

## THE IMPLICATIONS OF THE COMPTON (GRO) OBSERVATIONS FOR COSMOLOGICAL GAMMA-RAY BURSTS

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### ABSTRACT

The BATSE observations (Fishman et al.; Meegan et al.) have shown that the angular distribution of  $\gamma$ -ray sources in the sky is isotropic and that  $\langle V/V_{\max} \rangle = 0.35$ . We show that these observations are consistent with a cosmological distribution of sources. In fact the observed distribution of  $V/V_{\max}$  values is the one toward which a cosmological distribution converges. If there is even a weak cosmological time dependence of the event rate, then the bursts originate at modest  $z$  values ( $z_{\max} \approx 1$ ). For flat cosmological models, this requires  $10^{50}$  ergs or less. The energy and the observed event rate are in full agreement with the binary neutron star merger model.

*Subject headings:* cosmology: observations — gamma rays: bursts

### 1. INTRODUCTION

The recent observations of the BATSE detector (Fishman et al. 1991; Meegan et al. 1992) have shown that the angular distribution of  $\gamma$ -ray sources in the sky is isotropic and that  $\langle V/V_{\max} \rangle = 0.35$ . Galactic sources can fit these observations with a halo population with typical distances of more than 20 kpc (Paczynski 1991a, b; Hakkila 1991), a population that is not observed in any other way, or if there is a combination of two classes of bursts, which differ in luminosity by  $\approx 10^5$  (Lingenfelter 1991). An alternative possibility, which we consider here, is that  $\gamma$ -ray bursts originate at cosmological distances (Prilutski & Usov 1975; Usov & Chibisov 1975; Goodman 1986; Paczyński 1986; Eichler et al. 1989). Observational evidence in favor of this interpretation appeared almost 10 years ago when van den Bergh (1983) asserted that the isotropy of the distribution of the 46  $\gamma$ -ray bursts positions known at that time, shows that  $\gamma$ -ray bursts originate either at very close distances or at cosmological distances.

We show here that the  $V/V_{\max}$  distribution observed by BATSE is compatible with a  $V/V_{\max}$  distribution resulting from a cosmological population of sources. We discuss the constraints, imposed by the BATSE observations, on the required energy, the time evolution and other features of cosmological sources. In § 2, we discuss the formalism for calculating  $V/V_{\max}$  for a cosmological distribution of burst sources. In § 3, we compare the calculated distribution to the observed one, and we discuss the general implication for any cosmological population of sources.

Among cosmological models, generation of  $\gamma$ -ray bursts during neutron star binary coalescence (Eichler et al. 1989; Piran 1990; Piran, Narayan, & Shemi 1991) seems to be the most promising: It is based on a population (binary neutron stars) and a process (orbit decay leading to merger) for which we have direct evidence in our own Galaxy (Hulse & Taylor 1975). At the end of this phase the neutron stars coalesce, releasing  $\approx 10^{54}$  ergs, much larger than the energy required for cosmological bursts. Clearly, neutron star mergers cannot account for the three soft  $\gamma$ -ray repeaters (e.g., Higdon & Lingenfelter 1990), and they might be incompatible with the observations of the cyclotron lines seen by a small fraction of the bursts (Mazet et al. 1981; Murakami et al. 1988) and to a lesser extent with soft X-ray emission (Murakami et al. 1991); thus, they might not explain all observed bursts. Still, it is worthwhile to study the implications of the above conclusions to this model. In § 4, we show that the rate of neutron star coalescence, 1 per  $10^6$  yr per galaxy, (Narayan, Piran, & Shemi 1991; Phinney 1991) is comparable to the observed rate of  $\gamma$ -ray bursts. The  $\gamma$ -ray energy required for detection is  $\approx 10^{-4}$  to  $10^{-3}$  of the total energy released. Our conclusions and some suggestions for further tests of the cosmological origin of  $\gamma$ -ray bursts are discussed in § 5.

### 2. COSMOLOGICAL $V/V_{\max}$

The  $V/V_{\max}$  distribution in a Euclidean region depends only on the spatial distribution of sources. The expansion of the universe and the resulting redshift modify, however, the  $V/V_{\max}$  of cosmological distributions (Schmidt 1968). We examine several populations of sources in cosmological models with and without a cosmological constant.

