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LONG-TERM PULSAR FLUX MONITORING AND REFRACTIVE INTERSTELLAR SCINTILLATION

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ABSTRACT

We have observed the flux densities of 14 pulsars for over 1 year at an observing frequency of 610 MHz, using an NRAO 26 m telescope in Green Bank, West Virginia. The observed time scales for long-term flux variation for seven pulsars are in good agreement with refractive scintillation theory. Four pulsars, including the Vela pulsar, have observed time scales that are in poor agreement with theoretical predictions. Pulsars predicted to have extremely long refractive scintillation time scales are observed to have stable fluxes. These pulsars therefore have intrinsically stable luminosities. In spite of our sample having a wide range of dispersion measures (43–475 pc cm^{-3}), we find no scintillation modulation depth greater than 0.4. Our observations preclude models for the interstellar medium having a spectrum for electron density inhomogeneities $P(q) = C_n^2 q^{-\beta}$ with β greater than 4, and are inconsistent with the presence of an inner scale as large as 10^9 m. Our results are best described by a Kolmogorov spectrum of electron density inhomogeneities.

Subject headings: ISM: general — pulsars: general

1. INTRODUCTION

Variations of pulsar flux densities at radio frequencies over long periods of time can be attributed to intrinsic luminosity fluctuations or to effects of propagation through an inhomogeneous interstellar medium, or to some combination of both. Flux variations on short time scales have long been interpreted as extrinsic to the pulsar and have been identified as being due to diffractive interstellar scintillation (Scheuer 1968). This phenomenon, and its effect on pulsar fluxes, has been studied extensively (see Cordes, Weisberg, & Boriakoff 1985, hereafter CWB, and references therein). For years, long-term pulsar flux variations were attributed solely to the pulsar emission mechanism. In 1982, Seiber noticed that the time scales of these modulations varied with dispersion measure (Seiber 1982). This remark clearly identified at least some component of the long-term modulation as being due to propagation effects, and resulted in sudden interest in refractive interstellar scintillations (Rickett, Coles, & Bourgois 1984). For a good review of the subject, see Rickett (1990).

Physical interpretations for the related yet dissimilar effects are as follows. Diffractive interstellar scintillation, characterized by pulsar intensity fluctuations on time scales of minutes and with a small fractional bandwidth, is due to interference among rays in the ray bundle that reaches the telescope. These rays arrive from slightly different directions because of multipath scattering off electron density fluctuations in the interstellar medium on scales of 10^7–10^9 m. Refractive interstellar scintillation, on the other hand, is due to focusing or defocusing of the entire ray bundle by inhomogeneities of spatial scales of 10^9–10^{11} m and causes broad-band intensity fluctuations on time scales of weeks.

Detailed analyses of both effects provide valuable information about the distribution of the electron density along the line of sight between the Earth and the pulsar and contribute to the development of models of the interstellar medium. Unlike diffractive effects, no consensus has yet been reached on the exact nature of the inhomogeneities in the interstellar medium that result in refractive effects.

Inhomogeneities in the interstellar medium are commonly characterized by a power-law spectrum of the form $P(q) = C_n^2 q^{-\beta}$, where β is thought to be in the range $3 < \beta < 5$ (the value for Kolmogorov turbulence is $\beta = 11/3$) and where $q = 2\pi/L$ is the wavevector associated with a spatial size L. C_n^2 is a measure of turbulence along a particular line of sight labeled by n. The Kolmogorov value for β is generally assumed, mainly because of the analogy with the well-understood theory of neutral gas turbulence; however, steeper spectra have been proposed as explanations of large observed modulation indices (Blandford & Narayan 1985) and observed quasi-periodicities in pulsar dynamic spectra (Roberts & Ables 1982). Although a power-law spectrum, whether Kolmogorov or steeper, is generally accepted, the range over which such a power-law spectrum is valid is debated. Armstrong, Cordes, & Rickett (1981) as well as Rickett, Coles, & Bourgois (1984) have suggested that it extends over a wide range of scales, that is, from scales of less than 10^8 m to at least 10^{13} m. Coles et al. (1987) argue that an "inner scale" must be present around 10^9 m (Coles et al. 1987) such that $P(q) = 0$ for larger wavenumbers. Although the Kolmogorov power spectrum and steeper power spectra (steep generally means $\beta > 4$) have similar analytical forms, their physical interpretations may be quite different. Specifically, spectra having $\beta < 4$ are consistent with a turbulent cascade of energy from large to small scales. Inner scales added to such spectra imply that dissipative processes convert macroscopic motions to heat on scales of the inner scale. Spectra having $\beta > 4$ can result from a superposition of large objects, like interstellar clouds and do not have turbulence connotations.

Although some observational work has been done to measure time scales and modulation indices of refractive scintillation in pulsar fluxes, previously, the longest project specifically designed to study refractive interstellar scintillation ran for only 43 days (Stinebring & Condon 1990, hereafter SC). This was not long enough to observe significant refractive modulation in most of the pulsars studied. Rickett & Lyne (1990, hereafter RL) have done a very thorough analysis of 300 days of data for the Crab pulsar, but the nebular contribution to the refractive fluctuations complicates this system. A long-term systematic study of refractive effects in many different directions is clearly long awaited.

In this paper, we report the first results of a long-term pulsar flux monitoring project. We address two key issues: first, we consider the validity of assuming pulsars are intrinsically stable continuum sources. Clearly, if pulsars having large predicted refractive time scales were to show variations on much shorter time scales, the utility of analyzing flux time series to study refractive scintillation could be called into question. Second, we use our observations of refractive scintillation to draw conclusions about the steepness and range of the density inhomogeneity spectrum in the interstellar medium.

Our observing procedure and data acquisition system are described in § 2. We present our data, including 14 flux time series and corresponding structure functions in § 3. Discussion of these results is found in § 4, and we present our conclusions in § 5.

2. OBSERVING PROCEDURE AND DATA ACQUISITION

A 26 m radio telescope at Green Bank, West Virginia,[1] has been continually monitoring pulsar fluxes since February 1989, as part of a pulsar timing program (Nice 1990). The pulsar monitoring program runs whenever the telescope is not being used by the US Naval Observatory.

We observe a fixed schedule of 35 pulsars each day, with integration times depending on the strength of the source, but generally on the order of several minutes. Typically, a total of 40 minutes is spent observing a given pulsar, with the total integration time divided into "scans" of fixed length. The total observation and scan durations are longer for weaker sources.

The pulsars in the schedule were selected based on signal-to-noise ratio. Nearby pulsars, which have large diffractive bandwidths and time scales, are not useful for flux monitoring since diffractive scintillation acts as noise and obscures the refractive effects. Many of the pulsars in our sample of 35 are in this category and are therefore not mentioned in this paper, but are useful to the timing project.

Two orthogonal linearly polarized signals at 610 MHz are fed into a filter bank spectrometer having 16 × 1 MHz channels per polarization. The channels are individually square-law detected, the two polarizations for each frequency channel are summed, and the signals are fed into a multichannel data acquisition system (Stinebring et al. 1992). The signals are averaged synchronously with the pulsar period, and the resulting profiles are stored on disk and transferred to magnetic tape once every week.

