THE ASTROPHYSICAL JOURNAL, 391:550–559, 1992 June 1

ACTIVE GALACTIC NUCLEI. IV. SUPPLYING BLACK HOLE CLUSTERS BY TIDAL DISRUPTION AND BY TIDAL CAPTURE OF STARS

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Received 1991 April 1; accepted 1991 November 26

ABSTRACT

We study the extent to which individual holes in a cluster of black holes with a mass spectrum can liberate and accrete the resulting material by tidally disrupting stars they encounter, or by capturing stars as binary companions. We find that the smaller black holes in "the halo" of such clusters can adequately supply themselves to the level $\dot{M}_h > 10^{-4}(\dot{M}_h)_{\rm crit}$, and up to $0.05(\dot{M}_h)_{\rm crit}$ for the smallest holes, by tidal disruption, as long as the cluster is embedded in a distribution of stars of relatively high density, $\rho_s \geq 0.1 M_{\rm cl}$ pc^{-3}, and as long as the entire cluster of stars is not too compact (not <0.5 pc). Here $\dot{M}_h$ is the accretion rate onto the individual holes of the cluster, $(\dot{M}_h)_{\rm crit}$ the critical accretion rate onto the individual holes, and $M_{\rm cl}$ the mass of the entire cluster, stars and black holes. To reach this level of supply, the stellar cluster cannot be too extended, either—not larger than a few parsecs in radius. These holes in the outer parts of black hole clusters are precisely the ones which are severely underfed by an external accretion flow because of "the velocity problem," as we have shown in an earlier paper.

We briefly discuss the modifications this "internal" mode of supply introduces in the spectrum emitted by such black hole clusters, and the current status of their viability as models for AGNs and QSOs in light of recent dynamical studies by Quinlan and Shapiro. In particular, if the dominant supply mechanism is the tidal disruption of stars, then the accretion rate onto each of the component black holes in the cluster will vary as $\dot{M}_h \propto M_h^\gamma$, where $\gamma \geq 4/3$. Here M_h is the mass of the individual holes. By fitting to AGN and QSO spectra this leads to cluster models with mass spectral indices $\beta > 3.77$, tantalizingly consistent with the $\beta \cong 4$ Quinlan and Shapiro obtain for the clusters they produce in their evolutionary computer simulations. The limits on tidal capture of binary companions in these clusters is much more severe. There will be no such captures by component black holes unless the velocity dispersion of the stars in the cluster is $\lesssim 270$ km s^{-1}—that is, only for clusters which are not too compact and/or too massive. There may, however, be a significant number of binaries in these clusters due to either the survival of binaries from star formation processes, or binary captures by holes in earlier, less extreme phases of cluster evolution.

Subject headings: black hole physics — celestial mechanics, stellar dynamics — galaxies: active — galaxies: nuclei — quasars: general

1. INTRODUCTION

In Paper III (Pacholczyk, Stepinski, & Stoeger 1989) we discussed powering black hole clusters—conceived as models for QSOs and AGNs—with gas from outside the cluster which flows uniformly and spherically symmetrically onto it. We examined how that accreting material filtered through the black holes in the outer part of the cluster into the cluster core, which we have defined as the interior region of the cluster where the flow loses its spherical symmetry and/or becomes chaotic. We found that, due to the high relative velocity between most of the black holes in the cluster "halo" and the spherically symmetric flow there, they would accrete very little of it. Most of the energy emitted in this scenario would emanate from the cluster core.

In that case, then, it will be the black holes in the "cluster core"—which, from the point of view of the observations, would be given by the variability radius—which provide the entire luminosity of the AGN or QSO. And their mass distribution would determine the spectrum. Although we know very little about the details of the processes which dominate in this core, we supposed that the continuum radiation due to accretion onto the individual holes, ranging from stellar to supermassive size, dominate that due to magnetic fields, shock structures, etc. We also supposed that the specific accretion rate onto all the holes was the same, that is, independent of their mass. These are all assumptions which we shall have to examine much more carefully. We do this in our next paper.

Here we look at how stars internally supply mass to the component black holes of our clusters. We shall find that, when the black hole cluster is embedded in a star cluster of relatively high density, tidal disruption of stars by the individual black holes will be sufficient to render the entire cluster—not just the core—an efficient source of radiation. Thus, in such a case the entire black hole cluster is an acceptable model for QSOs and AGNs, as in Paper I (Pacholczyk & Stoeger 1986)—not just the cluster core, as in the case of a smooth, spherically symmetric flow from outside the cluster (Paper III). When the accretion process is dominated by tidal disruption and capture of stellar material by the component black holes, we automatically have a definite model for the accretion rate onto the individual holes—such that $\dot{M}_h \propto M_h^\gamma$, where $\gamma \geq 4/3$. Here M_h is the mass of the individual black holes, and $\dot{M}_h$ is the accretion rate onto them. When we fit such models to QSO and AGN spectra, we shall obtain for our cluster models mass spectral indices $\beta > 3.77$—clusters dynamically dominated by the holes of smallest mass, with relatively small masses for their largest holes.

Concerning possible sources of internal supply in general, of course, there are several mechanisms which must be examined: mass outflow from the stars themselves (stellar winds), stellar collisions, black hole–star collisions, tidal disruption of stars by cluster black holes, binary or multiple systems, involving a black hole and one or more stars. Mass outflow will generally contribute very little to mass supply, compared with these other mechanisms, and can be subsumed under the external uniform accretion supply treated in Paper III. The other mechanisms overlap one another somewhat. By collisions we mean inelastic encounters—in which the colliding objects coalesce or bind to one another. In clusters of black holes there will be collisions between stars, and, more importantly, collisions involving a star and a black hole.

Through collisions, too, binary or multiple systems may be formed, involving a black hole and one or more stars. But, as we shall see, tidal capture binary formation will not be important except when the velocity dispersion of the stars in the cluster is $\lesssim 270$ km s^{-1}. For larger velocity dispersions than this, which will commonly characterize most of our clusters, the encounters are two rapid for tidal capture to take place, unless they are close enough for disruption. Some smaller, stellar-size black holes in the cluster may have stellar companions ab initio. It is very difficult to estimate what the present remnant of this initial "binary star" fraction would be in our clusters. Due to the high density of stars and black holes in them, we would expect it to be quite low: many black holes originally with a companion would have lost it due to stripping. Those that would remain would most likely be close or contact binaries, efficiently supplying material to the black holes. The larger black holes—above 20 $M_\odot$, say—are formed mainly by coalescence of smaller black holes, whose possible companions would have been consumed or liberated in the process. Obviously, if there is a large fraction of small black holes with stellar companions, then these stars will be a significant source of material for the large holes which formed in the collision of smaller holes. The number of black holes in a dense cluster with one or more stellar companions, is, then, difficult to determine.

