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THE EFFECT OF HELIUM DIFFUSION ON THE AGES OF GLOBULAR CLUSTERS

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ABSTRACT

We have calculated evolutionary tracks for halo stars and constructed isochrones which included the effects of microscopic diffusion of helium. The isochrones were fitted to a metal poor (M92) and a moderately metal rich (NGC 288) globular cluster using an updated version of the Revised Yale Isochrone color calibration. Ages of the two clusters were also determined using the difference between the turnoff magnitude and horizontal branch magnitude, $\Delta V(\mathrm{TO} - \mathrm{HB})$, and the difference in color between the main-sequence turnoff and lower giant branch, $\Delta(B-V)$. Considering all methods and constraints, our best estimate is that diffusion reduces the derived ages of M92 and NGC 288 by 0.5–1 Gyr. The maximum age reduction that diffusion could cause is 3 Gyr. Age estimates including diffusion indicate that M92 is 16 ± 2 Gyr old, and that M92 is ~ 3 Gyr older than NGC 288, assuming that the clusters have the same [O/Fe] of +0.4. The observations of Li abundances can be used as a tracer of the degree of helium diffusion. Both our standard and our diffusive models produce acceptable fits to observations of Li in halo stars. Acceptably fitting diffusive models have a primordial Li abundance (2.24) that is only 0.09 dex higher than that of the standard models.

Subject headings: diffusion — globular clusters: general — stars: evolution — stars: interiors

1. INTRODUCTION

The absolute and relative ages of the globular clusters are of great importance in establishing the age of the universe, and in determining the formation history of our Galaxy. Ages of the globular clusters are found by comparing stellar evolutionary tracks to cluster color magnitude diagrams (CMDs). Clearly, the age estimate is accurate only if the stellar evolution calculations are a good representation of the true evolution of a star. As such, it is important to include all physical effects which can influence the evolution of a star.

Gravitational settling and thermal diffusion cause heavier elements to sink relative to hydrogen; these processes are not considered in standard stellar evolutionary models. Helium diffusion is particularly important in affecting the structure (Vauclair et al. 1974) and evolution (Noerdlinger & Arigo 1980) of a star because helium is very abundant and its diffusion time scale is relatively short. Diffusion occurs throughout the models, and different age indicators are sensitive to diffusion in different parts of the model. Diffusion of helium into the core, and the corresponding displacement of hydrogen out of the core, shortens the main-sequence lifetime and reduces the luminosity of the main-sequence turnoff. This will affect age estimates from all of the methods we consider here, including the $\Delta V(\mathrm{TO} - \mathrm{HB})$ method (Iben & Renzini 1984). Diffusion of helium out of the envelope increases the envelope opacity, which increases the model radius and can therefore affect the shape of isochrones and the color of the turnoff. This effect is particularly important for metal-poor stars with thin convection zones. Diffusion of helium out of the envelope will affect age estimates which are sensitive to model radii, such as isochrone fitting (Carney 1980), and the $\Delta(B-V)$ method (Sarajedini & Demarque 1990; VandenBerg, Bolte, & Stetson 1990). Envelope diffusion can also reduce the helium abundance for HB stars, which affects ages determined from the $\Delta V(\mathrm{TO} - \mathrm{HB})$ method.

It has long been recognized that microscopic diffusion of helium may have important effects on the age estimates of globular clusters (Noerdlinger & Arigo 1980). On the basis of stellar evolutionary tracks, and isochrones in the theoretical H-R diagram, it has been suggested that helium diffusion may reduce the age estimates of globular clusters by as much as 20%–30% (Noerdlinger & Arigo 1980; Stringfellow et al. 1983). In these early models, most of the diffusion took place in the deep interior and near the center; it had the effect of increasing the central depletion of hydrogen, thus speeding up the core exhaustion process which is responsible for the main-sequence turnoff. Diffusion in the outer layers was treated crudely and found to be small. However, more detailed studies show that diffusion, primarily gravitational settling, can take place in the outer layers of solar-type stars, and can modify the structure of these outer layers (Vauclair, Vauclair, & Pamjatnikh 1974; Michaud, Fontaine, & Beaudet 1984). When applied to complete stellar models, this in turn leads to a marked increase of their radii. Using the Michaud et al. (1984) formalism, Deliyannis, Demarque, & Kawaler (1990) have found that evolutionary tracks that include envelope helium diffusion have systematically cooler turnoffs than nondiffused tracks, thus suggesting a decrease in globular cluster ages of possibly several gigayears. Models by Proffitt & Michaud (1991a) included a more detailed treatment of diffusion which solved the diffusive equations throughout the entire star and found a decrease in globular cluster ages of several gigayears. However, in order to directly compare observation to theory, it is necessary to construct *isochrones* in the *observed* color magnitude plane.

In assessing the importance of helium diffusion on the evolution of a star, one must consider the uncertainties in the diffusion coefficients which are $\sim 30\%$ (Bahcall & Loeb 1990). In addition, the degree of diffusion depends on the detailed model structure, especially in the envelope. Uncertainties in the opa-

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cities, abundances, and convection theory can therefore affect the degree of diffusion. Furthermore, it is possible that rotationally induced mixing might render diffusion inefficient in stellar interiors. These uncertainties make it difficult to determine the effect of helium diffusion on age estimates of the globular clusters, especially for diffusion in the envelope.

Helium is not directly observed in normal halo dwarfs, so theory and observations are difficult to compare directly. Nevertheless, since neither helium nor lithium experience radiative levitation in halo dwarfs, the effects of helium diffusion can be constrained by observations of Li abundances in halo dwarfs. In particular, the nearly constant plateau in Li abundance for $T_{\rm eff}$ from 5500 to 6300 K (Spite & Spite 1982) constrains the amount of diffusion allowed, as diffusion can produce a large depletion in surface abundances among the hotter dwarfs (Proffitt, Michaud, & Richer 1990), which is clearly not observed.

The details of the stellar evolution program and the diffusion calculation are presented in § 2. In § 3, we present diffusive isochrones in the $V-(B-V)$ plane which fit the observed CMDs of the globular clusters M92 and NGC 288 and imply an age reduction of 0.5–1 Gyr as compared to standard isochrones. These same models also produce an acceptable fit to the observed Li abundances which is presented in § 4. The major conclusions of this work may be found in § 5. A portion of this work was presented at the 1991 January meeting of the American Astronomical Society (Chaboyer et al. 1991).

2. STELLAR EVOLUTIONARY TRACKS WITH DIFFUSION

The stellar evolutionary models used in this paper were calculated using the Yale stellar evolution code in its nonrotating configuration (Guenther, Jaffe, & Demarque 1989; Guenther et al. 1992), modified to include the effects of helium diffusion. Scaled Grevesse (1984) mixtures with and without enhanced oxygen were used. We choose [O/Fe] = +0.4 because observations of halo stars indicate this level of oxygen enhancement (Bond & Luck 1988; Gratton & Ortolani 1989; Lambert 1989; Gratton 1991; Brown et al. 1991; Spiesman & Wallerstein 1991; see, however, Abia & Rebolo 1989, who determined a higher [O/Fe] in halo dwarfs). Opacities were calculated using the Los Alamos Opacity Library (LAOL; Huebner et al. 1977). The LAOL do not include opacities below 10^4 K; for this temperature range Cox & Stewart (1970) opacities were used. The Cox & Stewart opacities are calculated for a different heavy element mixture (but the same Z) than the LAOL; however, the heavy elements contribute to the opacities in this temperature range only through their contribution to electron density and so this difference in mixtures should not cause a significant error. For temperatures above 10^6 K, a relativistic degenerate, fully ionized equation of state is used. Below 10^6 K, the single ionization of H, the first ionization of the metals and both ionizations of helium are taken into account via the Saha equation.

