or lenticulars (Faber & Jackson 1976; de Vaucouleurs & Olson 1982) and $L \propto v^{2.6}$ for spirals (Tully & Fisher 1977) in the B band. From equations (12) and (22), we find that the differential optical depth of lensing in traversing dz_L with the angular separation beteen ϕ and $\phi + d\phi$ is given by

$$\frac{d^2\tau}{dz_L\,d\phi}\,d\phi\,dz_L = F^*(1+z_L)^3\left(\frac{D_{OL}D_{LS}}{R_0 D_{OS}}\right)^2 \frac{1}{R_0}\frac{c\,dt}{dz_L}\frac{2}{\Gamma(\alpha+2)}\left(\frac{D_{OS}}{D_{LS}}\phi\right)^{2\alpha+4}\exp\left[-\left(\frac{D_{OS}}{D_{LS}}\phi\right)^2\right]\frac{d\phi}{\phi}\,dz_L \tag{23}$$

for ellipticals (lenticulars), where $\phi = \Delta\theta/8\pi(v^*/c)^2$ with v^* the velocity dispersion corresponding to the characteristic luminosity L^* in (22) an $F^* = F(v = v^*)$. After integrating over the lens redshift z_L, we obtain the angular separation distribution

$$\frac{dP}{d\phi} = \frac{1}{\tau(z_S)}\int_0^{z_S}\frac{d^2\tau}{dz_L\,d\phi}\,dz_L\,, \tag{24}$$

which is shown in Figure 9, together with that for spirals obtained in a similar way. (To write this figure we used parameters specified in eq. [26] below, and α in eq. [22] is set equal to -1.1.) For a flat universe (24) reduces to equation (16) of FT which uses the variable $D = (1 + z_L)D_{OL}/(1 + z_S)D_{OS}$.

4.2.2. *Point Mass Case*

Top two panels of Figure 10 show the mean image separations averaged over impact parameter for point mass lens model as a function of z_L with use of the standard and the DR distances, respectively. With the standard distance the mean image separation

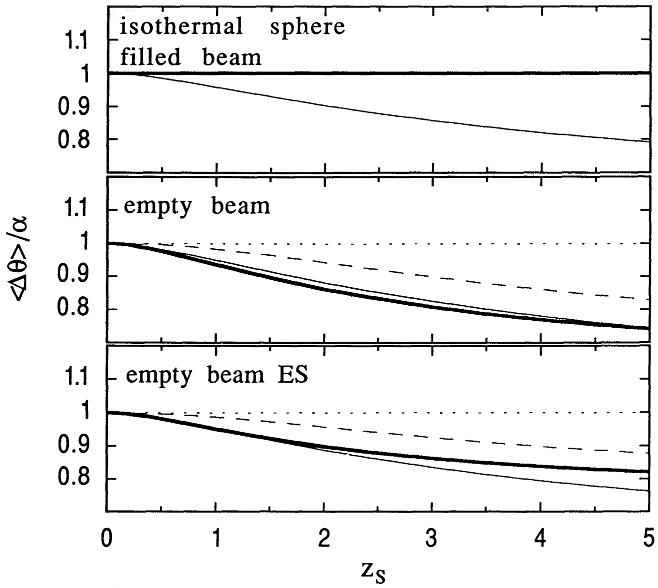

FIG. 8.—Image separation angle averaged over a uniform distribution of isothermal sphere lenses as a function of z_S. Curves are given for the three formalisms and normalized by the intrinsic bend angle α. The meaning of curves is the same as Fig. 4.

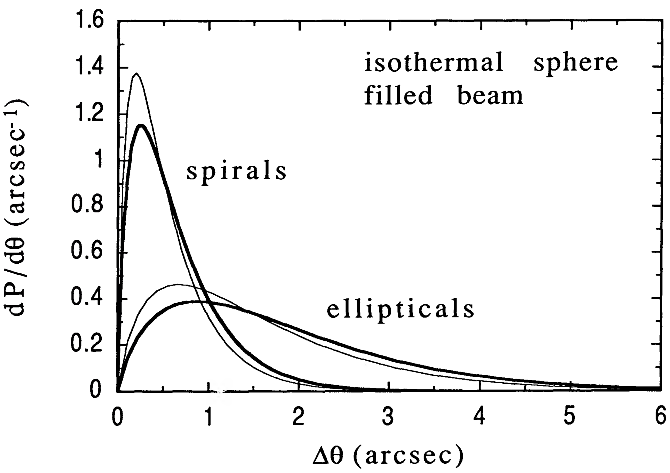

FIG. 9a

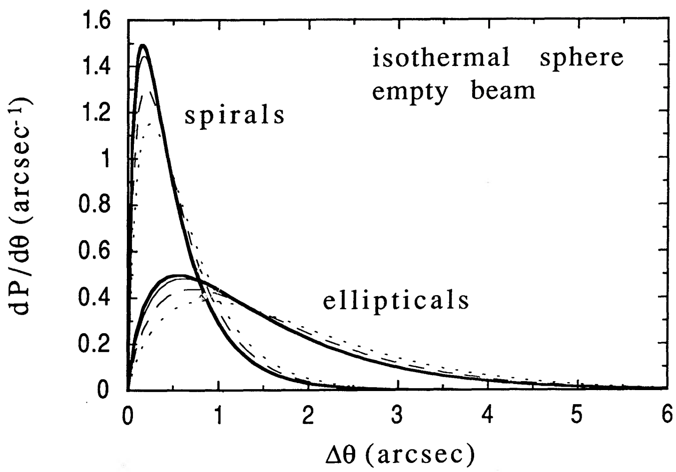

FIG. 9b

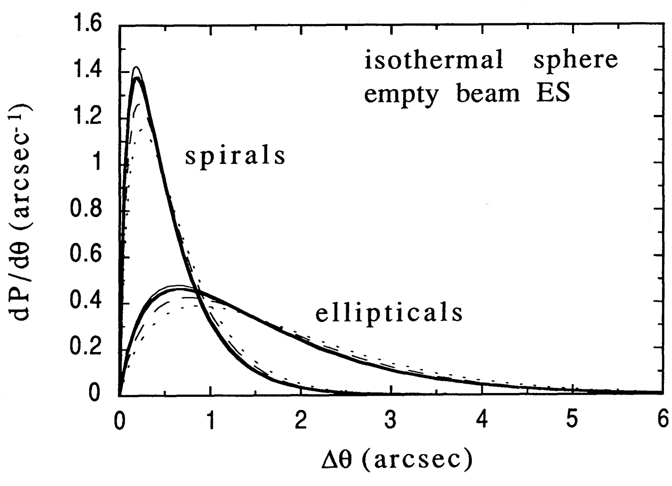

FIG. 9c

FIG. 9.—Distribution of the image separation angle for isothermal sphere lenses for ellipticals and for spirals for a source at $z_S = 3$. In (*a*) (filled beam) curves for models A, C, and D, i.e., $\Omega_0 + \lambda_0 = 1$, are degenerated and are represented by a thick line. In (*b*) and (*c*) the curve for model D is identical with that for $\Omega_0 + \lambda_0 = 1$ in (*a*) and may be used as a reference for comparison.