We now turn to consider the important process of the tidal disruption of stars. By tidal disruption we mean an encounter of a star with a black hole in which part or all of the star is torn apart by passing within the black hole's tidal radius R_t, but in which the liberated material will not necessarily be captured by the liberating hole. This failure of the disrupting black hole to accrete the material does not occur, of course, in the single black hole model of QSOs and AGNs, but it will frequently be the case for individual black holes in our black hole clusters, except for the largest black holes in the core of the cluster and the smaller black holes in a cluster of only low or moderate mass. There are two key issues here for black hole clusters: (1) How much material is liberated by encounters and collisions within the cluster? and (2) How much of the material liberated from a star by a black hole in a given encounter will be accreted by that black hole? Or, within the entire cluster, what percentage of the material liberated by tidal disruption will be accreted by the disrupting holes themselves? This latter question is crucial, because material liberated but not captured by a hole, along with the material liberated in stellar collisions, will be added to the overall accretion flow within the cluster, and generally very little of that will be accreted by any hole—because of "the velocity problem" discussed in Paper III—until it reaches the cluster core.

We shall treat the supply of the black holes in the "halo" of the cluster by tidal disruption of stars, first, in § 3. Then we shall turn to treat briefly the capture of a stellar companion in a close tidal encounter in § 4. But first we shall spend a few paragraphs describing some important radial features of our black hole clusters.

2. DESCRIPTION OF STARS IN THE FIELD OF THE CLUSTER

Studying the tidal disruption of stars by the component black holes in the cluster requires some consideration of how the cluster of black holes affects the distribution and the dynamics of the distribution of stars in which it is embedded. Such study was originally done for a single massive black hole in the center of a cluster of stars (Frank & Rees 1976; Lightman & Shapiro 1978; Hills 1975; Hills & Day 1976). We slightly modify these results to be applicable to an extended stellar distribution around a cluster of black holes rather than the single massive hole. The compact size of our black hole cluster, smaller than most important dynamical radii around it, makes such application possible. We now consider several radii around the cluster of black holes which will be important in determining which processes dominate in providing material to the cluster from the stars and the density profile of the stars near the cluster.

Assume that a given AGN or QSO has a distribution of stars spread uniformly up to the radius $r_{\rm cl}$ of the order of few parsecs, with density n_s and velocity dispersion v_s. These are observable quantities. Embedded in the core of this distribution is our black hole cluster, with a total mass $M_{\rm bh}$ between 10^4 and 10^9 $M_\odot$, and a radius $r_{\rm bh}$ between 10^{14} and 10^{18} cm (less than a parsec), depending on the luminosity and especially the temporal variability of the object in question. If a typical cluster member (star or black hole) has a mass M and we may assume that (Frank & Rees 1976) $M_{\rm cl} = n_s r_{\rm cl}^3 M \gg M_{\rm bh} \gg M_*$, then we can estimate the characteristic velocity dispersion v_s from the virial theorem,

$$v_s \approx (GMn_s r_{\rm cl}^2)^{1/2} . \tag{1}$$

The cluster of black holes dominates the dynamics of stars out to a radius r_a given by

$$r_a = \frac{GM_{\rm bh}}{v_s^2} . \tag{2}$$

Within r_a, which for conditions under consideration is larger than r_{bh} but smaller than r_{cl}, the number of stars may be enhanced resulting in the formation of a power-law cusp in the stellar density. Bahcall & Wolf (1976) proposed that the density of stars between a certain r_{min} and r_a is

$$n(r) = n_c \left(\frac{r_a}{r}\right)^{7/4} . \quad (3)$$

The stars in the cusp move in an elliptical orbit around the center of a cluster, changing gradually their energy and angular momentum as a result of encounters with other stars and/or black holes. Within r_{min} equation (3) no longer, holds, the velocity distribution becomes anisotropic, and stars are quickly flowing into the cluster or a hole. We now need to find the approximate value of r_{min}. Following Frank & Rees (1976), we adopt for the value of r_{min} the largest among following characteristic dynamical radii: the collision radius r_{coll}, the tidal radius r_t, and the critical radius r_{crit}.

The radius at which the velocity dispersion is comparable to the escape velocity from typical stars, or black holes, is called the "collision radius" and denoted by r_{coll}. As Frank & Rees (1976) point out, stellar encounters responsible for relaxation of the velocity distribution can be treated as elastic only outside r_{coll}. For star-star collisions

$$r_{coll} \approx R_* \left(\frac{M_{bh}}{M}\right) , \quad (4)$$

where R_* is the stellar radius. For collisions between a star and a black hole r_{coll} is smaller by a factor $(R_g/R_*)^{1/2}$, where R_g is the Schwarzschild radius, because the escape velocity from a black hole is larger than the escape velocity from a star of the same mass and radius R_*. If r_{min} is determined by r_{coll}, then the mass supply rate to the cluster is dominated by collisions.

The radius inside which an approaching star would be disrupted by tidal forces exerted by all the mass within it is called the tidal radius of the cluster and denoted by r_t. It is given by Hills (1975) as

$$r_t = \left(\frac{6M_{bh}}{\pi\rho}\right)^{1/3} , \quad (5)$$

where ρ is the density of the stars. If r_{min} is determined by r_t, then the mass supply rate is dominated by tidal disruption of stars by the entire cluster. However, our clusters are quite massive, and for such masses r_t are always smaller than r_{coll}. Thus tidal disruption of stars by an entire cluster, considered as a single gravitating object and not as a collection of stars and holes, is not an important source of cluster mass supply.

Last, there is the "critical radius" r_{crit} (Frank & Rees 1976; Lightman & Shapiro 1978), such that stars on orbits with $r < r_{crit}$ diffuse into very eccentric (low angular momentum) orbits, pass through the black hole cluster, are disrupted, and thus removed from the cusp. In estimating the values of r_{coll} and r_t we have followed the original, single hole calculations. However, estimation of r_{crit} requires a key modification to be applicable to our clusters. For a single black hole, any star on an orbit with low enough angular momentum to take it to $r = r_t$ will be disrupted and removed from the cluster. For a black hole cluster immersed in a cluster of stars, in contrast, we have to say that any star on an orbit taking it to $r < r_{bh}$, the radius of the black hole cluster, is in danger of being removed—if in passing through the black hole cluster it comes within the tidal radius of any of the component black holes. Thus in the estimation of r_{crit} we have to use r_{bh} instead of r_t. Because $r_{bh} > r_t$ in our clusters, stars on relatively higher angular momentum orbits can be removed from the cluster, resulting in an r_{crit} larger than that for a single black hole of an equivalent mass. If r_{min} is determined by r_{crit} then the mass supply is dominated by tidal disruptions of stars by individual holes in a cluster. Following Frank & Rees (1976) and replacing r_t by r_{bh} we have

$$\frac{r_{crit}}{1\ \mathrm{pc}} = \begin{cases} 623\left(\dfrac{M_{bh}}{10^8\ M_\odot}\right)^{1/9}\left(\dfrac{n_s}{10^7\ \mathrm{pc}^{-3}}\right)^{1/3}\left(\dfrac{r_{cl}}{1\ \mathrm{pc}}\right)^{14/9}\left(\dfrac{r_{bh}}{1\ \mathrm{pc}}\right)^{4/9} & \text{for } r_{crit} < r_a \\ 210\left(\dfrac{M_{bh}}{10^8\ M_\odot}\right)^{1/3}\left(\dfrac{r_{cl}}{1\ \mathrm{pc}}\right)^{2/3}\left(\dfrac{r_{bh}}{1\ \mathrm{pc}}\right)^{1/3} & \text{for } r_{crit} > r_a \end{cases} . \quad (6)$$