The spectrum of the observed bursts (see, e.g., Higdon & Lingenfelter 1990; Band, Matteson, & Ford 1991) is steep so the low-energy  $\gamma$ -rays dominate the count rate. In this range the spectrum can be approximated by a power law,  $n(E) \propto E^{-s}$ , characterized by a spectral index  $s$ . The integrated number of counts observed by a detector from a burst at a cosmological redshift,  $z$ , is

$$C = \frac{A(1+z)^{(3-s)}}{R_l^2(z)}, \quad (1)$$

where  $A$  depends on luminosity of the source and  $R_l(z)$  is the “luminosity distance”:

$$R_l(z) = \frac{c}{H_0} \bar{R}_l = \frac{c}{H_0} (1+z) \int_0^z \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_M(1+z')^3}}. \quad (2)$$

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$\Omega_M$  is the closure parameter of the matter density and  $\Omega_\Lambda$  is the value of the cosmological constant measured in units of the closure density (for simplicity we use  $\Omega_\Lambda + \Omega_M = 1$ ). If  $\Omega_\Lambda = 0$  (i.e., if the universe is flat and the cosmological constant vanishes), we have  $R_l = (2c/H_0)[1 + z - (1 + z)^{1/2}]$ .

For a detector whose triggering depends on the total count (denoted “ $C$ ”) we find that

$$\left(\frac{V}{V_{\max}}\right)_C = \left(\frac{C_{\max}}{C}\right)^{3/2} = \frac{(1+z)^{3(s-3)/2} R_l^3(z)}{(1+z_{\max})^{3(s-3)/2} R_l^3(z_{\max})}, \quad (3a)$$

where  $z_{\max}$  is the maximal  $z$  value (corresponding to a maximal distance) from which the burst triggers the detector.

If the triggering depends on the count rate (denoted “ $R$ ”) rather than on the total number of counts, we have to add a  $(1+z)$  factor:

$$\left(\frac{V}{V_{\max}}\right)_R = \left[\frac{C_{\max}/(1+z_{\max})}{C/(1+z)}\right]^{3/2} = \frac{(1+z)^{3(s-2)/2} R_l^3(z)}{(1+z_{\max})^{3(s-2)/2} R_l^3(z_{\max})}. \quad (3b)$$

The observations of a “ $C$ ” detector of a source with a spectral index  $s$  can be related to those of an “ $R$ ” detector of a source with a spectral index  $s-1$ . We define the effective spectral index  $\tilde{s}$ :  $\tilde{s} = s$  for a “ $R$ ” detector and  $\tilde{s} = s-1$  for a “ $C$ ” detector. The observed data suggest that  $s \approx 1$  (see, e.g., Higdon & Lingenfelter 1990; Band et al. 1991), and we present here models with  $\tilde{s} = 1$  and  $\tilde{s} = 2$ .

We integrate equation (3a) over the source distribution to obtain  $\langle V/\bar{V}_{\max} \rangle$ , the average value of  $V/V_{\max}$ . For cosmological sources we expect a uniform distribution in (comoving) space and a time-dependent burst rate  $R_e(t)$ . We characterize this dependence as  $R_e = R_{e0}(t/t_0)^\beta$  where  $R_{e0}$  is the current event rate (at  $t = t_0$ ). The event rate,  $R_e$ , is redshifted and the observed rate,  $R_{\text{obs}}$  satisfies  $R_{\text{obs}} = R_e/(1+z) \propto t^\beta/(1+z)$ . Using this time dependence we obtain:

$$\left\langle \frac{V}{V_{\max}} \right\rangle = \frac{\int_0^{z_{\max}} dz' R_l^{\tilde{s}}(z') \{d[R_l/(1+z')]/dz'\} t^\beta(z')(1+z')^{3(\tilde{s}-4)/2}}{(1+z_{\max})^{3(\tilde{s}-2)/2} R_l^{\tilde{s}}(z_{\max}) \int_0^{z_{\max}} dz' R_l^{\tilde{s}}(z') \{d[R_l/(1+z')]/dz'\} t^\beta(z')(1+z')^{-3}}. \quad (4)$$

Equation (4) should be integrated over the luminosity function of the source, which, in turn, corresponds to a weighted integration over  $z_{\max}$ . We assume in the following that we can approximate the bursts' source population by a standard candle with a fixed luminosity.