Obtaining accurate fluxes requires careful calibration of the strength of the signal. Variations in fluxes caused by changes in the system temperature due to sky temperature, telescope pointing positions, or various systematic effects have been eliminated using the following technique. At the start of observations of a new source, a variable gain amplifier is used to reset the signal level to a predetermined value. The scan that follows consists of a 5 K noise diode turned on by a 40 Hz square wave for 1000 periods of the square wave. The strength of the calibration signal is recorded in each of the 16 channels. This calibrator deflection serves as our signal strength reference. The stability of the calibrator on time scales of minutes and hours has been verified by leaving the calibrator running for several hours. Further, we have observed no long-term trends common to two or more pulsar flux time series. Hence we conclude that the calibrator is stable to within the variation of our most stable pulsar time series. We compare the calibrator to a strong continuum source once every day to fix an absolute flux scale. Interference, the large beam size of the telescope, and the disparity between pulsar and continuum source fluxes have limited the accuracy of this procedure. Therefore, our absolute flux values are rough estimates only.

In order to extract an accurate flux density measurement in spite of modest signal-to-noise ratios and occasional interference, we fitted data for each frequency channel to a high signal-to-noise average profile. The data are assumed to be of the form

$$p(j) = a + b \times s(j - \tau) + n(j) \tag{1}$$

where s is the average profile and n is a noise background. The parameters a, b, and τ are adjusted iteratively to find the minimum chi-squared value. Here, a is a constant offset, b is a scale factor, and τ is the time lag, or the phase difference between the peak of the pulse in the standard profile and the peak in the data. The scale factor b, which is computed along with its uncertainty, is a measure of the relative peak flux density for that integration (Taylor 1990). The reported flux value, in arbitrary units, is equal to b divided by the calibration value calculated from the calibration scan.

In this way, we obtain some 100 measurements (number of channels times number of individual integrations) for each source every day. We have found that the distribution of flux measurements is roughly Gaussian, with a small number of outlying points due to interference. We eliminate points that deviate by more than 3 σ from the mean of all points from that day. With the outliers removed, we calculate the mean and standard deviation of the remaining points. Typically, one such flux density estimate represents the average of 90 measurements.

Because our observing system is fully automated, spurious points can get included in the data sets in spite of careful logs kept by operators at the telescope. Our criteria for removal of such points are as follows: if the pulse time of arrival has an unusually large residual and has independently been discarded by the timing analysis software, we delete the corresponding flux value (D. Nice, private communication). This technique is not sufficient to remove all spurious flux values since it is possible to establish a reliable time of arrival in moderately strong interference that would not allow any reasonable flux measurement. Two further criteria are used to determine if a flux measurement is reliable. First, if there are unusual flux values for two or more different pulsars in one day, those flux values are deleted. Second, if any unusual data remains, the raw data for those observations is retrieved from the archive and is examined for interference or other problems. In practice, $\sim 0.5\%$ of all of our data has to be retrieved for inspection, and of that, approximately half ends up being deleted.

[1] The National Radio Astronomy Observatory is operated by Associated Universities, Inc., under contract with the National Science Foundation.

3. DATA AND RESULTS

We present our results in two formats: we have plotted flux versus day number, and corresponding structure functions for each of the 14 pulsars in Figure 1. We have included lines connecting neighboring days to guide the eye in regions of rapid flux variation. US Naval Observatory observations, time reserved for maintenance, and occasional interference have resulted in useful flux time series of varied lengths. Table 1 contains the duration of each data set. We provide in Table 2 our measurements of mean absolute peak fluxes in janskys along with their associated uncertainties. We quote a catalog value for rough comparison (Manchester & Taylor 1981, and references therein). Note that this catalog value has been obtained using a flux value at 400 MHz and assuming a spectral index of -1.5 for all pulsars. Although there are discrepancies which may be due either to the limitations of our rough absolute calibration method or to inaccuracies in the catalog and in the spectral index estimate, the agreement is sufficient for our purposes.

We have obtained the structure functions presented in Figure 1 in the manner described in RL and detailed in the Appendix. The structure functions are plotted on log-log scales to facilitate analysis, and, more important, to help in the recognition of the three-structure function regimes: a noise regime at small lags, a structure regime characterized by a

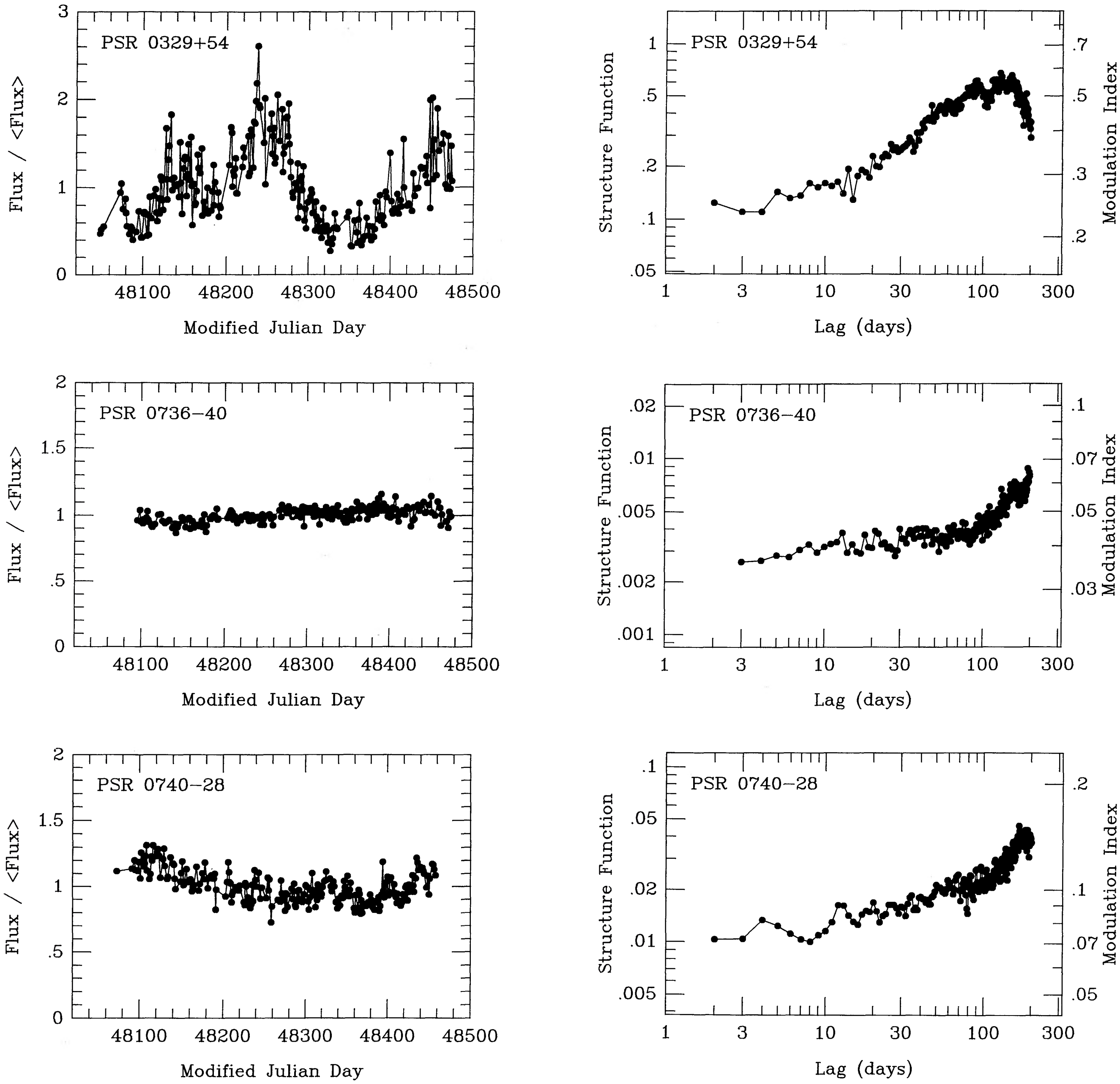

FIG. 1.—Flux time series and structure functions for the 14 pulsars. The fluxes plotted have been divided by the mean value for that time series. The right-hand ordinate axis on the structure function plots shows the modulation index corresponding to the value of the structure function.

