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ON THE INFLUENCE OF THE CONVECTIVE EFFICIENCY ON THE DETERMINATION OF THE ATMOSPHERIC PARAMETERS OF DA WHITE DWARFS

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ABSTRACT

We present a detailed calculation of model atmospheres for DA white dwarfs where several versions of the mixing-length theory, with different associated convective efficiencies, are used. The predicted emergent fluxes, color indices, and equivalent widths are most sensitive to the assumed parameterization of the theory in the range $T_{\rm eff} \sim 8000$–15,000 K. This, it turns out, is also the region where the Balmer jump is a most useful gravity discriminant. We discuss the implications of our calculations for previous determinations of atmospheric parameters of DA white dwarfs, and show that these results are much more model-dependent than previously believed.

Subject headings: convection — stars: atmospheres — white dwarfs

1. INTRODUCTION

White dwarf stars with hydrogen lines, the so-called DA stars, represent the vast majority of the white dwarfs observed. As such, this rather homogeneous group has provided the bulk of the observational material on white dwarf stars, and of our knowledge of the fundamental parameters of these objects. Measurements of stellar mass, radius, and effective temperature all provide important and valuable information for, and constraints on, current theories of stellar evolution and white dwarf cooling.

Despite their often attributed simplicity, the atmospheres of DA stars hold several complications in store for us. Because the atmospheres exhibit only hydrogen lines, and because downward element settling is expected to be extremely efficient in white dwarfs, a pure, or almost pure, composition was adopted in most model atmosphere calculations of these objects, and this for the whole temperature range where these stars are found. This question of the purity of the atmospheres of the hydrogen-line stars was recently reconsidered by Bergeron et al. (1990) and Bergeron, Wesemael, & Fontaine (1991c), who discuss evidence which suggests that the atmospheres of DA stars below 12,000 K could well be enriched in helium, while retaining their DA spectral type. This helium originates in the underlying convective helium envelope, and is brought to the surface when a significant hydrogen convection zone develops at the surface at an effective temperature below 11,000–13,500 K, depending on the assumed convective efficiency (see Tassoul, Fontaine, & Winget 1990 for a description of this process, and further references to earlier work).

A further complication is caused by the presence of convective energy transport in the atmospheres, even at temperatures above those of mixing. This range of temperature ($T_{\rm eff} \lesssim$ 15,000 K), it turns out, is the one where most DA stars for which atmospheric parameters have been obtained are found. For example, the most comprehensive determination of the mass distribution for DA white dwarfs comes from the analysis of Weidemann & Koester (1984) of a sample of 70 DA stars with $8000 \lesssim T_{\rm eff} \lesssim 16{,}000$ K. Similarly, more than 70% of the DA stars analyzed by Shipman (1979) have convective atmospheres. Also, the photometric analyses of Koester, Schulz, & Weidemann (1979), Wegner (1979), Shipman & Sass (1980), and Fontaine et al. (1985) make use of the sensitivity of the Balmer jump to log g *exactly in the range where convection is important.* One thus suspects that the usual uncertainties associated with the description of convective energy transport are likely to affect both the reliability of the models and the conclusions of these analyses.

This regime of temperature is also the one where an important group of DA stars lies, the pulsating DA white dwarfs, or ZZ Ceti stars. Interestingly enough, pulsation studies of these objects seem to impose some constraints on the efficiency of convection. It has been repeatedly emphasized in the past that a match to the observed location of the blue edge requires that a relatively large convective efficiency be used in the calculation of the envelope structure (Winget & Fontaine 1982; Fontaine, Tassoul, & Wesemael 1984; Tassoul et al. 1990). However, little attention has been paid to this requirement when time has come to determine observationally the location of the blue edge: model atmosphere calculations used in the determination of the hot boundary of the instability strip (whether narrow-band photometry, ultraviolet and optical spectrophotometry, or spectroscopy) have consistently made use of what one could term a conventional convective efficiency, akin to the ML1 theory (see Tassoul et al. 1990). There is thus a small, and up to now unavoidable, internal inconsistency in the procedure of defining observationally the blue edge of the instability strip which must, ultimately, be removed if observations are to provide stringent tests of pulsation calculations.