### 3. COMPARISON WITH OBSERVATIONS—GENERAL CONSIDERATIONS

For fixed luminosity sources  $\langle V/V_{\max} \rangle$  is a function of four parameters:  $\tilde{s}$ ,  $\beta$ ,  $z_{\max}$ , and  $\Omega_M$ . We consider two cosmological models: a flat universe with  $\Omega_M = 1$  and a universe with a low matter density  $\Omega_M = 0.1$  and a cosmological constant (Peebles 1984). The latter results represent to a large extent the situation in a low- $\Omega_M$  universe without a cosmological constant. In both cases the distances to a given  $z$  value are larger than those in a flat, matter-dominated universe.

The prediction of the theoretical models should be compared with the observed average value of the distribution:  $\langle V/V_{\max} \rangle = 0.35$  and with  $N_{\text{obs}}(V/V_{\max})$ , the observed distribution of number of events with a given  $V/V_{\max}$  value (Meegan et al. 1992). Figure 1 depicts  $z_{\max}$  versus  $\beta$  for distributions satisfying  $\langle V/V_{\max} \rangle = 0.35$ . The average  $z$  values,  $z_{\text{av}}$  vary between 0.55 and 0.65 of  $z_{\max}$ . Two calculated distributions of  $N(V/V_{\max})$  are shown in Figure 2. The generic distribution, toward which  $N(V/V_{\max})$  converges, fits  $N_{\text{obs}}(V/V_{\max})$  with a surprising accuracy. It peaks at  $V/V_{\max} \approx 0$  and decreases sharply toward a plateau at  $V/V_{\max} > 0.3$ . This distribution is produced in most of the parameter phase space that we have examined. The curves in Figure 1 are continued only in regions in which this  $N(V/V_{\max})$  appears.

The second distribution shown on Figure 2 also has  $\langle V/V_{\max} \rangle = 0.35$ , but it is qualitatively different from the first one. It does not peak around  $V/V_{\max} = 0$ , and it is significantly different from  $N_{\text{obs}}(V/V_{\max})$ . Such distributions appear at (1) low effective spectral index, (small  $\tilde{s}$ ), (2) when the rate of bursts does not increase with time ( $\beta < 0$ ), or (3) in  $\Omega_M \ll 1$  universes (these effects increase the number of events observed at a high  $z$  values relative to the number observed at a low  $z$  values).

$z_{\max}$  ranges between 0.5 (for  $\tilde{s} = 1$  and  $\Omega_M = 1$ ) and 1.4 (for  $\tilde{s} = 0$  and  $\Omega_M = 0.1$ ) if the event rate depends strongly on time ( $\beta > 1$ ).  $z_{\max}$  becomes very sensitive to  $\tilde{s}$  and to  $\Omega_M$ , and it varies between 1.5 (for  $\tilde{s} = 1$  and  $\Omega_M = 1$ ), and 8 (for  $\tilde{s} = 0$ ) if the event rate is time-independent ( $\beta = 0$ ). Even higher values are reached for  $\Omega_M = 0.1$ . However, the upper range of the  $z_{\max}$  values, in which  $N(V/V_{\max})$  does not have the right shape, can be excluded.

In some physical models the burst cannot take place prior to some initial moment, i.e., there were no bursts at  $z > z_{\text{cut}}$ . Specifically, bursts from any source that depends on stellar evolution cannot take place before galaxies form at  $z \approx 2-3$ . When we add an upper  $z$  cutoff to the models described earlier, we find that  $\langle V/V_{\max} \rangle = 0.35$  for  $z_{\max}$  slightly larger than  $z_{\text{cut}}$ , independently of the values of the other parameters (see Fig. 1). However, in these cases there are no events in the high range of  $V/V_{\max}$  values and  $N(V/V_{\max})$  is significantly different from  $N_{\text{obs}}(V/V_{\max})$ . The cutoff argument can be turned around and used in reverse. We can rule out the region in parameter space that requires  $z_{\max} > z_{\text{cut}}$ , for all models that require that such a cutoff exists.