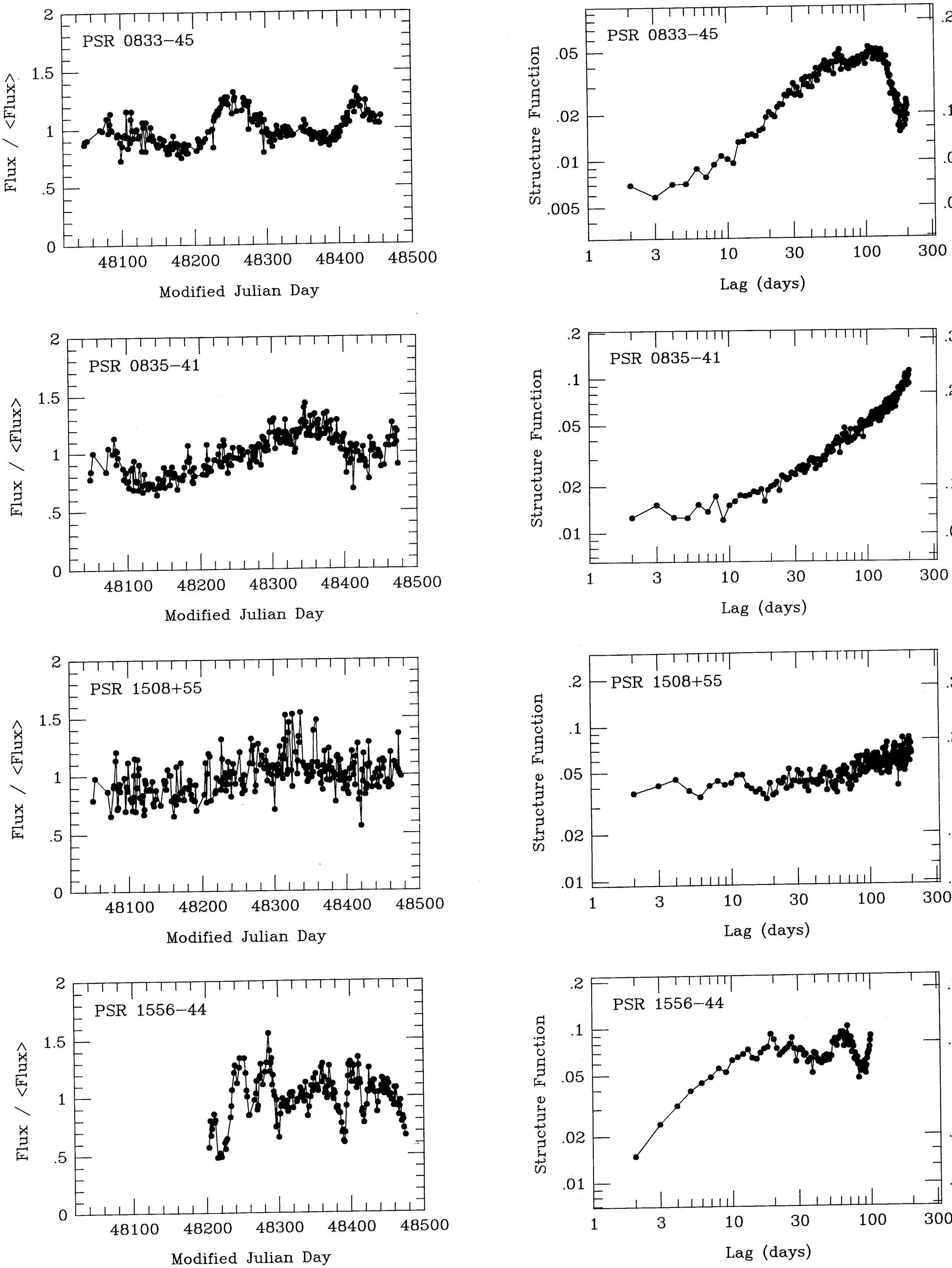

FIG. 1—*Continued*

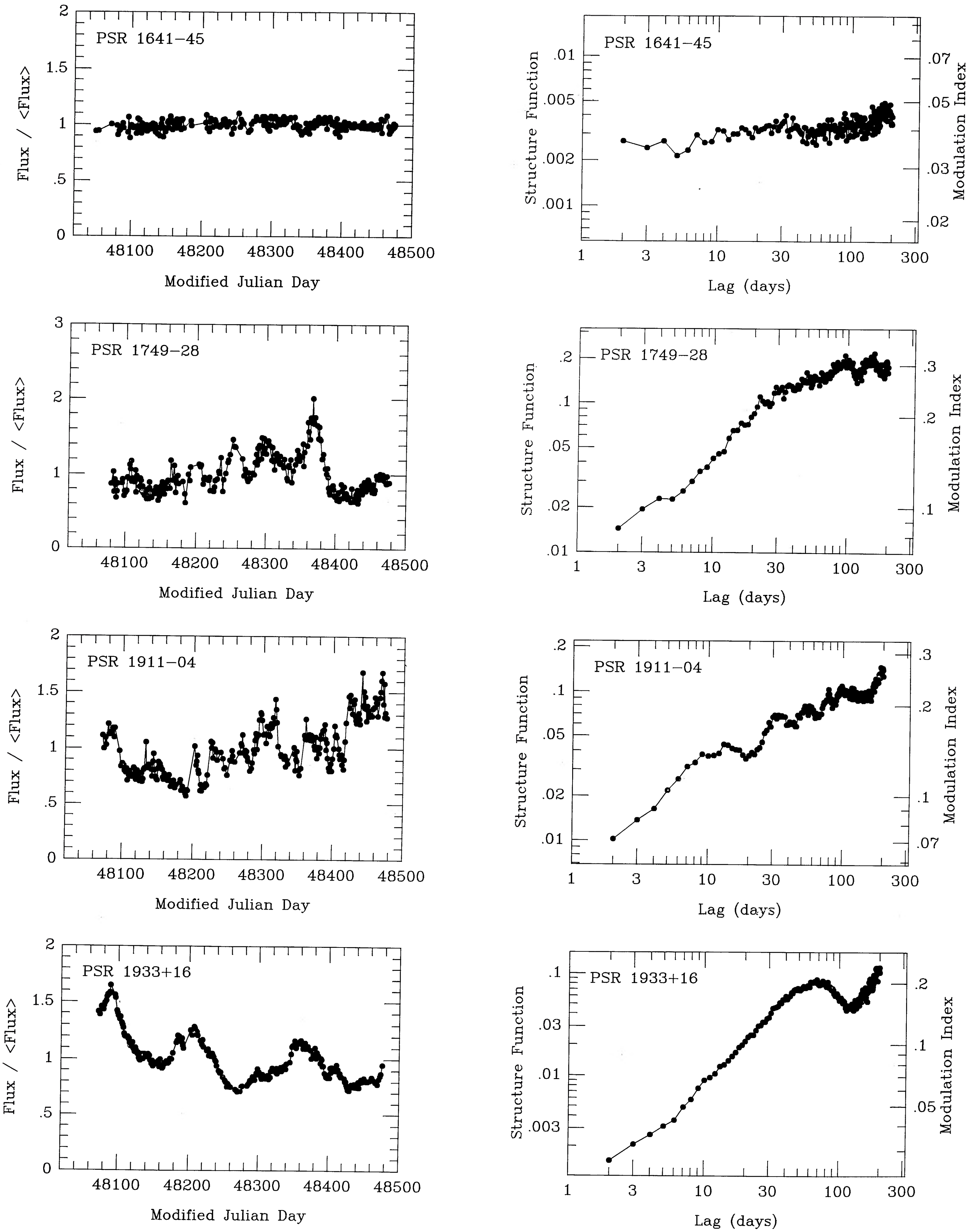

FIG. 1—*Continued*

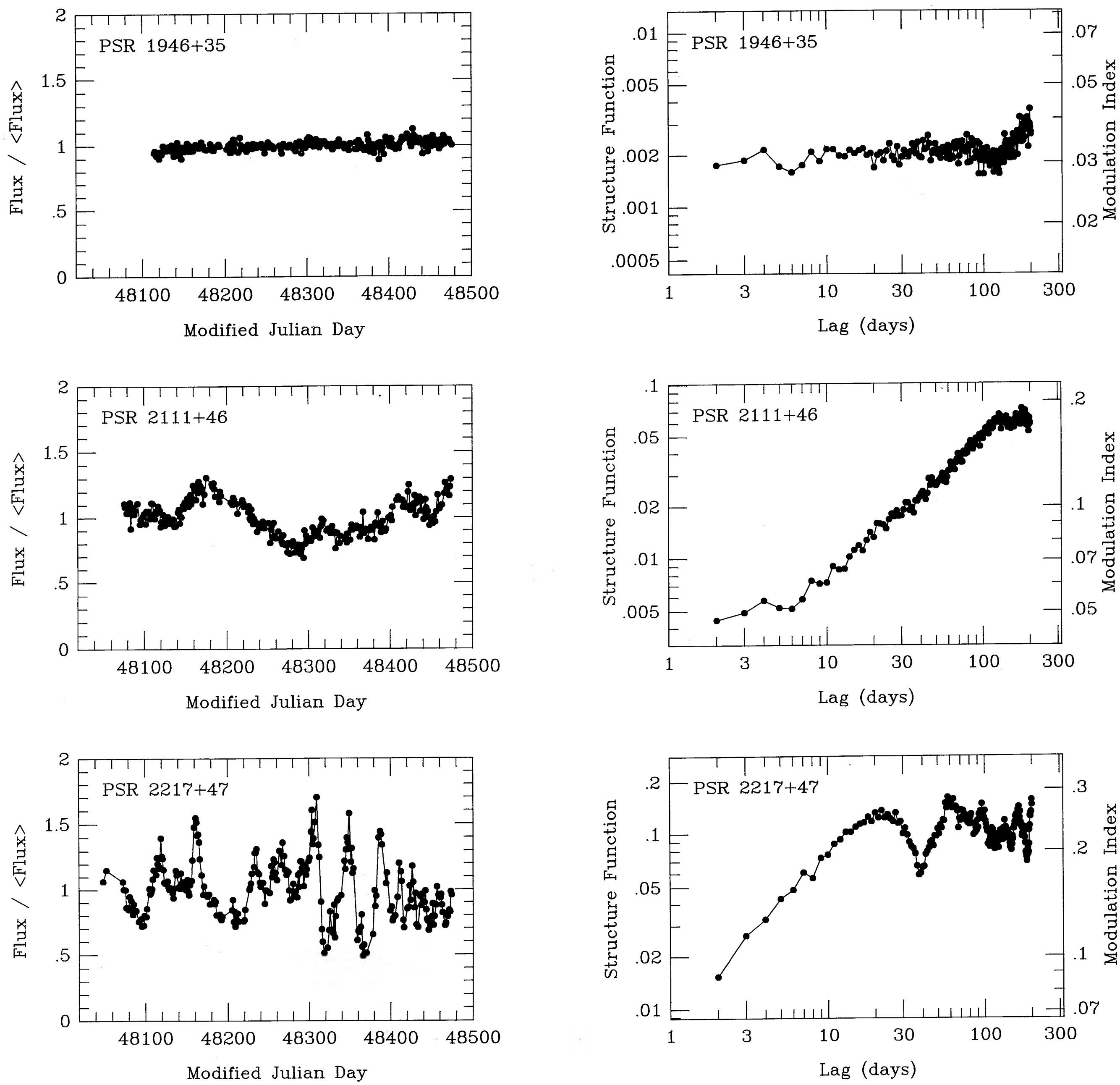

FIG. 1—*Continued*

linear slope in the log-log plot, and finally, a saturation regime where the structure function flattens out. For a more detailed description of structure functions, see SC and references therein. We use the structure functions to calculate our measured refractive parameters. We have used the RL definitions of refractive time scale and modulation index: the refractive time scale T_r is taken to be the lag corresponding to half the saturation value S of the structure function (see Fig. 2), and the modulation index is the square root of half the saturation value. The modulation index corresponding to the value of the structure function is shown on the right-hand ordinate axis in Figure 1. Minor differences in the details of RL's error analysis and ours are described in the Appendix.

Eight of the 14 pulsars studied show clear saturation regions in their structure functions. For these pulsars, we have calculated refractive time scales and modulation indices, and these can be found in Table 3. In addition, for pulsars having both well-defined