The only previous effort made to consider the uncertainties brought about by the adopted theory of convection in the modeling of DA stars is that of Shipman (1979).[1] Variations in colors and monochromatic flux at 5500 Å, H_{5500}, are presented for a single DA model at 10,000 K computed within the mixing-length theory with various values adopted for the ratio of the mixing length to the pressure scale height: variations in ℓ/H from 0.3 to 1.5 are allowed. At that temperature, a 7% decrease in the monochromatic flux is observed at V when ℓ/H

[1] Note that Thejll, Vennes, & Shipman (1991) have recently studied the sensitivity to the adopted version of the mixing-length theory of the ultraviolet energy distribution of hot DB stars.

is reduced from the standard value of 1.0 to 0.3. An increase of ℓ/H to a value of 1.5 produces a negligible effect ($\sim 1\%$) at that wavelength. Variations of the same magnitude are reported at 6000 K. In Shipman's (1979) analysis, the stellar radius of DA stars below 12,000 K would be underestimated by $\sim 3.5\%$ if the convective efficiency in these stars were characterized by a value of $\ell/H = 0.3$.

With the renewed effort at using the location of the blue edge to constrain pulsation calculations (e.g., Wesemael et al. 1991), the time appeared ripe to investigate anew, at least qualitatively, the influence of this neglected parameter on model atmosphere calculations of lukewarm ($T_{\rm eff} \sim$ 10,000–15,000 K) DA stars. We tackle this problem by carrying out a series of model atmosphere calculations for DA stars below 17,000 K which incorporate different efficiencies for the convective energy transport. All these variations are made within the formalism of the mixing-length theory, a simplification used for two reasons: first, this is the theory used in all white dwarf model atmosphere calculations up to date, and we felt it was important to use a formalism easily connected to previous work in the field. Second, the mixing-length theory is fairly easy to implement in model atmosphere codes, a quality which no doubt has contributed to its popularity. There is currently renewed interest in the application of alternate theories of convection (Canuto & Mazzitelli 1991) in the envelopes of white dwarf stars (Mazzitelli & D'Antona 1991). However, it appears premature to apply these developments to *atmosphere* calculations, until these new formalisms have had the opportunity to be further tested in the more standard context of stellar structure.

No attempt is made in this paper to rederive a mean mass for lukewarm DA stars with convective atmospheres, or to redetermine the boundaries of the ZZ Ceti instability strip while taking into account the newly found sensitivity to the convective efficiency. These essential, but rather complex, investigations are in progress, and their results will be reported in due course.

2. THEORETICAL FRAMEWORK

We have calculated a grid of LTE model atmospheres for DA white dwarfs covering the range of $T_{\rm eff} = 8000\ (500)\ 17{,}000$ K and $\log g = 7.50\ (0.25)\ 8.50$. These are hydrogen-line blanketed, assume a pure hydrogen composition, and include convection treated within the mixing-length theory. Details of our numerical procedure can be found in Bergeron (1988), and Bergeron et al. (1991c). We go beyond our earlier work, here, by considering different efficiencies for the convective energy transport.

Within the mixing-length theory, the convective flux (which requires an expression for the convective velocity) is given by

$$F_c = \frac{bC_p\rho T\ell^2}{H_p}\left(\frac{agQ}{H_p}\right)^{1/2}(\nabla - \nabla')^{3/2}\,, \tag{1}$$

where $(\nabla - \nabla')$ is obtained from the solution of

$$(\nabla - \nabla')^{1/2} = -\frac{B}{2} + \left(\frac{B^2}{4} + \nabla - \nabla_{\rm ad}\right)^{1/2}\,, \tag{2}$$

and B is given by

$$B \equiv \frac{\nabla' - \nabla_{\rm ad}}{(\nabla - \nabla')^{1/2}} = \frac{\sigma T^3 d}{\rho\ell\tau_e C_p}\left(\frac{H_p}{agQ}\right)^{1/2}\,. \tag{3}$$

TABLE 1

NUMERICAL CONSTANTS OF THE MIXING-LENGTH THEORY

Version	a	b	c	ℓ/H_p
ML1	1/8	1/2	24	1
ML2	1	2	16	1
ML3	1	2	16	2

In the last equation, we have allowed the convective cell to be optically thin by introducing the parameter d defined as

$$d = \frac{8\tau_e^2}{1 + (8\tau_e^2/c)}\,, \tag{4}$$

where τ_e is the optical depth of a convective cell given by

$$\tau_e = \kappa\ell\rho\,. \tag{5}$$

The parameter d defined this way recovers equations (38) of Tassoul et al. (1990) and (7–72) of Mihalas (1978) at large and small optical depths, respectively. In all equations above, the different symbols have their usual meanings (see, e.g., Cox & Giuli 1968). In equations (1)–(4), the numerical constants a, b, and c parameterize the efficiency of convection and depend on the assumed geometry of the convective cells. The value of the mixing length, ℓ, is also considered as a free parameter.