Once  $z_{\max}$  is known the observed limiting fluence,  $F \approx 10^{-7}$  ergs  $\text{cm}^{-2}$ , can be converted to energy emitted by the source. If the bulk of the energy is emitted significantly above the lower limit of the detector's window (as is the case for  $\gamma$ -ray bursts and BATSE) and if the sources are emitting isotropically, we have

$$E = 4\pi R_l^2(z_{\max})(1+z_{\max})F \approx 1.3 \times 10^{50} \text{ ergs } \bar{R}_l^2(z_{\max})(1+z_{\max})h_0^{-2}F_{-7}, \quad (5)$$

where  $h_0$  is the Hubble constant in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$  and  $F_{-7}$  is the minimal observed fluence in units of  $10^{-7}$  ergs. The required energy,  $E$ , which depends only on  $z_{\max}$  and on the cosmological model, is shown in Figure 3a. Moderate amounts of energy

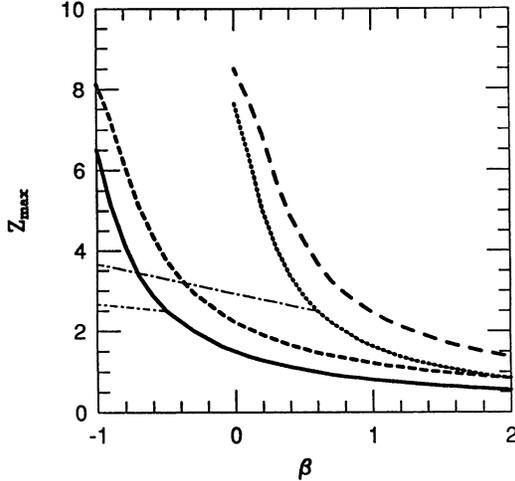


FIG. 1

FIG. 1.— $z_{\max}$  vs.  $\beta$  for models with  $\langle V/V_{\max} \rangle = 0.35$ .  $\tilde{s} = 1$ ,  $\Omega_M = 1$  (solid curve);  $\tilde{s} = 0$ ,  $\Omega_M = 1$  (dotted curve);  $\tilde{s} = 1$ ,  $\Omega_M = 0.1$ ,  $\Omega_\Lambda = 0.9$  (short-dashed curve);  $\tilde{s} = 1$ ,  $\Omega_M = 0.1$ ,  $\Omega_\Lambda = 0.9$  (long-dashed curve);  $\tilde{s} = 1$ ,  $\Omega_M = 1$ ,  $z_{\text{cut}} = 2.5$  (dot-long-dashed curve);  $\tilde{s} = 0$ ,  $\Omega_M = 1$ ,  $z_{\text{cut}} = 2.5$  (dot-short-dashed curve). The curves do not continue to values for which  $N(V/V_{\max})$  does not fit  $N_{\text{obs}}(V/V_{\max})$ .

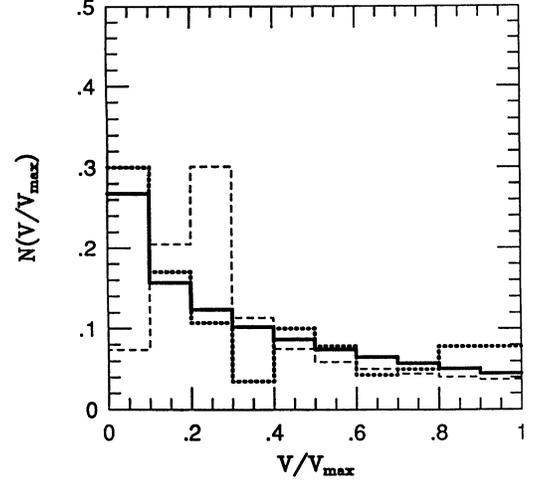


FIG. 2

FIG. 2.— $N(V/V_{\max})$  distribution (solid curve) which fits the observed data from Meegan et al. (1992) (dotted curve) and a typical  $N(V/V_{\max})$  distribution that does not fit it (dashed curve).

