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USING GAMMA-RAY BURSTS TO DETECT A COSMOLOGICAL DENSITY OF COMPACT OBJECTS

O. M. BLAES AND R. L. WEBSTER[1]
Canadian Institute for Theoretical Astrophysics, University of Toronto, 60 St. George Street, Toronto, Ontario, Canada M5S 1A7
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ABSTRACT

If gamma-ray bursts come from cosmological distances, then a significant fraction of them will be lensed. Multiple images will be detected by the time delays between the two images. The shortest bursts are sensitive to lens masses $\gtrsim 250\ M_\odot$. The observed fraction of double images provides a direct measure of $\Omega_{\rm compact}$ as a function of the assumed maximum redshifts of the burst sources. The results depend weakly on the cosmological model and the burst spectra.

Subject headings: cosmology: theory — dark matter — gamma rays: bursts — gravitational lensing

1. INTRODUCTION

Recent results from the BATSE experiment on board the *Compton Gamma Ray Observatory* (Meegan et al. 1992) show that gamma-ray burst (GRB) source counts are flatter than expected for a homogeneous population in Euclidean space and yet are still isotropic on the sky. As argued forcefully by Paczyński (1991b), a plausible explanation is that the GRB sources are cosmological, with redshifts $z_s \sim 1$. Assuming isotropic emission, the sources then emit $\sim 10^{51}$ ergs in gamma rays, making them energetically comparable to supernovae. An alternative explanation in terms of two more local populations, Galactic disk and halo neutron stars, has been put forward by Lingenfelter & Higdon (1992). The latter is perhaps easier to reconcile with the observations of soft X-ray emission and so-called cyclotron lines from some GRBs (e.g., Higdon & Lingenfelter 1990, and references therein). Nevertheless the cosmological hypothesis is an exciting possibility, and perhaps a point in its favor is the fact that the current problems with Galactic source models were predicted before BATSE started collecting data (Paczyński 1991a).

In an elegant paper, Press & Gunn (1973) suggested that a cosmologically significant density of dark compact objects in the universe could be detected by gravitational lensing of more distant sources. The probability of lensing depends purely on $\Omega_{\rm compact}$, the fraction of the critical density in compact objects, and is independent of the individual lensing mass M_d. In contrast, the image separations and time delays associated with each lens depend mainly on M_d, with only a weak dependence on $\Omega_{\rm compact}$. Using this method, optical observations of quasars have imposed density limits of $\Omega < 0.25$ on compact objects with masses exceeding $\sim 10^{10.3}\ M_\odot$ (Nemiroff 1991). In addition, VLBI observations of compact radio sources have put a limit of $\Omega < 0.4$ in the mass range 10^7–$10^9\ M_\odot$ (Kassiola, Kovner, & Blandford 1991).

As first discussed by Paczyński (1986), if GRBs are indeed cosmological, they provide an important new tool in the search for dark matter in the form of compact objects. Although the angular resolution of gamma-ray detectors is very poor, the short durations of GRBs (between 10^{-2} and 10^3 s, Higdon & Lingenfelter 1990), unique as far as we know among extragalactic sources, enable them to probe mass scales in the range 10^3–$10^8\ M_\odot$ by the time delay between the lensed images of the burst. GRBs are also perhaps the ideal sources for gravitational lensing because the universe is transparent in the MeV range out to high redshifts (e.g., Zdziarski & Svensson 1989), and hence they will not be obscured as quasars might be by dust, say, in the optical. Averaged over time, BATSE covers all portions of the sky with approximately equal sensitivity, so there are no constraints imposed by finite angular coverage of the sky as there are in quasar catalogs. The primary disadvantage with GRBs compared to, say, quasars is, however, that their redshifts and intrinsic luminosities are unknown.

Theoretical studies of lensing of GRBs are not new. Paczyński (1986, 1987) proposed that the soft repeaters GBS 1900+14 and GBS 1806−20 were produced by gravitational lensing and microlensing respectively of single intrinsic bursts at cosmological distances. More recently Gould (1992) proposed that interference between lensed images of a GRB with very short time delays could be used to probe dark matter mass scales in the range 10^{-13} to $10^{-16}\ M_\odot$. Paczyński (1991b) has argued that lensing of bursts by foreground galaxies would produce repetitions on a ~ 1 month time scale, and that this could be used to test the hypothesis of a cosmological origin of GRBs. Mao (1992) has calculated the expected distribution of time delays for sources at a number of different redshifts due to foreground galaxies and point masses, and estimates a probability of repetition of 0.04%–0.4% in the former case.