In this exploratory analysis, we have restricted our calculations to the versions of the mixing-length theory usually quoted in the context of white dwarf envelopes. The values for the different constants are given in Table 1, following the nomenclature of Fontaine, Villeneuve, & Wilson (1981). The ML1 and ML2 versions correspond to the versions of the mixing-length theory of Böhm-Vitense (1958) and Böhm & Cassinelli (1971), respectively. As discussed by Tassoul et al. (1990), the ML2 parameterization decreases the horizontal energy loss rate and consequently, increases the convective efficiency. The ML3 version is identical to the ML2 version but with a value of $\ell/H = 2$, which makes it even more efficient. However, there is no fundamental reason to expect that convection in real stars should be restricted to the range of efficiency considered here.

Our ML1 grid of model atmospheres is similar to that used by Daou et al. (1990) in the determination of the atmospheric parameters of ZZ Ceti stars. The ML1 version is also the one widely used in previous model grids. In particular, it is the standard version used by Koester et al. (1979) and is roughly equivalent to that adopted in the ATLAS code used by Shipman (1977). These grids have been used repeatedly in the past to determine atmospheric parameters for DA stars (see e.g., Koester et al. 1979; Shipman 1979; Wegner 1979; Shipman & Sass 1980; Weidemann & Koester 1984; Fontaine et al. 1985).

3. INFLUENCE OF THE CONVECTIVE EFFICIENCY

3.1. *Emergent Fluxes*

The emergent fluxes for the complete grid of model atmospheres are calculated using the procedure described in Bergeron et al. (1991c). Some results of our calculations are displayed in Figure 1 for the different convective efficiencies considered above. The differences in the emergent flux between the models are small at high effective temperatures where a negligible fraction of the energy is transported by convection, and at low effective temperatures where convection becomes

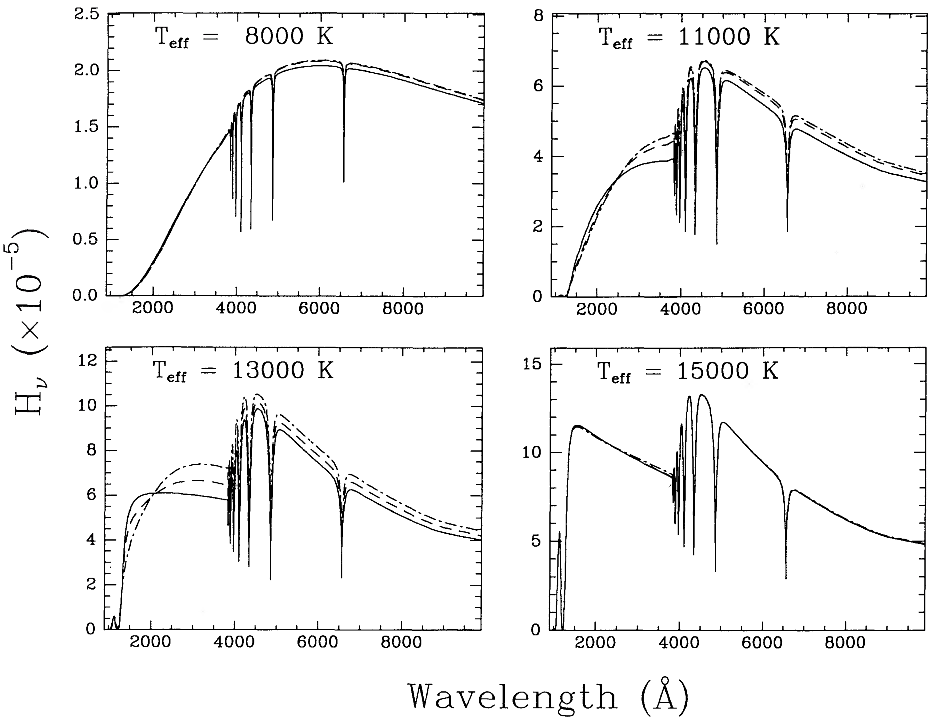

FIG. 1.—Influence of the convective efficiency on the emergent fluxes of DA models at log $g = 8.0$ for various effective temperatures. The efficiencies used are ML1 (*solid lines*), ML2 (*dashed lines*), and ML3 (*dash-dot lines*).

