

COSMOLOGICAL “SEED” MAGNETIC FIELD FROM INFLATION

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ABSTRACT

We extend the canonical exponential potential scalar field inflation modified hot big bang model by adding an inflation epoch coupling, between the scalar field Φ responsible for inflation and an Abelian gauge field A_μ , $\propto e^{\alpha\Phi} F_{\mu\nu} F^{\mu\nu}$ (where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and α is a parameter). Requiring that the homogeneous part of the scalar field “freeze” at a constant value at the inflation-radiation (reheating) transition, we find that it is possible to generate a present intergalactic magnetic field of strength (depending upon the values of parameters of the model) between $\lesssim 10^{-65}$ and $\gtrsim 10^{-10}$ G, on a scale of 1/1000 the present Hubble radius. The larger values provide an example of a primeval seed magnetic field that might be amplified by anisotropic protogalactic collapse and galactic differential rotation to produce the observed galactic magnetic fields.

Subject headings: cosmology: theory — early universe

1. INTRODUCTION

There is persuasive observational evidence for large-scale intragalactic magnetic fields of μ G (microgauss) strength uniform on the ~ 10 kpc scale (Sofue, Fujimoto, & Wielebinski 1986; Asseo & Sol 1987; Ruzmaikin, Shukurov, & Sokoloff 1988; Hummel et al. 1988; Krause, Beck, & Hummel 1989a; Krause, Hummel, & Beck 1989b and references therein). There is evidence for somewhat larger scale magnetic fields of similar strength within clusters (Perley 1990; Kim, Tribble, & Kronberg 1991 and references therein). For observational studies of still larger scale magnetic fields see Giovannini et al. (1990) and Vallée (1990), and for the evidence for μ G magnetic fields in protogalactic objects at larger redshifts ($\lesssim 3$), see Watson & Perry (1991), Kronberg, Perry, & Zukowski (1992), and Wolfe, Lanzetta, & Oren (1992). These magnetic fields are believed to result from the amplification of earlier, small, “seed” magnetic fields.

The two most studied phenomenological processes for the amplification of a large-scale seed magnetic field are the amplification by anisotropic protogalactic collapse and differential rotation model (Piddington 1970, 1981; Kulsrud 1986, 1988, 1990 and references therein); and the dynamo amplification model (Parker 1971, 1979; Vainshtein & Ruzmaikin 1972a, b and references therein). Both amplification models need an initial seed magnetic field to feed on; the phenomenological dynamo model is a more effective amplifier and so needs a smaller initial seed field. An intergalactic (seed) magnetic field of $\sim 10^{-13}$ to 10^{-12} G is needed for the collapse and differential rotation amplification model while the dynamo model might need a seed field (scaled to the present time) of $\sim 10^{-22}$ to 10^{-16} G. Galactic seed fields might be produced on scales very much smaller than typical galactic scales (in stars, star clusters, etc.) or on scales very much larger than typical galactic scales (on cosmological scales); in this *Letter* we focus on the primeval cosmological seed magnetic field model.

An early epoch of inflation would generate small-scale quantum mechanical magnetic field spatial irregularities, just as it generates energy density, peculiar velocity, and gravitational wave spatial irregularities. The rapid expansion of the universe during inflation would then stretch these small-scale magnetic field irregularities to cosmological scales, in exactly

the same manner as other spatial irregularities are stretched. Phase transitions in the early universe might also generate a cosmological “seed” magnetic field.

The QCD phase transition might generate a magnetic field of present strength $\sim 6 \times 10^{-38}$ G on a present scale of ~ 100 kpc (Quashnock, Loeb, & Spergel 1989), while the electroweak transition might generate a field of $\sim 10^{-30}$ G on a similar scale (Vachaspati 1991). Adding an Abelian (Maxwell) gauge field to the inflation modified hot big bang model results in a present magnetic field of $\sim 10^{-59}$ G on a scale of 1/1000 the Hubble radius (Ratra 1991d), while explicitly breaking gauge-invariance by giving the “photon” a mass (which is a function of the Ricci tensor) makes it possible to produce a present magnetic field of $\sim 10^{-10}$ G on a scale of a megaparsec (Turner & Widrow 1988) (gauge-invariant couplings of the photon to curvature might result in a magnetic field of $\sim 10^{-40}$ G on a similar scale; Turner & Widrow 1988). The primeval magnetic fields in these gauge-invariant models would seem to be too small to be the needed seed field.