Here r_{crit} dominates r_{coll} and thus becomes an effective r_{min} when the following relations hold:

$$1 < \begin{cases} 274\left(\dfrac{M_{bh}}{10^8\ M_\odot}\right)^{-8/9}\left(\dfrac{n_s}{10^7\ \mathrm{pc}^{-3}}\right)^{1/3}\left(\dfrac{r_{cl}}{1\ \mathrm{pc}}\right)^{14/9}\left(\dfrac{r_{bh}}{1\ \mathrm{pc}}\right)^{4/9} & \text{when } r_{crit} < r_a \\ 93\left(\dfrac{M_{bh}}{10^8\ M_\odot}\right)^{-2/3}\left(\dfrac{r_{cl}}{1\ \mathrm{pc}}\right)^{2/3}\left(\dfrac{r_{bh}}{1\ \mathrm{pc}}\right)^{1/3} & \text{when } r_{crit} > r_a \end{cases} . \quad (7)$$

For a representative case with $r_{cl} = 1$ pc, and $r_{bh} = 0.01 r_{cl}$, the above relations hold for masses up to $10^{10}\ M_\odot$ when the density of stars n_s exceeds 10^7 pc^{-3}. For lower densities the above relations are not fulfilled only when a relatively massive cluster is embedded in a star cluster with relatively low star density. Altogether we can conclude that, for the overwhelming majority of physically relevant cases r_{crit} is the largest of the three discussed radii. So the mass supply to the black hole cluster is determined by stars which, having diffused to low angular momentum orbits, entered it and were tidally disrupted by component black holes. Figure 1 shows how these dynamical radii change with the mass of the central black hole cluster which extends up to 10^{-2} pc from the center and is

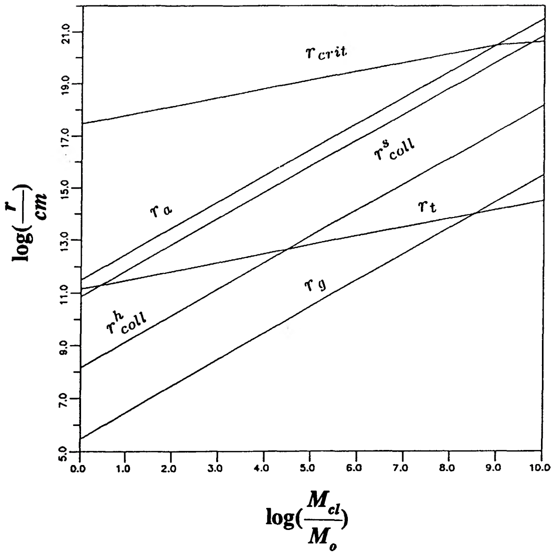

FIG. 1.—Relative positions of the various dynamical radii described in § 2, as functions of cluster mass, for cluster stars of stellar type. $r_{\rm crit}$ is given for a black hole cluster which extends to 10^{-2} pc from the center and is embedded in a star cluster with a density of 10^7 pc^{-3}. For this reason, the lower segment of the $r_{\rm crit}$ curve (lower $M_{\rm bh}$ values) does not represent a consistent cluster configuration.

embedded in a star cluster with density 10^7 pc^{-3} which extends up to 1 pc from the center. Note that only for most massive clusters is $r_{\rm crit}$ actually smaller than r_a, so that a stellar cusp forms. For most cases $r_{\rm crit} > r_a$, with no cusp, the conditions at $r_{\rm crit}$ determine the mass supply rate to the cluster.

That concludes a semiquantitative description of the overall structure of such central black hole–dominated clusters. Our main concern was to determine the most likely regime under which internal supply of material to the cluster operates. Our focus on relatively high stellar densities (10^7 pc^{-3} or more) reflects our desire to provide enough material to feed the component holes to a level of a percent or a few percent of their critical accretion rate. In the next section we shall show that such high densities are required in order to achieve our objectives. Are such high densities possible in the center of galactic nuclei? Although the detection of such dense stellar cores within 0.01–0.1 pc of the centers of galactic nuclei has been nearly impossible until now (Shapiro 1986), such measurements should be forthcoming with the *Hubble Space Telescope*. Then we shall know if such stellar density conditions are in fact present in AGNs. There is already some indication that they are. Schwarzschild (1973) estimated that the density of stars in the central 3.5 pc of NGC 4151 is 2×10^9 $M_\odot$ pc^{-3}, and Light, Danielson, & Schwarzschild (1974) calculate that within the central 0.5 pc of M31 there may be 10^8 $M_\odot$.

3. TIDAL DISRUPTION OF STARS BY COMPONENT CLUSTER BLACK HOLES

We are primarily interested in how much material can be tidally captured from cluster stars by component black holes. We are also interested in how much material will be liberated by tidal disruption, but not necessarily captured by the disrupting hole. This will be a measure of the severity of "the velocity problem" (see Paper III) for a given cluster. The fraction escaping capture will be added to the overall accretion flow sifting down through the black hole cluster.

In determining the amount of material liberated by a component black hole tidally disrupting nearby stars, the key result is that given by Hills (1975). The rate at which materal is liberated from stars passing within the tidal radius of a black hole of mass M_h, when there is no cusp (see Frank & Rees 1976), is

$$\left(\frac{dM}{dt}\right)_L = 1.9 \times 10^{-15}\left(\frac{M_\odot}{\rm yr}\right)\left(\frac{\rho_s}{M_\odot\ {\rm pc}^{-3}}\right)\left(\frac{\rm g\ cm^{-3}}{\rho}\right)^{1/3}\left(\frac{M_h}{M_\odot}\right)^{4/3}\left(\frac{\rm km\ s^{-1}}{v_h}\right), \tag{8}$$

where ρ_s is the stellar mass density in the vicinity of the black hole; ρ is the mean mass density of the stars being broken up—we take $\rho = 2$ g cm^{-3}; and v_h is the rms velocity of the stars with respect to the given black hole.

Although equation (8) has been used primarily in giving the tidal disruption and capture rate of stars swarming around a single supermassive black hole, it was in fact first derived in order to give the average total amount liberated (but *not* necessary captured) by tidal disruption in collisions between field stars and test stars, or equivalently between stars and black holes, assuming a Maxwellian distribution for their *relative* velocities (see Hills & Day 1976 and Hills 1978 [especially p. 523]—the derivation of eq. [8] is given partially in the former and completed in the latter).

The two important parameters determining $(dM/dt)_L$ in equation (8) are ρ_s and v_h, which, in turn, are constrained and depend on, respectively, the mass of the entire cluster $M_{\rm cl}$ and its radius $r_{\rm cl}$. We shall take

$$v_h^2 = \frac{GM_{\rm cl}}{2r_h}, \tag{9}$$

where r_h is the radius containing $M_{\rm cl}/2$.