adiabatic. In the latter case, the thermodynamic stratification is almost completely specified by the gradient $\nabla \sim \nabla' \sim \nabla_{ad}$, and the hydrostatic equilibrium equation. The convective flux then becomes independent of the mixing-length theory. Figure 1 indicates that at $T_{eff} = 8000$ K, convection is completely adiabatic in the ML2 and ML3 models, but not completely in the ML1 model. Additional computations show that, at that temperature, a parameterization more efficient than that provided by ML3 no longer affects the temperature stratification. We have also verified that in a model at $T_{eff} = 6000$ K, the ML1, ML2, and ML3 stratifications are identical.

At intermediate temperatures ($8000 \lesssim T_{eff} \lesssim 15{,}000$ K), however, the differences in the emergent flux between the ML1 and ML3 versions become appreciably large, reaching a maximum around $T_{eff} = 13{,}000$ K; at log $g = 8.0$, the flux in the ML3 model is 8 (25)% larger at 5500 (3500) Å.[2] The only spectral region where the emergent flux of the ML1 models is larger than that of the ML3 models is in the far-ultraviolet. The smaller amount of energy transported by convection in the ML1 models forces the temperature gradient to be steeper. This results in an increased emergent flux in the optically thin regions, which, in our models, lie in the $\lambda\lambda$1300–2000 region.[3]

[2] Note that a change from ML1 to ML3 involves *both* a change in the geometrical constants *a*, *b*, and *c* of the mixing-length theory and an increase in the mixing length from 1 to 2 pressure scale heights. Shipman's (1979) result that small variations (~ 1%) in H_{5500} are observed at 10,000 K when ℓ/H is increased from 1 to 1.5 is consistent with the variations (~ 1.3%) we observe between ML2 and ML3 at that temperature.

[3] Although it may appear from Fig. 1 that, at a given effective temperature, the *total flux* in the ML3 models is larger than in the ML1 models, the same plot made on a *frequency* scale clearly shows that the total flux, given by $\int H_\nu \, d\nu$, is identical.

Our calculations indicate that, in the whole range of temperatures considered above, the *absolute fluxes* are sensitive to the parameterization of the mixing-length theory. However, absolute fluxes are not always used as such to determine atmospheric parameters of white dwarfs, since effective temperatures are often derived from the slope of the energy distribution, while gravities result from either the size of the Balmer absorption edge or from line profiles and equivalent widths. These effects are discussed below. One significant exception arises when use is made of trigonometric parallaxes (Shipman 1979; Koester et al. 1979). In that case, the stellar radius, *R*, is given by

$$R = D(f_\nu / 4\pi H_\nu)^{1/2} , \tag{6}$$

where *D* is the distance, H_ν is the Eddington flux, and f_ν is the flux measured at the top of the Earth's atmosphere. Shipman (1979) derives a relationship between colors and the Eddington flux H_{5500} which allows him to obtain *R* once some colors, the *V*-magnitude, and the parallax are available. His H_{5500} versus colors relationship is based on models at log $g = 8$ with standard (ML1-like) convection. Here, we use the H_{5500} versus $(b - y)$ relationship for log $g = 8$, and let the convective efficiency vary. Since, at a given $(b - y)$, the flux H_{5500} predicted for models with more efficient convection is larger than that predicted in models making use of ML1, the radii obtained in analyses with ML2 or ML3 models will be correspondingly smaller, and the masses larger. For ML2, we obtain $M_{ML2}/M_{ML1} = 1.026$, 1.079, and 1.046 for stars around $T_{eff} =$ 13,000, 10,000, and 8000 K, respectively. For the ML3 grid, we obtain $M_{ML3}/M_{ML1} = 1.048$, 1.108, and 1.050 for the same three effective temperatures. In studies of the parallax sample, the *maximal* uncertainty in stellar mass brought about by *increasing* the convective efficiency from ML1 to ML3 is thus ~11% near 10,000 K. The related uncertainty on the stellar radius (a 7% effect near 10,000 K for ML3 convection) complements that reported by Shipman (1979), who considered only the effects of a *reduced* energy transport efficiency.