In this *Letter* we describe the effects of gauge-invariantly coupling the scalar field Φ responsible for inflation to the Maxwell field A_μ through a term $\propto e^{\alpha\Phi} F_{\mu\nu} F^{\mu\nu}$ (where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and α is a parameter). We find, in the exponential potential scalar field inflation modified hot big bang model (Ratra & Peebles 1988; Ratra 1989a, b and references therein), that it is possible to generate an intergalactic magnetic field whose present strength (depending on values of parameters of the model) lies between $\lesssim 10^{-65}$ and $\gtrsim 10^{-10}$ G (and possibly even as large as $\sim 10^{-9}$ G) on a scale of 1/1000 the Hubble scale. The model is described in the next section, and the derivation of the large time magnetic field two-point correlation function is described in § 3.

2. THE MODEL

The inflation epoch model we study is characterized by the scalar field potential

$$V(\Phi) = \left(\frac{6-q}{3} \right) \frac{16\pi}{m_p^2} \rho_{b\Phi}^{(0)} \exp \left[-\sqrt{\frac{q}{2}} (\Phi - \Phi^{(0)}) \right], \quad (1)$$

where m_p is the Planck mass, $\Phi^{(0)}$ is the value of the scalar field,

and $\rho_{\Phi}^{(0)}$ is the homogeneous scalar field energy density when the FLRW scale factor $a = a_0$, and q is a parameter ($0 < q < 2$). The postinflation model is divided into a radiation-dominated epoch and a baryon-dominated epoch, the potential $V(\Phi)$ vanishes after the transition from inflation to radiation dominance (at redshift $z = z_{R\Phi}$), and the transitions between epochs are instantaneous. The spatial momentum (k) dependence of the baryon epoch large-scale fractional energy density perturbation (δ) power spectrum is given by $\langle \delta(k)\delta^*(k) \rangle \propto k^n$, where $n = (2 - 3q)/(2 - q)$ (which implies $-3 < n < 1$); requiring that the baryon epoch departure velocity from Hubble expansion agree with the observational data results in an expression for the free parameter $z_{R\Phi}$ as a function of the free parameter n (Ratra 1989a).

Parenthetically, we note that the exponential potential scalar field inflation model is a generic representative of the inflation scenario, has the least number of free parameters of any generic inflation model considered to date, and is not “fine-tuned” (Ratra 1989a, c, 1990, 1991b, c).

We take the Maxwell-scalar field coupling Lagrangian density during inflation to be $\propto \bar{e}^{-2}(-g)^{1/2}e^{\alpha\Phi}F_{\mu\nu}F^{\mu\nu}$ (where $\bar{e}$ is the Maxwell coupling constant during inflation, g is the determinant of the metric tensor, and α is an additional free parameter), and we assume that at the inflation-radiation transition the scalar field potential is chosen to ensure that this new term reduces to the standard Maxwell term $\propto e^{-2}(-g)^{1/2}F_{\mu\nu}F^{\mu\nu}$ (where e is the standard Maxwell coupling constant). (During inflation the Maxwell field behaves as though it were in a medium with a time-dependent effective dielectric “constant” and magnetic permeability, and with an appropriate choice of parameters one can arrange for a large magnetic field which does not contribute excessively to the energy density.)

To mimic the generation of charged particles at the inflation-radiation transition, we assume that the conductivity instantaneously jumps to a large value at this transition (from a negligibly small value during inflation—this is an assumption that must be justified by a microphysical analysis) (Turner & Widrow 1988; Ratra 1991d). We find that this jump in conductivity forces the electric field to drop to an insignificant value at this transition while the magnetic field is not affected.