saturation and structure regimes, we have calculated the logarithmic slope of the structure function at small lags using a least-squares fitting routine. The observed slopes are also presented in Table 3. The structure regime, for the purposes of calculating a slope, is taken as lying between T_1 and T_2 as shown in Figure 2. Because of the variation of the transitions between regimes among the structure functions, it was necessary to estimate the positions of T_1 and T_2 for each case by eye.

TABLE 1

DATA SPAN DURATIONS

Pulsar	Duration (days)
0329+54.........	426
0736−40.........	380
0740−28.........	386
0833−45.........	410
0835−41.........	426
1508+55.........	425
1556−44.........	273
1641−45.........	426
1749−28.........	397
1911−04.........	405
1933+16.........	405
1946+35.........	363
2111+46.........	399
2217+47.........	426

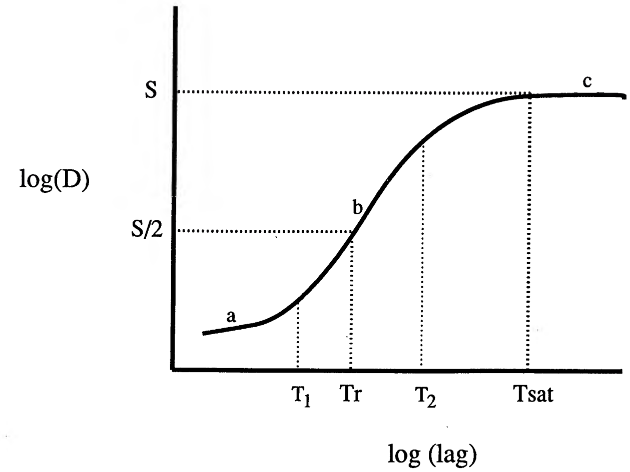

FIG. 2.—Schematic of the log-log plot of structure vs. lag. S is the saturation value, and T_r is the refractive time scale. The modulation index is equal to the square root of half the saturation value. The three structure function regimes are indicated: a is the noise regime, b is the structure regime, and c is the saturation regime.