The choice of the mixing length is a difficult one. Previously, the mixing length has often been calibrated by requiring a solar model to have a solar radius at the age of the sun. However, it is not clear if a solar calibrated mixing length is appropriate for halo stars. This is particularly true for the diffusive isochrones because the amount of diffusion is a function of the mixing length (for more discussion, see Deliyannis & Demarque 1991a). For these reasons, we decided to use two different values of the mixing length to explore its effects on our calculations.

The Yale stellar evolution code has been modified to include the effects of He diffusion from gravitational settling and thermal diffusion (Bahcall & Pinsonneault 1991) using the method of Bahcall & Loeb (1990).[2] Radiative acceleration is only important when an atom is not fully ionized, from our models we know that helium in old metal-poor stars is always fully ionized below the surface convection zone (where the diffusion occurs), and so we do not include radiative acceleration. In this formulation, the hydrogen mass fraction, X, satisfies the equation

$$\frac{\partial X}{\partial t}=\frac{1}{\rho r^2}\frac{\partial}{\partial r}\left[\frac{r^2 X T^{5/2}\xi(r)}{\ln\Lambda}\right] \tag{1}$$

where

$$\xi(r)=\frac{5(1-X)}{4}\frac{\partial\ln P}{\partial r}+\frac{\partial}{\partial r}\ln\left[\frac{X(1+X)}{(3+5X)^2}\right]+\frac{6(1-X)(X+0.32)}{(1.8-0.9X)(3+5X)}\frac{\partial\ln T}{\partial r}\,, \tag{2}$$

and radius, temperature, and density are in nondimensional units defined by $r=r'/R_\odot$, $T=T'/10^7$ K, $\rho=\rho'/100$ g cm^{-3}, and $t=\tau/10^{13}$ yr. The ln Λ term which appears in equation (1) is the Coulomb logarithm. We consider two different formulations for the Coulomb logarithm, one by Loeb (1989):

$$\ln\Lambda_{\rm L}=-19.11-\tfrac{1}{2}\ln\rho'+\tfrac{3}{2}\ln T'\,, \tag{3}$$

and one by Noerdlinger (1977):

$$\ln\Lambda_{\rm N}=-19.7-\frac{1}{2}\ln\left(\frac{4}{3X+1}\right)-\frac{1}{2}\ln\rho'+\frac{3}{2}\ln T'\,. \tag{4}$$

In general, for the halo models we have run, $\ln\Lambda_{\rm L}\sim 3.0$ and $\ln\Lambda_{\rm N}\sim 2.2$. However, near the center of the model, the two formulations of ln Λ may differ by up to 100%. The run of ln Λ with radius is shown in Figure 1. As a function of radius, the two different formulations of ln Λ have very similar shapes, and differ only in their zero point. This uncertainty in the Coulomb logarithm is the key reason why diffusion coefficients are only accurate to 10%–30% (Bahcall & Loeb 1990). In order to explore the uncertainty in our isochrones, we have calculated stellar evolutionary tracks using both formulations of ln Λ.

We have constructed a stellar model with [Fe/H] = −2.29, [O/Fe] = +0.8, $M=0.75\ M_\odot$, $\alpha=1.5$ and $Y=0.25$ to compare with Proffitt & Michaud (1991a, hereafter PM). The differences in our models at the main-sequence turnoff (MSTO) are summarized in Table 1. In addition, after the first dredge-up on the red giant branch, PM report their diffusive models to have 0.015 lower surface helium abundance than their standard model, while we obtain 0.009 ($\Lambda_{\rm L}$), 0.012 ($\Lambda_{\rm N}$).

The differences shown in Table 1 are caused by both differences in the standard model structure and differences in the diffusion coefficients. A direct comparison to the PM diffusion velocities is possible when we neglect thermal diffusion and assume $\partial X/\partial r=0$. We have computed the relative diffusion velocities from the Bahcall & Loeb (1990) approach appropriate for the core and the envelope of a star and plot our results as a function of X in Figure 2. We see that, in general, our diffusion velocities are somewhat smaller than PM. This is why

[2] Although Bahcall & Loeb stated in their paper that their diffusion equations were only valid for $X > 1/3$, this is not the case. The diffusion equations are valid for all X (A. Loeb 1990, private communication).

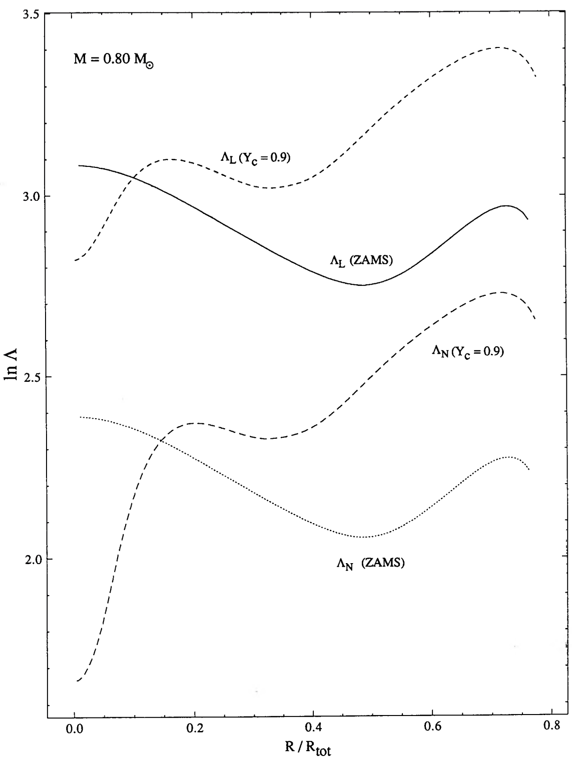

FIG. 1.—The Coulomb logarithm as a function of radius in the nonconvective region of a 0.80 $M_\odot$ star. The solid line is Λ_L for a ZAMS model, the short-dashed line is Λ_N for a ZAMS model. The medium-dashed and long-dashed lines are Λ_L and Λ_N, respectively, for a model with $Y_{cen} = 0.90$ (age $\approx$ 12.6 Gyr).

Bahcall & Pinsonneault (1991) found 20% less diffusion in the core of a solar model than Proffitt & Michaud (1991b).

However, a more important difference is that our standard models differ substantially from the PM standard models. At the MSTO, our standard model has a surface convection zone which is 3 times as massive as those in the PM standard models. Since the time scale for diffusion out of the surface convection zone is directly proportional to the surface convection zone mass (Michaud et al. 1984), this explains much of the differences in our surface helium abundances. This difference in standard models is most likely due to the different equations of state, the different mixtures, and to the different low temperature opacities used in the codes. PM use the Eggleton, Faulkner, & Flannery (1973) equation of state, which they have attempted to correct for the unphysically large amounts of singly ionized helium which is present at several millions of degrees. As PM state, there is no physical justification for their correction term. PM use a different heavy element mixture (Ross & Aller 1976) then the current work. In addition, PM use Alexander (1985) opacities below 10^4 K. As Proffitt & Michaud note, the Alexander (1985) opacities are *lower* than the Cox & Stewart (1970) opacities above 6645 K, despite the fact that the Alexander (1985) opacities include the effect of molecules not considered by Cox & Stewart (see Cox & Cahn 1988 for a discussion of the Alexander opacities). Finally, we note that the Bahcall & Loeb (1990) approach takes into account the thermal diffusion self-consistently, while PM calculate thermal diffusion of helium in the trace element approximation. Proffitt & Vandenberg (1991) have recently constructed isochrones using models similar to PM; we will discuss the Proffitt & Vandenberg isochrones at the end of this paper.

