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CONSTRAINTS ON THE SURFACE MAGNETIC FIELDS OF HOT STARS WITH WINDS

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ABSTRACT

Several constraints on the surface (i.e., photospheric) magnetic fields of rotating stars with winds are discussed. It is shown that there are two allowed ranges for the strengths of surface radial magnetic fields, which we shall call the "strong field" and "weak field" ranges. In a previous paper (Maheswaran & Cassinelli) it was shown that, in a fast magnetic rotator (FMR), the radial component must be strong with a lower bound determined by the speed of subsurface motions. In the present paper, rotating hot stars with winds and weaker surface magnetic fields are considered. If there is appreciable mass loss caused, say, by radiation forces, the surface magnetic field may have a weak radial component which has an upper bound that varies directly as the mass-loss rate and the lower bound of the strong field. Since the strong-field lower bound increases with rotation speed so does the weak-field upper bound. For stars with moderate or rapid rotation, this upper bound allows us to predict a minimum mass-loss rate when the strength of a weak radial field can be estimated. For the O-type main-sequence star 9 Sgr and the B supergiant star ζ^1 Sco, the upper bound for a weak field is found to be of order 1 G, which is consistent with recent interpretations of radio observations. For a Wolf-Rayet star, with stellar parameters similar to those of CV Ser, it is found to be ~20 G, at faster rotation speeds. The average strength of a surface radial field, in a hot star with moderate or rapid rotation, cannot lie between the lower bound for the "strong field" and the upper bound for the "weak field." There is some observational support for the claim that a surface radial field should belong to one of these two distinct intervals. The condition used by Strittmatter & Norris for a radial magnetic field to withstand the effect of rotationally driven circulations is also discussed. This is different from the one used by Maheswaran & Cassinelli. The Strittmatter-Norris condition would require the azimuthal magnetic field strength to be almost equal to the maximum allowed by the condition of hydrostatic support, and the radial field would have to be much larger than the upper limit derived by Nerney, for FMR stars. Hence, the Strittmatter-Norris condition is overly restrictive and would not be valid for FMR stars. From the requirement of hydrostatic equilibrium in the interior, we derive a condition for a star to be an "extreme magnetic rotator," which is similar to the Eddington limit for stars with radiative forces.

Subject headings: stars: early-type — stars: magnetic fields — stars: mass loss — stars: rotation — stars: Wolf-Rayet

1. INTRODUCTION

Magnetic fields play an important role in the study of winds from early-type stars. They affect the flow dynamics and, also, the emergent radiation field. Although, direct measurement of fields (through Zeeman splitting of spectral lines) is usually not possible in the types of stars that we shall consider, estimates of their strength have been made from their effects on some of the observational properties. One possible consequence of fields in rotating stars is the production of fast magnetic rotator (FMR) winds from the equatorial region of the star (e.g., Poe, Friend, & Cassinelli 1989). In a previous paper, Maheswaran & Cassinelli (1988) (hereinafter Paper I) discussed bounds on surface magnetic fields of early-type stars, with FMR winds. In §§ 3, 4, and 5 of the present paper, we consider further constraints on surface fields of hot stars with winds and do not restrict the discussion to FMR models. We show that in rotating stars with appreciable (non–FMR) winds, a weak radial component, $B_{r,*}$, of the magnetic field may be present at the surface, and we derive an upper limit on the strength of this "weak" radial field. In the case of very slow rotators the distinction between "strong" and "weak" radial fields disappears, and the radial component at the surface may take any value less than the maximum allowed by the condition for hydrostatic equilibrium in the subphotospheric region.

For the purpose of this paper, we use the term "surface" of the star to mean the photosphere, i.e., the surface along which $\tau = 1$, where τ denotes optical depth. We shall be concerned with the large-scale field of the star, and we shall take $B_{r,*}$ to mean the average value of the magnitude of the radial component at the photosphere, with field lines emerging from an interior region at optical depth of order 10. The discussion of theoretical aspects contained in §§ 3, 4, and 6 of this paper refer to "surface region" which will be taken to mean an interior region where the optical depth takes values from 1 to a value of order 10. The expression "hydrostatic equilibrium," in a surface region, will be used to describe mechanical equilibrium that includes the centrifugal force due to rotation, and the Lorentz force due to a magnetic field. Such equilibrium is assumed to be valid, at least, to the first order in the ratio of the perturbing forces (caused by rotation or by magnetic fields) to gravity, at the equator. Also, we assume that any meridional motion occurring in the surface region is extremely slow in comparison with the rotation speed and that the Coriolis force which arises due to the presence of such motion is negligible. The "wind zone" will be taken to mean the region above the photosphere where line radiation forces, or other forces, are

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important in accelerating an outflow. In order to distinguish between results derived from interior considerations and from wind theory, we shall use subscripts * and 0, respectively, to denote values at the photosphere and at the base of the wind zone. Our goal is to understand the influence of the subphotospheric conditions on the properties needed to describe the "base of the wind."