Six pulsars show no saturation in their structure functions. For these pulsars, we have estimated lower limits for the observed modulation index and the observed refractive time scale by assuming that the modulation in the data would eventually reach the level of the least modulated time series that did reach saturation. The details of our estimation method are in the Appendix. The estimates are in Table 4.

Table 5 contains previously published scintillation parameters for all 14 pulsars. Values for the diffractive time scale T_d, decorrelation bandwidth $\Delta\nu$, scintillation velocity $V_{\rm iss}$ and C_n^2 are taken from CWB and Cordes (1986, hereafter C86). Where values from the two sources conflicted, we chose the value given by C86. Scaling of T_d and $\Delta\nu$ to our observing frequency of 610 MHz was done using the standard $\nu^{1.2}$ and $\nu^{4.4}$ dependencies, respectively, valid for a Kolmogorov exponent in the power spectrum of inhomogeneities; scaling for a different value of β is not appreciably different over the small extrapolation used here. Blank entries in the table occur where no published value is available. The distance estimates in Table 5 are from a recent distance model (J. Taylor, private communication). The diffractive scintillation parameters for 0740−28 were provided by A. Lyne (private communication), except for C_n^2 which we calculated using the formula from C86,

$$C_n^2 \approx 0.002\nu^{11/3}D^{-11/6}\Delta\nu^{-5/6}\,, \tag{2}$$

with the observing frequency ν in the GHz, the distance to the pulsar D in kpc, and $\Delta\nu$ in MHz.

In order to compare our observed refractive time scales, with theory, we estimated refractive time scales using the formula

$$T_r = K\,\frac{\sqrt{D/\Delta\nu}}{V_{\rm iss}/100}\,. \tag{3}$$

Here $\Delta\nu$ is the decorrelation bandwidth in MHz, $V_{\rm iss}$ is in km s^{-1}, and T_r is in days. Blandford & Narayan (1985) derive this formula for a spectrum having Kolmogorov exponent with $K \approx 8$. The physical significance of K lies in the assumed geometry of the pulsar-screen-Earth system. Blandford & Narayan (1985) derive equation (3) formally in the weak deflec-

TABLE 2

MEAN ABSOLUTE PEAK FLUXES

Pulsar	Flux (Jy)	δ Flux (Jy)	Catalog Value (Jy)
0329+54.........	87	52	61
0736−40.........	3.3	0.9	1.4
0740−28.........	4.3	1.4	3.5
0833−45.........	82	31	91
0835−41.........	11	3	9.0
1508+55.........	1.5	0.5	3.8
1556−44.........	3.8	1.4	2.1
1641−45.........	5.2	1.5	9.8
1749−28.........	35	11	52
1911−04.........	3.9	1.5	7.0
1933+16.........	6.2	1.7	7.6
1946+35.........	3.7	1.0	1.9
2111+46.........	1.2	0.4	6.4
2217+47.........	3.1	1.2	2.1

TABLE 3

MEASURED REFRACTIVE PARAMETERS

Pulsar	m	δm	$T_{\rm ref}$ (days)	$\delta T_{\rm ref}$ (days)	Slope	δSlope
0329+54......	0.39	0.07	10	7	1.01	0.03
0833−45......	0.11	0.01	11	4	1.08	0.05
1556−44......	0.16	0.02	3.1	0.5	0.98	0.07
1749−28......	0.26	0.05	18	3	1.43	0.05
1911−04......	0.20	0.03	13	8	1.41	0.07
1933+16......	0.18	0.04	28	5	1.31	0.02
2111+46......	0.16	0.05	44	20	1.00	0.01
2217+47......	0.21	0.02	5.4	1.0	1.05	0.04

TABLE 4

MEASURED LOWER LIMITS ON REFRACTIVE PARAMETERS

Pulsar	Modulation Index	$T_{\rm ref}$ (days)
0736−40.........	0.039	1042
0740−28.........	0.1	424
0835−41.........	0.062	749
1508+55.........	0.11	424
1641−45.........	0.026	1824
1946+35.........	0.021	1858

TABLE 5

KNOWN SCINTILLATION PARAMETERS

Pulsar	DM (pc cm^{-3})	Distance (kpc)	log ($\Delta\nu$) (MHz)	T_d (sec)	V_{iss} (km/s)	log C_n^2 (m$^{-20/3}$)	Reference
0329+54.........	27	1.35	−0.36	310	68	−3.86	1
0736−40.........	161	5.55	...	...	...	...	...
0740−28.........	74	0.76	−0.51	137	165	−2.84	2, 3
0833−45.........	69	0.55	−3.89	3	53	0.31	1
0835−41.........	148	3.19	−2.72	...	...	−2.35	3
1508+55.........	20	1.38	−0.57	71	128	−2.76	1
1556−44.........	59	1.25	...	...	...	...	...
1641−45.........	475	>5.32	−5.80	...	...	−0.01	3
1749−28.........	51	1.20	−1.29	320	15	−2.41	1
1911−04.........	89	2.29	−2.72	13	125	−2.16	1
1933+16.........	159	7.80	−2.74	18	120	−2.63	1
1946+35.........	129	7.39	−4.30	...	...	−1.36	3
2111+46.........	142	5.22	−2.57	28	80	−2.52	1
2217+47.........	44	2.31	−0.58	124	103	−3.30	1

REFERENCES.—(1) C86; (2) AGL; (3) CWB.

tion limit by integrating the received intensity over the solid angle subtended by the phase-changing screen, then assuming that the Earth moves relative to the screen. T_r is then the half-power width of the autocorrelation function for intensity fluctuations. SC derive the same formula, but with $K \approx 1.6$, using simple geometric arguments and rough estimates of the size of the electron-density fluctuations causing the scintillation. For simplicity, our predictions have $K = 1.0$.

For pulsars for which no scintillation velocity is known, we used $V_{iss} = 100$ km s^{-1}, but predictions arrived at in this way are to be regarded as extremely rough. The predicted modulation indices have been estimated using the expression

$$m \approx 1.10\left(\frac{\Delta\nu}{\nu}\right)^{0.17}, \tag{4}$$

valid to first order for a Kolmogorov spectrum (Romani, Narayan, & Blandford 1986). Predictions for modulation indices and refractive time scales arrived at in this way are given in Table 6. We stress that our predictions are to be regarded as order-of-magnitude estimates only, especially since most of the published diffractive scintillation parameters have large uncertainties.

Since our sample of pulsars includes a wide range of refractive time scales and modulation indices, it is most useful to consider each source separately and then draw general conclusions about the comparison between observations and theory. In each case, we refer to the flux time series and structure functions found in Figure 1.