TABLE 1

COMPARISON OF MSTO PARAMETERS BETWEEN STANDARD AND DIFFUSIVE MODELS

Author	ΔL	ΔT_{eff}	ΔAge (Gyr)	Y_{surf} (diffusive)
PM	10%	3.4%	0.96	0.086
This work, Λ_L	6%	2.1%	0.67	0.147
This work, Λ_N	8%	2.5%	0.91	0.130

3. ISOCHRONES

Stellar evolutionary tracks with and without diffusion were constructed for masses ranging from 0.60 $M_\odot$ to 0.95 $M_\odot$. Models were calculated using an initial helium abundance of $Y = 0.24$, two values of the mixing length, $\alpha = 1.3$ (near solar) and $\alpha = 1.7$, and metallicities of [Fe/H] $= -2.29$ and -1.29 with [O/Fe] $= +0.4$ and 0.0. For a given stellar model, including helium diffusion will change both the lifetime and the evolutionary path in the HR diagram (Fig. 3). Such tracks by themselves, however, may provide a misleading guide to the impact of diffusion on cluster age estimates. To determine globular cluster ages one compares isochrones to observations. Diffusion will change the shape of the isochrones in a different way than it changes the shape of individual tracks because

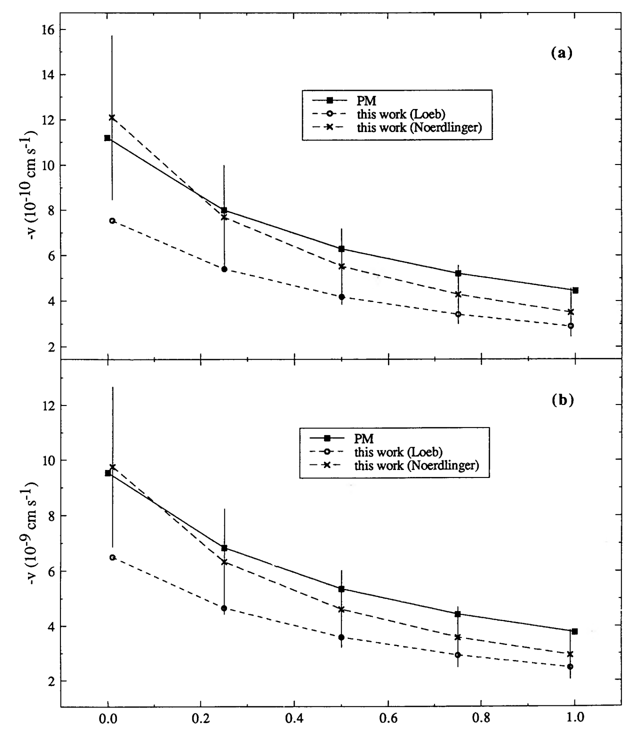

FIG. 2.—A comparison of the gravitational settling velocities used in this work (for Λ_N and Λ_L), and in PM. These velocities do not include the thermal diffusion term of the concentration gradient term. They were calculated for conditions appropriate in the core of a star in panel (*a*) ($T' = 1.5 \times 10^7$ K, $\rho' = 100$ g cm^{-3}, $g = 1.25 \times 10^5$ cm s^2), while panel (*b*) shows the envelope diffusion velocities ($T' = 2.0 \times 10^6$ K, $\rho' = 0.1$ g cm^{-3}, $g = 2.5 \times 10^4$ cm s^2). Error bars on the Λ_N velocities represent 30% errors, as quoted by Bahcall & Loeb (1990).

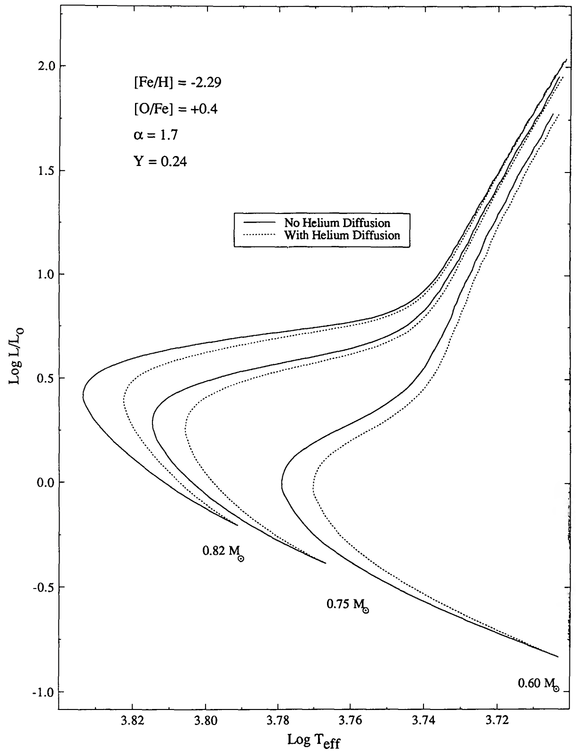

FIG. 3.—Stellar evolutionary tracks with (*dashed lines*) and without (*solid lines*) diffusion for [Fe/H] $= -2.29$, [O/Fe] $= +0.4$, $\alpha = 1.7$. The masses plotted are 0.60, 0.75, and 0.82 $M_{\odot}$.

models with different masses experience different degrees of diffusion.

Isochrones were calculated from the stellar evolutionary tracks using the method of equal evolutionary points (Prather 1976) with an updated version (E. M. Green, private communication) of the Revised Yale Isochrone color calibration (Green, Demarque, & King 1987; Green 1988) for the transformation from the log L–$T_{\rm eff}$ plane to the M_V–$(B-V)_0$ plane. A subset of the isochrones we calculated are displayed in Figure 4. The effects of diffusion are clear—the MSTO is redder and slightly less luminous for the diffused isochrones than the nondiffused isochrones. The diffusive 14 Gyr, [Fe/H] $= -1.29$ [O/Fe] $= +0.4$ isochrone has a MSTO which is 0.051 mag fainter and 0.022 mag redder than the corresponding standard isochrone. The red giant branches are nearly identical in all cases. Thus, diffusion is a small, but nonnegligible effect. In order to examine its effect on age estimates, it is clear that excellent globular cluster photometry is required.

Different age indicators utilize different features of the isochrones. As we will show, diffusion changes age estimates obtained from different techniques by different amounts. Using [O/Fe] $= +0.4$, $\alpha = 1.7$, we calculate the effects of diffusion on three different age indicators: isochrone fitting (§ 3.1); $\Delta(B-V)$ (§ 3.2) and ΔV(TO − HB) (§ 3.3). To investigate the sensitivity of diffusion to metallicity, we compare isochrones to the metal poor cluster M92, and a moderately metal rich cluster, NGC 288. The sensitivity of our results to [O/Fe], α and the diffusion coefficients is presented in § 3.4. In § 3.5, we present our best estimate for the effect of diffusion on the ages of M92 and NGC 288, along with a brief discussion of the absolute and relative ages of the two clusters.