It is important to distinguish between the various criteria for constraining the allowed surface field strengths. The lower bound derived in Paper I stated that, *for a star to have an FMR wind*, it must have a $B_{r,*}$ that is stronger than the lower bound determined by the speed of a large-scale internal motions, such as the Eddington-Vogt circulation currents. This limiting condition may not be relevant for stars with winds driven by other mechanisms, such as radiative forces. Radial fields with strengths less than the lower bound derived in Paper I can appear in such stars. Weak magnetic fields are of interest because their presence has been inferred from observed nonthermal radio emission and X-ray emission from hot stars, even though the fields may be too weak to affect the acceleration of winds.

In § 4, we derive constraints on rotation and magnetic fields, in relation to the extreme case when the equatorial rotation speed approaches the critical rotation speed. These would be relevant for studies of the evolution of rotation and magnetic fields of massive stars as they contract in their near-terminal Wolf-Rayet phases (e.g., Cassinelli et al. 1989; Maheswaran & Cassinelli 1991).

The upper limit for the surface field, derived in Paper I, pertains to the azimuthal component, $B_{\phi,*}$. This upper limit is obtained from the radial component of the equation of hydrostatic support and, in axially summetric magnetic rotator models for winds, only B_ϕ enters the Lorentz force component (e.g., Weber & Davis 1967). In § 5 of this paper, we consider a way of combining the upper limit on B_ϕ, obtained from interior constraints, with an equation for B_r/B_ϕ at the base of the wind zone, to obtain an approximate value for the upper limit on $B_{r,*}$.

In § 6, we discuss the lower bound on the surface magnetic field, derived by Strittmatter & Norris (1971) for the case of A-type stars, and show that this lower bound would be on $B_{\phi,*}$ when applied to an axially symmetric magnetic rotator model. They used the condition that the magnetic *force* must be stronger than the centrifugal *force*, for a surface radial field to survive the effect of rotationally driven circulation. In the case of FMR models, $B_{r,*}$ would have to be much larger than $B_{\phi,*}$ (e.g., Weber & Davis 1967; MacGregor & Friend 1987). Since, in fast rotators, $B_{\phi,*}$ would have to be almost equal to the maximum permitted by the condition of hydrostatic equilibrium, the Strittmatter-Norris condition would require that $B_{r,*}$ be very much larger than the upper limit allowed by Nerney's formula (Nerney 1980) for FMRs.

Applications of the constraints derived here are considered in § 7. In particular, the strengths of the weak radial fields of the B supergiant ζ^1 Sco and the main-sequence O-type star 9 Sgr are discussed with reference to their rotation speeds and mass-loss rates.

2. SURFACE FIELDS ESTIMATED FROM NONTHERMAL EMISSION EFFECTS

Bieging, Abbott, & Churchwell (1989) have summarized the nonthermal emission observed from early-type stars, at radio wavelengths. There are two classes of stars with radio emission associated with a magnetic field: "magnetospheric" and "stellar wind" thermal emitters. The "magnetospheric" class of stars are associated with helium strong Bp type (Drake et al. 1987) or with the strong fields that have been discovered in eclipsing binary systems (Stewart et al. 1989). These stars have relatively weak winds and have very large radio brightness temperatures (10^9–10^{10} K), and the emission can be explained as optically thick gyrosynchrotron radiation from closed field line regions extending out to 20 R_* from stars that have rotation speeds with $V \sin i$ of the order of 100 km s^{-1}. The field strengths at the surface of these stars are estimated to be in the range from 300 to 10,000 G. This range is consistent with the theoretical bounds derived in Paper I. The "stellar wind" class of nonthermal emitters are quite different. The OB stars detected by Bieging et al. (1989) are among the most luminous in the Galaxy with $L > 10^6 L_\odot$. They have very large mass-loss rates, and their winds are optically thick at radio wavelengths because of free-free opacity such that their radio photospheres have radii of over 100 R_*. The nonthermal radiation that can be detected is, therefore, assumed to originate in the outer wind regions. The nonthermal character of the emission is detected as a time-varying radio excess, which is about an order of magnitude larger than the expected free-free emission. Also, the spectral index a (i.e., $S_\nu \sim \nu^a$) tends to be flat or have a negative value, instead of the value $a = +0.6$ expected from free-free thermal emission (Wright & Barlow 1975). White (1985) presented an explanation of stellar wind nonthermal emission as optically thin synchrotron emission of low brightness temperature that produces an observable flux because of the large volume associated with the outer wind. The relativistic electrons which produce the radiation are accelerated by the Fermi mechanism operating at the shocks that are believed to permeate the wind. The surface field required to explain the nonthermal emission in the stellar wind is very small, in the stars studied by Bieging et al. (1989), and is of order 1 G. Some questions about the validity of White's model have been raised because it fails to explain negative spectral indices that are sometimes observed. Chen & White (1991) have summarized arguments in favor of the basic picture. Shocks in stellar winds were originally proposed to explain the X-ray emission observed in hot stars. Chen & White have used the shock model of Lucy (1982) to reanalyze the X-ray observations of the OB supergiants in the belt of Orion, obtained by the high spectral resolution instrument SSS in the *Einstein* satellite (Cassinelli & Swank 1983). Chen & White have explained the anomalously high flux in the high-energy tail of the X-ray specta, as arising from the inverse Compton scattering of the relativistic particles accelerated in the neighborhood of the shock. They have also estimated that the magnetic field in the wind is consistent with a surface field of 1 G.