3. THE LARGE TIME MAGNETIC FIELD

In this section we present a derivation of our results. During inflation, linearizing about a spatially flat (synchronous gauge) FLRW model, a spatially homogeneous scalar field, and a vanishing spatially homogeneous (temporal gauge) vector potential, the only new linear perturbation equations of motion (compared to those of Ratra 1989a) are the equations for the vector potential perturbation,

$$\ddot{A}_i + \left(\frac{\dot{a}}{a} + \alpha\dot{\Phi}\right)\dot{A}_i - \frac{1}{a^2}(\partial_k\partial_k A_i - \partial_i\partial_k A_k) = 0, \quad (2)$$

$$\partial_i \dot{A}_i = 0. \quad (3)$$

The proper electric and magnetic field perturbations are given by

$$a^{-1}E_i = -a^{-1}\dot{A}_i, \quad (4)$$

$$a^{-1}B_i = a^{-2}\epsilon_{ijk}\partial_j A_k, \quad (5)$$

where E_i and B_i are the coordinate electric and magnetic fields and ϵ_{ijk} is the totally antisymmetric tensor density ($\epsilon_{123} = 1$).

Solving equations (2) and (3) (and using eqs. [4] and [5]), we

find during inflation that the longitudinal electric and magnetic field perturbations vanish while the transverse coordinate electric and magnetic fields are

$$E_I^{(\Phi)}(k, t) = Wka^{-[1+(2-q)\hat{\nu}/2]}H_{\hat{\nu}-1}^{(1)}\left(\frac{2}{2-q}\frac{k}{aH}\right), \quad (6)$$

$$B_I^{(\Phi)}(k, t) = -i(-1)^I Wka^{-[1+(2-q)\hat{\nu}/2]}H_{\hat{\nu}}^{(1)}\left(\frac{2}{2-q}\frac{k}{aH}\right). \quad (7)$$

Here $I = 2, 3$ (we have chosen the x^1 axis to lie along the wavevector $\mathbf{k}$ of the perturbation), H is the Hubble parameter during inflation, $H_{\hat{\nu}}^{(1)}$ and $H_{\hat{\nu}-1}^{(1)}$ are Hankel functions,

$$\hat{\nu} = \frac{1}{2} + \alpha\frac{\sqrt{2q}}{2-q}, \quad (8)$$

and we have defined

$$W = \left(\frac{16\pi}{m_p^2}\right)^{1/2} \bar{e} \left[\frac{\pi}{2(2-q)}\right]^{1/2} e^{i(\hat{\nu}+1/2)\pi/2} \times \left[\frac{8\pi}{3m_p^2} \rho_{\Phi}^{(0)}(a_0)^q\right]^{-1/4} [e^{-\Phi^{(0)}}(a_0)^{\sqrt{2q}}]^{\alpha/2}. \quad (9)$$

We have determined the constants of integration in the general solution of equations (2) and (3) by requiring that the small-scale (early time) part of the appropriately rescaled vector potential solution agree with the flat spacetime case. These initial conditions are motivated by the fact that the large spatial momentum modes of the field do not recognize as special the length scale set by the spacetime curvature (Ratra 1985, 1989a, b and references therein).

From equations (4)–(7) we find that the large-scale transverse proper electric field scales like $a^{-(3-q/2)}$ (when $1 \leq \hat{\nu}$) and like $a^{-[2+\alpha(2q)^{1/2}/2]}$ (when $\hat{\nu} \leq 1$), while the large-scale transverse proper magnetic field scales like a^{-2} (when $0 \leq \hat{\nu}$) and like $a^{-[6-q+2\alpha(2q)^{1/2}/2]}$ (when $\hat{\nu} \leq 0$). The large-scale transverse proper magnetic field scales like $k^{1-\hat{\nu}}$ (when $\hat{\nu} > 0$) and like $k^{1+\hat{\nu}}$ (when $\hat{\nu} < 0$). The energy density spectrum of the transverse Maxwell perturbations is proportional to the sum of the squares of the transverse proper electric and magnetic fields multiplied by $e^{\alpha\Phi}$ (Ratra 1991d); when $\alpha > 0$, the large-scale Maxwell energy density scales like $a^{-[4-\alpha(2q)^{1/2}]}$ and when $\alpha < 0$ like $a^{-[4+\alpha(2q)^{1/2}]}$ (in both cases the energy density increases slower than a^{-4} as a is decreased).

In the radiation and baryon epochs the only new linear perturbation equations (compared to those of Peebles 1980, §§ 82 and 85) are the vector potential perturbation equations for an electrically conducting fluid,

$$\ddot{A}_i + \left(\frac{\dot{a}}{a} + 4\pi\sigma\right)\dot{A}_i - \frac{1}{a^2}(\partial_k\partial_k A_i - \partial_i\partial_k A_k) = 0, \quad (10)$$

$$\partial_i \dot{A}_i = 0, \quad (11)$$

where σ is the conductivity.