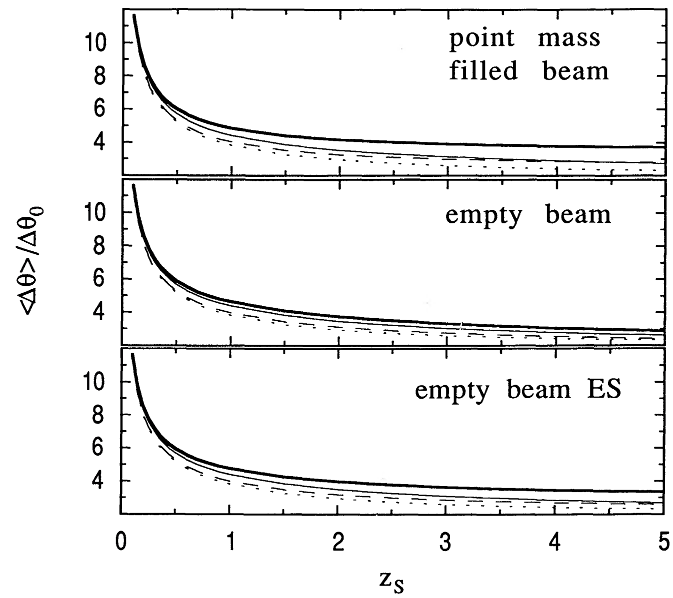

FIG. 10.—The same as Fig. 8 for point mass lenses. The separation angle is normalized by the ratio of the Schwarzschild radius to the Hubble distance $\Delta\theta_0 = 2GM/c^2R_0$.

angle for case C, i.e., the case with the cosmological constant, falls in the middle of cases A and B for any z_S and z_L. With the DR distance the separation angles for cases B and C almost coincide. The image splitting thus is not a quantity suitable to test the cosmological constant.

The image splitting angle does depend sensitively on the lens model. In the point mass model the averaged angle diverges as $z_S \to 0$, while they stay close to the constant α in the isothermal sphere model. Thus statistical studies of image splitting are more suitable to explore models for the lensing object than to test the cosmological constant.

4.3. *Gravitational Lensing Frequencies*

We now predict lensing frequencies, assuming that lensing objects are essentially galaxies and that properties of galaxies are given by those of the local sample (Turner 1990; FT). We first assume that the mass distribution of galaxies is described well by singular isothermal spheres. We also discuss the selection biases and the effect for the departures from singular isothermal sphere models as discussed in FT (see also Kochanek 1991a) for more general cases.

The lensing frequency is basically controlled by the F parameter introduced in equation (13). An estimate of F with the best modern data from the local sample of galaxies was given in FT, which yielded

$$F = \begin{cases} 0.019 \pm 0.008 \text{ for E} \\ 0.021 \pm 0.009 \text{ for S0} \\ 0.007 \pm 0.002 \text{ for S} \end{cases}, \tag{25}$$

for the morphological composition E:S0:S = $12 \pm 2 : 19 \pm 4 : 69 \pm 4$. The errors include uncertainties in the local luminosity function and those in the luminosity-velocity relation. The velocity v^* for a galaxy with the characteristic luminosity L^* was also estimated to be

$$v^* = \begin{cases} 276^{+15}_{-24} \text{ km s}^{-1} \text{ for E} \\ 252^{+15}_{-24} \text{ km s}^{-1} \text{ for S0} \\ 134^{+7}_{-12} \text{ km s}^{-1} \text{ for S} \end{cases}. \tag{26}$$

The expected number of lenses is given by

$$N_L = \int dz_S\, \tau(z_S) \frac{dN_Q(z_S)}{dz_S}, \tag{27}$$

where dN_Q/dz_S is the redshift distribution of quasars. As examples of the predictions, we take two quasar samples given by Hewitt & Burbidge (1987; 1989; hereafter HB) and by Boyle et al. (1990, hereafter BFSP) (aee also Boyle, Shanks, & Peterson, hereafter BSP). The former contains 4250 quasars, but it is not a homogeneous survey for lenses. The latter is more homogeneous as a quasar survey, but contains only 420 quasars. Furthermore, there is a bias against lens images with certain $\Delta\theta$ in the BFSP survey. The reported $dN_Q(z_S)/dz_S$ for HB (BFSP) sample rises till $z_S = 2.3(2.2)$ and then sharply drops beyond this redshift. Table 1 shows our prediction of the expected number of lensing event with the three formulations and for the four cosmological parameter choices. Here corrections or selection effects are not taken into account.

TABLE 1

NOMINALLY EXPECTED NUMBER OF LENSES WITH $F = 0.047$

Distance Formula	Case A: $\Omega_0 = 1$, $\lambda_0 = 0$	Case B: $\Omega_0 = 0.1$, $\lambda_0 = 0$	Case C: $\Omega_0 = 0.1$, $\lambda_0 = 0.9$	Case D: $\Omega_0 = 0$, $\lambda_0 = 1$
HB Sample (4250 quasars)				
Filled beam (standard distance)	2.9	5.0	17.9	45.9
Empty beam (DR distance)	2.3	4.8	16.1	45.9
Empty beam (ES)	3.7	5.2	19.7	45.9
BFSP Sample (420 quasars)				
Filled beam (standard distance)	0.25	0.41	1.32	2.74
Empty beam (DR distance)	0.20	0.40	1.21	2.74
Empty beam (ES)	0.29	0.42	1.40	2.74