3.1. *Isochrone Fitting*

To examine the effects of diffusion on isochrone fitting of a metal-poor cluster, we choose M92. This cluster has a metallicity of [Fe/H] $= -2.24 \pm 0.08$ (Zinn & West 1984) and a very small reddening of $E(B-V) = 0.02$ (Zinn 1985). M92 was chosen since excellent photometry, well below the MSTO, is available from Stetson (1991). It is also one of the oldest known clusters, and as such, provides a useful minimum age for the

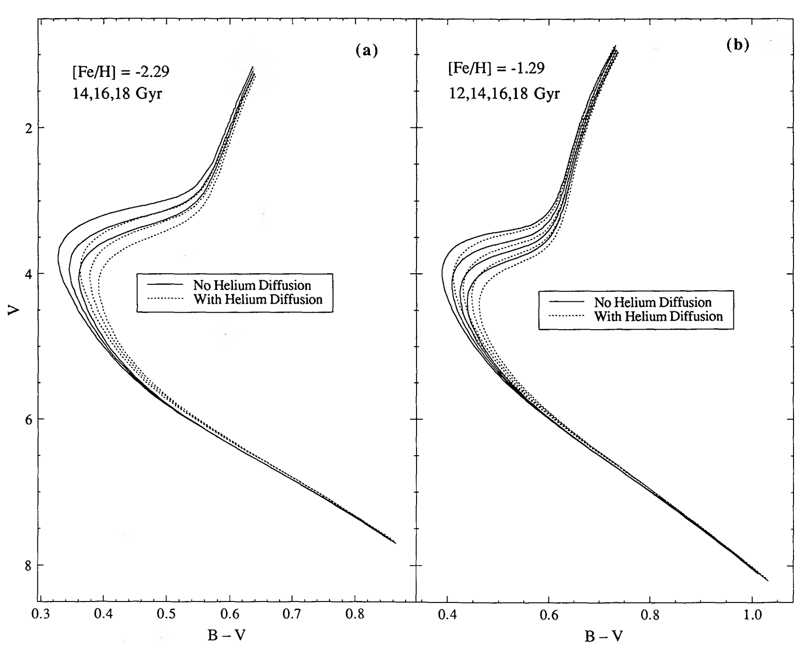

FIG. 4.—Comparison of standard (*solid lines*) and diffusive isochrones (*dashed lines*) for [O/Fe] $= +0.4$, $\alpha = 1.7$. Isochrones are 14, 16, and 18 Gyr for [Fe/H] $= -2.29$ in (*a*). Panel (*b*) shows the [Fe/H] $= -1.29$ isochrones for 12–18 Gyr in 2 Gyr intervals.

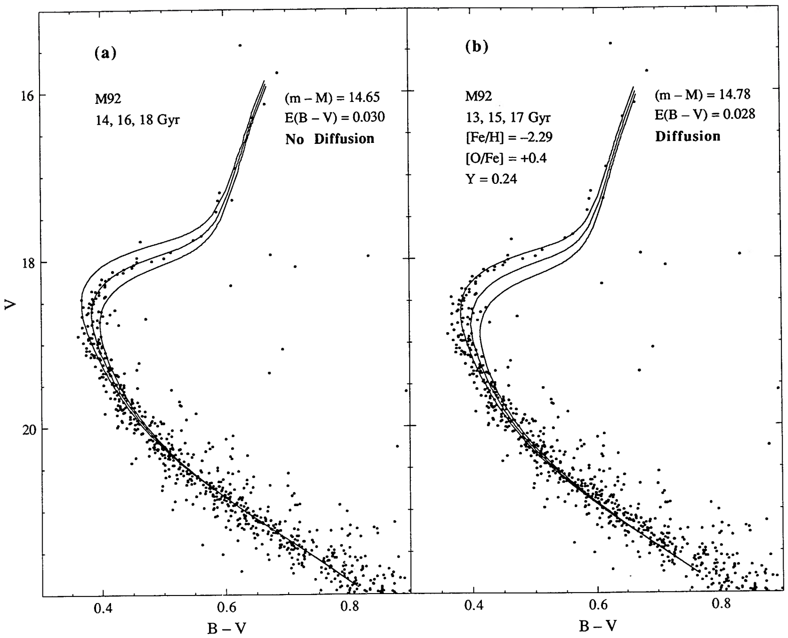

FIG. 5.—Isochrone fits to M92, photometry from Stetson (1991). (*a*) Standard isochrones. (*b*) Diffusive isochrones.

universe. Figure 5 shows our diffusive and standard fits to the CMD of M92 for [Fe/H] = -2.29, [O/Fe] = $+0.04$, and $\alpha = 1.7$. The fits to the data were obtained by attempting to match as much of the cluster locus as possible. This is a rather subjective procedure. It is quite easy to obtain fits of nearly the same quality (with slightly different distant moduli, and reddening) which imply ages that differ by as much as 2 Gyr. Both the standard and diffusive isochrones provide excellent fits to the upper main-sequence, turnoffs, subgiant branches, and giant branches. However, it appears that our diffusive isochrones are too faint on the lower main sequence. The reddening we derive, of 0.030 (standard) and 0.028 (diffusive) is in good agreement with the observations, since the observed reddening and the zero point of the photometric calibration are uncertain by $\sim$0.01–0.02 mag. The standard isochrones yield an age estimate of 16 Gyr, while the diffusive isochrones yield an age estimate of 13 Gyr.

We also fitted our isochrones to NGC 288, which has [Fe/H] = -1.40 ± 0.12 (Zinn & West 1984) and $E(B-V)$ of 0.04 (Zinn 1985). This cluster was chosen since it too has excellent photometry available, well below the MSTO (Bolte 1990). The standard and diffusive fits are shown in Figure 6. Once again, the fits to the data are good, with the diffusive isochrone fit being of slightly lower quality than the nondiffusive fit. The

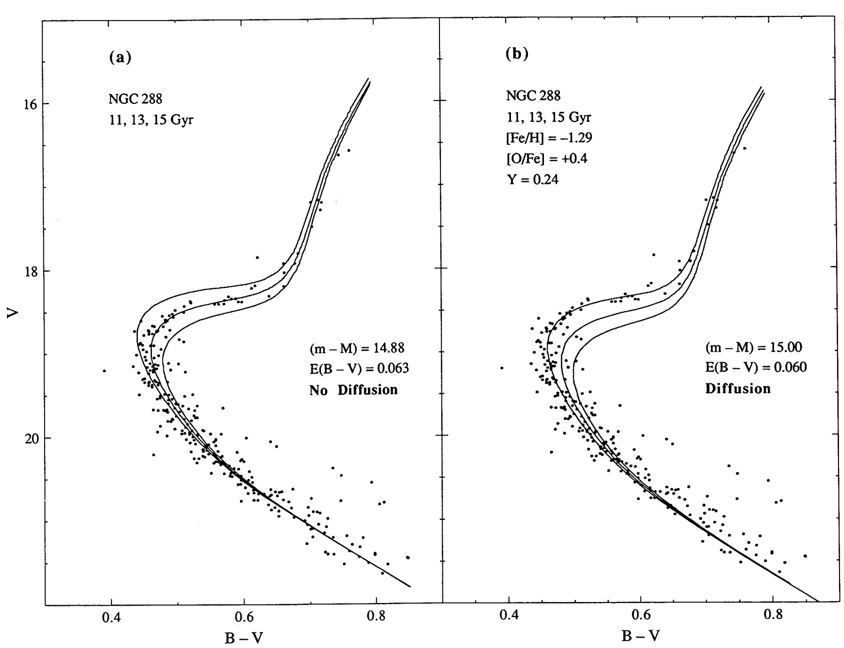

FIG. 6.—Isochrone fits to NGC 288, photometry from Bolte (1990). (*a*) Standard isochrones. (*b*) Diffusive isochrones.

reddening we derive of 0.063 (standard) and 0.060 (diffusive) is slightly larger than that which is observed. The standard isochrones yield an age estimate of 13 Gyr, while the diffusive isochrones yield an age estimate of 11 Gyr.