TABLE 6

PREDICTED REFRACTIVE PARAMETERS

Pulsar	Modulation Index	T_{ref} (days)
0329+54.........	0.32	3
0736−40.........	...	...
0740−28.........	0.30	1
0833−45.........	0.081	120
0835−41.........	0.13	41
1508+55.........	0.30	2
1556−44.........	...	...
1641−45.........	0.038	1800
1749−28.........	0.22	32
1911−04.........	0.13	28
1933+16.........	0.13	55
1946+35.........	0.069	1900
2111+46.........	0.14	55
2217+47.........	0.29	3

3.1. *Discussion of Individual Sources*

3.1.1. *Stable Flux Time Series*

PSR 1641−45.—The structure function for this source may not even be out of the noise regime, indicating that the refractive time scale must be extremely long, in excellent agreement with the prediction of 1800 days. This unambiguously demonstrates that PSR 1641−45 is a stable continuum source on time scales of the order of the duration of our observations.

PSR 1946+35.—Like PSR 1641−45, this pulsar has a structure that still appears to be in the noise regime. The implied extremely long refractive time scale is again in good qualitative agreement with the prediction of 1900 days. Therefore, this pulsar is also a stable continuum source on time scales of the order of the duration of our observations.

3.1.2. *Pulsars in Good Agreement with Theory*

PSR 1749−28.—This structure function is clearly saturated. Its observed and predicted refractive time scales are in good agreement, as are its observed and predicted modulation indices.

PSR 1911−04.—Saturation is apparent in the structure function of this pulsar, but there are also several plateaus due to estimation noise in the structure function. The observed refractive time scale is in good agreement with the prediction of 28 days, and the observed modulation index agrees with the prediction.

PSR 1933+16.—This structure function has or is nearly saturated. The observed and predicted refractive time scales are in good agreement. The modulation observed is slightly larger than our prediction. We note the possible presence of a much longer term trend in the data.

PSR 2111+46.—The structure function appears to have just reached saturation. There is good agreement between the predicted and observed refractive time scales, and also between the predicted and observed modulation indices.

PSR 2217+47.—Saturation has also clearly occurred for this pulsar. There is excellent agreement between the predicted and observed refractive time scales, although the observed modulation is slightly lower than that predicted by theory. The dip in the structure function around lag of 30 days is due to the notable periodicity in the flux time series.

3.1.3. *Pulsars in Poor Agreement with Theory*

PSR 0329+54.—This pulsar has a relatively long diffractive time scale and a correspondingly short predicted refractive time scale. The structure function has clearly saturated. The measured refractive time scale, although having a large uncertainty is in fair agreement with the prediction. We note, however, the much longer observed time scale that is apparent in the flux time series. It is not seen in the structure function because of the relatively short duration of our data set. This longer term modulation is not predicted by theory. It may be explained in two ways. The predicted refractive time scale of 3 days is quite close to our minimum observable time scale: due to our sample rate of once per day the shortest time scale we are sensitive to is 2 days. Therefore, if the predicted time scale were correct, we would not expect it to be strongly confirmed by our observations. If the true refractive time scale were 3 days for this pulsar, then the longer time scale observed is of an unknown origin and may be due either to intrinsic flux changes or to an unknown interstellar medium effect. Alternatively, the longer time scale may in fact be the refractive time scale, and the discrepancy between it and the theoretical value is real and sizable.

PSR 0740−28.—No clear saturation is observed yet, but the noticeable modulation seen in the flux time series and the possible turn-over in the structure function at long lags suggest that this pulsar's refractive time scale is roughly equal to the duration of our observations. The predicted refractive time scale of 1 day when compared with our data, as in the case of PSR 0329+54, implies that either the refractive scintillation is buried in our noise and therefore that any modulation on much longer time scales is of unknown origin, or that the predicted refractive time scale is discrepant with our observations. The predicted modulation index for this pulsar is also much larger than what we observe; however, if the refractive time scale really is 1 day, then the refractive modulation is in the noise of the structure function and is much smaller than that predicted.

PSR 0833−45.—This structure function is clearly saturated, but the refractive time scale measured is not in agreement with the predicted value, and furthermore, does not agree with the time scale one would estimate by just looking at the time series. The latter does agree qualitatively with the predicted value. This may signal a problem with the method used to quantify the observed refractive time scale, but is more likely because of the relatively short duration of our observation. This is the only pulsar in our sample for which our technique of measuring the refractive time scale does not jibe at all with that estimated by eye. The predicted modulation index for this pulsar is much smaller than that observed. We note that this pulsar is atypical since it is imbedded in the Vela supernova remnant, and therefore has an environment different from the other pulsars in our sample, and quite different from the underlying assumptions that went into the theoretical predictions (see § 4).

PSR 1508+55.—Like the structure function of PSR 0740−28, the structure function in this case does not appear to be saturated. The predicted time scale is just below our minimum observable time scale, and so, if it really is 2 days, we do not expect to be able to observe it. Again, this implies that the apparent longer time scale modulation is of unknown origin. Alternatively, the longer time scale modultion may in fact be the true refractive time scale, and then our observed refractive time scale is more than 2 orders of magnitude different from the prediction.

3.1.4. *Pulsars Without Diffractive Parameters*

PSR 0835−41.—In this case, no clear saturation is observed but there is noticeable modulation in the flux time series suggesting that this pulsar's refractive time scale is roughly equal to the duration of our observations. This is not in good agreement with the predicted value. However, this pulsar's scintillation velocity is not known and so we may use the discrepancy to deduce that this pulsar's scintillation velocity is considerably less than 100 km s^{-1}. We do not yet have enough data to properly compare the predicted and observed modulation indices.

PSR 0736−40.—No saturation is observed in the structure function for this source. The structure function has begun to increase appreciably at longer lags, indicating that it is in the structure regime. There are no published diffractive scintillation parameters for this pulsar, and so no comparison with theoretical predictions is possible.

PSR 1556−44.—Saturation is evident in the structure function. Like PSR 0736−40, no diffractive scintillation parameters are available, and so no comparison can be made with theory.

To summarize, our observed refractive time scales and modulation indices for seven of our sample of 14, PSRs 1641−45, 1749−28, 1911−04, 1933+16, 1946+35, 2111+46, and 2217+47 are in good qualitative agreement with theory. Five of these are in good quantitative agreement, with the other two not having been observed long enough to allow a quantitative comparison. Two of the 14, PSRs 0736−40 and 1556−44 do not have the necessary parameters to allow any comparison, and PSR 0835−41's predicted refractive time scale is at best a very rough estimate. Four pulsars from our sample show effects that are unexpected but not necessarily in contradiction with refractive scintillation theory. PSRs 0329+54, 0740−28 and 1508+55 all have predicted refractive scintillation time scales that are close to our minimum observable time scale but show modulation at time scales much longer than would be predicted by refractive scintillation theory. PSR 0833−45 has a shorter refractive time scale than predicted and a larger modulation index than was expected.

4. DISCUSSION

The agreement between the predicted and the observed refractive time scales for PSRs 1749−28, 1911−04, 1933+16, 2111+46, and 2217+47 is surprisingly good, especially in light of the large uncertainties associated with many of the previously measured parameters used to calculate the predicted refractive values (see C86 and CWB). The predicted time scales in Table 6 were calculated using a value of 1.0 for the constant K in equation (3). We chose this value for simplicity, since the formulae for refractive time scale given by Blandford & Narayan (1985) and SC agree up to this constant. For the above five pulsars, we find that the best value for the constant K is ~ 1.5. This value is in good agreement with the value of 1.6 estimated by SC using heuristic arguments. The rough qualitative agreement with the theory demonstrates that in spite of the many possible complications such as those due to an extended phase-changing medium, variations in the power spectrum on different lines of sight, spatial variations of inner scales, or even the existence of deterministic structures along certain lines of sight, the simple model of the thin phase-changing screen half-way between the observer and a source of plane waves well represents the basic physics involved.