4.3.1. *Corrections and Selection Effects*

The main effects considered by FT are (1) finite core radii of galaxies, (2) angular resolution selection effects, and (3) magnification bias selection effects. For elliptical galaxies core radii are generally quite small (Lauer 1985; FT). In Lauer's study only 14 out of 42 ellipticals have resolved cores, and more than a half (23) have unresolved cores. FT argued that the core of some of ellipticals might be extremely small ($\lesssim 3$ pc). Under this circumstance we assume that one-third of ellipticals obey the regression line $r_c = -140h^{-1}(M_{B_T^0} - 5 \log h + 19.9) + 160h^{-1}$ obtained from 14 galaxies of Lauer's study (FT). With this relation the ratio of the core radius to the critical radius of singular isothermal sphere $a_{\rm cr} = 4\pi v^2 D_{LS} D_{OL}/D_{OS}$ that controls the suppression of lensing cross section is fairly constant as the luminosity of galaxies changes ($L \rightarrow 4L^*$ modifies $r_c/a_{\rm cr}$ only by 16%). We assume that another one-third have a core with $r_c \sim 10$ pc and the remaining one-third have, somewhat arbitrarily, $\frac{1}{3}$ of r_c prescribed by the above relation. Using the model cross section by Hinshaw & Krauss (1987) we obtain the reduction factor $c \sim 0.65$. By changing the assumptions in a reasonable range we found that c is always in the range 0.5–0.7. We assume that the suppression factor for S0's is the same as for E's. On the other hand, most of spirals have larger cores $\gtrsim 0.5$–1 kpc as inferred from the rotation curve by, e.g., Rubin, Whitmore & Ford (1988; see their Fig. 3), and this makes only 0.1%–1% of lensing event surviving the finite core radius effect. The study of Rubin et al., however, suggests that $\sim 10\%$ of spirals may have unresolved core. Taking this into account, we may estimate that $c \sim 6\%$–8% assuming that 10% of spirals have cores smaller than 10 pc.

For the selection effect on angular image separation, we assume that the lensing event with the separation angle larger than ϕ_c is observed. The fraction of a lens giving an image separation larger than ϕ_c is given by

$$P(>\phi_c) = \int_{\phi_c}^{\infty} \frac{dP}{d\phi} d\phi = \frac{F}{\tau(z_S)} \int_0^{z_S} (1 + z_L)^3 \left(\frac{D_{OL} D_{LS}}{R_0 D_{OS}}\right)^2 \frac{1}{R_0} \frac{c\,dt}{dz_L} \frac{\Gamma[\alpha + 2, (\phi_c D_{OS}/D_{LS})^2]}{\Gamma(\alpha + 2)} dz_L , \tag{28}$$

where $\Gamma(a, x)$ is the incomplete gamma function defined by $\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$. In Figure 11 we plot $P(>\phi_c)$ for sources at $z_S = 3$ for elliptical and spiral lenses. The curves do not change with z_S or with Ω_0 for the flat ($\Omega_0 + \lambda_0 = 1$) universe (FT). For the case of the HB sample, we calculated the factor $P(>2'')$ in Table 2. The factor $P(>2'')$ for the BFSP sample does not differ from that for the HB sample by more than 5% and is therefore omitted from the table. The factor is basically determined by $P(>\phi_c, z_S \sim 1.5$–$2)$, where the number of quasars reaches its maximum. The cut at 2″ leaves 30%–35% of the elliptical lenses for these samples (this increases to 40%–50% if the cut is made at 1″.5). For spiral lenses only a very small fraction ($<1\%$ for $\phi_c = 2''$) survives this cut. Together with the strong suppression by the finite core radius effect, we conclude that spiral lenses should be rather unlikely to be found in a survey.

The magnification bias effect as formulated in FT does not depend on the world geometry nor on the formulations discussed here. (Let us note that the formula given by eq. [3.5] of TOG contains an error; the $1/A$ factor in their equation should be omitted; see also FT.) It is given by

$$B(m) = \int_0^\infty \frac{dN_Q}{dm} (m + \Delta) P(\Delta) d\Delta \Bigg/ \frac{dN_Q}{dm} \tag{29}$$

at apparent magnitude m. Here $\Delta = 2.5 \log A$ (A = magnification factor) and $P(\Delta) = 7.37 \times 10^{-0.8\Delta}$ for $\Delta \geq 2.5 \log 2$. An explicit evaluation of $B(m)$ with the standard dN_Q/dm (Hartwick & Schade 1990) is given in equation (22) of FT, and we adopt their estimate.

TABLE 2

FRACTION OF LENSING THAT SURVIVES ANGULAR RESOLUTION SELECTION EFFECTS ($\phi > 2''$): AN EXAMPLE FOR THE HB SAMPLE

Distance Formula	Case A: $\Omega_0 = 1$, $\lambda_0 = 0$	Case B: $\Omega_0 = 0.1$, $\lambda_0 = 0$	Case C: $\Omega_0 = 0.1$, $\lambda_0 = 0.9$	Case D: $\Omega_0 = 0$, $\lambda_0 = 1$
Filled beam (standard distance)	0.28	0.23	0.28	0.28
Empty beam (DR distance)	0.21	0.22	0.25	0.28
Empty beam (ES)	0.23	0.22	0.25	0.28