Using equations (11) and (12) along with equation (10) of Paper III, we can work out what M_{cl} is in terms of parameters describing the combined mass spectrum of black holes and stars

$$f(m) = Bm^{-\beta}\, dm \,, \tag{10}$$

where m is the mass of the individual cluster member in solar masses. We can work out as well what r_h will be. We find that, for $\beta > 2$,

$$M_{cl} \cong \left(\frac{B}{\beta - 2}\right) m_1^{2-\beta} \,, \tag{11}$$

where m_1 is the mass of the smallest star in the cluster. The value of r_h is given by

$$\frac{3r_h^{2\beta-4}}{(7/2 - \beta)r_{cl}^{2\beta-4}}\left[1 - \frac{(2\beta - 4)r_h^{7-2\beta}}{2r_{cl}^{7-2\beta}}\right] = 1 \,. \tag{12}$$

For $\beta > 3.5$, we find that

$$r_h \cong 0.79 r_{cl} \,. \tag{13a}$$

For $2.5 < \beta < 3.5$,

$$r_h \cong (1/2)^{1/2\beta-4} r_{cl} \,. \tag{13b}$$

We need these values of r_h to calculate v_h for our clusters.

Returning to equation (8), we can calculate the total amount of mass tidally liberated by black holes in the cluster by integrating it over the mass distribution given in equation (10). Using this result, with equation (11) and the fact that the critical accretion for the entire cluster is $(\dot{M}_{cl})_{crit} = 3 \times 10^{-8} m_{cl} M_\odot \text{ yr}^{-1}$, where m_{cl} is M_{cl} in solar masses, we have

$$\dot{M}_{cl} = 6.3 \times 10^{-8} \frac{\beta - 2}{\beta - 7/3}\left(\frac{\rho_s}{M_\odot \text{ pc}^{-3}}\right)\frac{\text{km s}^{-1}}{v_h}(\dot{M}_{cl})_{crit} \,, \tag{14}$$

for $\rho = 2 \text{ g cm}^{-3}$ and $m_1 = 2$. For a single supermassive black hole tidally disrupting stars, this result will be, instead,

$$\dot{M}_{SH} = 5.03 \times 10^{-8}\left(\frac{\rho_s}{M_\odot \text{ pc}^{-3}}\right)\left(\frac{\text{km s}^{-1}}{v_h}\right) m_{SH}^{1/3}(\dot{M}_{SH})_{crit} \,, \tag{15}$$

for $\rho = 2 \text{ g cm}^{-3}$. Because of the extra factor $m_{SH}^{1/3}$ it is quite a bit easier to supply a single supermassive black hole by tidal disruption than it is a cluster of black holes. This is a direct result of the relatively greater efficiency of large holes in tidally disrupting stars—the factor of $m^{4/3}$ in equation (8). In both cases, however, it is clear that very high stellar densities are needed, and that it is much more difficult with very massive, very compact stellar distributions, because of their very high v_h.

Equation (8) can be particularized for 2 and 10 $M_\odot$ black holes in the cluster, again for $\rho = 2 \text{ g cm}^{-3}$:

$$(\dot{M}_{2M_\odot})_L = 3.33 \times 10^{-8}\left(\frac{\rho_s}{M_\odot \text{ pc}^{-3}}\right)\left(\frac{\text{km s}^{-1}}{v_h}\right)(\dot{M}_{2M_\odot})_{crit} \,, \tag{16a}$$

and

$$(\dot{M}_{10M_\odot})_L = 1.07 \times 10^{-7}\left(\frac{\rho_s}{M_\odot \text{ pc}^{-3}}\right)\left(\frac{\text{km s}^{-1}}{v_h}\right)(\dot{M}_{10M_\odot})_{crit} \,, \tag{16b}$$

which clearly shows this difference in efficiency again.

Using equations (16a) and (16b) the amount of material tidally liberated (but not necessarily captured!) by individual component black holes of 2 and 10 $M_\odot$, respectively, is given in Figure 2 as a function of ρ_s for clusters with $r_{cl} = 1$ pc and $r_{cl} = 10$ pc, and for various m_{cl}. For massive clusters with a radius much less than $r_{cl} = 1$ pc, the amounts liberated will be very small, due to the high velocity dispersions—even for very high stellar densities. The range of ρ_s represented for $r_{cl} = 10$ pc is less than that for $r_{cl} = 1$ pc; we have only included values of ρ_s consistent with M_{bh} for the given volume. By comparing these figures, we see the effect of increasing the volume of the cluster.

We now turn to the question of consumption: How much of the material liberated by a given black hole in the cluster is consumed by that hole? This is where "the velocity problem" enters the calculation. For the smaller black holes in a dense, high-mass cluster of stars and holes, the relative velocities will be too high for the holes to capture (and accrete) all the material they liberate from stars by tidal disruption.

To calculate this, we must integrate the rate coefficient $\gamma = \sigma v$ (see, e.g., Hills & Day 1976), where σ is the tidal collision cross section, over the Maxwellian relative velocity distribution (Spitzer 1968) with the upper limit set at v_{max} instead of at infinity; v_{max}