Requiring that the equations of motion remain singularity-free at the transition spatial hypersurfaces (Ratra 1991a), we find that the joining conditions at the inflation-radiation transition at $t = t_{R\Phi}$ (where we assume that the homogeneous scalar field “freezes” and the conductivity jumps to a finite value) are

$$E_i^{(R)}(t_{R\Phi}, \mathbf{x}) = e^{-4\pi\sigma t_{R\Phi}} E_i^{(\Phi)}(t_{R\Phi}, \mathbf{x}), \quad (12)$$

$$B_i^{(R)}(t_{R\Phi}, \mathbf{x}) = B_i^{(\Phi)}(t_{R\Phi}, \mathbf{x}). \quad (13)$$

The conductivity does not jump at the radiation-baryon transition so the electric field and magnetic field perturbations do not jump at this transition.

In the large-conductivity limit it may be seen, from equation (10), that the radiation and baryon epoch transverse coordinate magnetic fields are $\propto a^{-1}$ (and so the transverse proper magnetic fields are $\propto a^{-2}$; eq. [5]); the transverse coordinate electric fields are suppressed by a factor of σ^{-1} .

We solve equations (10) and (11) and use equations (4) and (5) to determine the radiation and baryon epoch electric and magnetic fields and use the joining conditions (eqs. [12] and [13] and the radiation-baryon transition joining conditions) to determine the constants of integration. The baryon epoch transverse proper magnetic field two-point correlation function is $\propto k^{\hat{n}} a^{-4}$, $-3 < \hat{n} < 2$ (where $\hat{n}$ is a function of n and α and the lower limit follows from requiring that the Fourier transform to position space not diverge in the infrared).

We use a Gaussian window function to smooth the large spatial momentum contribution to the Fourier transform of the two-point correlation function. Defining

$$r = \sqrt{(1-n)(3-n)}, \quad (14)$$

we find that the baryon epoch, large-conductivity, mean square transverse proper magnetic field, on a proper scale $\mathcal{R}$, at the present time t_{now} , is given by, for $-1 < \alpha r < 4$ (where the second inequality is a consequence of requiring that the Fourier transform be infrared finite and the first is a rewriting of $\hat{v} > 0$),

$$\begin{aligned} & \langle \{[a(t_{\text{now}})]^{-1} B_I^{(B)}(t_{\text{now}} | \mathcal{R})\}^2 \rangle \\ &= \frac{2}{\pi^2} \left(\frac{4}{3-n} \right)^{\alpha r} \Gamma\left(2 - \frac{\alpha r}{2}\right) \Gamma^2\left(\frac{1}{2} + \frac{\alpha r}{2}\right) \\ & \quad \times (1 + z_{R\Phi})^{\alpha r} (H_{BR})^4 (1 + z_{BR})^{-(6+\alpha r/2)} \\ & \quad \times (\mathcal{R} H_{\text{now}})^{-(4-\alpha r)}, \end{aligned} \quad (15)$$

and for $-6 < \alpha r < -1$ (where the first inequality is a consequence of requiring infrared finiteness and the second is a rewriting of $\hat{v} < 0$),

$$\begin{aligned} & \langle \{[a(t_{\text{now}})]^{-1} B_I^{(B)}(t_{\text{now}} | \mathcal{R})\}^2 \rangle \\ &= \frac{2}{\pi^2} \left(\frac{4}{3-n} \right)^{-(2+\alpha r)} \Gamma\left(3 + \frac{\alpha r}{2}\right) \Gamma^2\left(-\frac{1}{2} - \frac{\alpha r}{2}\right) \\ & \quad \times (1 + z_{R\Phi})^{-(2+\alpha r)} (H_{BR})^4 (1 + z_{BR})^{-(5-\alpha r/2)} \\ & \quad \times (\mathcal{R} H_{\text{now}})^{-(6+\alpha r)}, \end{aligned} \quad (16)$$

Here H_{BR} and H_{now} are the values of the Hubble parameter at the radiation-baryon transition and now, $z_{R\Phi}$ and z_{BR} are the redshifts of the inflation-radiation and radiation-baryon transitions, and n is the energy density power spectrum index ($-3 < n < 1$). We have retained the contributions of only those modes that were on scales larger than the Hubble radius at the inflation-radiation transition (the large-scale expansion bifurcates at $\hat{v} = 0$; this is the reason for the two different expressions, eqs. [15] and [16]). The expressions in equations (15) and (16) are for one transverse component of the magnetic field; the total magnetic field contribution is a factor of 2 larger.