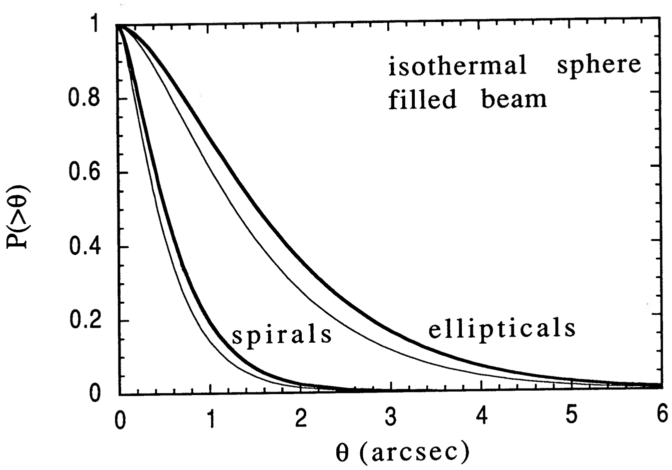

FIG. 11*a*

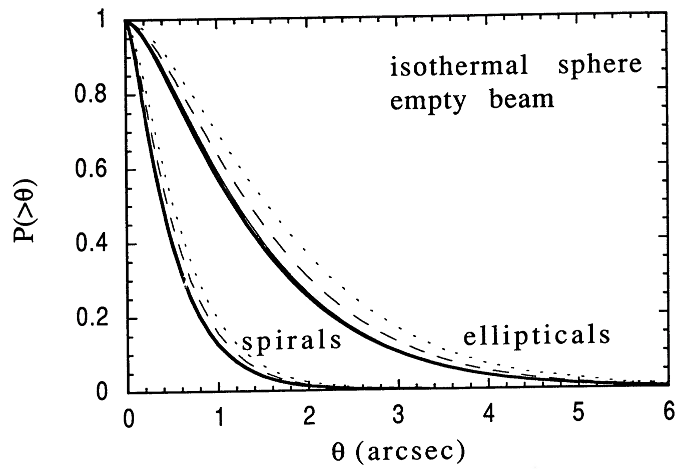

FIG. 11*b*

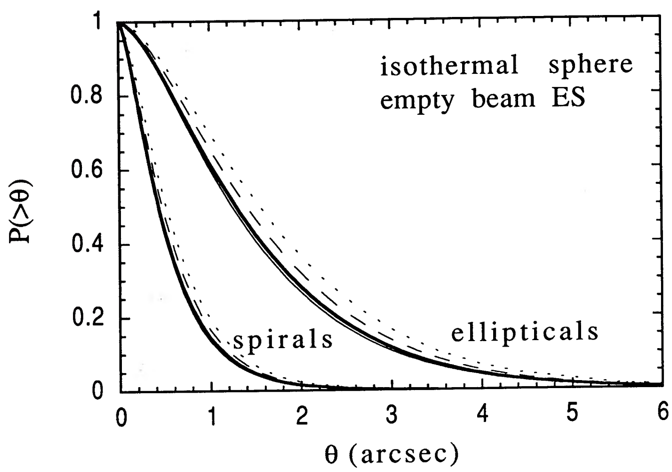

FIG. 11*c*

FIG. 11.—(*a–c*) The fraction of lensing events that survives the cutoff of the specified angular separation for singular isothermal sphere lenses. The source is assumed at $z_S = 3$. Note that the curve for model D always coincides with models A, C, and D in the case with the standard distance (*a*), serving to give a reference.

In Table 3 we present a summary of multiplication factors for the corrections discussed here, as well as the average probability for finding lensing events for a given quasar in various samples. Here the reduction factors for finite core effects and angular selection functions are calculated by

$$f = \sum_i F_i c_i P_i(\phi \geq 2'') , \tag{30}$$

where the summation is taken over E, S0, and S. Because $c_S \lesssim 0.1$ and $P_S \lesssim 0.01$, however, we can almost ignore the spiral contributions. As we assume that $c_E \approx c_{S0}$, we write

$$f = c \sum_{i=E,S0} F_i P_i(\phi \geq 2'') , \tag{31}$$

and c and $\sum F_i P_i$ are tabulated separately. We show here only the case for the filled beam. In this table we also included two more quasar samples, those by Schneider, Schmidt, & Gunn (1989a, b, hereafter SSG) and by Schmidt & Green (1983, hereafter BQS). The former is characterized by large redshifts ($z > 4$), which enhance the lensing probability, and the latter by bright magnitudes. In case for BQS the magnification bias amounts to a factor of 38. The size of these samples, however, is too small for our purpose.

The final numbers of expected lensing events are given in Table 4 for the HB and BFSP samples. The uncertainty for these numbers is about $\pm 45\%$. Complete detectability is assumed for lenses that satisfy $\phi > 2''$ for both cases. We note that the systematic formulation uncertainties are small in the final predictions. In particular, the use of the standard distance yields results virtually identical with those given by the ES formalism, if lenses with $\phi > 2''$ are selected. The power of lensing statistics for discriminating among cosmological models is apparent in this table. We need, however, a well-studied sample in excess of 10^3 quasars for a decisive result.

4.4. *Comparison with Observation*

The present status of lens observations is somewhat confusing. The HB sample contains nine reasonably convincing candidate lens systems (see Table 5 in Appendix). It has been argued, however (Blandford & Kochanek 1990; Kochanek 1991b), that only four

TABLE 3

NORMALIZED LENSING PROBABILITIES (PER GALAXY) AND CORRECTION FACTORS

				Case A					Case B					Case C				
Sample	Size	$\langle z \rangle$	B mag Range	Nominal Probability	Core Radii	Angular Selection	Mag. Bias	Final Probability	Nominal Probability	Core Radii	Angular Selection	Mag. Bias	Final Probability	Nominal Probability	Core Radii	Angular Selection	Mag. Bias	Final Probability
HB	4250	1.55	~14–22.5	0.66×10^{-3}	0.62	0.28	7.5	0.86×10^{-3}	1.1×10^{-3}	0.63	0.23	7.5	1.2×10^{-3}	4.0×10^{-3}	0.68	0.28	7.5	5.7×10^{-3}
BFSP	420	1.42	17.6–20.9	0.57×10^{-3}	0.62	0.28	2.9	0.29×10^{-3}	0.93×10^{-3}	0.63	0.24	2.9	0.40×10^{-3}	3.0×10^{-3}	0.68	0.28	2.9	1.6×10^{-3}
SSG	11	4.24	20.2–23.6	2.2×10^{-3}	0.64	0.28	2.5	1.0×10^{-3}	4.2×10^{-3}	0.65	0.19	2.5	1.3×10^{-3}	20×10^{-3}	0.71	0.28	2.5	10×10^{-3}
BQS	114	0.41	12.9–16.4	0.11×10^{-3}	0.50	0.28	38	0.58×10^{-3}	0.17×10^{-3}	0.51	0.25	38	0.82×10^{-3}	0.50×10^{-3}	0.54	0.28	38	2.9×10^{-3}

TABLE 4

EXPECTED NUMBER OF LENSES INCLUDING CORRECTIONS AND SELECTION EFFECTS

Distance Formula	Case A: $\Omega_0 = 1$, $\lambda_0 = 0$	Case B: $\Omega_0 = 0.1$, $\lambda_0 = 0$	Case C: $\Omega_0 = 0.1$, $\lambda_0 = 0.9$	Case D: $\Omega_0 = 0$, $\lambda_0 = 1$
HB Sample (4250 quasars)				
Filled beam (standard distance)	3.7	5.2	24.5	64.8
Empty beam (DR distance)	2.1	4.8	19.5	64.8
Empty beam (ES)	3.6	5.2	24.0	64.8
BFSP Sample (420 quasars)				
Filled beam (standard distance)	0.12	0.17	0.69	1.48
Empty beam (DR distance)	0.07	0.16	0.57	1.48
Empty beam (ES)	0.12	0.17	0.68	1.48

NOTES.—(1) 50% error should be implied for numbers listed in this table. (2) A survival fraction given in Table 3 is adopted for the finite core radius suppression. (3) A cut $\phi > \phi_c = 2''$ is applied for angular image separation. (4) Magnification bias is estimated following FT. (5) Complete detectability is assumed for lenses that satisfy (2).

of them could be accounted for by the sort of isolated single galaxy lensing events modeled here, and furthermore two of four are lensed by spiral galaxies (see Table 5) (which should rarely occur, according to our argument above). Therefore, it can be argued that the relevant observed number for the HB sample is between 9 and 2 (FT). On the other hand, the HB catalog was not intended to be a complete list of lens systems, and it seems at least possible that one-third of the lens systems, say, might be missed. Therefore, we may have to compare the number of lens candidates with numbers in Table 3 after some reductions, multiplying by a factor $\frac{2}{3}$, say (FT). We plot in Figure 12 the expected number of lenses for the HB sample as a function of λ_0 for the three choices of Ω_0 in order to see the dependence on these two cosmological parameters. The curves are drawn with the standard distance, and the error bars represent for the uncertainties in the lensing efficiency parameter F, the correction for finite core radii, as well as the ambiguity arising from the choice of the distance formula. Consulting with Table 3 and Figure 12, we may conclude that $\lambda > 0.95$ is probably excluded with case C being marginally allowed for the flat universe, the same conclusions obtained in FT.