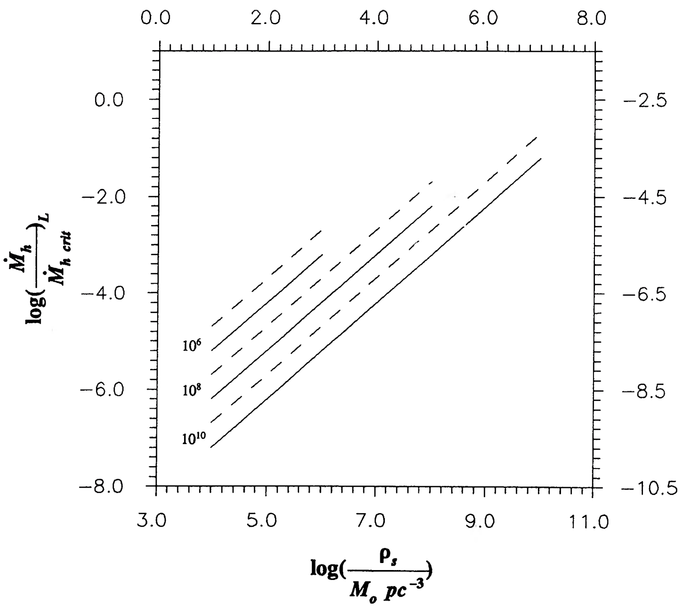

FIG. 2

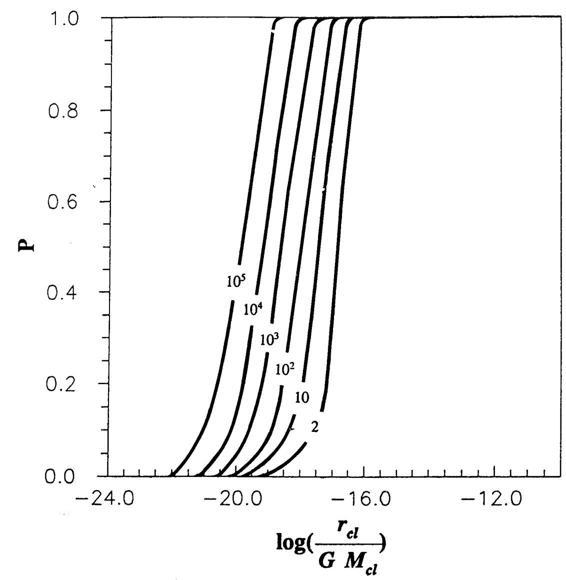

FIG. 3

FIG. 2.—The amount of material $(\dot{M}_h)_L$ tidally liberated from stars having a stellar density of 2 g cm^{-3} by individual black holes in a dense star–black hole cluster, as a function of the density of stars ρ_s in the cluster. Results are given for different cluster masses $M_{\rm cl}$. We assume that $\beta > 3.5$ for the mass spectrum of the cluster. So $r_h \approx 0.79 r_{\rm cl}$. Solid lines are for 2 $M_\odot$ black holes; dashed lines are for 10 $M_\odot$ holes. Left and bottom scales pertain to clusters with $r_{\rm cl} = 1$ pc; right and top scales pertain to those with $r_{\rm cl} = 10$ pc.

FIG. 3.—The fraction of tidally liberated material P (see eq. [19] and [21]) which is captured by the disrupting black hole in a star–black hole cluster, as a function of $(r_{\rm cl}/GM_{\rm cl})$, which is essentially v_h^{-2}. Here again we have assumed $\beta > 3.5$. Only for massive clusters will there be material liberated which will not be captured by the individual holes.

represents the maximum relative velocity in a tidal disruption event for capture of all the liberated material. Obviously, it will be related to the escape velocity $v_{\rm esc} = (2GM_h/r)^{1/2}$ from the black hole in question. Here r is the distance from the center of the hole. As Hills (1975) and others point out, in the process of tidal disruption an amount of energy equivalent to a velocity of about $k = 5.35 \times 10^7$ cm s^{-1} is expended. So, incoming stars tidally disrupted by a black hole which have a velocity v with respect to the hole such that

$$v < (2GM_h/r)^{1/2} + k = v_{\rm max} \tag{17}$$

will have all their material captured by the hole. Those with velocities greater than $v_{\rm max}$ will be disrupted by the hole, if they come within $r_{\rm ht}$—the tidal radius of a single black hole—but the material freed in this encounter will escape to enrich the overall accretion flow within the cluster.

Finally, since the upper limit on the velocity integral depends on r, through equation (17), the result of the velocity integration of γ over the Maxwellian must be integrated over r between the limits 0 and $r_{\rm ht}$, with the proper normalization. The dominant term in γ will be then (see Hills & Day 1976)

$$\langle \gamma \rangle = \frac{4l^3}{\pi^{1/2}} \int_0^{r_{\rm ht}} \int_0^{v_{\rm max}} v^3 \frac{v_{\rm ht}^2}{v^2} \frac{3r^2}{r_{\rm ht}^3} e^{-l^2 v^2} dv\, dr , \tag{18}$$

where $v_{\rm ht}^2 \cong 2GM_h/r_{\rm ht}$ and $l^2 \cong (3/2) v_h^{-2}$. Performing the v-integration in equation (18), and the simple component of the r-integration, we obtain

$$\langle \gamma \rangle = \frac{2 l v_{\rm ht}^2}{\pi^{1/2}} P , \tag{19a}$$

where

$$P \equiv 1 - \frac{3}{r_{\rm ht}^3} e^{-l^2 k^2} \int_0^{r_{\rm ht}} r^2 \exp\,[-l^2 g(r)] dr , \tag{19b}$$

with

$$g(r) \equiv \frac{2GM_h}{r} + 2k \left(\frac{2GM_h}{r} \right)^{1/2} . \tag{19c}$$

The fraction multiplying P in equation (19a) is just that given by Hills (1975), that is, equation (8) is simply, in this formalism,

$$\left(\frac{dM}{dt} \right) = \rho_s \frac{2 l v_{\rm ht}^2}{\pi^{1/2}} . \tag{20}$$

The complicated integral term in equation (19b) gives the fraction of this liberated material which will escape from the disrupting hole.

The integral in equation (19b) can be integrated approximately for different values of (r_{cl}/GM_{bh}), which determines the value of l^2, that is, the velocity dispersion of the cluster of stars within which the black hole cluster resides. For clusters which are not very dense nor very massive the exponents in equation (19b) will be large and negative, and the term will be zero, or very, very small. For more and more massive dense clusters the integral term will gradually approach unity, that is, more and more of the liberated material will escape. Furthermore, it is clear that for the same (r_{cl}/GM_{bh}) value, a larger component black hole will capture a larger percentage of the material it liberates than a smaller one will. P, the *percentage of disrupted material captured*, which is given in equation (19b), is plotted in Figure 3 as a function of (r_{cl}/GM_{bh}) for component black holes of different masses. The physics of the situation can be understood as follows. When v_{max} lies in the high-velocity limit of the Maxwellian distribution, almost all liberated material is consumed by the black hole. As v_{max} moves into and through the maximum of the distribution, there is a rapid drop in the percentage of the liberated matter consumed. Finally, when v_{max} lies in a low-velocity tail, P is close to zero as the liberated material has too high a velocity to be accreted by the hole.

Therefore, the amount of tidally disrupted stellar material captured by a disrupting component black hole is just given by the right-hand-side of equation (8), multiplied by P. That is,

$$\left(\frac{dM_h}{dt}\right)_C = 1.9 \times 10^{-15} P \left(\frac{M_\odot}{\text{yr}}\right)\left(\frac{\rho_s}{M_\odot\ \text{pc}^{-3}}\right)\left(\frac{\text{g cm}^{-3}}{\rho}\right)^{1/2}\left(\frac{M_h}{M_\odot}\right)^{4/3}\left(\frac{\text{km s}^{-1}}{v_h}\right). \tag{21}$$

This is plotted in Figures 4 and 5 for clusters with $r_{cl} = 1$ pc and $r_s = 10$ pc, respectively.

From these results, one sees, for instance, that for 2 $M_\odot$ black holes in a cluster of radius 1 pc with very high stellar densities, the tidal disruption and capture of stars can yield accretion rates up to from 1 to 7 $\times 10^{-3}\dot{M}_{crit}$ for $M_{cl} = 10^6$–10^{11} $M_\odot$. For 10 $M_\odot$ black holes the range is 1–5 $\times 10^{-2}\dot{M}_{crit}$. For larger mass black holes in the cluster, the mass supply is even much better. It is true that for the smallest mass holes these rates are somewhat smaller than the $0.1\dot{M}_{crit}$ we assumed for all the holes in Paper I. But it is still enough to power these holes adequately so that the clusters of which they are a part, including the cluster "halos" as we have characterized them, can model many AGNs, and QSOs.

In these cases, fitting will yield cluster models with mass spectra $\beta > 3.77$—instead of $\beta \approx 2$ under the assumption of Paper I. This means that they will have many more small holes, and fewer large ones, and a much smaller maximum-mass hole than clusters with lower β. Because of the lower accretion rates for these small holes, these models essentially put 15 to 50 small holes in the cluster for every small hole in the $\beta \approx 2$ models.