Though the weak magnetic fields associated with the type of winds described above do not fall within the "strong" range derived in Paper I for FMR model stars, we show in § 3 that the presence of a wind with significant mass loss can account for such a weak radial field.

3. "STRONG" AND "WEAK" RADIAL SURFACE FIELDS

In the preceding section, we noted that "weak" magnetic fields can contribute to interesting observational effects in hot stars. Here, we consider conditions under which weak surface radial fields could be present in rotating stars. We find that hot stars with radiatively driven (non–FMR) winds and appreciable mass loss may possess such small fields, even when the rotation is moderate or fast. In Paper I, we noted that, in FMR model stars, $B_{r,*}$ should, at least, be strong enough to withstand distortion by rotationally driven meridional currents. The condition that the Alfvén speed should be larger than the speed of meridional currents gives us a lower bound, B_L, on $B_{r,*}$. There are two possible reasons why a residual radial field, weaker than B_L, may exist at the surface of a star. First, if the initial field is weaker than our lower bound, in a star where the magnetic Reynolds number, $R_m = lV/\eta$, is extremely large, then advection of the field by large-scale meridional currents will cause the field lines to align themselves with the streamlines of the subsurface motion, as required by equation (2) below. Here, l is the length scale of the field, V is the characteristic speed of the currents, and η is the magnetic diffusivity of the stellar material. η is inversely proportional to the local electrical conductivity. In most main-sequence A or B type stars, R_m is of order 10 or 100 near the surface, for moderately fast rotation, and this leads to some relaxation of the perfect conductor approximation in the surface regions of such stars (Maheswaran 1969). However, in hot stars, η is quite small near the surface and R_m can be of order 10^4 or larger, requiring the magnetic field lines to be almost parallel to the direction of flow. Thus, when the meridional component of the magnetic field has strength less than the minimum for FMR stars, the radial component, $B_{r,*}$, which would be permitted to appear above the surface due to ohmic diffusion, will be negligible.

On the other hand, in stars with winds and appreciable mass loss, there must be a small, but nonzero, radial velocity $V_{r,*}$ in the photosphere, and its presence affects the surface magnetic field. The continuity condition for mass gives

$$V_{r,*} = \dot{M}/4\pi R^2 \rho_* , \qquad (1)$$

where R is the equatorial radius and ρ_* is the mass density at the stellar surface (optical depth = 1). $\dot{M}$ is the rate of mass loss from the star. When R_m is large, as in a hot star, the MHD induction equation reduces to curl $(\boldsymbol{V} \times \boldsymbol{B}) = 0$, in a steady state (the ohmic diffusion term is far too small and has been omitted). In an axially symmetric system, this equation requires that (e.g., Mestel 1965)

$$\boldsymbol{B}_m = \kappa \, \boldsymbol{V}_m , \qquad (2)$$

where κ is a scalar function and $\boldsymbol{B}_m$ and $\boldsymbol{V}_m$ are, respectively, the meridional components of the magnetic field and local fluid velocity. If we use suffixes r and t to denote radial and tangential components, respectively, equation (2) gives

$$B_{r,*}/B_{t,*} = V_{r,*}/V_{t,*} .$$

This, together with equation (1), yields the estimate

$$B_{r,*} = \dot{M} B_{t,*}/4\pi R^2 \rho_* V_{t,*} . \qquad (3)$$

Equation (3) shows that hot stars with reasonably high mass-loss rates, but whose surface magnetic field strengths are below the minimum required to overcome the effects of circulation currents, could have weak radial components at the surface, while the tangential component parallel to the streamlines of internal currents may be somewhat larger.