3.2. $\Delta(B-V)$

Another way to estimate ages of globular clusters is the $\Delta(B-V)$ method (Sarajedini & Demarque 1990; see also VandenBerg, Bolte, & Stetson 1990 for a different implementation of this method). This method compares the observed difference in color between the MSTO and the red giant branch to that obtained from our theoretical isochrones. It has the advantage of being independent of the reddening and distance modulus and is only slightly dependent on metallicity. As this method uses the colors to determine the age (which are determined by the model radii), it is most sensitive to the effects of helium diffusion in the envelope (which modifies the model radii). For M92, we obtain ages of 18.4 ± 1.1 (standard) and 15.3 ± 0.9 (diffusive). The quoted errors are due to the uncertainties in determining the observational quantities. The errors do not reflect any uncertainties in the stellar models. The ages derived from $\Delta(B-V)$ are systematically higher than that which is obtained from isochrone fitting, but the age difference which is inferred due to diffusion is similar. For NGC 288, we obtain ages of 13.6 ± 0.9 (standard), 11.8 ± 0.8 (diffusive). Once again, these values are systematically higher than that which is obtained from isochrone fitting. We emphasize that the age difference inferred by diffusion is similar for the two methods. This systematic shift in absolute age is likely caused by the fact that isochrone fitting uses the entire shape of the cluster CMD, but does this in a qualitative way. The $\Delta(B-V)$ method uses only a part of the CMD, but does so in a quantitative way.

3.3. ΔV(TO − HB)

The ΔV(TO − HB) method compares the observed difference in magnitude between the MSTO and the horizontal branch (HB) to that obtained from theoretical calculations. It would appear that this method has the advantage of depending only on the luminosity, and so is not influenced by uncertainties in the structure of the envelope. Observationally, ΔV(TO − HB) is independent of the interstellar reddening and of the distance modulus, though the theoretical calibration of ΔV(TO − HB) in terms of ages does depend on the absolute magnitudes of the RR Lyrae variables. From the theoretical point of view, ΔV(TO − HB) has been considered to be the preferred technique to estimate the age of globular clusters because the MSTO luminosity is a relatively well determined theoretical quantity. However, theoretical HB models are sensitive to the assumed surface helium abundance, which (due to dredge-up on the red giant branch) is different from the ZAMS value and depends on the structure of the envelope (although not as much as the colors of the models do).

It is important to reiterate that calibrating the ΔV(TO − HB) requires accurate knowledge of the absolute luminosity of HB stars, which is more difficult to determine theoretically. Uncertainties in the basic physics of nuclear reaction rates, opacities, the equation of state, and structural effects due to rotation and associated mixing on the giant branch, could easily combine to change the mass M_c of the helium core at the helium flash by 0.02 $M_\odot$. Decreasing M_c by this amount can correspond to a decrease in ZAHB of 0.2 mag or more, depending on the composition of the model (Sweigart & Gross 1976). The same physics uncertainties are also present on the HB and could introduce further adjustments of the same order of magnitude in the luminosity of the models.

Furthermore, the magnitude difference between the MSTO and the HB is difficult to determine observationally. The MSTO is nearly vertical, and so the MSTO luminosity is an ill-defined quantity. In addition, the HB has a finite vertical height, which makes the HB luminosity difficult to determine. Thus, the intrinsic (observational) errors associated with this method are much larger than the $\Delta(B-V)$ method.

Diffusion of helium into the core lowers the MSTO luminosity, which is shown by our isochrones. Compared to the colors of the models, the MSTO luminosity is much less sensitive to the uncertainties in the opacities, abundances and convection theory. In addition, rotationally induced mixing (which may affect the amount of diffusion in the envelope) is inhibited by composition gradients (e.g., Mestel 1953; Zahn 1974), and so is much weaker in the core than in the envelope.

Diffusion will also lower the luminosity of the HB, because less helium will be dredged up on the red giant branch, causing the surface He abundance to be lower for the diffusive models. For a 0.80 $M_\odot$ star with [Fe/H] = −2.29, [O/Fe] = +0.4 we find that our standard models have a surface helium abundance of 0.247 after maximum dredge-up on the red giant branch, while diffusive models have a surface helium abundance of 0.240. These numbers do depend on the structure of the envelope, so not even the ΔV(TO − HB) method of age determination is completely free from the uncertainties in the envelope physics. However, if we are overestimating the effects of diffusion in the envelope (and hence, the age reduction we derive from $\Delta(B-V)$ and isochrone fitting) then the reduction in the HB luminosity we infer from the surface helium abundance on the red giant branch is too large. To a certain degree, this will compensate for any overestimate of diffusion into the core. For the above reasons, we consider the estimate for the age reduction caused by diffusion provided by the ΔV(TO − HB) age determination technique is the most reliable one considered in this paper (however, see Sarajedini 1991).

We have not computed self-consistent HB models. The change in surface helium abundance on the HB luminosity were therefore taken into account by using the Lee, Demarque, & Zinn (1990) relationships for HB luminosity as a function of helium abundance (eqs. [4] and [6]). However, the input physics of the Lee et al. (1990) calculations are somewhat different than those used in this paper. In addition, the Lee et al. work assumed [O/Fe] = 0.0. We simulated the enhanced oxygen abundance by changing the [Fe/H] to the value appropriate for the oxygen, since, at low-metallicity, HB models are primarily affected by the CNO abundance. Thus, the absolute ages we derive will be revised when we compute self-consistent horizontal branch models. It is unlikely that the age reduction we derive for diffusion will change. From the ΔV(TO − HB) age determination method, we find ages for M92 of 18.3 ± 1.8 Gyr (standard), 17.7 ± 1.9 Gyr (diffusive). For NGC 288, we find 15.7 ± 1.7 Gyr (standard), 15.0 ± 1.6 Gyr (diffusive). The quoted errors are due to the uncertainties in determining the observational quantities and do not reflect the uncertainties in the stellar models.

The ΔV(TO − HB) method suggests that diffusion will cause a decrease in the estimated ages by ~0.7 Gyr. Clearly, a large number of clusters must be studied in order to get a more accurate estimate of the effect of diffusion on the ΔV(TO − HB) age determination method. We plan to carry out such a study in the future. The absolute ages are systematically higher than that found by isochrone fitting. It is important to realize that the absolute ages derived by the ΔV(TO − HB) method depend sensitively on the absolute magnitudes of the RR Lyrae variables. The absolute magnitudes of the RR Lyrae variables, and the dependence on metallicity of these magnitudes, are rather uncertain. For example, the Baade-Wesselink method applied to the field RR Lyrae (Liu & Janes 1990; see also the discussion of Carney, Storm, & Jones 1991), has internal errors of ~0.1 mag, but yields a zero-point for the magnitude-metallicity relation that is ~0.2 mag fainter than the theoretical relationship of Lee et al. (1990). Changing the zero point by 0.2 mag causes the ages inferred by the ΔV(TO − HB) method to change by ~2 Gyr. Lee (1990) has since pointed out that agreement could be achieved by using the low value of 0.20 for the initial helium content Y on the main sequence. More refined red giant and HB models are needed to remove this large uncertainty in the ΔV(TO − HB) method.

3.4. *Parameter Variations*

In order to investigate the reliability of the age reduction we infer due to diffusion, we have explored the effects of varying α, [O/Fe] and the diffusion coefficients. The various age estimates for M92 are shown in Table 2, those for NGC 288 in Table 3. The $\alpha = 1.3$ case required negative reddening $[E(B-V) \sim -0.01]$ in *all* of the isochrone fits, which indicates that such a small α is probably unrealistic. In addition, models with $\alpha = 1.3$ do not fit the Li observations as well as the $\alpha = 1.7$ models (see § 4). In all cases, the age reduction (due to diffusion) determined by isochrone fitting, the $\Delta(B-V)$ and ΔV(TO − HB) methods implied by the $\alpha = 1.3$ isochrones are similar to those found with the $\alpha = 1.7$ isochrones. Thus, we are confident that even though the value of α is uncertain, it does not affect our conclusion regarding the reduction in the age estimate caused by diffusion.