Figure 1 also shows that the ultraviolet energy distributions will be strongly affected by an increase of the convective efficiency. At a given temperature, the normalized ultraviolet energy distributions are flatter in the ML3 models than in the ML1 models. Therefore, effective temperatures using *IUE* data will depend on the assumed convective efficiency: comparisons of typical models for ZZ Ceti stars indicate that fits based on ML3 models would yield temperatures that are ~1000 K hotter than those based on ML1 models. Wesemael, Lamontagne, & Fontaine (1986) have analyzed *IUE* data of ZZ Ceti stars and determined effective temperatures using exclusively the energy distribution slope in the SWP camera, and model atmospheres calculated with a mixing length to pressure scale height ratio of 1.5 (Nelan & Wegner 1985) and 1.0 (Koester et al. 1985); they would have found higher temperatures had they used models characterized by a more efficient convective transport.

3.2. *Color Indices*

We have also explored the influence of the convective efficiency on the morphology of color-color diagrams, a tool which can be used for a much larger sample of DA white dwarfs and, in particular, for the sample of DA stars without parallaxes. Our results are presented in Figure 2 for Strömgren colors calculated with the sensitivity functions of Olson (1974) and the calibration of Schulz (1978). Again, differences between models computed with various convective efficiencies are negligible at both low and high effective temperatures, but become important in the intermediate regime, *just where the sensitivity of optical colors to* log *g is the largest*. Also shown in Figure 2 is the Strömgren photometry of Fontaine et al. (1985), which is representative of a set of homogeneous colors for a sample of bright DA white dwarfs. Determinations of both effective temperatures and surface gravities would be affected were an efficiency different from ML1 used in the data analysis. As an illustrative example, the blue edge of the ZZ Ceti instability strip is shifted downward, from 13,500 K with ML1 to a cooler 12,500 K with ML3. This effect goes in the opposite direction to that observed earlier with the ultraviolet energy distribution slope, a fact which underscores the need for a better internal consistency in the determination of these boundaries. Note also that, because of the differing slopes of the isotemperature lines in the two-color diagram, the relative ordering of stars in the instability strip may be slightly affected by the use of different convective efficiencies. In addition, although the photometric data follow approximately a line of constant surface gravity for all three grids of models, the *absolute value* of the mean surface gravity depends strongly on the assumed convective efficiency. This effect is substantial: for example, the ML3 calibration implies a mean surface gravity for DA white dwarfs as low as log $g \sim 7.6$.

The atmospheric parameters determined from color-color diagrams are thus very sensitive to the parameterization of the mixing-length theory. Such color-color diagrams have been used in the past to estimate the mean mass for DA white dwarfs (e.g., Shipman & Sass 1980). From their Strömgren sample, Shipman & Sass (1980) derive an average surface gravity of 7.86 ± 0.25, where the error estimate is associated, in most part, with uncertainties in the photometric calibration. The corresponding mean mass is $M/M_\odot = 0.50$. On the basis of Figure 2, an average surface gravity lower by ~0.25 would probably have been derived had a more efficient theory of convection, like ML3, been used to analyze the data.