The case for BFSP sample is given for an illustrative purpose to show how many lenses are expected for such a sample. The actual sample is not only too small in its size to use for selecting cosmology models, but also suffers from a selection bias against the lens systems because of the selection criteria imposed on the COSMOS survey searching for stellar images (Boyle 1991).[6]

Crampton et al. (1989) and Surdej (1989) suggested a 20%–23% lensing rate in two quasar samples. Our calculation shows that these numbers are at least one order of magnitude larger than are expected even if the lensing search was made down to a very small image angular separation.

5. OTHER STATISTICAL LENSING EFFECTS AND THE COSMOLOGICAL CONSTANT

5.1. *The Quasar-Galaxy Correlation*

WHHW reported a significant excess of quasars in the vicinity ($\theta < 6''$) of foreground galaxies and suggested that it be ascribed to gravitational lensing. It has then been found, however, that the enhancement by lensing should be smaller than the reported value ($\sim$4.2) by, at least, a factor of 2 (Narayan 1989; Kovner 1989). Here we discuss what amount of enhancement is generally expected in the galaxy-quasar correlation in the presence of the cosmological constant.

[6] We thank Brian Boyle for clarifying this point.

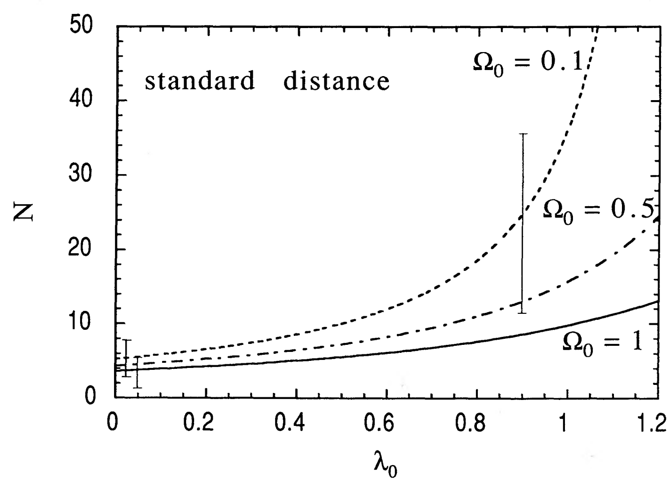

FIG. 12.—Expected number of lenses for the HB sample as a function of λ_0 for a given Ω_0 for the case of a filled beam. The sample completeness is assumed. Error bars are uncertainties of the prediction.

Narayan has also shown that if M is a magnification associated with a gravitational lens and B_0 represents the limiting magnitude of the quasar survey under question, then the enhancement factor is given by

$$q(M, B_0) = \frac{1}{M} \frac{N_Q[<(B_0 + 2.5 \log M)]}{N_Q(<B_0)} , \tag{32}$$

where $N_Q(<B)$ represents the number of quasars per square degree brighter than magnitude B. The factor M^{-1} is due to the reduction of the effective area by lensing.

For isothermal sphere lenses, the magnification factor M reads

$$M = 1 + 4\pi\left(\frac{v}{c}\right)^2 \frac{D_{LS}}{D_{OS}} \left\langle\frac{1}{\theta_S}\right\rangle , \tag{33}$$

where θ_S is the angular position of the source, and

$$\left\langle\frac{1}{\theta_S}\right\rangle = \frac{2}{\theta_L - \theta_+} \tag{34}$$

with $\theta_L = 6''$ and $\theta_+ = 4\pi(v/c)^2 D_{LS}/D_{OS}$. In the WHHW observation the brightness of the galaxies is $B_J = 20.5$–21 mag. This magnitude is translated into the mean redshift of $z_L = 0.3$ (case A), 0.28 (case B), and 0.26 (case C) using the canonical luminosity evolution model of galaxies (Arimoto & Yoshii 1986; Fukugita et al. 1990b). With typical source redshift $z_S \simeq 1.5$, we estimate $\langle M \rangle \simeq 1.6$ (case A), 1.7 (case B), and 1.8 (case C) for typical L^* of elliptical galaxies with the filled beam formalism. Therefore, the enhancement factor q is expected to be 1.2–1.8 (A), 1.2–1.8 (B), and 1.2–2.0 (C) where the range accounts for uncertainties in the estimate of $N(B_0)$ (Hartwick & Schade 1990; BSP) as well as for a possible incompleteness of the WHHW survey close to the nominal limiting magnitude $B_0 = 18.9$ mag (see Narayan 1989). With the largest estimate the difference between the prediction and the WHHW observation is still 3 σ (for cases A and B) to 2.5 σ (for case C). This means that the inclusion of a finite cosmological constant improves the situation only slightly.

5.2. *Anisotropies in the Cosmic Microwave Background Radiation*

There has been some recent interest in the possible effect of gravitational lensing upon fluctuations in CBR. The suggestion was made that the lensing might decrease the fluctuations by random scattering (Kashlinsky 1988; Tomita 1988), but it was recognized soon that gravitational lensing leads rather to enhancement of the temperature anisotropies on scales smaller than the intrinsic coherence angle of the CBR anisotropy (Cole & Efstathiou 1989; Sasaki 1989). It was also found that the effect is too small to be observationally relevent. We re-examine the effect here, since we found that the cosmological constant greatly increases the lensing optical depth for a high-redshift source.[7]

Let us define the autocorrelation function $C(\theta)$ of the temperature field $T(\boldsymbol{x}_1)$

$$C(\theta) = \langle T(\boldsymbol{x}_1)T(\boldsymbol{x}_2)\rangle \tag{35}$$

where θ is the angle between the directions $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$. When the temperature field is perturbed by the gravitational deflection, the perturbed autocorrelation function $\tilde{C}(\theta)$ is given approximately by (Blanchard & Schneider 1987; Cole & Efstathiou 1989)

$$\tilde{C}(\theta) \approx C(\theta) - \frac{\sigma^2(\theta)}{\theta_c^2} C(0) , \tag{36}$$

where $\theta_c^{-2} = -[1/C(0)][d^2C(\theta)/d\theta^2]_{\theta=0}$ and $\sigma = \langle(\tilde{\theta} - \theta)^2\rangle^{1/2}/2^{1/2}$ is the one-dimensional dispersion of the change of the angle θ under the influence of the gravitational deflection. This approximation should be valid for angles $\theta \ll \theta_c$.