( $3 \times 10^{50}$  ergs for  $\tilde{s} = 0$ ,  $\beta = 0.8$  and  $\Omega_M = 1$ ) are sufficient for time-independent populations.  $E$  decreases by an order of magnitude if the time dependence of the event rate is stronger and if  $\tilde{s} = 1$ . In the latter case the bursts are taking place at lower  $z$  values and at nearer distances.  $E$  increases to  $10^{51}$  ergs (for  $\tilde{s} = 0$ ,  $\beta = 1$  and  $\Omega_M = 0.1$ ) if  $\Omega_M$  is low. A stronger time dependence can reduce  $E$ , but not by much ( $2 \times 10^{50}$  ergs for  $\tilde{s} = 0$ ,  $\beta = 2$ ,  $\Omega_M = 0.1$ ).

When we vary  $z_{\max}$  we also vary the volume covered by the detector. This determines also the rate of burst (per unit volume per unit time) that fits the observed number of events per year:  $\mathcal{R}_{\text{obs}} \approx 10^3 \text{ yr}^{-1}$ . Comparison with the theoretical model yields

$$\mathcal{R}_{\text{obs}} = R_{\text{e0}} \left( \frac{c}{H_0} \right)^3 \int_0^{z_{\max}} dz' \tilde{R}_i^2(z') \left\{ \frac{d[\tilde{R}_i/(1+z')]}{dz'} \right\} \left[ \frac{t(z')}{t_0} \right]^\beta (1+z')^{-3} \equiv R_{\text{e0}} \left( \frac{c}{H_0} \right)^3 v(z_{\max}, \beta, \Omega_M, \Omega_\Lambda). \quad (6a)$$

$v(z_{\max}, \beta, \Omega_M, \Omega_\Lambda)$  is shown in Figure 3b. A typical value for  $v$  for models in a flat, matter-dominated universe is  $\approx 0.01$ . Equation (6a) can be translated to observable quantities as

$$R_{\text{e0}} = 0.3 \times 10^{-6} (\text{yr}^{-1} \text{ galaxy}^{-1}) \left( \frac{\mathcal{R}_{\text{obs}}}{1000 \text{ yr}^{-1}} \right) \left[ \frac{10^{-2}}{v(z_{\max}, \beta, \Omega_M, \Omega_\Lambda)} \right], \quad (6b)$$

where we used a galaxy density of  $10^{-2} h^3 \text{ Mpc}^{-3}$  (Kirshner et al. 1983).

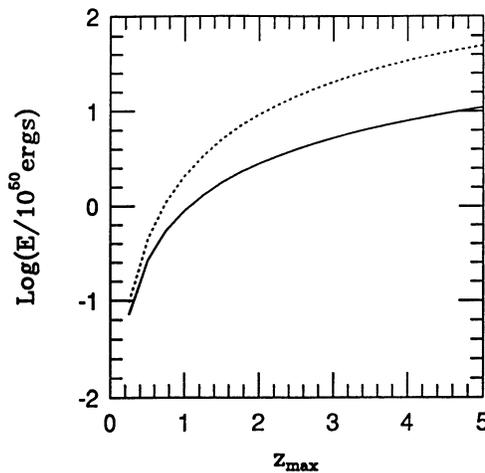


FIG. 3a

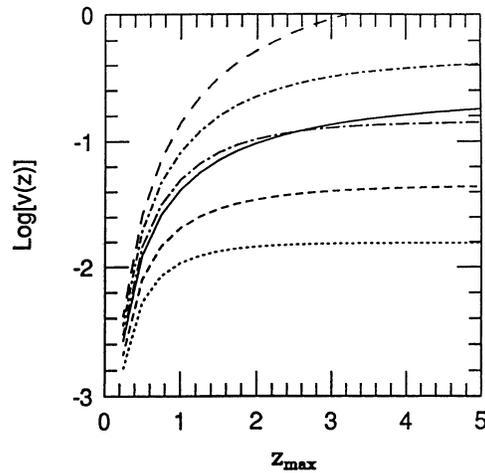


FIG. 3b

FIG. 3.— $\log(E/10^{50} \text{ ergs})$  vs.  $z_{\max}$  for  $\Omega_M = 1$  (solid curve) and  $\Omega_M = 0.1$  and  $\Omega_\Lambda = 0.9$  (dotted curve) (Fig. 3a).  $\log[v(z)]$  vs.  $z_{\max}$  for  $\Omega_M = 1$  and  $\beta = 0, 1, 2$  (solid, short-dashed, and dotted curves); and for  $\Omega_M = 0.1$  and  $\Omega_\Lambda = 0.9$  and  $\beta = 0, 1, 2$  (long-dashed, dot-short-dashed, and dot-long-dashed curves) (Fig. 3b).