TABLE 2

AGE DETERMINATIONS FOR M92

Method	[O/Fe]	α	Diffusion	Age (Gyr)	ΔAge (Gyr)
Isochrone fit	0.0	1.7	No	18 ± 2	...
	0.0	1.7	Yes	15 ± 2	3
$\Delta(B-V)$	0.0	1.7	No	18.9 ± 1.2	...
	0.0	1.7	Yes	15.7 ± 0.9	3.2
ΔV(TO − HB)	0.0	1.7	No	18.3 ± 1.8	...
	0.0	1.7	Yes	17.5 ± 1.7	0.8
Isochrone fit	+0.4	1.7	No	16 ± 2	...
	+0.4	1.7	Yes	13 ± 2	3
$\Delta(B-V)$	+0.4	1.7	No	18.4 ± 1.1	...
	+0.4	1.7	Yes, N_L	15.3 ± 0.9	3.1
	+0.4	1.7	Yes, Λ_N	14.7 ± 0.9	3.7
ΔV(TO − HB)	−0.4	1.7	No	18.3 ± 1.8	...
	+0.4	1.7	Yes, Λ_L	17.7 ± 1.9	0.6
	+0.4	1.7	Yes, Λ_N	17.8 ± 1.8	0.5
Isochrone fit	+0.4	1.3	No	20 ± 2	...
	+0.4	1.3	Yes	17 ± 2	3
$\Delta(B-V)$	+0.4	1.3	No	21.9 ± 1.1	...
	+0.4	1.3	Yes	18.3 ± 1.1	3.6
ΔV(TO − HB)	+0.4	1.3	No	18.0 ± 1.8	...
	+0.4	1.3	Yes	17.1 ± 1.8	0.9

TABLE 3

AGE DETERMINATIONS FOR NGC 288

Method	[O/Fe]	α	Diffusion	Age (Gyr)	ΔAge (Gyr)
Isochrone fit	0.0	1.7	No	14 ± 2	...
	0.0	1.7	Yes	11 ± 2	3
$\Delta(B-V)$	0.0	1.7	No	15.5 ± 1.0	...
	0.0	1.7	Yes	13.0 ± 0.8	2.5
ΔV(TO − HB)	0.0	1.7	No	15.8 ± 1.7	...
	0.0	1.7	Yes	15.0 ± 1.6	0.8
Isochrone fit	+0.4	1.7	No	13 ± 2	...
	+0.4	1.7	Yes, Λ_L	11 ± 2	2
	+0.4	1.7	Yes, Λ_N	11 ± 2	2
$\Delta(B-V)$	+0.4	1.7	No	13.6 ± 0.9	...
	+0.4	1.7	Yes, Λ_L	11.8 ± 0.8	1.8
	+0.4	1.7	Yes, Λ_N	11.6 ± 0.8	2.0
ΔV(TO − HB)	+0.4	1.7	No	15.7 ± 1.7	...
	+0.4	1.7	Yes, Λ_L	15.0 ± 1.6	0.7
	+0.4	1.7	Yes, Λ_N	14.9 ± 1.6	0.8
Isochrone fit	+0.4	1.3	No	17 ± 2	...
	+0.4	1.3	Yes	14 ± 2	3
$\Delta(B-V)$	+0.4	1.3	No	16.9 ± 1.0	...
	+0.4	1.3	Yes	13.9 ± 0.8	3.0
ΔV(TO − HB)	+0.4	1.3	No	15.0 ± 1.5	...
	+0.4	1.3	Yes	14.2 ± 1.5	0.8

We have also calculated isochrones using [O/Fe] = 0.0 ($\alpha = 1.7$). Once again, we get very similar results for the age reduction implied by diffusion. Thus, our conclusions regarding the age reduction caused by diffusion are unaffected by variations in [O/Fe] in the regime that we have tested.

In order to investigate the effects of the uncertainties in the diffusion coefficients, we have calculated isochrones using the Noerdlinger formulation of Λ (for [Fe/H] = −2.29 and [Fe/H] = −1.29 with [O/Fe] = +0.4, and $\alpha = 1.7$). The difference between the two diffusive isochrones is rather small, with the Λ_N 14 Gyr isochrone having a MSTO is 0.009 mag fainter and 0.006 redder than the Λ_L isochrone (see Fig. 7). The use of Λ_N reduces our age estimate derived from the $\Delta(B-V)$ method by 0.6 Gyr for NGC 288 and 0.2 Gyr for M92 compared to the ages derived using Λ_L. However, the ΔV(TO − HB) age estimate is *increased* by 0.1 Gyr for M92, and decreased by 0.1 Gyr for NGC 288. This small change in derived ages regardless of the diffusion coefficients used is due to the fact that the decrease in the MSTO luminosity (due to enhanced diffusion in the core) is counterbalanced by the decrease in the HB luminosity (due to the enhanced envelope diffusion, which causes a lower helium abundance on the HB). Using isochrone fitting, we get the same age as was derived using Λ_L.

This clearly demonstrates how the relatively large uncertainty in diffusion coefficients translates into only a small uncertainty in

TABLE 4

AGE REDUCTION DUE TO DIFFUSION

Method	M92 ΔAge (Gyr)	NGC 288 ΔAge (Gyr)
Isochrone fit	3	2
$\Delta(B-V)$	3.4	1.9
ΔV(TO − HB)	0.6	0.7

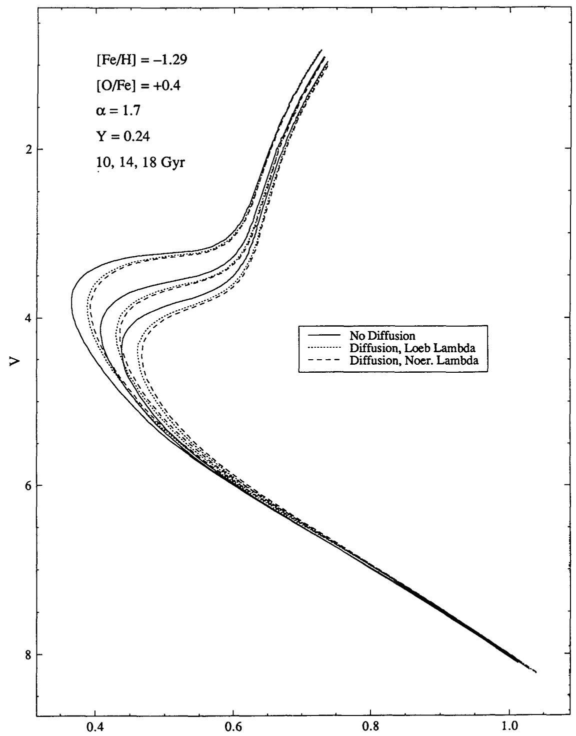

FIG. 7.—Ten, 14, and 18 Gyr standard (*solid lines*) and diffusive isochrones for [Fe/H] = -1.29, [O/Fe] = $+0.4$, $\alpha = 1.7$. Diffusive isochrones using Λ_L are the dashed lines, those calculated with Λ_N are the dashed-dotted lines.

the age estimates. This also indicates that differences between this work and PM are mostly due to differences in the standard physics employed, not due to differences in the diffusion coefficients.