To evaluate the expressions in equations (15) and (16) we choose the standard hot big bang model values $H_{\text{now}} = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$, $z_{BR} = 4 \times 10^4 h^2$, $H_{BR} = 1.4 \times 10^{-54} h^4 m_p$,

and the values of the redshift of reheating determined by comparing the theoretical expression for the mean square departure velocity from Hubble expansion to observational data (Ratra 1989a)

$$z_{R\Phi} \simeq \begin{cases} 5.4 \times 10^{29} \sqrt{\epsilon} h^{\epsilon/2} & (n = 1 - 2\epsilon^2) \\ 3.8 \times 10^{26} h^{1/9} & (n = \frac{1}{2}) \\ 2.2 \times 10^{24} h^{1/5} & (n = 0) \\ 2.9 \times 10^{22} h^{3/11} & (n = -\frac{1}{2}) \\ 1.2 \times 10^{21} h^{1/3} & (n = -1), \end{cases} \quad (17)$$

where ϵ is a small positive parameter used to “regulate” the de Sitter inflation limit divergence of the late time energy density power spectrum (Ratra 1989a). We find for the “free” case (when $\alpha = 0$) that the present RMS magnetic field on a scale of 1/1000 the Hubble scale is $\sim 10^{-59} \text{ G}$ and that for most of the allowed range of n and α the magnetic field is well below the theoretically desired value for a collapse and differential rotation amplification “seed” field (the smallest magnetic field we find on this scale is $\sim 10^{-65} \text{ G}$ [Ratra 1991d]). However, when $\alpha r = 4 - 2\bar{\epsilon}$ (where $\bar{\epsilon}$ is a small positive parameter) and $n = 1 - 2\epsilon^2$ (de Sitter inflation and scale-invariant energy density perturbations [Harrison 1970; Peebles & Yu 1970; Zel’dovich 1972]), we find an RMS magnetic field $\simeq 6.5 \times 10^{-10} \text{ G}$ (where we have taken $\epsilon = 0.1$ [so $\alpha = 20$] and $\bar{\epsilon} = 0.01$; the total magnetic field is a factor of $2^{1/2}$ larger). From the relations in the paragraph below equation (9) we note that in this case the inflation epoch, large-scale, Maxwell energy density scales like $\sim a^0$. [When $\alpha r > 0$ we find from the inflation epoch solution given above that the inflation epoch, large-scale, transverse proper magnetic field scales as $k^{(1-\alpha r)/2} a^{-2}$; the large value of the present magnetic field (relative to the $\alpha = 0$ case) when $\alpha r = 4 - 2\bar{\epsilon}$ is a result of magnetic field power being shifted to the infrared by the inflation epoch scalar field coupling. We also note that in the case discussed above the inflation epoch large-scale proper transverse electric field is down from the large-scale proper transverse magnetic field by a factor of $k/(aH)$.] Values of the magnetic field for other values of α and n are tabulated in Ratra (1991d).

4. CONCLUSION

We have studied a simple, inflation-based, mechanism that can generate a present intergalactic magnetic field that might be as large as $\sim 10^{-9} \text{ G}$. We note that for the larger values of the present large-scale magnetic field the effective gauge coupling constant during inflation, $\bar{e}/\langle e^{\alpha\Phi_0/2} \rangle$, is large (Ratra 1991d). We emphasize that we do not make use of an expansion in this coupling constant to derive our results.

Since we have had to make a number of simplifying approximations, our results should be considered to be preliminary. In particular we have not taken account of the electroweak transition (which might raise the magnitude of the magnetic field by a factor of $2^{1/2}$; Ratra 1991d), the assumption that the conductivity during inflation is negligibly small needs to be justified by a microphysical analysis, and our model of the inflation-radiation transition needs to be improved upon.

We emphasize that it remains to be seen whether this mechanism for generating an intergalactic magnetic field can be put to use in a more realistic model.