In this paper we describe in detail how to use GRBs to explore cosmological clumped dark matter. We discuss the individual lensing effects of such matter in § 2. In § 3 we point out the useful properties of the GRB source population which may be inferred or plausibly assumed from the observations, assuming that they are at cosmological distances. We estimate the probability that detectable lensing will occur in § 4, and we also discuss what may be deduced about the lens and the source should such lensing be seen. This in fact may already have occurred, and we suggest some possible candidates.

2. THE LENS POPULATION

Consider for simplicity a cosmologically nonevolving population of massive clumps with present-day density, scaled with the critical density, equal to $\Omega_d(M_d)dM_d$ in the mass range M_d

[1] Current address: School of Physics, University of Melbourne, Parkville, Victoria, Australia 3052.

to $M_d + dM_d$. It is this function that we wish to determine by observations of GRBs. It is normalized such that $\int_0^\infty \Omega_d(M_d)dM_d = \Omega_{\rm compact} = (1 - \tilde{\alpha})\Omega$, where Ω is the current value of the density parameter of the universe and $\tilde{\alpha}$ is the fraction of mass which is homogeneously distributed throughout the universe.

Define $D_{\rm os}$, $D_{\rm od}$, and $D_{\rm ds}$ to be the angular diameter distances between the observer and the source, the observer and the lens (deflector), and the lens and the source, respectively. The dependence of these distances on redshift z is given by integrating the equation (Dyer & Roeder 1973),

$$(z+1)(\Omega z+1)\frac{d^2D}{dz^2} + \left(\frac{7}{2}\Omega z + \frac{1}{2}\Omega + 3\right)\frac{dD}{dz} + \frac{3}{2}\tilde{\alpha}\Omega D = 0 \tag{1}$$

from z_i to z_j, with initial conditions

$$D_{ii} = 0 \quad \text{and} \quad \left(\frac{dD_{ij}}{dz}\right)_{z_j=z_i} = \frac{c}{H_0}\frac{\text{sgn}\,(z_j - z_i)}{(z_i+1)^2(\Omega z_i + 1)^{1/2}}\,. \tag{2}$$

The nature of the dark matter is unknown, in particular the mass distribution inside any putative clumps. For the moment we again opt for simplicity and model the clumps as ideal point masses (black holes). In this case all sources at greater distances than the clump will generally produce two images with a well-defined intensity ratio and time delay if the source emission is isotropic. If the source is directly behind the lens, then an Einstein ring is produced with angular radius $\alpha_{\rm E} = (2R_{\rm S}D_{\rm ds}/D_{\rm os}D_{\rm od})^{1/2}$, where $R_{\rm S} = 2GM_d/c^2$ is the Schwarzschild radius of the lens.

Define $\theta_s \equiv \alpha_{\rm E}\, y$ to be the angle between the true lines of sight to the source and the lens. Then the magnifications of the two images are

$$\mu_{1,2} = |1 - y_\mp^4|^{-1}\,, \quad y_\pm \equiv \frac{(y^2+4)^{1/2} \pm y}{2}\,, \tag{3}$$

where the subscripts 1 and 2 refer to the brighter and fainter of the images, respectively. The delay between burst arrival times along the two image directions is

$$\Delta t = \frac{R_{\rm S}}{c}(1+z_d)\left[y_+^2 - y_-^2 + \ln\left(\frac{y_+^2}{y_-^2}\right)\right]. \tag{4}$$

Typically, $\Delta t \sim 4R_{\rm S}/c$. Since the shortest duration bursts are ~ 10 ms, the smallest mass which can be probed is $\sim 250\, M_\odot$.

The number of lenses with redshifts around z_d and masses around M_d is (cf. Ehlers & Schneider 1986)

$$dN_d = 3\Omega_d(M_d)\frac{(1+z_d)}{(1+\Omega z_d)^{1/2}}\frac{D_{\rm od1}^2 H_0}{R_{\rm S}c}\,dz_d\,dM_d\,, \tag{5}$$

where $D_{\rm od1}$ is the angular diameter distance to a source at redshift z_d in a smooth universe with no clumping ($\tilde{\alpha} = 1$ in eq. [1]).

3. THE SOURCE POPULATION

Equations (3)–(5) contain all the information necessary to determine the effects of a population of point-mass lenses on the observed properties of more distant sources with known intrinsic properties. Unfortunately, we know very little about the intrinsic properties of GRBs, in particular their individual redshifts (if indeed they are cosmological!). We are therefore forced to make some assumptions concerning the source population in order to place constraints on the unknown lensing population.