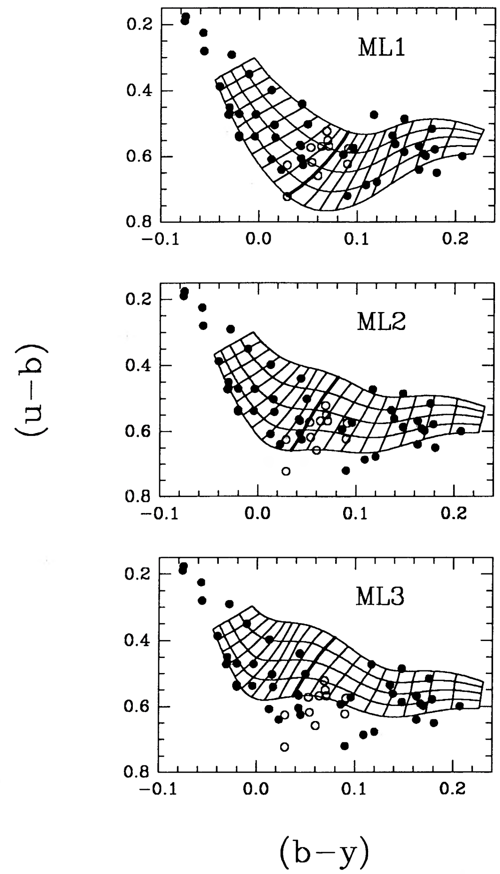

FIG. 2.—Influence of the convective efficiency on the morphology of the Strömgren diagram of DA stars. The effective temperatures range (from right to left) from 8000 to 17,000 K by steps of 500 K, and the values of log g (from bottom to top) from 7.5 to 8.5 by steps of 0.25. As a reference, the models at 12,500 K have been highlighted. Dots represent the Strömgren photometry of Fontaine et al. (1985); open circles are ZZ Ceti stars.

3.3. *Equivalent Widths*

The use of optical line profiles and equivalent widths to determine atmospheric parameters of DA white dwarfs has a long history as well. The most recent investigations include those of Weidemann & Koester (1980), Schulz & Wegner (1981), Bergeron et al. (1990), Daou et al. (1990), and Bergeron, Saffer, & Liebert (1991a, b). Because the local continuum flux is sensitive to the convective efficiency, the strengths and profiles of *all* Balmer lines will be affected by the particular choice of convective efficiency in the 8000–16,000 K temperature range. We have calculated equivalent widths for our three grids of models at log $g = 8.0$, the results of which are reported in Figure 3. Although the continuum flux in the optical increases with more efficient convection (see above), the *normalized* line profiles become narrower, and the equivalent widths smaller. Another interesting result is the shifting in temperature of the maximum of the equivalent widths. The ML1 grid gives a maximum of the equivalent widths in the region where ZZ Ceti

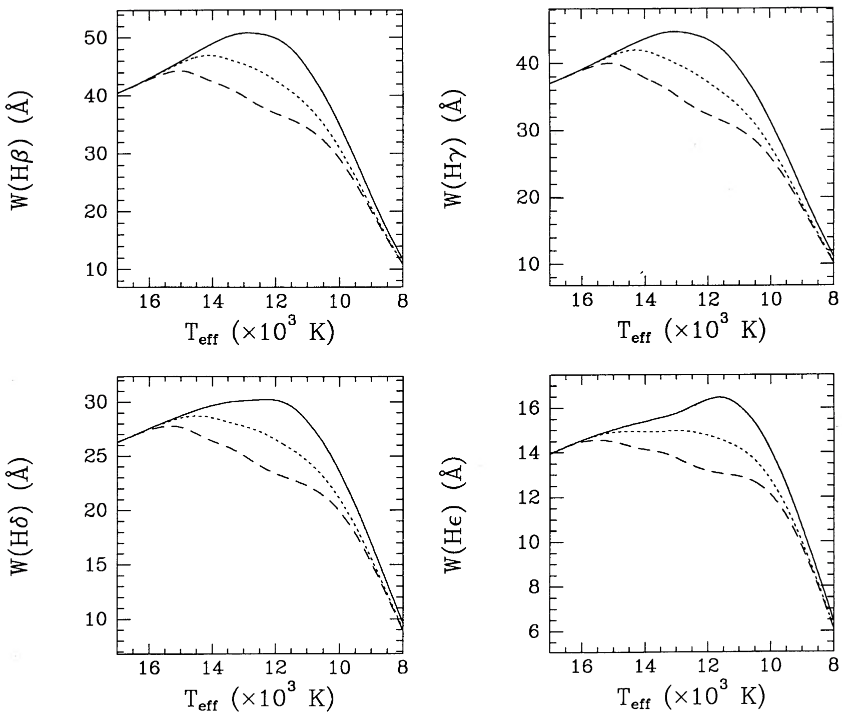

FIG. 3.—Variation of the equivalent widths of Hβ to Hϵ as a function of effective temperature in models at log g = 8.0. Models are calculated with the ML1 (*solid line*), ML2 (*dotted line*), and ML3 (*dashed line*) versions of the mixing-length theory.