We calculate σ according to Cole & Efstathiou,

$$\sigma^2(\theta, z) = \frac{64\pi^2 G^2}{c^2 H_0^2 d(0, z)} \int_0^z \frac{d(z', z)\bar{\rho}^2(z')}{(1 + z')^4(1 + \Omega z')} dz' \int_0^{x(z')} \ln\left(\frac{x}{l}\right) l\, dl \int_{-\infty}^{\infty} \xi(l, \Delta, z')d\Delta \tag{37}$$

where $x(z') = \theta d(0, z')$, $\xi(r, z) = (1 + z)^{-3}(r/r_0)^{-\gamma}$ with $\gamma = 1.8$, $r_0 = 5.4h^{-1}$ Mpc, and we ignore the cutoff in ξ which has a negligible effect for small θ. The result is given in Figure 13 for σ/Ω_L. The standard distance formula is adopted in this calculation and the last scattering surface is assumed to be at $z = 1000$. For a small θ, σ is proportional to $\theta^{0.6}$, and typical values at $\theta = 1''$ are $\sigma/\Omega_L = 2''\!.1$ for case A, $2''\!.7$ for case (B), $6''\!.8$ for case C. When normalized by Ω_L, the dispersion (σ/Ω_L) increases with the cosmological constant, as is expected (If Ω_L is identified with Ω_0, σ itself rather decreases from case A to case C, however.)

In the presence of lensing the mean square temperature fluctuation as measured in a two beam switching experiment $\langle(\Delta T/T)^2\rangle = 2[C(0) - C(\theta)]$, acquires the additional term,

$$\delta^2 = 2C(0)[\sigma(\theta)/\theta_c]^2 . \tag{38}$$

We note that θ_c and $C(0)$ are also functions of Ω_0 and λ_0, the evaluation of which requires model calculations for structure formation. Without detailed model calculations, however, we can argue that the effect of a nonzero cosmological constant does not increase the importance of the extra fluctuations induced by lensing: since θ_c corresponds to the coherence length of the last

[7] See Durrer & Kovner (1990) for a quite different approach to this problem.

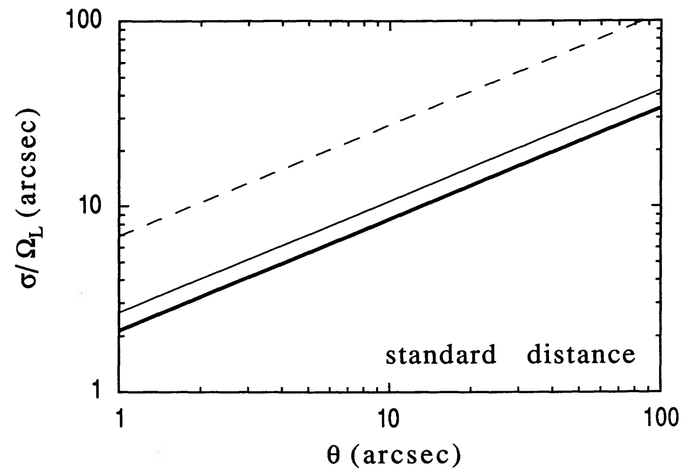

FIG. 13.—The dispersion $\sigma(\theta)$ of deflection angles normalized by the density of lensing objects (see eq. [36]) as a function of the beam separation θ for three cases A, B, and C using the standard distance.

scattering surface, roughly speaking θ_c is inversely proportional to the angular diameter distance to that surface. For three cases that we consider, $d(0, 1000)$ is $0.0019R_0$ (Case A), $0.0165R_0$ (Case B) and $0.0049R_0$ (Case C). The change of σ hence is almost compensated by the change of θ_c, and the contribution of δ^2 relative to the asymptotic value $\langle(\Delta T/T)^2\rangle_{\theta\to\infty} = C(0)$ depends only weakly on the cosmological parameters. Therefore, the conclusion for the $\Omega_0 = 1$ case derived by Cole & Efstathiou (1989) that the effect of gravitational lensing is quite small in various models also holds in the presence of a nonzero cosmological constant.

6. CONCLUSIONS

We have studied statistical properties of gravitational lenses, especially in the presence of the cosmological constant. A particular emphasis was given to systematic uncertainties arising from the various possible formulations of lensing calculations. In the present study we have employed two different distance formulae, the standard distance and the DR distance, and two different formulations of the lensing probability, TOG and ES. We have found that there generally exist substantial uncertainties associated with the adopted formulations in the prediction of the optical depth of lenses. For $z_S \gtrsim 2.5$, the uncertainties becomes large so that discrimination among the models with different cosmological mass densities (e.g., $\Omega_0 = 1$ and $\Omega_0 = 0.1$) is difficult. However, the cosmological constant strongly affects the optical depth, so that its effect is much larger than the ambiguity arising from the uncertainty of the formalism for any z_S. Therefore, the optical depth, i.e., the probability of finding lenses in a given quasar sample, provides us with a simple and very useful means to test the cosmological constant. For $z_S \lesssim 2$–2.5 the uncertainties are rather modest and all four cases of cosmological models can probably be discriminated by the optical depth given large, homogeneous lens samples. An estimate is also made of selection effects and corrections to the model in order to predict lensing frequencies for a realistic case. Our provisional conclusion drawn from the comparison of the prediction with the current observation indicates that an excessively large Λ (e.g., $\lambda_0 \gtrsim 0.95$ in the flat geometry) is excluded, in agreement with FT.

We have also examined the effect of the cosmological constant on the lensing effect with respect to the observed quasar-galaxy correlation, and we found that it is not sufficiently large to alleviate the difficulty in explaining the reported large correlation. The effect of the cosmological constant on the lensing effect for temperature fluctuations in CBR is also found to be unimportant.