Consider a fast rotator with a wind such that the speed of mass loss at the surface is finite, though small. If we combine the arguments given in Paper I for a lower bound B_L, for the radial field in the "strong" regime, together with the result in equation (3), we find that $B_{r,*}$ must either be larger than B_L or

$$B_{r,*} < B_L \dot{M}/4\pi R^2 \rho_* V_{t,*} . \qquad (4)$$

We shall call the former a "strong field" and the latter a "weak field." The right-hand side of condition (4) imposes an upper bound on the weak radial field. Note that this varies directly as the mass-loss rate and the strong-field lower bound. Thus, it increases with increasing rotation speed.

In stars that do not rotate fast, so that $V_{t,*} \leq V_{r,*}$, the radial component $B_{r,*}$ of the surface magnetic field can take any value up to the maximum allowed by other criteria, such as hydrostatic equilibrium. These will be very slow rotators and will not have a separation into the distinct categories of strong and weak, with respect to their surface magnetic fields.

Conditions (3) and (4), derived above, provide the theoretical basis for computation of the weak-field upper bound on $B_{r,*}$. However, in practice, it is a complicated task to determine $V_{t,*}$ (e.g., see Paper I). However, if we focus our attention on field lines that pass through the outer boundary, S_d, of the radiative diffusion zone, which would be at an optical depth of order 10, or larger, we could obtain a reasonable approximation to $B_{r,*}$ using the value $B_{r,d}$, where subscript d is used to denote quantities evaluated at S_d. Corresponding to condition (4), we have

$$B_{r,d} < B_L \dot{M}/4\pi R_d^2 \rho_d V_{t,d} . \qquad (5)$$

The right-hand side in condition (5) can be evaluated using the results given in Paper I. Here, $V_{t,d}$ would be the speed of large-scale circulation, at S_d. The method of computation of $V_{t,d}$ was discussed in § 3 of Paper I. In each application, it is necessary to check whether the limiting speed of circulation is reached interior to the diffusion zone boundary, S_d, and an appropriate adjustment should be made. Further reference is made to this aspect of the problem in § 7.

Since we have chosen to focus on the average value of the radial component of the large-scale field near the surface, we may assume that B_r varies as $1/r^k$, where $k \geq 2$ is an index that describes the behavior of the radial field in the surface regions. For early-type stars, we have, to a high degree of accuracy, $R_d = R$, so that the upper bound on $B_{r,*}$ would be approximately the same as that on $B_{r,d}$, which can be computed using condition (5). Note that the only field lines for which this approximation may not be valid will be those that lie outside the surface S_d and cross the photospheric surface S. However, since the region between S_d and S is extremely narrow (with thickness probably of order 10^{-5} R or less), it is unlikely that there will be a concentration of field lines that cross S but do not cross S_d. Thus, $B_{r,*}$ will not be significantly different from $B_{r,d}$.

We should point out that for typical hot stars with mass-loss rate of order 10^{-5} $M_\odot$ yr^{-1} or less, $V_{r,*}$ is small when compared with the equatorial rotation speed, or with the local sound speed. Also, $V_{r,d}$ is about one order of magnitude smaller than $V_{r,*}$, e.g., in the case of the stars 9 Sgr and ζ^1 Sco, shown in Table 1, $V_{r,*}$ is approximately 2×10^4 cm s^{-1} when the rotation parameter α (ratio of rotation speed to critical rotation speed) is equal to 0.4. Thus, the radial motion due to mass loss will not contribute to a Coriolis force that could affect hydrostatic equilibrium in the surface region of the star. In the case of both these stars, $V_{r,d}$ is slightly smaller than the circulation speed $V_{t,d}$, at the diffusion zone boundary, S_d.

We may understand the surface magnetic field in rapid rotators as follows: If the initial surface field has Alfvén speed larger than $V_{t,*}$, then the meridional currents will not be able to drag the field beneath the surface. This is the "strong" field case. If the initial field has Alfvén speed less than $V_{t,*}$, the

TABLE 1

PARAMETERS OF STELLAR MODELS CONSIDERED[a]

Parameter	Star 1	Star 2
Star name	9 SGR	ζ^1 SCO
Spectral type	O4 V	B1 Ia
Mass $M/M_\odot$	98	73
Luminosity $L/L_\odot$	1.55×10^6	1.74×10^6
Effective temperature $T_{\rm eff}$ K	50,100	20,400
Radius $R/R_\odot$	16	103
Mass-Loss Rate $\dot{M}/(M_\odot\ {\rm yr}^{-1})$	1.1×10^{-5}	1.3×10^{-5}
Terminal speed of wind $V_\infty/({\rm km\ s}^{-1})$	3400	500

[a] Taken from Bieging et al. 1989, and mass-loss rates from the empirically derived formula (eq. [1]) in Garmany & Conti 1984.

meridional currents will drag the field beneath the surface but the slower radial motion of the wind will be able to draw out a "weak" radial component.