This discussion, it should be noticed, neglects the fact that many of the small holes will be in the cluster "core," where it is likely that they would be supplied more abundantly from the overall flow of accretion material to the center (see Paper III). We shall not pursue this problem here; a succeeding paper will deal with the cluster core in more detail. As we shall see, too, in § 4, the supply from disrupted stars may well be supplemented by that from binary companions of the black holes. Finally, the larger holes in these

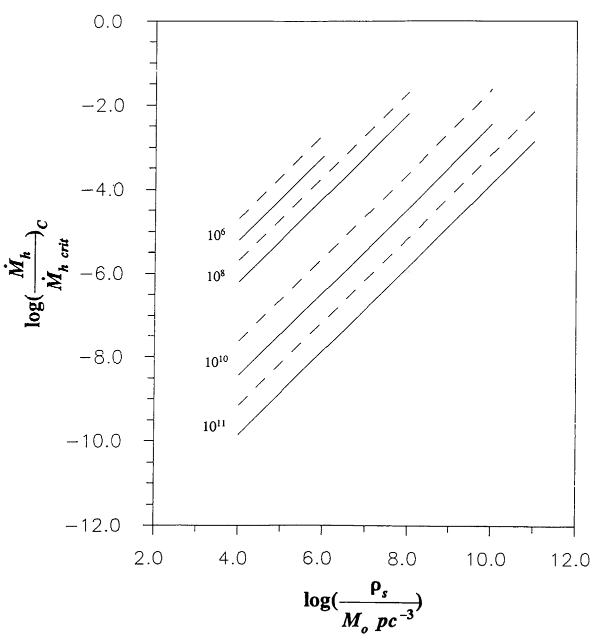

FIG. 4

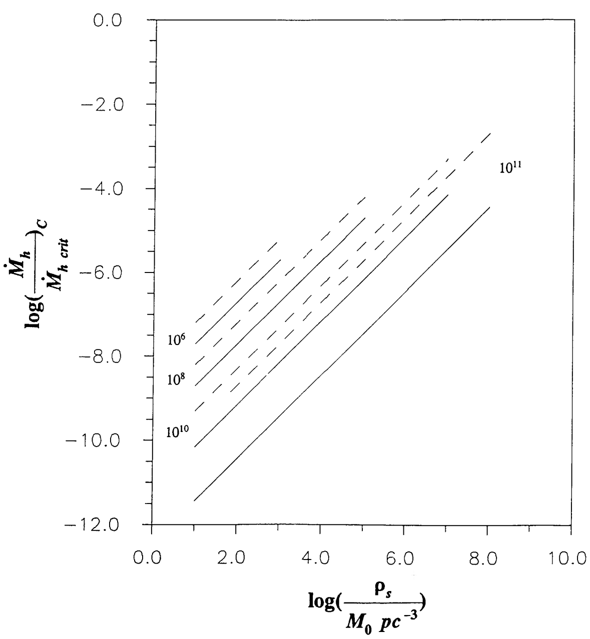

FIG. 5

FIG. 4.—The amount of tidally disrupted material captured by the disrupting hole, $(\dot{M}_h)_C$, in a star–black hole cluster with $\beta > 3.5$ and with $r_{cl} = 1$ pc, as a function of ρ_s, for clusters of different masses M_{cl}. Again, solid lines represent $2\,M_\odot$ black holes and dashed lines $10\,M_\odot$ holes. The velocity problem in more massive clusters is manifested by the displacement of lines toward the right (compare Fig. 2).

FIG. 5.—The same as for Fig. 4, but with $r_{cl} = 10$ pc

clusters will be fed much more efficiently by tidal disruption than the smaller holes, as we have already pointed out. In fact, some of them will be critically fed.

It should also be noted that our calculation of the fraction of tidal stellar debris captured by the black holes is only adequate for the smaller black holes, those less than $10^6\ M_\odot$. But, as is clear by now, these are the ones of primary interest to us. For very massive black holes, other considerations show that half of the liberated material will be captured in very tightly bound orbits and half of it will be ejected at very high velocities (of the order of 10^4 km s^{-1}) (Rees 1988; Hills 1988). (We thank an anonymous referee for pointing out these limitations to us and for providing us with these references.)

There is another limitation to this study which should also be mentioned. We have assumed a Maxwellian distribution for the relative velocities of the holes and the stars, and for the holes and the stars separately. But there may be several important effects which distort the velocity distribution significantly from Maxwellian. First, due to the escape of high-velocity holes and stars from the cluster, the real distribution will have a high-velocity cutoff; so there will be no high-velocity tail. This would give a larger percentage of relative velocities less than $v_{\max}$ for a Maxwellian distribution, and allow us to feed all the holes more efficiently by tidal disruption. Second, there will, of course, be orbit segregation and mass segregation effects which would introduce anisotropies in the velocity distribution. It is unclear, intuitively, how these latter considerations would affect our results.

If we just consider tidal disruption, and neglect the possibility of larger accretion rates in the cluster "core" and the likelihood of a significant contribution of material to the individual holes from their binary companions, then we can see from equation (8), and from equations (19) and (21), that the accretion rates on the individual holes will go as

$$\dot{M}_h \propto M_h^\gamma , \tag{22}$$

where $\gamma > 4/3$. When $P < 1$, terms going as M_h and $M_h^{5/3}$ will be added to the $M_h^{4/3}$ dependence.

For clusters of black holes which do not have a dense stellar environment and are not dominated by binaries, the results of Paper III continue to hold, as we have already implied. In these cases, the cluster "cores" will provide all the luminosity with the consequent high-energy break in the spectrum. In *both* cases, there may be an optically thick core, as well (see Paper II).

4. ESTIMATED EFFECTS OF BINARY AND MULTIPLE STAR SYSTEMS ON THE LUMINOSITY OF BLACK HOLE CLUSTERS

In discussing binary and multiple star systems in clusters of black holes, we are primarily interested, of course, in those where the black holes in the cluster each have one or more stars as companions which feed them adequately, and therefore enable them to radiate near their full potential. If most of the black holes were found to have such stellar companions, the black hole cluster would have a high enough luminosity even from the holes in its halo to model quasars and active galactic nuclei—without relying either solely on the cluster core, or on material from tidally disrupted stars in the halo.

We shall concentrate here on trying to establish whether or not it is plausible that a good number of the component cluster black holes might have companions captured by the tidal dissipation process. We are, as indicated above, mainly interested in these small holes, because they can be one of the dominant sources of X-rays in our model and because they dominate the environs of the cluster halo. If it can be plausibly argued that a large percentage of these small holes have companions, then it seems relatively straightforward to extend the conclusion to the larger holes. This is simply because the larger holes will have formed from the coalescence of smaller ones. And if most of the colliding black holes have close stellar companions, then at least a fair number of the larger black holes will have either stellar companions or significant remnants of the stellar companions of their smaller black ancestors on which they can feed.