stars are found, that is, roughly between 11,000 and 13,000 K. The models of Koester et al. (1979) yield the same result (see Greenstein 1986). This, however, appears to be at odds with the results of Greenstein (1986) who shows, in his Figure 4, that there are several DA white dwarfs, not known to be variable, that have an Hβ line *stronger* than that observed in the ZZ Ceti stars. A similar result was reported by Fontaine et al. (1985); their plot of the Strömgren m_1 index (which measures the strength of Hδ) against $(b - y)$ indicates that the maximum in m_1 is *blueward* of the ZZ Ceti range. In light of Figure 3, this could be an indication that convection in DA stars is more efficient than that provided by the ML1 calibration of the mixing-length theory.

4. DISCUSSION

The results of our calculations show that the predicted absolute fluxes, color indices, and equivalent widths are sensitive to the efficiency of convection in the range $T_{\rm eff} \sim$ 8000–15,000 K, with a maximum sensitivity around 13,000 K. The effects, which seem to have gone largely unnoticed in the past, are, in fact, substantial. However, we find that these effects affect different analyses in different ways: for example, in photometric analyses, a more efficient convection leads to a lower average surface gravity, but that trend is reversed if one uses parallaxes instead. There, a high convective efficiency yields larger emergent fluxes in the optical and therefore, for a given observed V-magnitude, a smaller estimated radius (or higher surface gravity and mass). Clearly, since most previous mass, radius, and surface gravity determinations concentrated in regions where model atmospheres are sensitive to the assumed convective efficiency, *the results of these determinations must be considered model-dependent.*

Recently Bergeron et al. (1991a, b) have used line profile fitting techniques to determine atmospheric parameters for DA stars above $T_{\rm eff}$ = 15,000 K, where convection is negligible. Their motivation for such an investigation was precisely to avoid the uncertainties related to the atmospheric composition and to the modeling of convection in the atmospheres of cooler objects. Although the results reported are preliminary, their analysis should provide an estimate of the mass distribution for DA white dwarfs which is independent of the particular theoretical uncertainties discussed in the present investigation. In turn, their result can ultimately be used to *calibrate* the mixing-length theory in the atmospheres of DA stars by extending the same mass distribution into the cooler convection-dependent temperature range. It remains to be seen if this calibration will, for the first time, give a consistent picture between the observations and the requirements of non-adiabatic pulsation calculations as to the location of the blue edge of the ZZ Ceti instability strip.

How efficient is convective mixing in white dwarf atmospheres? There is no quick answer to this question, and only a thorough analysis of all the available observational data with a set of models similar to that developed here can lead to a consistent answer. We are currently investigating the effects of convective efficiency on the determinations of atmospheric parameters of ZZ Ceti and other lukewarm DA stars. On the basis of a preliminary analysis of the internal consistency between photometric and spectroscopic analyses of lukewarm DA stars, Wesemael et al. (1991) concluded that the ML3 parameterization could be excluded. Similar conclusions can be reached if the mass distribution of lukewarm DA stars is compared to that of stars at higher effective temperature. Interestingly enough, however, a recent comparison of homogeneous, high-quality spectra of ZZ Ceti stars with our latest grid of models for these stars suggests that ML1 is equally inadequate to the task. Clearly, the problem of finding the convective efficiency which is most appropriate to white dwarf atmospheres is difficult and will require careful consideration. The results of our ongoing analysis will be reported elsewhere.

Further implications bear on eventual *mode identifications* in ZZ Ceti pulsators. Indeed, the period spectrum of a ZZ Ceti star depends on many parameters, whose individual effects cannot be easily untangled (see, e.g., Brassard et al. 1991). For example, the period of a given mode increases if the mass of the outer hydrogen layer is decreased, but the same effect can be obtained if the total mass of the star is decreased, the effective temperature is decreased, or the convective efficiency of the model is increased. Thus, it is highly desirable (if not essential) to obtain *independent* estimates of stellar parameters. Model atmosphere studies allow, in principle, a determination of the effective temperature and the mass (via a suitable mass-radius relationship) of a star, but, as we have seen, the effects of convective transport must be properly taken into account.

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