APPENDIX

While our prime interest is in statistical lensing, it is often useful to give cosmological factors for known lens systems. In Table 5 we provide D_{OL}, D_{OS}, D_{LS}, D_{LS}/D_{OS}, $D_{OL}D_{LS}/D_{OS}$ and $D_{LS}/D_{OL}D_{OS}$ for reasonably convincing candidate lens systems (Turner 1989). The ratio D_{LS}/D_{OS} controls the effective bending angle, and the quantity $D_{OL}D_{LS}/D_{OS}$ is the inverse of the critical surface density for the uniform-density lens model. The last quantity $D_{LS}/D_{OL}D_{OS}$ (see Fig. 14) appears in the differential time delay, which is used to derive the Hubble constant from lens systems (Refsdal 1964; Falco, Gorenstein & Shapiro 1985). From Table 5 it is seen that the choice of world model little affects derived values of the Hubble constant. For the best-studied 0957+561 case A → case B makes H_0 7.5% larger and the change caused by case A → case C is even smaller (H_0 becomes 3.5% larger); the large change in D_{LS} is canceled by that in D_{OL} and D_{OS}. We note that there is also a strong dependence on the distance formulation, especially for a large z_S and z_L. This uncertainty may be as large as 14% for 0957+561. (For the 0142−100 and 2016+112 systems, the uncertainty increases to 30% and 60%, respectively.) This means that the arrival time difference is not a quantity useful for discrimination among cosmological models, even if the Hubble constant were known by other means.

TABLE 5

ANGULAR DIAMETER DISTANCES AND THEIR RATIOS FOR KNOWN LENS CANDIDATE SYSTEMS

Name	z_S	z_L	Case	D_{OL}	D_{OS}	D_{LS}	D_{LS}/D_{OS}	$D_{OL}D_{LS}/D_{OS}$	$D_{LS}/D_{OL}D_{OS}$	Notes
0142−100.........	2.7	0.49	A	0.243 (0.252)	0.260 (0.385)	0.162 (0.197)	0.624 (0.513)	0.151 (0.129)	2.570 (2.031)	Isolated galaxy lens
			B	0.271 (0.272)	0.421 (0.451)	0.256 (0.267)	0.609 (0.591)	0.165 (0.161)	2.246 (2.173)	
			C	0.314 (0.316)	0.503 (0.599)	0.376 (0.422)	0.749 (0.704)	0.235 (0.223)	2.384 (2.228)	
0957+561.........	1.4	0.36	A	0.210 (0.215)	0.295 (0.355)	0.177 (0.191)	0.598 (0.538)	0.125 (0.116)	2.854 (2.510)	Large cluster
			B	0.227 (0.228)	0.394 (0.405)	0.238 (0.241)	0.603 (0.595)	0.137 (0.136)	2.653 (2.609)	Perturbation
			C	0.256 (0.257)	0.491 (0.520)	0.346 (0.357)	0.704 (0.687)	0.181 (0.177)	2.749 (2.672)	
1115+080.........	1.7	?								Isolated galaxy lens (spiral?)
1120+019.........	1.5	0.6	A	0.262 (0.277)	0.294 (0.360)	0.127 (0.133)	0.430 (0.370)	0.113 (0.102)	1.643 (1.337)	Very large splitting
			B	0.299 (0.301)	0.399 (0.412)	0.175 (0.176)	0.438 (0.429)	0.131 (0.129)	1.463 (1.423)	For isolated galaxy
			C	0.353 (0.357)	0.498 (0.531)	0.272 (0.278)	0.546 (0.524)	0.193 (0.187)	1.544 (1.467)	
1413+117.........	2.6	?								Isolated galaxy lens
1635+267.........	2.0	?								No lens detected
2016+112.........	3.3	1.0	A	0.293 (0.329)	0.241 (0.390)	0.105 (0.121)	0.434 (0.310)	0.127 (0.102)	1.483 (0.940)	Two lensing galaxies;
			B	0.363 (0.369)	0.420 (0.460)	0.175 (0.181)	0.417 (0.394)	0.151 (0.145)	1.147 (1.068)	One dominant?
			C	0.447 (0.460)	0.484 (0.613)	0.276 (0.306)	0.571 (0.499)	0.255 (0.230)	1.277 (1.085)	
2237+031.........	1.7	0.04	A	0.037 (0.037)	0.290 (0.367)	0.276 (0.342)	0.950 (0.934)	0.035 (0.035)	25.45 (25.00)	Isolated galaxy lens
			B	0.038 (0.038)	0.407 (0.422)	0.386 (0.400)	0.949 (0.947)	0.036 (0.036)	25.19 (25.13)	(spiral)
			C	0.038 (0.038)	0.506 (0.549)	0.492 (0.530)	0.971 (0.967)	0.037 (0.037)	25.32 (25.23)	
2345+007.........	2.2	?								Cluster?

NOTES.—Case A $\Omega_0 = 1, \lambda_0 = 0$; case B $\Omega_0 = 0.1, \lambda_0 = 0$; case C $\Omega_0 = 0.1, \lambda_0 = 0.9$. All distances are normalized by $R_0 \equiv c/H_0$.

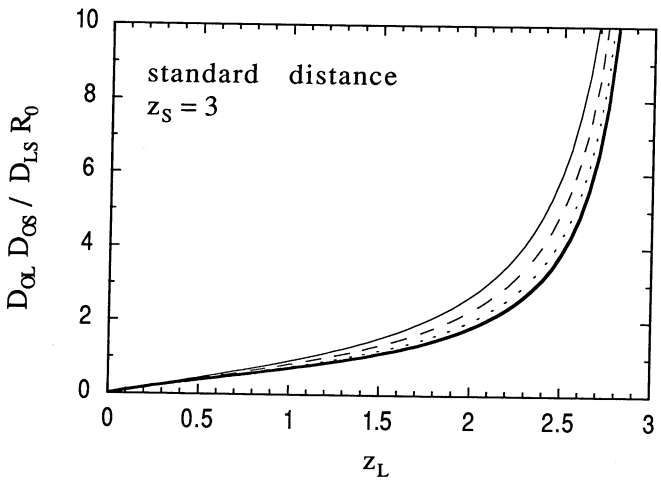

FIG. 14a

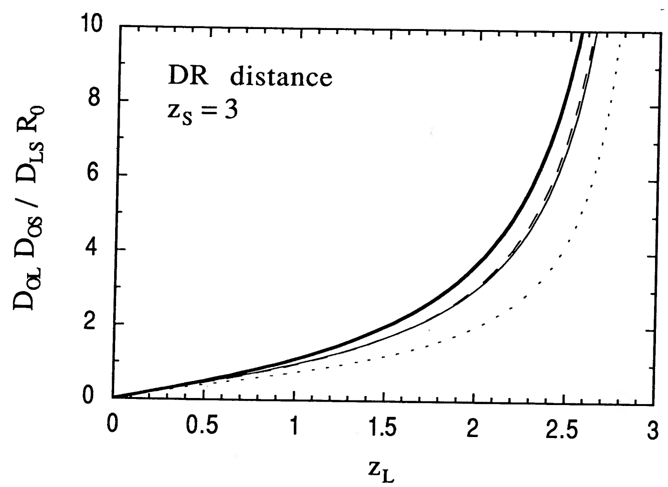

FIG. 14b

FIG. 14.—Combination $D_{OL}D_{OS}/D_{LS}$ as a function of z_L for $z_S = 3$; (a) the standard distance, (b) the DR distance. The meaning of curves is the same as Fig. 1