3.5. *Derived Ages*

It is clear that various uncertainties in the input physics contribute little to the reduction in age caused by diffusion, but they do generate uncertainties in the inferred *absolute* ages. Table 4 shows the various age reductions inferred from the [O/Fe] = $+0.4$, $\alpha = 1.7$ isochrones (which are likely to be the best representation of the clusters considered here). We have averaged the age reductions determined using Λ_L and Λ_N. It is readily apparent from Table 4 that isochrone fitting and the $\Delta(B-V)$ techniques yield very similar age reductions. However, the ΔV(TO − HB) technique suggests a substantially smaller age reduction. The reasons for this were discussed in § 3.4, where we concluded that the ΔV(TO − HB) method would yield the most reliable age reduction estimates. Thus, we conclude that diffusion will cause the age estimates to decrease by 0.5–1 Gyr for globular clusters. This corresponds to an age reduction due to diffusion of 3%–6%, which is considerably less than previous work.

Although the age reduction due to diffusion is relatively insensitive to variations in the parameters, a glance at Tables 2 and 3 reveals a wide variety of absolute ages for M92 and NGC 288. Including diffusion, age estimates for M92 vary from 15 to 18 Gyr, while those for NGC 288 range from 11 to 15 Gyr. This gives an indication of how uncertainties in the input physics of our stellar models translate into uncertainties in the absolute ages. A reasonable estimate of the total error associated with the absolute age of a globular cluster is ± 2 Gyr (assuming [O/Fe] is known to within 0.4 dex). We conclude that M92 is 16 ± 2 Gyr old, while NGC 288 is 13 ± 2 Gyr old. The above absolute age determination method suggests that M92 is 3 Gyr older than NGC 288, although the possibility that the two clusters are coeval is not excluded.

A better estimate of the relative ages of the two clusters is found if we make a direct comparison of the various age estimates for the two clusters. This is shown in Table 5 where we see that with diffusion, the relative age difference between M92 and NGC 288 varies from 2 to 4 Gyr. Hence, we conclude that if M92 has the same oxygen to iron ratio as NGC 288, then the two clusters differ in age by 3 ± 1 Gyr. This demonstrates the fact that relative ages are much better determined than absolute ages.

4. CONSTRAINTS FROM THE Li OBSERVATIONS

There is no direct test for helium diffusion in normal halo dwarfs since helium is not observed in these stars. However, because Li is expected to diffuse readily, and since extreme halo stars are thought to provide a sample with uniform initial Li abundance, observations of Li constrain the amount of helium diffusion (Deliyannis & Demarque 1991a). The efficiency of envelope diffusion is generally higher in models with thinner surface convection zones. This causes a downward curvature toward hotter $T_{\rm eff}$ in a diffusive Li isochrone. Therefore, the observed flatness (or perhaps even slightly increasing slope toward higher $T_{\rm eff}$) of the Li–$T_{\rm eff}$ relation constrains the effects of helium diffusion.

We have constructed Li isochrones by following the pre–main-sequence evolution in detail (Deliyannis et al. 1990), to take into account nuclear destruction of Li. We then took advantage of the results of PM that He and Li diffuse at nearly identical rates on the MS, to estimate the amount of Li depletion caused by diffusion. The Li isochrones were calculated for [Fe/H] = -2.29, [O/Fe] = $+0.4$ at an age of (18, 20) Gyr for the standard isochrones (α = 1.7, 1.3) and (16, 18) Gyr for the diffusive isochrones. These ages were chosen by taking the average age determination of M92 from the isochrone fitting

TABLE 5

AGE DIFFERENCE BETWEEN M92 AND NGC 288

Method	[O/Fe]	α	ΔAge (Gyr)
Isochrone fit	0.0	1.7	4.0
$\Delta(B-V)$	0.0	1.7	3.4
ΔV(TO − HB)	0.0	1.7	2.5
Isochrone fit	+0.4	1.7	3.0
$\Delta(B-V)$	+0.4	1.7	4.8
ΔV(TO − HB)	+0.4	1.7	2.6
Isochrone fit	+0.4	1.3	3.0
$\Delta(B-V)$	+0.4	1.3	5.0
ΔV(TO − HB)	+0.4	1.3	3.0
Diffusion			
Isochrone fit	0.0	1.7	4.0
$\Delta(B-V)$	0.0	1.7	2.7
ΔV(TO − HB)	0.0	1.7	2.5
Isochrone fit	+0.4	1.7	3.0
$\Delta(B-V)$	+0.4	1.7	3.5
ΔV(TO − HB)	+0.4	1.7	2.7
Isochrone fit	+0.4	1.3	3.0
$\Delta(B-V)$	+0.4	1.3	4.4
ΔV(TO − HB)	+0.4	1.3	2.9

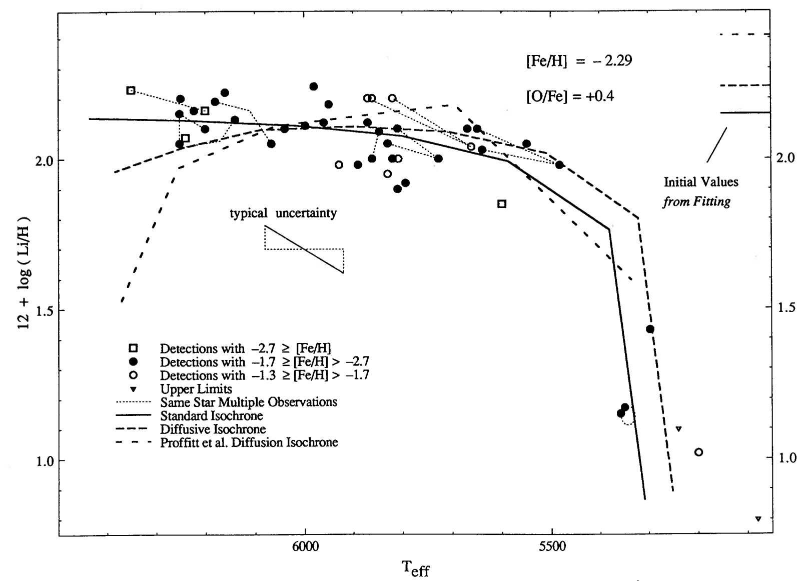

FIG. 8.—Weighted least-squares fits of Li isochrones to observations of Li in extreme halo dwarfs. Shown is our best standard isochrone (*solid line*), our best diffusive isochrone (*long dashes*), and the best diffusive isochrone of Proffitt et al. (1990; *short dashes*). Also indicated on the right are the initial abundances derived from the fits.

and $\Delta(B-V)$ techniques. In Figure 8, we plot the Li abundance in extreme halo stars and our standard and diffusive Li isochrones. To avoid the possibility of including stars already enriched in Li from galactic sources, we have restricted the sample to include only the most chemically and kinematically extreme stars ([Fe/H] ≤ -1.3, $V_{\rm LRS} \geq 160$ km s^{-1}). These stars have been compiled in Deliyannis et al. (1990) and Deliyannis & Demarque (1991b) from the observations of Spite & Spite (1982, 1986), Boesgaard (1985), Hobbs & Duncan (1987), Hobbs & Pilachowski (1988), Rebolo, Beckman, & Molaro (1987), Rebolo, Molaro, & Beckman (1988), Spite, Maillard, & Spite (1984), and Spite et al. (1987). It is important to restrict the sample to dwarfs, since the Li abundance in subgiants may be complicated by Li dredge-up, dilution, and other effects.