The BATSE source counts are measured in terms of the observable $\tilde{C}$, defined as the peak photon count rate per unit area, $C_{\rm max}$, divided by the minimum, $C_{\rm min}$, which would have been detectable at the time. Both are measured in the fixed energy range $E_1 = 50$ keV to $E_2 = 300$ keV of the BATSE trigger window (Meegan et al. 1992). The observed faint differential counts distribution may adequately be described by a power law of the form $dN/d\tilde{C} \propto \tilde{C}^{\gamma-1}$, with $\gamma \sim -0.8$ and with bounds $\tilde{C}_{\rm max} \simeq 100$ and $\tilde{C}_{\rm min} = 1$ (Meegan et al. 1992). The counts for the rarer, brighter GRBs above $\tilde{C}_{\rm max}$ steepen to $\gamma \sim -1.5$. Mao (1992) estimates that amplification bias by any putative lenses may have moderately important effects on the source counts. We believe, however, that the observed flatness of the source counts indicates that amplification bias is unimportant. Very briefly (cf. Schneider 1992; Webster 1992), if dN_o and dN_i are the observed and intrinsic source counts, respectively, then $dN_o(\tilde{C}) = \int_0^\infty dN_i(\tilde{C}/\mu)p(\mu)/\mu\, d\mu = dN_i(\tilde{C})\int_0^\infty \mu^{-\gamma}p(\mu)d\mu \sim dN_i(\tilde{C})$ if $\gamma \gtrsim -2$, where $p(\mu)$ is the amplification function which for high-amplification events is generically dominated by sources near fold caustics [$p(\mu) \sim \mu^{-3}$]. In other words, the luminosity function and spatio-temporal distribution of GRB sources is such that they would produce the observed counts independently of the clumpiness in the universe.

If we know the intrinsic photon emission rate of a source, then we can calculate the redshift necessary to produce, on average over all positions on the sky, an observed peak count rate, independent of the clumpiness of the universe (Weinberg 1976). Because there is no amplification bias, the observed source counts can be used to infer the source redshift distribution, *independent* of any further assumptions about the redshift dependence of the number or burst rate of the sources. This is important because all known populations of sources are known to evolve with redshift.

We still need to adopt an intrinsic photon emission rate for the source population, and for the purposes of simplicity and illustration we assume that the sources are standard candles with identical power-law spectra (photons s^{-1} keV^{-1}) in photon energy E, $\dot{N}_E = \dot{N}_0 E^{-\alpha}$. In reality the observed spectra vary widely both between and within bursts. For definiteness we adopt a fixed observing window between photon energies E_1 and E_2. Here we may take α to lie between 0.8 and 1.5 (Higdon & Lingenfelter 1990). Our results turn out not to depend too sensitively on the spectral shape, and in any case the calculation could be improved by folding in an ensemble of observed spectra at the time of the peak count rates.

More serious is the fixed choice of scale $\dot{N}_0$. It may turn out that like supernovae, GRB sources do make good standard candles, but the enormous variety in their light curves suggests a broad intrinsic spread at least in their peak count rates. Standard candles again serve to illustrate the calculation, however, and we return to the possibility of a broad luminosity function below.

With the above assumptions, the observed count rate per unit area at BATSE, averaged over all positions on the sky, is

$$\tilde{C} \equiv \frac{C_{\rm max}}{C_{\rm min}} = \frac{1}{4\pi D_{\rm so1}^2 C_{\rm min}}\int_{(1+z_s)E_1}^{(1+z_s)E_2} dE\,\frac{\dot{N}_0 E^{-\alpha}}{(1+z_s)}\,, \tag{6}$$

or

$$\tilde{C} = \tilde{C}_0(1+z_s)^{-\alpha-2}\left(\frac{c}{H_0 D_{\rm os1}}\right)^2 = \left(\frac{1+z_{s\max}}{1+z_s}\right)^{\alpha+2}\frac{D_{\rm os1}^2(z_{s\max})}{D_{\rm os1}^2}\,. \tag{7}$$

Here

$$\tilde{C}_0 \equiv \frac{\dot{N}_0}{(\alpha-1)}\,(E_1^{1-\alpha} - E_2^{1-\alpha})\,\frac{H_0^2}{4\pi c^2 C_{\min}}\,, \tag{8}$$

$D_{\rm os1}$ is the angular diameter distance to the source in a smooth $\tilde{\alpha} = 1$ universe, and $z_{\rm smax}$ is the maximum source redshift detectable, on average, by BATSE. Equation (7) may be used to convert the observed source counts to a redshift distribution. The main uncertainty lies in choosing $z_{\rm smax}$ or equivalently the intrinsic source emission rate $\tilde{C}_0$.