REFERENCES

Aaronson, M., et al. 1986, ApJ, 302, 536
Alcaino, G., Liller, W., & Alvarado, F. 1988, ApJ, 330, 569
Alcock, C., & Anderson, N. 1986, ApJ, 302, 43
Arimoto, N., & Yoshii, Y. 1986, A&A, 164, 260
Blanchard, A., & Schneider, J. 1987, A&A, 184, 1
Blandford, R. D., & Kochanek, C. S. 1987, in Dark Matter in the Universe, Proc. 4th Jerusalem Winter School 1986–1987, ed. J. N. Bahcall, T. Piran, & S. Weinberg (Singapore: World Scientific), 133
———. 1990, private communication
Boyle, B. J. 1991, private communication
Boyle, B. J., Shanks, T., & Peterson, B. A. 1988, MNRAS, 235, 935 (BSP)
Boyle, B. J., Fong, R., Shanks, T., & Peterson, B. A. 1990, MNRAS, 243, 1 (BFSP)
Canizares, C. R. 1982, ApJ, 263, 508
———. 1987, in IAU Symp. 124 Observational Cosmology, ed. A. Hewitt, G. Burbidge, & L. Z. Fang (Dordrecht: Reidel), 729
Cole, S., & Efstathiou, G. 1989, MNRAS, 239, 195
Crampton, D., McClure, R. D., Fletcher, J. M., & Hutchings, J. B. 1989, AJ, 98, 1188
Dashevskii, V. M., & Slysh, V. I. 1966, Soviet Astron, AJ., 9, 671
de Vaucouleurs, G., & Olson, D. W. 1982, ApJ, 256, 346
Durrer, R., & Kovner, I. 1990, ApJ, 356, 49
Dyer, C. C., & Roeder, R. C. 1972, ApJ, 174, L115 (DR)
———. 1973, ApJ, 180, L31 (DR)
Ehlers, J., & Schneider, P. 1986, A&A, 168, 57 (ES)
Faber, S. M., & Jackson, R. E. 1976, ApJ, 204, 668
Falco, E. E., Gorenstein, M. V., & Shapiro, I. I. 1985, ApJ, 289, L1
Fugmann, W. 1988, A&A, 204, 73
Fukugita, M., Futamase, T., & Kasai, M. 1990a, MNRAS, 246, 24p
Fukugita, M., & Hogan, C. 1991, ApJ, 368, L11
Fukugita, M., Okamura, S., Tarusawa, K., Rood, H. J., & Williams, B. A. 1991, ApJ, 376, 8
Fukugita, M., Takahara, F., Yamashita, K., & Yoshii, Y. 1990b, ApJ, 361, L1
Fukugita, M., & Turner, E. L. 1991, MNRAS, 253, 99 (FT)
Futamase, T., & Sasaki, M. 1989, Phys. Rev., D40, 2502
Gott, J. E., III. 1987, in IAU Symp. 117, Dark Matter in the Universe, ed. J. Kormendy & G. R. Knapp (Dordrecht: Reidel), 219
Gott, J. E., III, Park, M.-G., & Lee, H. M. 1989, ApJ, 338, 1
Hartwick, F. D. A., & Schade, D. 1990, ARA&A, 28, 437
Hewitt, J. N., & Burbidge, G. 1987, ApJS, 63, 1 (HB)
———. 1989, ApJS, 69, 1 (HB)
Hinshaw, G., & Krauss, L. M. 1987, ApJ, 320, 468

Iben, I., Jr., & Renzini, A. 1984, Phys. Rep., 105, 331
Jacoby, G. H., Ciardullo, R., & Ford, H. C. 1990, ApJ, 356, 332
Kasai, M., Futamase, T., & Takahara, F. 1990, Phys. Lett., A147, 97
Kashlinsky, A. 1988, ApJ, 331, L1
Kochanek, C. S. 1991a, ApJ, 379, 517
———. 1991b, preprint
Kovner, I. 1989, ApJ, 341, L1
Lahav, O., Lilje, P. B., Primack, J. R., & Rees, M. J. 1991, MNRAS, 251, 128
Lauer, T. R. 1985, ApJ, 292, 104
Lilly, S. J., Cowie, L. L., & Gardner, J. P. 1991, ApJ, 369, 79
Narayan, R. 1989, ApJ, 339, L53
Paczyński, B., & Gorski, K. 1981, ApJ, 248, L101
Peebles, P. J. E. 1984, ApJ, 284, 439
Pierce, M. J., & Tully, R. B. 1988, ApJ, 330, 579
Press, W. H., & Gunn, J. E. 1973, ApJ, 185, 397
Refsdal, S. 1964, MNRAS, 128, 295
Rubin, V. C., Whitmore, B. C., & Ford, W. K., Jr. 1988, ApJ, 333, 522
Sasaki, M. 1989, MNRAS, 240, 415
Schmidt, M., & Green, R. F. 1983, ApJ, 269, 352 (BQS)
Schneider, D. P., Schmidt, M., & Gunn, J. E. 1989a, AJ, 98, 1507 (SSG)
———. 1989b, AJ, 98, 1951 (SSG)
Surdej, J. 1989, in Gravitational Lensing, ed. Y. Meller et al. (Berlin: Springer) 88
Tomita, K. 1988, PASJ, 40, 751
Tonry, J. L. 1991, ApJ, 373, L1
Tonry, J. L., Ajhar, E. A., & Luppino, G. A. 1989, ApJ, 346, L57
Tully, R. B., & Fisher, J. R. 1977, A&A, 54, 661
Turner, E. L. 1989, in Proc. 14th Texas Symp. on Relativistic Astrophysics ed. E. J. Fenyves (New York: New York Acad. Sci.), 319
———. 1990, ApJ, 365, L43
Turner, E. L., Ostriker, J. P., & Gott, J. R., III. 1984, ApJ, 284, 1 (TOG)
Tyson, J. A. 1988, AJ, 96, 1
VandenBerg, D. A. 1983, ApJS, 51, 29
Vietri, M., & Ostriker, J. P. 1983, ApJ, 267, 488
Watanabe, K., & Tomita, K. 1990, ApJ, 355, 1
Webster, R. L., Hewett, P. C., Harding, M. E., & Wegner, G. A. 1988, Nature, 336, 358 (WHHW)
Zel'dovich, Y. 1964, Soviet Astron. AJ, 8, 13