Unfortunately, tidal capture of binaries will not be significant in our black hole cluster, unless the dispersion velocities are less than about 270 km s^{-1}. In many cases, the velocities of periastron passage will be too high and at distances which must be larger than r_t (otherwise, the star would be disrupted, not captured intact as a binary companion). Not enough kinetic energy will be tidally dissipated in these rapid, relatively distant encounters. In the original Fabian, Pringle, & Rees paper (1975) on tidal capture, they give the criterion for tidal capture as

$$x \equiv \frac{R_{\min}}{R_*} \lesssim f^{1/3}\left[\frac{GM_*}{R_* V_*^2}\,\frac{M_h(M_h + M_*)}{M_*^2}\right]^{1/6} , \tag{23}$$

where M_* and M_h are the masses of the individual stars and black holes, respectively, $R_{\min}$ is the periastron distance, R_* is the stellar radius, V_* is the velocity dispersion of the stars with respect to the black hole, and f is a dynamical quantity with a value of about 3. If we take $R_* = R_\odot$, $M_* = M_\odot$, and $M_h = 2\ M_\odot$, our smallest black holes, and set $x > 2.3$, corresponding to the value of r_t, we find that equation (23) gives $V_* < 264$ km s^{-1} for tidal capture.

Thus, only in clusters with V_* smaller than this limit will tidal capture be significant—that is, only in clusters which are not very massive nor very centrally condensed. For instance, a cluster of $10^7\ M_\odot$ must occupy at least a volume of 1 lt-yr^3 for this criterion to be fulfilled. This, in turn, means that such clusters must not vary significantly on time scales less than a year. So, for more rapidly fluctuating QSOs and AGNs supplying the black holes from recently tidally captured binaries will not work.

However, it may be that a significant number of binaries are present in such clusters on other grounds. There may be a high percentage of the original binary fraction of stars which survives their evolution into a compact black hole cluster—especially the close and contact binaries which are of the greatest interest here. Furthermore, the process of tidal capture itself, operating throughout the birth and evolution of such a cluster, may be adequate to ensure that many of the small and intermediate black holes have stellar companions. In our discussion above we gave the criterion for tidal capture and applied it to black hole clusters already in their relaxed, compact, and mass-segregated states, with massive cores and usually very high velocity dispersions. Earlier configurations of the cluster would not have had such high velocity dispersions and may have provided a favorable environment for binary capture. It is impossible to assess quantitatively either of these possibilities with any confidence, nor the rate at which close binary systems would be dissociated in the evolution of such clusters.

5. CONCLUSIONS

We have shown how the black holes in the halo of a black hole cluster can be efficiently supplied with material via the tidal disruption of stars and the capture of this liberated material, but not via the tidal capture of stars by the component black holes, except in clusters with relatively low velocity dispersions. The essential requirements for supplying black holes by tidal disruption are that there be a very high density of stars around the black hole cluster, $\gtrsim 10^7\ M_\odot/\mathrm{pc}^3$, which seems likely to be realized in the cores of many galactic nuclei, and that the black hole cluster itself not be so massive or so compact that the velocity of stars as they encounter the black holes is too high for the tidal dissipation and tidal capture processes to operate efficiently. This means that the largest clusters, with $M_{\mathrm{cl}} \approx 10^{10}\ M_\odot$, should not have stellar cluster radii much less than about 10^{18} cm for a stellar density of between 10^8 to $10^{10}\ M_\odot\ \mathrm{pc}^{-3}$. The smallest clusters, with $M_{\mathrm{cl}} \approx 10^6\ M_\odot$ and with, say, $r_{\mathrm{cl}} \approx 10^{18}$ cm, would need a stellar density of about 10^6–$10^7\ M_\odot\ \mathrm{pc}^{-3}$, depending. In practice, of course, the black hole cluster radius would be given observationally by the variability of the source at X-ray frequencies.

Although the process of tidal capture of binary companions will be limited to clusters in which the velocity dispersion is smaller than about 270 km s^{-1}, it is still possible, as we have discussed, that supply of the component black holes from binaries will make a significant contribution in more compact clusters. This will be the case, if most binaries survive the evolution into a cluster and/or tidal capture is favored in its earlier stages of development.

When the black hole cluster halo can be so supplied, the entire cluster then becomes an efficient source for the central engine of at least low and moderate luminosity AGNs and QSOs. When, on the other hand, the holes in the cluster halo cannot be so supplied, it is only the general accretion of material onto the cluster core—defined phenomenologically as that central part of the cluster where the spherical symmetry of an originally symmetric flow breaks down, the flow becomes chaotic, and/or the accretion rate onto the individual black holes can no longer be described by a simple application of the continuity equation—which can model AGNs and QSOs, as presented in Paper III.

The effect on the spectrum issuing from such an "internally supplied" cluster of black holes will be to straighten out the break in th spectrum predicted for the case where the bulk of the radiation emanates from the cluster core as we have just described it (see Fig. 3 in Paper III). That break would be due to the decreased proportion of smaller holes to larger holes in the core relative to their proportion in the entire cluster, leading to a relative decrease in radiation of higher frequency, which in our models comes from the accretion disks of the smaller holes. With a more efficient supply of material by tidal capture and in some cases from binary companions to the smaller holes in the cluster halo, there is now an increased production of radiation in the far-ultraviolet and X-ray regions of the spectrum from the disk around these smaller holes, and the spectral break is "washed out."

As we have pointed out above, if the tidal disruption of stars is the dominant form of mass supply to the black holes, so that $\dot{M}_h \propto M_h^\gamma$, $\gamma > 4/3$, giving us a natural model for the accretion rate onto the holes, we end up with AGN and QSO cluster models with spectral indices $\beta \geq 3.8$. These will have much smaller central masses than those described in Paper I (with spectral indices $\beta \approx 2$) and will be dynamically dominated by the smaller holes and stars. Quinlan & Shapiro's (1989a) evolutionary simulations predict compact-object clusters with $\beta \approx 4$ (see below).

Of course, as pointed out in Paper III, there is still the possibility that the cluster core may be optically thick, radiating an optical-ultraviolet continuum. In that case, the integrated spectrum of these smaller holes in the halo would be added to the emission from the optically thick core. In fact, as is well known, enhanced ultraviolet emission is a hallmark of AGNs and QSOs. It is from such a core also that one might expect regions of outflow—as the accretion flow into the cluster reaches a point where it becomes critical with respect to the mass interior to that radius, which may be already deep within the cluster.

The tidal disruption mechanism of supplying a cluster of black holes can easily explain how AGNs and QSOs eventually "turn off." If star formation in the central regions of a galactic nucleus cannot keep up with the destruction of stars in the core, then the density of stars will rapidly decrease. As is clear from our calculations, that would cause the luminosity of the black hole cluster to fade dramatically, once it had used up the material from disrupted stars and from binary companions. This, of course, will also be true for a single supermassive black hole. As we saw above, however, it will be somewhat more noticeable for a black hole cluster.