4. LENSED BURSTS

We are now ready to calculate the fraction of BATSE bursts with lensed images. To be detectable, the second image must be bright enough to be above the minimum detectable count rate, that is $\mu_2 D_{\rm os1}^2 \tilde{C}/D_{\rm os}^2 > 1$. Hence from equation (3), the line of sight to the source must pass sufficiently close to the lens, with

$$y < y_{\max}(z_s) \equiv \left(1 + \frac{D_{\rm os1}^2\,\tilde{C}}{D_{\rm os}^2}\right)^{1/4} - \left(1 + \frac{D_{\rm os1}^2\,\tilde{C}}{D_{\rm os}^2}\right)^{-1/4}. \tag{9}$$

Furthermore, we also need to ensure that the time delay between the two images is larger than Δt_m, the larger of the instrumental time resolution and the burst width. This implies that the source cannot be too close to the lens, that is, $y > y_{\min}(M_d, z_d)$, determined from equation (4). Each lens therefore has a cross section on the source sphere $z = z_s$ to produce detectable lensed images given by $\sigma = \pi \alpha_{\rm E}^2 D_{\rm os}^2 (y_{\max}^2 - y_{\min}^2) H(y_{\max} - y_{\min})$, where H is the unit step function.

Assuming that the probability of lensing is sufficiently small that the individual lens cross sections do not overlap, then the fraction of sources with double peaks caused by lenses of mass around M_d is given by integrating $\sigma dN_d/4\pi D_{\rm os1}^2$, or

$$f(M_d)dM_d = \Omega_d(M_d)dM_d\,\frac{3\gamma}{2(\tilde{C}_{\max}^{\gamma} - 1)}\int_1^{\tilde{C}_{\max}} d\tilde{C}\tilde{C}^{\gamma-1}$$
$$\times \int_0^{z_s} dz_d\,\frac{(1+z_d)}{(1+\Omega z_d)^{1/2}}\,\frac{H_0 D_{\rm ds} D_{\rm os}}{c D_{\rm od}}\,\frac{D_{\rm od1}^2}{D_{\rm os1}^2}$$
$$\times (y_{\max}^2 - y_{\min}^2) H(y_{\max} - y_{\min})\,. \tag{10}$$

We have restricted consideration to the faint bursts with power-law counts, although the actual counts over the entire observed sample could be used. This equation implies that $f(M_d)/\Omega_d(M_d)$ has a weak dependence on lensing mass due to the finite time resolution Δt_m of the observations (Press & Gunn 1973). The effect of this is to produce a cutoff in the detectable lensed fraction for sources with $c\Delta t_m > R_{\rm S}$.

Assuming for the moment that all clumps have $R_{\rm S} \gg c\Delta t_m$, we may integrate equation (10) over M_d to find the total fraction F of lensed bursts as a function of $\Omega_{\rm compact}$. This is depicted in Figure 1 for various source parameters. If GRBs are in fact cosmological, that is, $z_{\rm smax} \gtrsim 1$, then a substantial portion of them ($> 10^{-3}$) will be lensed by any cosmologically significant ($\Omega_{\rm compact} \gtrsim 0.1$) population of clumps. Measuring this fraction will place constraints on both $\Omega_{\rm compact}$ and $z_{\rm smax}$. Note that replacing standard candles with broad luminosity function sources will tend to increase F, because then some of the bright observed sources will be at higher redshift and therefore will tend to have a larger probability of being lensed.

Gravitational lensing provides a possible explanation for the difficult problem as to how cosmological GRBs can produce light curves consisting of two or more peaks separated by some tens of seconds with no detectable emission. Indeed, because they are relatively clean, those GRBs with double-peaked light curves are the best ones to search for evidence of lensing. A corollary of the lensing interpretation is that many bursts should be seen with just single spikes, which is indeed the case.