The four pulsars whose modulation does not seem to conform well with the theory cannot be ignored, however. Aside from the Vela pulsar for which the presence of a surrounding nebula most certainly complicates matters, it is probably not coincidence that the three shortest predicted time scales are the three that differ most radically from what we observe. As discussed previously, it is possible that the predicted refractive time scales for these pulsars are genuinely in error by two orders of magnitude; however, given the successes of the other predictions, it seems more likely that the predicted refractive time scales are correct or even slightly longer than the actual ones, and that our sampling of once per day precludes our being able to measure them. This does not explain the much longer modulations we observe in each case. Intrinsic fluctuation is a possibility, but the extremely stable fluxes observed in the time series of 1641−45 and 1946+11 suggest that pulsar fluxes are intrinsically stable. Although there may be a selection effect due to our being able to detect only extremely luminous pulsars at large distances, there is no compelling physical or empirical reason to expect luminous pulsars to have more stable fluxes than less luminous pulsars. Also, we have found no correlations between either refractive time scale or modulation index with intrinsic pulsar parameters such as period derivative. It may be the case that the diffractive parameters used to calculate the predicted values for these pulsars are in error. A remaining explanation for the unexpected modulation seen in PSRs 0329+54, 0740−28 and 1508+55 is that we have detected a new propagation effect that is characterized by a time scale much longer than the diffractive and refractive time scales that would only be detectable for these three pulsars given the duration of our present data span.

The modulation indices, in principle, are very useful for extracting information about the nature of the power spectrum of electron density inhomogeneities, since they are predicted to have very different behavior depending on whether β is less than or greater than 4. If β is greater than 4, the modulation indices are expected to be large, that is, near unity and independent of frequency and distance to the source (Goodman & Narayan 1985). Also, if β is less than 4 but there is a limiting inner scale of the order of 10^9 m, refractive scintillation is greatly enhanced, with modulation depths very similar to those obtained with the steep spectrum models (Coles et al. 1987). All the modulation indices we observed are modest in depth, with none greater than 0.4. This observation is in agreement with the findings of SC. This is strong evidence against both the steep spectrum and the presence of an inner scale on the order of 10^9 m. In addition, all our flux variations are smooth; that is, we observe no spikes or cusps in the flux time series. We therefore find no evidence for the strong focusing effects or "caustics" (Goodman et al. 1987) that are associated only with steep spectra.

Following the scheme of RL, we have plotted the logarithm of our observed modulation indices versus the strength of scattering parameter log $(\Delta\nu/\nu)$ in Figure 3. Lower limits to modulation indices for pulsars whose structure functions have not yet saturated are represented by arrows. The solid line indicates the prediction for a Kolmogorov spectrum, and the short-dashed lines indicate the predictions for the same spec-

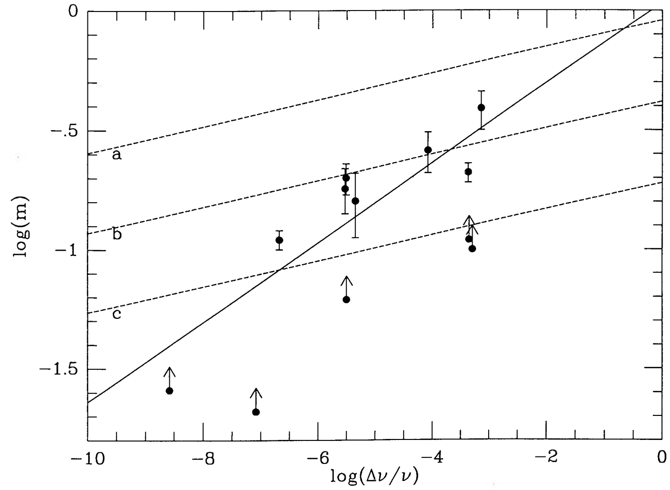

FIG. 3.—Modulation index vs. scattering parameter. The solid line corresponds to a Kolmogorov medium, while the dashed lines show predictions for inner scales of (*a*) 10^9 m, (*b*) 10^7 m, and (*c*) 10^5 m. Pulsars with unsaturated structure functions have lower limits on modulation indices indicated by arrows.

trum having various inner scales. Our data is clearly inconsistent with the inner scale of 10^9 m. The agreement with the Kolmogorov prediction is best, although we note that most of the established points lie slightly above it.

We have plotted modulation index versus distance in Figure 4 for those pulsars with known diffractive parameters and measured modulation indices. PSR 0833−45 has not been included on this plot. On the basis of this plot alone, we cannot exclude power spectrum exponents β greater than 4, since there is not a clear correlation between modulation index and distance. Fitting a line to these points on the log-log plot yields a slope of -0.22 ± 0.16. The modulation index for a power-law spectrum of exponent $\beta < 4$ has a dependence on distance to the source D that is given by

$$m \propto D^{-\beta(4-\beta)/2(\beta-2)} \tag{5}$$

(Blandford & Narayan 1985). We can use this expression and

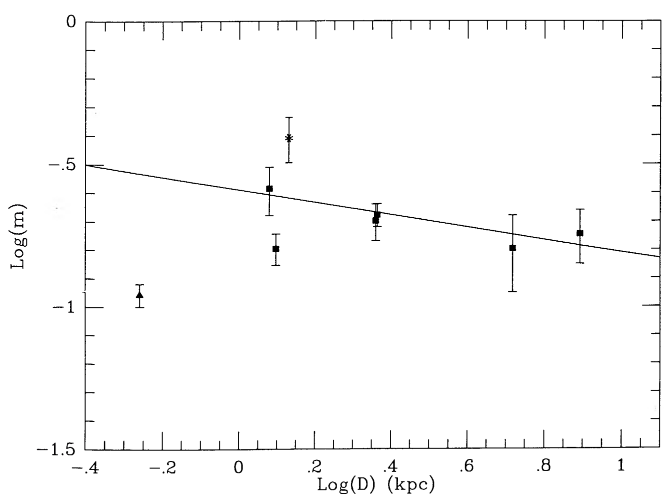

FIG. 4.—Modulation index vs. distance. The line indicates the best fit including all points except the triangle, which is PSR 0833−45. The star is PSR 0329+54.

our measured slope to calculate β. In this way, we obtain a value of $\beta = 3.7 \pm 0.1$, in excellent agreement with the Kolmogorov value of 11/3. However, the error on the measured slope is quite large, and if the measured modulation index for PSR 0329+54 (indicated by star on the plot) is excluded from the fit then the data is consistent with the modulation indices being independent of distance. Clearly, any correlation between modulation index and distance would be complicated by variations in the strength of scattering along different lines of sight.

The measured slopes of the structure functions in the structure regime for pulsars with a well-defined structure regime, given by Table 3, are consistent with a value of one. According to Blandford et al. (1986), the slope of the structure function is a signature of the nature of the medium: it is 1.0 for an extended medium and 2.0 for a single thin screen. Thus, the slopes of our structure functions are all consistent with that expected from an extended medium, with three exceptions: PSRs 1749−28, 1911−04, and 1933+16 have larger slopes, although still less than two. This suggests that the scattering medium along those lines of sight is clumpier than along the others, and therefore that they are better described by the thin screen model.