## 4. IMPLICATIONS FOR THE NEUTRON STAR MERGER MODEL

The neutron star merger model asserts that the bursts appear during the merger of binary neutron star pairs (Eichler et al. 1989; Piran 1990; Piran et al. 1991). As with all models based on stellar evolution, the neutron star merger model has a natural  $z_{\text{cut}} \approx 2.5$ . Thus, it requires a modest time dependence of the burst rate ( $\beta > 0.6$  for  $\tilde{s} = 0$  and  $\beta > 1$  for  $\tilde{s} = 0$  and  $\Omega_M = 0.1$ ). No time dependence is needed if  $\tilde{s} = 1$ .

The observed event rate per galaxy agrees to a surprising accuracy with the rate of binary mergers calculated by Narayan et al. (1991) and Phinney (1991),  $\approx 1$  per  $10^6$  yr per galaxy. These estimates are based on only three binary pulsars, and they are uncertain by about one order of magnitude.

The energies needed (for  $\Omega_M = 1$ ) are a factor of 3 to 30 below the canonical number obtained naively for a cosmological source using  $E = 4\pi(c/H_0)^2 F \approx 10^{51}$  ergs. The energy required is always small compared with the  $5 \times 10^{53}$ – $10^{54}$  ergs released in a neutron star binary merger. Still, this additional factor might be important in working out the energy balance of these sources, as it is unknown what fraction of the total energy can be channeled to  $\gamma$ -rays.

Is it possible to estimate the time dependence of the neutron star merger model? The event rate,  $R_e$ , depends on the birth rate of neutron star binaries and on their lifetime. Both can be estimated, in principle, from binary pulsar observations.  $\tau_{\text{gr}}$ , the lifetime for gravitational radiation decay of the orbit, depends on two parameters:  $a_i$ , the initial orbital separation ( $\tau_{\text{gr}} \propto a_i^4$ ) and  $e_i$ , the initial ellipticity. If the probability of the initial orbital separation  $P(a_i)$  is a power law:  $P \propto a_i^q$  then:  $P(\tau_{\text{gr}}) \propto \tau_{\text{gr}}^{(q-3)/4}$ . Assuming that the birth rate of these systems is independent of time, and ignoring the distribution in  $e_i$ , we find that  $\beta = (q + 1)/4$ . If the birth rate peaks at some early time, as suggested by evidence for an early epoch of extensive star formation, we find that  $\beta = (q - 3)/4$ . These suggest a weak time dependence unless  $P(a_i)$  is a very steep function. Using data from regular binaries (Abt 1983), for which  $q = -1$ , we obtain  $R_e(t) \propto \ln(t)$  for the first case and  $\beta = -1/4$  for the second case. These will be consistent only with  $\tilde{s} = 1$ . However, this conclusion should be taken with caution: we have neglected the possible variation in the initial ellipticity (which amounts to a difference of a factor of 10 between the lifetime of PSR 1913+16 and PSR 2303+46). Moreover, binary neutron stars might not have the same distribution of orbital separations as regular binaries.

## 5. CONCLUSIONS

We have shown that the observations of  $V/V_{\text{max}}$  of  $\gamma$ -ray bursts by BATSE are compatible with a cosmological population of sources with modest  $z_{\text{max}}$  values ( $z_{\text{max}} \approx 1$ ). The shape of  $N_{\text{obs}}(V/V_{\text{max}})$  is not the one expected for a Euclidean population with an edge, for which there is a depletion in the high  $V/V_{\text{max}}$  values. However, it is the one toward which cosmological  $V/V_{\text{max}}$  distributions converge. A wide range of parameters of cosmological populations fits the observed data. The average  $z$  values are about  $0.6z_{\text{max}}$ ; thus, it is not alarming that optical galaxies are not observed at the location of the bursts.