In this paper and in Paper III, we have treated only the problem of supplying the component black holes of the cluster. There remain a large number of other issues to resolve before we can assess the overall suitability of black hole clusters as models of AGNs and QSOs. Some of the more important are these: expected modifications of the spectrum from the accretion disks due to absorption and/or reprocessing of radiation; the occurrence of shocks and other sources of high-energy electrons within the accretion flow, whether in the cluster halo or in the core (a preliminary treatment of this has been given in Paper II); the likely conditions within the cluster core itself—which are the dominant processes there? We shall examine these questions in succeeding papers.

A crucial issue for these black hole cluster models is that of their dynamics—whether or not such clusters can form with power law–like mass-spectra from clusters of stars and stellar-size compact objects, and whether or not their lifetimes will be long enough to be viable models of AGNs and QSOs. Recently Quinlan & Shapiro (1987, 1989a, b) have begun to study these questions in some detail. Starting with a cluster of about 10^8 stellar-mass compact objects, they (Quinlan & Shapiro 1989a) follow the evolution of the cluster using multicomponent Fokker-Planck simulations until black holes of masses $\gtrsim 100\ M_\odot$ have formed. At this point their cluster cores (not to be identified with our definition of "cluster core"!) have shrunk so much that the Fokker-Planck equations are no longer valid. They find that the cluster evolution is dominated by binary mergers of the compact stars to form larger black holes, which then rapidly sink to the center of the cluster because of mass-segregation processes. The principal dissipative process is gravitational radiation, which accelerates the hardening and coalescense of the binary compact objects. This becomes more pronounced in the core itself.

Although the beginning of the evolutionary sequence takes considerable time as the first larger black holes form, and is dominated by binary mergers, by the time the central potential has increased by a factor of 5, mass segregation becomes very important and there is a sudden *increase* in the core density and in the mean stellar mass in the core, along with a sudden *decrease* in the core mass, the core radius (again as defined by Quinlan and Shapiro, not as we have used these terms above), and the central velocity dispersion.

Quinlan and Shapiro stopped their simulations at this point—when the core contained only a few black holes of about 100 $M_\odot$—actually about 70 black holes with a mean mass of 90 $M_\odot$. At this point the central redshift is large enough for a relativistic instability to cause this core to collapse on a dynamical time scale (Quinlan & Shapiro 1989a).

Several things are worth noting in relating these important results to our black hole clusters. First of all, Quinlan and Shapiro's clusters do develop a power-law spectrum, with index 4 (Quinlan & Shapiro 1989a, b), close to our limit of 3.8 which we have derived for clusters whose mass supply is provided by tidal disruption of stars, instead of the index 2 which we derived from a preliminary fitting to AGN and ESO observations (Paper I). This reflects, among other things, the fact that at the stage Quinlan and Shapiro stop their evolution simulations, their clusters contain very many smaller black holes which still dominate the mass of the cluster and only of few larger black holes, each of which is only two orders of magnitude more massive than the original components. In our clusters of Paper I (see also Paper III) the largest black holes were many orders of magnitude larger than the smaller ones and contained most of the mass of the cluster.

Second, closely related to this, when the core collapses in one of Quinlan and Shapiro's clusters, we have a core containing a total mass of about $6 \times 10^3\ M_\odot$ in one or several large black holes, surrounded by a "halo" of many smaller compact objects containing the bulk of the $10^8\ M_\odot$ in the cluster. These smaller holes and other compact objects would then be merging with one another and the resulting larger ones migrating to smaller radii. The central collapsed core would then gradually grow by capturing these newly formed larger holes. This is not significantly different from the type of cluster we have been envisioning in our work so far, particularly in having many smaller holes surrounding a central core.

In this paper and in Paper III we have been examining the conditions under which these smaller holes in the "halos" of such a clusters can contribute significantly to the overall luminosity emanating from it, and thus to its observational characteristics. The very fact that the clusters emerging from Quinlan and Shapiro's evolution studies are mass-dominated by their halos, and not by their collapsed cores, is important. They are considerably different from a single supermassive black hole—and their observational characteristics will be determined by the considerations we have been examining in this series of papers. As we have seen here, one of the conditions for such a cluster to manifest its halo of smaller black holes—and not only just its central core or central massive black hole—is that it be embedded in a dense distribution of stars which can supply them with adequate accretable material. Once these stars are significantly depleted, the central cluster will be starved of material, and the AGN or QSO mechanism will turn off.

There remains, of course, the question of how long such a cluster with a collapsed core of, say, 10^3–$10^4\ M_\odot$ would take to evolve to single supermassive black hole of 10^7–$10^8\ M_\odot$. It would be very worthwhile to begin from a simplified version of the clusters Quinlan and Shapiro end up with—a $6 \times 10^3\ M_\odot$ central black hole surrounded by a large number of smaller holes and stars, with a mass spectral index of 4 and see how this configuration evolves through consumption by the central hole, coalescence of the smaller holes with one another, and evaporation. Particularly, it would be important to see how long it would take for this configuration to evolve to a cluster which is mass-dominated by its core, like our clusters of Paper I, and then how much longer still it would take for the cluster to evolve to a single supermassive black hole.

We thank an anonymous referee for several suggestions which have improved this paper and for correcting a serious error in our original calculation of the rate of binary capture in our clusters. A portion of this research was done while T. S. was a staff scientist at the Lunar and Planetary Institute, which is operated by the Universities Space Research Association under contract NASW-4574 with the National Aeronautic and Space Administration. This is Lunar and Planetary Institute Contribution No. 779.

REFERENCES

Bahcall, J. N., & Wolf, R. A. 1976, ApJ, 209, 214
Fabian, A. C., Pringle, J. E., & Rees, M. J. 1975, MNRAS, 172, 15P
Frank, J., & Rees, M. J. 1976, MNRAS, 176, 633
Hills, J. G. 1975, Nature, 254, 295
———. 1978, MNRAS, 182, 517
———. 1988, Nature, 331, 687
Hills, J. G., & Day, C. A. 1976, Ap. Lett., 17, 1976
Light, E. S., Danielson, R. E., & Schwarzschild, M. 1974, ApJ, 194, 257
Lightman, A. P., & Shapiro, S. L. 1977, ApJ, 211, 244
———. 1978, Rev. Mod. Phys., 50, 437
Pacholczyk, A. G., & Stepinski, T. S. 1988, ApJ, 324, 695 (Paper II)
Pacholczyk, A. G., Stepinski, T. S., & Stoeger, W. R. 1989, ApJ, 343, 563 (Paper III)
Pacholczyk, A. G., & Stoeger, W. R. 1986, ApJ, 303, 76 (Paper I)
Quinlan, G. D., & Shapiro, S. L. 1987, ApJ, 321, 199
———. 1989a, ApJ, 343, 725
———. 1989b, in Dynamics of Dense Stellar Systems, ed. D. Merritt (Cambridge: Cambridge Univ. Press), 113
Rees, M. J. 1988, Nature, 333, 523
Schwarzschild, M. 1973, ApJ, 182, 357
Shapiro, S. L. 1986, in Structure and Evolution of Active Galactic Nuclei, ed. G. Giuricin, F. Mardirossian, M. Mezzetti, & M. Ramella (Dordrecht: Reidel), 129
Spitzer, L., Jr. 1968, in Diffuse Matter in Space (New York: Wiley Interscience), 88