A glance at Figure 8 shows that the standard Li isochrone reproduces in detail the trends defined by the observations. This isochrone was obtained by taking the initial Li abundance to be unknown and performing a weighted least-squares fit normalized to render this isochrone an ideal fitting function to the plateau stars. Doing this required $\sigma_{\rm Li} \sim 0.085$, which is very close to the observer's typical value of 0.1, i.e., the excellent quality of fit of our standard isochrone is consistent with the possibility that the observers may have overestimated their uncertainties slightly. For further discussion of the choice of relative weights for the different stars and of the sample selection see Deliyannis & Demarque (1991b). The χ^2 value of the fit may be used to test quantitatively the quality of the fit from the isochrones. Those in the 2 σ confidence interval are deemed to be acceptable. Although it possesses some curvature, our $\alpha = 1.7$ diffusive Li isochrone is acceptable at the 1.5 σ level. Thus, so is the age reduction of 0.5–1 Gyr found in § 3. These are the first diffusive evolutionary calculations to pass the Li test; previous work has overestimated the amount of diffusion (see discussion in Deliyannis & Demarque 1991a). Our $\alpha = 1.3$ isochrone is more curved and is not acceptable at the 2.4 σ level. Furthermore, this isochrone's turnoff is significantly cooler than the edge of the plateau, and it misses the cool star Li detections. The Li observations imply that the age reduction of 3 Gyr [obtained from isochrone fitting and $\Delta(B-V)$] is the maximal amount allowed. To compare with the Montreal group, we have fitted the $\alpha = 1.5$, 15 Gyr isochrone of Proffitt, Michaud, & Richer (1990) to the same stars and find it to yield an unacceptable fit at the 5.5 σ level (Fig. 8). PM also present an $\alpha = 1.7$ and an $\alpha = 1.5$ isochrone, both of which yield still worse fits (rejection at the 7 σ and $\gg 7$ σ levels, respectively). *Clearly, the models of Proffitt et al. and of PM overestimate the effects of diffusion.* The models of PM have too much envelope diffusion because they have much shallower convection zones than ours (see § 2), which causes more diffusion of He and Li out of the bottom of the convection zone. Furthermore, Deliyannis & Demarque (1991a) provide several arguments as to why PM have not been judicious in comparing with the observations.

It is interesting to note that our diffusive and standard Li isochrones have nearly identical initial Li abundances, with the initial value of Li from the diffusive isochrone being less than 0.1 dex higher than the standard value. *Thus, diffusion does not have a significant effect on estimates of the primordial Li abundance.*

We thus disagree with the conclusions of PM (see, e.g., their Abstract), that the initial abundance of the halo stars is at least 0.23 dex higher than their average, that the mere presence of Li in these stars implies that significant gravitational settling must have occurred, and that therefore a significant age reduction for globular clusters is unavoidable due to the effects of diffusion. The poor quality of the fits of their Li isochrones has already been emphasized. In addition, the turbulent diffusion coefficients used by PM (to simulate rotational mixing) have no physical basis. The more realistic treatment of rotation of Pinsonneault, Deliyannis, & Demarque (1992) is able to reproduce well the morphology of the Li observations, without any introduction of microscopic diffusion.

5. CONCLUSIONS

The standard and diffusive isochrones we have calculated provide good fits to the observed CMDs of M92 and NGC

288. The Li isochrone counterparts to the best fits in the CMD are also found to be in good agreement with the observed Li abundances in halo stars. Isochrone fitting and the $\Delta(B-V)$ age determination techniques suggest that the reduction in the age estimate of globular clusters caused by diffusion will be 2–3 Gyr. However, these estimates are sensitive to the colors of the models and to envelope diffusion, which depend on the detailed structure of the envelope. Due to uncertainties in the opacities and the treatment of convection, the envelopes are the least understood aspect of our models. Recent improved opacity tables give higher envelope opacities than the LAOL, which yield a deeper surface convection zone (Rogers & Iglesias 1991), and hence, less envelope diffusion. We are encouraged however, by the fact that isochrones calculated with different mixing lengths, yield similar estimates for the age reduction caused by diffusion.

In contrast, the $\Delta V(\mathrm{TO}-\mathrm{HB})$ age determination method suggests that diffusion will reduce the age estimates of globular clusters by 0.5–1 Gyr. This method depends mainly on the luminosity of the models, which is a better-known theoretical quantity, than the colors of the models. However, it is difficult to determine $\Delta V(\mathrm{TO}-\mathrm{HB})$ observationally, so the errors associated with this method are large.

Taking into account all three age determination methods, and their associated uncertainties, as well as the constraints imposed by the Li observations, we feel that the inclusion of diffusion in stellar evolutionary calculations is likely to reduce the age estimates of the globular clusters by ~ 1 Gyr. We find that M92 is 3 Gyr older than NGC 288, assuming that the clusters have similar [O/Fe], independent of the age determination method used. The uncertainties in the stellar models imply that the absolute ages are accurate to ± 2 Gyr. For the $\Delta V(\mathrm{TO}-\mathrm{HB})$ method, the uncertainty in the absolute magnitude of the RR Lyrae variables leads to an uncertainty in the zero point of the ages of globular clusters of ± 2 Gyr. For isochrone fitting and the $\Delta(B-V)$ method, uncertainties in the low temperature opacities and in the treatment of convection are the largest sources of error. Taking into account all of these factors, we find that M92 is 16 ± 2 Gyr old. In addition, we found that diffusion increases estimates for the initial Li abundance in halo stars by less than 0.1 dex compared to standard models.

A final note: as this paper was being completed, we learned that Proffitt & VandenBerg (1991, hereafter PV) have also recently calculated globular cluster isochrones that include the effects of diffusion. PV mention that we have employed "a less sophisticated method" to determine the diffusion rates. However, PV used the same diffusion coefficients as PM, which, as we have shown in Figure 2 are similar to the coefficients we have employed. More importantly, as we demostrated in § 3.4, changes in the diffusion coefficients by 30% have little effect on our derived age reduction. As was mentioned by PV, we find that the effects of diffusion on the turnoff age-luminosity relation are 30%–40% lower than PV. However, our derived age reduction of 0.5–1 Gyr due to diffusion is similar to what PV determined.

The significant difference between this work and PV is in the standard physics assumed. For example, PV employed a scaled solar T–τ relation for their model atmospheres, while we used a gray atmosphere. It is unclear whether an observationally determined T–τ relation for the present-day Sun is appropriate for halo stars. More research in stellar atmospheres and stellar convection theory will hopefully determine the correct boundary conditions to be used in stellar evolutionary calculations. In other respects, the PV models are similar to the PM models, which have significantly shallower surface convective zones than our models (see § 2). This leads PV to have substantially more diffusion in the envelopes of their models. This, combined with the pre–main-sequence depletion of Li in the cool stars, leads to the fact that the diffusive PV models do not fit the observed abundance of Li in halo dwarfs. In addition, the diffusive PV isochrones are a poor fit to the M92 photometry. In contrast, our diffusive Li isochrones fit the observations, and provide a good fit to the CMD of M92. Hence the importance of diffusion near the turnoff cannot be ruled out.

We agree with PV that diffusion will lower the luminosity of the HB and, as discussed in § 3.3, we have considered the effects of diffusion on the luminosity of the HB. PV's claim that Chaboyer et al. (1991) used $\Delta(B-V)$ to investigate the absolute ages is unfounded. All of the age indicators used by Chaboyer et al. (1991) were used in a differential sense to study the reduction in ages due to the inclusion of diffusion. Investigating the colors of the models [with $\Delta(B-V)$] allows us to probe the effects of envelope diffusion on the ages of globular clusters. We have looked at the colors of our models in relation to those of the observations and set useful constraints on the amount of helium diffusion in the envelope.

Finally, PV mention that heavier elements, notably iron, might undergo significant gravitational settling, which would alter the predicted T_{eff}'s of the models (and hence, the shape of the isochrones and ages derived via the $\Delta(B-V)$ technique). This would imply that turnoff stars have different metallicities than giants in the same cluster, which is an interesting possibility. We note that for the Hyades F stars, where similar models predict more diffusion than is expected for halo stars, there is no evidence for variations in [Fe/H] (Boesgaard & Budge 1988).

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