The steep spectrum and inner scale theories were developed in order to explain essentially four different observed phenomena: strong long-term modulation in pulsar fluxes (Cole, Hesse, & Page 1970; Huguenin, Taylor, & Helfand 1973; Rankin, Payne, & Cambell 1974; Helfand, Fowler, & Kuhlman 1977; McAdam 1981; Slee, Alurkar, & Bobra 1986), intensity spikes in pulsar fluxes (Cole, Hesse, & Page 1970; Helfand, Fowler, & Kuhlman 1977), multiple imaging and quasi-periodicities in pulsar dynamic spectra (Roberts & Ables 1982; Hewish, Wolszczan, & Graham 1985; Cordes & Wolszczan 1986; Wolszczan & Cordes 1987), and dramatic intensity fluctuations (Fiedler et al. 1987). The third can be understood by noting that steep spectrum models imply that the image on the scattering screen is patchy, having perhaps even a fractal nature and hence resulting in multiple images of the source. A Kolmogorov spectrum results in a smooth, Gaussian distributed image on the screen, and therefore is not expected to yield multiple images (Goodman & Narayan 1985).

With such a wealth of evidence suggesting the need for an enhanced refractive scintillation mechanism, the question naturally arises as to why our data, as well as that from other recent studies of pulsar flux time series (SC, RL) do not reproduce the old results, and instead find modest modulation depths. It is worth noting that in a few specific instances, data used to support a steep spectrum or an inner scale is actually rather ambiguous. Cole, Hesse, & Page (1970) observed several pulsars at 81.5 and 408 MHz, but do not report the bandwidth they used. Clearly, if their bandwidths were smaller, or on the order of the decorrelation bandwidths of their sources, diffractive scintillation would have significantly affected their results. For example, they observed PSR 1919+21, but the decorrelation bandwidth for this pulsar at 408 MHz is ~ 500 kHz, which could easily have been close to the size of their bandwidth. Rankin et al. (1974) measured large pulsar flux variations; however, the published data clearly indicate that the modulation depth is frequency dependent, which is inconsistent with $\beta > 4$. Slee, Alurkar, & Bobra (1986) report strong modulation of pulsar fluxes for observations from 1979 April to 1980 November but on average, they have no more than four independent observations of any pulsar, and so their results have large uncertainties.

Nevertheless, there is substantial evidence for either a steep spectrum, or an inner scale on the order of 10^9 m. In light of our results, we must therefore conclude that one or several of the following is true: (1) the inhomogeneities in the interstellar medium are anisotropic and observations along different lines of sight can yield very different results, (2) the actual spectrum is highly time variable and observations along the same line of sight at different times can yield very different results, or (3) some of the observations or the interpretations of some of the observations are inaccurate. The most plausible of these choices is (1), since it seems entirely possible that there exist localized regions in our Galaxy having enhanced turbulence, or regions containing aggregates of discrete objects, that could play a major role in refractive effects. Thus, the spectrum of electron density inhomogeneities in the interstellar medium may have a Kolmogorov exponent overall, but there exist regions that deviate from this description.

5. CONCLUSIONS

We have found that refractive interstellar scintillation contributes significantly to the flux modulation of pulsars on time scales predicted by theory. We have shown that strong pulsars at large dispersion measures are stable continuum sources. This may be true of pulsars in general. Our data are best described by a Kolmogorov power spectrum for electron density inhomogeneities in the interstellar medium, and we find no compelling reason to introduce an inner scale. We note that four of the pulsars in our sample show modulation on time scales not predicted by theory.

Our results should improve with time. While we continue to take data and lengthen our flux time series in order to better estimate refractive time scales and modulation indices, we remain sensitive to unusual and dramatic flux variations such as those observed in the flux time series for quasar 0954+658 in 1981 (Fiedler et al. 1987). We note that the absence of such enhanced scattering events in our data set is consistent with previous observations and underscores their rarity.

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APPENDIX

1. STRUCTURE FUNCTION ANALYSIS

We follow closely the method described by RL in the Appendix to their report. We define our structure function as

$$D_k = \frac{1}{M^2} \sum_{j=1}^{n} \frac{(I_j - I_{j+k})^2}{W_k}, \tag{6}$$

where W_k is the total number of data pairs with lag k days, and M is the mean of the flux time series. I_j is the value of the flux on the jth day. Note that our definition incorporates the mean M of the flux time series, which has an associated uncertainty δM. The refractive time scale T_r is the lag corresponding to half the saturation value S, which is the variance V:

$$D_{T_r} = S/2 = V . \tag{7}$$

The modulation index m is the square root of V. The saturation time $T_{\rm sat}$ is the time such that

$$D_{k > T_{\rm sat}} = S . \tag{8}$$

$T_{\rm obs}$ is the total duration of the time series.

In all our time series, we conservatively chose the fractional error in S to be

$$\frac{\delta S}{S} \approx \sqrt{\frac{2T_r}{T_{\rm obs}}}, \tag{9}$$

since $\sigma_s/(T_{\rm sat}/T_r)^{0.5}$, an estimate used by RL, was always smaller. Note that the fractional error in S is equal to the fractional error in V. The estimated error on the refractive time scale was found from the lags at $(S + \delta S)/2$ and $(S - \delta S)/2$.

Our error estimate for the modulation index m differs slightly from that used by RL. We conservatively estimate the fractional error in m as the sum of the errors due to fractional errors in V and M. We estimate the fractional error in M as

$$\frac{\delta M}{M} \approx \frac{m}{\sqrt{T_{\rm obs}/T_r}}, \tag{10}$$

and since we use equation (9) as the fractional error in V, our estimate for the fractional error in m is

$$\frac{\delta m}{m} \approx \sqrt{\frac{T_r}{T_{\rm obs}}}\left(m + \frac{1}{\sqrt{2}}\right). \tag{11}$$

Our method of measuring the refractive time scale, as described above, introduces a slight bias toward shorter refractive time scales. Prior to calculating the variance, we subtracted a correction for noise from the saturation value of the structure function. The correction for noise was taken to be the value of the structure function at one day of lag. Since the refractive time scale was measured to be the lag corresponding to half the saturation value minus the noise correction, and since the noise correction was not made to the structure function itself, the reading of refractive time scale was slightly biased toward the low end. The effect is most noticeable in pulsars with large noise corrections.

A further point to note regarding this analysis is that the values of the structure function at lags larger than $T_{\rm sat}$ are not true independent estimates of the saturation value; while structure function values are calculable using equation (6) up to the lag equal to $T_{\rm obs} - 1$, the information contained in each value decreases with increasing lag. We have chosen to consider only structure function values out to lags of 200 days since that is approximately half the duration of the time series.

2. ESTIMATING LOWER LIMITS OF REFRACTIVE PARAMETERS FOR PULSARS THAT HAVE NOT YET SATURATED

To obtain a lower limit for the refractive modulation index, we subtract the noise variance from the variance of the flux time series. The noise variance, due to diffractive scintillation and random noise, is estimated from half the value of the structure function at one day of lag. Thus, $m_<$, our lower limit on the refractive modulation index, is equal to

$$m_< = \sqrt{V - \frac{D(1)}{2}} . \tag{12}$$

As an estimate of a lower limit for the observed refractive time scale, we use

$$T_r > T_{\rm obs} \frac{m_r}{m_<}, \tag{13}$$

where we choose a value of 0.11 for m_r, which is the smallest modulation index we have observed for the pulsars that have structure functions that did saturate.

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