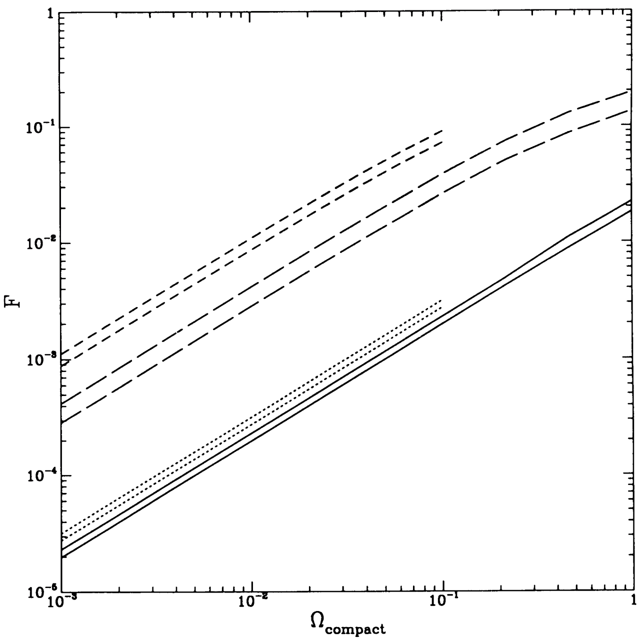

FIG. 1.—Fraction of bursts in which detectable lensing has occurred as a function of the present-day density parameter of compact objects. The burst durations are taken to be shorter than the time delays produced by the lenses. The upper and lower of each pair of curves correspond to $\alpha = 1.5$ and 0.8, respectively. Otherwise the different curves correspond to $z_{\rm smax} = 1$ and $\Omega = 1$ (*solid*), $z_{\rm smax} = 10$ and $\Omega = 1$ (*long dashed*), $z_{\rm smax} = 1$ and $\Omega = 0.1$ (*dotted*), and $z_{\rm smax} = 10$ and $\Omega = 0.1$ (*short dashed*).

If the source emission is isotropic, the point-mass lens produces a light curve in which the brighter image *always* precedes the fainter. Observed light curves in which the main, bright burst follows the fainter can be produced by more generic lens mass models. For example a spherical lens with a finite core radius produces a circular caustic in the source plane. Outside the caustic the source produces a single image, but as the source crosses the caustic, two additional images are born. At this point these new images are quite bright and are delayed relative to the fainter, primary image. Although rare, such a configuration could produce a burst light curve in which a faint precursor precedes two bright bursts which may or may not be resolved depending on the individual burst durations. An example might be GRB 910522, which consisted of a precursor which preceded the main burst by 110 s (G. J. Fishman 1991, private communication). More general, aspherical lenses can produce multiple images.

It may be difficult to distinguish intrinsic source variability from time delays between multiple lensed images. Ideally, if the source is an isotropic point emitter, then the images will be scaled (in count rate) exact copies of each other in both light curve and spectrum. If the time delays are longer than the duration of each image, then the case for lensing can therefore be made quite strong.

Provided the GRB source is smaller than the size of the Einstein ring projected on to the source plane, $\sim 10 M_{d6}^{1/2}$ pc with M_{d6} being the lens mass in units of $10^6\ M_\odot$, then the source can in fact be treated as pointlike. Whatever their origin, cosmological GRBs are likely to involve bulk relativistic motions with high Lorentz factor γ_L (Krolik & Pier 1991). An upper limit to the source size is $\sim\gamma_L^2 c\Delta t_v$, where Δt_v is the observed variability time scale which can be as short as a few ms (e.g., Higdon & Lingenfelter 1990). A sufficient condition for the source to be pointlike as far as lensing is concerned is therefore that $\gamma_L^2 \Delta t_v \lesssim 10^9 M_{d6}^{1/2}$ s. Even then the source might be anisotropic, and each image would then be effectively viewing sources with different luminosities and temporal behavior (cf. Babul, Paczyński, & Spergel 1987; Paczyński 1987). The angle between the two image rays at the source is $\sim M_{d6}^{1/2}$ milli-arcsec. Radiating particles will beam their emission into a cone of half-angle $1/\gamma_L$, implying that the source will be isotropic as far as lensing is concerned if $\gamma_L \lesssim 10^8 M_{d6}^{-1/2}$. Provided γ_L is not too high, the idealization of isotropic point sources is therefore justified.

The BATSE burst GRB 910503 (G. J. Fishman 1991, private communication) is a beautiful example of a double-peaked burst which might be a product of lensing. The first "image" precedes the second by approximately 45 s, and is a factor 4 brighter in peak count rate. The light curves of each "image" are very similar, although the first has more structure, and each lasts for 10 s. Comparison of the photon spectra should be made to strengthen (or weaken) the case, but in any case we can take this as an illustrative example. Defining R as the brightness ratio, and noting that $y_+ y_- = 1$, equation (3) implies $R = y_+^4$ or $y = R^{1/4} - R^{-1/4}$. Hence for a point-mass lens, the source position $y = 0.7$. Using equation (4), the time delay then implies a lensing mass of $\simeq 2 \times 10^6 (1 + z_d)^{-1}\ M_\odot$.

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