This suggests that we look for other tests of the cosmological hypothesis of the origin of the bursts. One immediate proposal is to look for a correlation between the duration of the bursts [or another temporal feature proportional to  $(1 + z)$ ] with the intensity or another feature corresponding to the total count rate [which is also a function of  $(1 + z)$ , although possibly a more complicated one]. Such correlations would be weak if  $z_{\text{max}} < 1$  and might be hidden by large intrinsic variations of the bursts. Still, such correlations must be eventually observed when enough data accumulates if the events are indeed cosmological.

The specific model of emission of  $\gamma$ -ray bursts during the coalescence of neutron star binaries agrees with the BATSE observations. This model has a natural cutoff of  $z_{\text{cut}} \approx 2.5$ . Even a time-independent population can easily produce the observed  $V/V_{\text{max}}$  distribution (with  $s = 1$  and a rate detector). Modest energy sources (of the order of  $10^{50}$  ergs or less) suffice for such a population. The observed rate of  $\gamma$ -ray bursts is in complete accord with the rate of neutron star binary mergers estimated from pulsar statistics. The energy requirements increase and the agreement between the two rates worsen in models with a low  $\Omega_M$ . A detection of a gravitational radiation signal just prior to a gamma-ray burst will provide an ultimate confirmation of this model.

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## REFERENCES

- Abt, H. A. 1983, *ARA&A*, 21, 343  
 Band, D. L., Matteson, J. L., & Ford, L. A. 1991, in Proc. of the Huntsville GRO Meeting, ed. W. S. Paciasas & G. J. Fishman (New York: AIP), in press  
 Eichler, D., Livio, M., Piran, T., & Schramm, D. N. 1989, *Nature*, 340, 126  
 Fishman, G. et al. 1991, presentation for BATSE at GRO Workshop in Annapolis  
 Goodman, J. 1986, *ApJ*, 308, L47  
 Hakkila, J. 1991, in Proc. of the Huntsville GRO Meeting, ed. W. S. Paciasas & G. J. Fishman (New York: AIP), in press  
 Higdon, J. C., & Lingefelter, R. E. 1990, *ARA&A*, 28, 401  
 Hulse, R. A., & Taylor, J. H. 1975, *ApJ*, 368, 504  
 Kirshner, R. P., Oemler, A., Schechter, P. L., & Sheckman, S. A. 1983, *AJ*, 88, 1285  
 Lingefelter, R. E. 1991, in Proc. of the Huntsville GRO Meeting, ed. W. S. Paciasas & G. J. Fishman (New York: AIP), in press  
 Mazet, E. P., et al. 1981, *Nature*, 290, 378  
 Meegan, C. A., et al. 1992, *Nature*, 355, 143  
 Murakami, T., et al. 1988, *Nature*, 335, 234  
 Murakami, T., et al. 1991, *Nature*, 350, 592  
 Narayan, R., Piran, T., & Shemi, A. 1991, *ApJ*, 379, L17  
 Paczyński, B. 1986, *ApJ*, 308, L51  
 ———. 1991a, in Proc. of the Huntsville GRO Meeting, ed. W. S. Paciasas & G. J. Fishman (New York: AIP), in press  
 ———. 1991b, *Acta Astron.*, 41, 157  
 Peebles, P. J. E. 1984, *ApJ*, 284, 439  
 Phinney, E. S. 1991, *ApJ*, 380, L17  
 Piran, T. 1990, in *Supernovae*, ed. J. A. Wheeler, T. Piran, & S. Weinberg (Singapore: World Scientific), 303  
 Piran, T., Narayan, R., & Shemi, A. 1991, talk given at the GRO Workshop, Huntsville  
 Prilutski, O. F., & Usov, V. V. 1975, *Ap&SS*, 34, 395  
 Schmidt, M. 1968, *ApJ*, 151, 393  
 Usov, V. V., & Chibisov, G. V. 1975, *Soviet Astron.*, 19, 115  
 van den Bergh, S. 1983, *Ap&SS*, 97, 385