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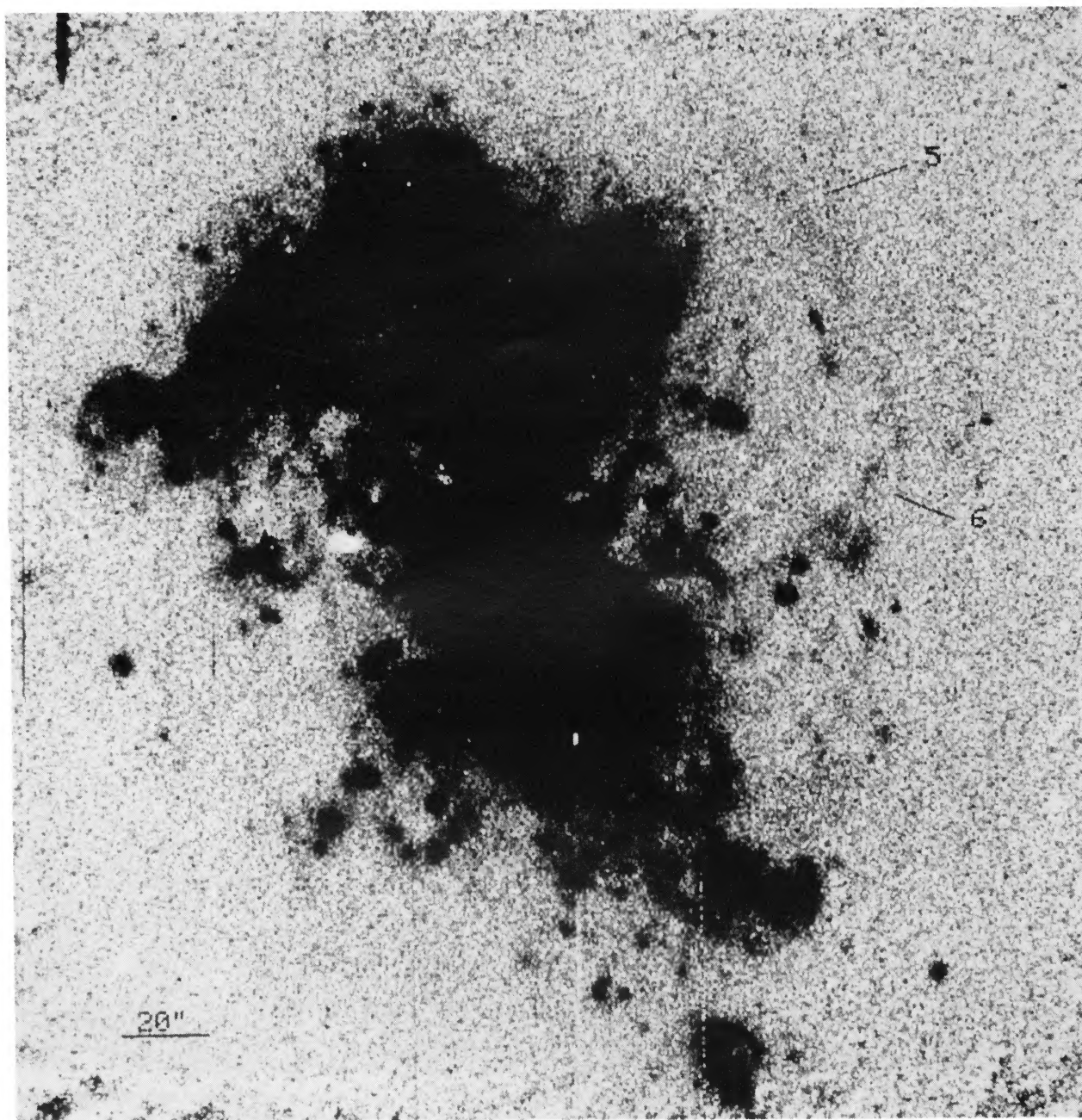


FIG. 1.—A deep H α image of NGC 4449 from which the stellar continuum has been subtracted. The structures discussed in the text are numbered, continuing the numbering scheme begun by Hunter & Gallagher (1990). East is to the left, and north is up.

HUNTER & GALLAGHER (see 391, L10)