When a magnetic field in the radiative envelope reaches a quasi-steady state, in the presence of large-scale circulation, the strength of the radial component in the polar regions may be an order of magnitude larger than the average strength of the radial component, taken over the surface (Maheswaran 1969, 1989). Also, the equatorial radial field, at the surface, could be stronger than the field in the middle latitudes, by a factor of 2 or 3, but will not be as strong as the polar field.

4. EXTREME MAGNETIC ROTATOR

The condition for hydrostatic support in the equatorial region, at the photosphere and below, in a hot star rotating with angular velocity Ω has radial component given by

$$-\frac{1}{\rho}\frac{dp}{dr} = \frac{GM}{r^2} - \frac{\Gamma GM}{r^2} - \Omega^2 r - \frac{(\text{curl}\ \boldsymbol{B} \times \boldsymbol{B})_r}{4\pi\rho}, \tag{6}$$

where G, M, p, and Γ are, respectively, the gravitational constant, stellar mass, pressure, and the ratio of the radiative force to gravity at the equator. If $B_\phi \sim 1/r^n$, with $n > 1$, near the surface equation (6) gives

$$-\frac{r}{\rho}\frac{dp}{dr} = \frac{GM(1-\Gamma)}{r} - \Omega^2 r^2 - \frac{(n-1)B_\phi^2}{4\pi\rho}.$$

If we evaluate this at the equator, with $r = R$, we get

$$-\frac{R}{\rho_*}\frac{dp}{dr}\bigg|_* = V_c^2 - V_{\rm rot}^2 - (n-1)V_{{\rm A},\phi,*}^2, \tag{7}$$

where $V_c^2 = GM(1-\Gamma)/R$ is the critical speed, $V_{\rm rot} = \Omega R$ is the equatorial rotation speed and $V_{{\rm A},\phi,*}^2 = B_\phi^2/4\pi\rho_*$ is the azimuthal Alfvén speed at the equator. In a star we must have

$$\frac{dp}{dr}\bigg|_* \leq 0,$$

so that equation (7) gives

$$V_{\rm rot}^2 + (n-1)V_{{\rm A},\phi,*}^2 \leq V_c^2. \tag{8}$$

The rotation rate can be specified by the parameter $\alpha = V_{\rm rot}/V_c$. Let $\beta = (n-1)^{1/2}V_{{\rm A},\phi,*}/V_c$, be the ratio of the radial component of the Lorentz force to gravity, at the equator. Then, condition (8) reduces to

$$\alpha^2 + \beta^2 \leq 1. \tag{9}$$

We can use the ratios α and β to rewrite equation (7), for hydrostatic equilibrium at the equator, so that it reads conveniently in the form

$$-\frac{1}{\rho}\frac{dp}{dr} = GM(1-\Gamma)\frac{(1-\alpha^2-\beta^2)}{R^2}. \tag{10}$$

Note that equations (6) and (10) will be valid in the surface region up to the photosphere. At higher levels, e.g., the wind zone, the line radiation force and other inertial terms will have to be included in the equation of motion (e.g., Friend & MacGregor 1984).

In Paper I we used the equation of hydrostatic support to derive an upper limit, B_u, on the surface magnetic field. This applies only to the azimuthal component, $B_{\phi,*}$. From equation (7) or (10), we get

$$(n-1)B_{\phi,*}^2 \leq B_u^2 = 4\pi GM(1-\Gamma)(1-\alpha^2)\rho_*/R. \tag{11}$$

A simpler version of this using the magnetic field parameter β, which allows for different values of $n > 1$, is derived from condition (9) in the form

$$\beta \leq (1-\alpha^2)^{1/2}. \tag{12}$$

Note that condition (9) states that arbitrarily large values of the rotation rate and surface magnetic field strength cannot be invoked in modeling. Using this condition, we may define an "extreme magnetic rotator" as a star in which $\alpha^2 + \beta^2 = 1$. Because of its derivation from the radial component of the hydrostatic equilibrium equation, it is similar to the Eddington limit for radiative acceleration.

5. UPPER LIMIT ON STRONG RADIAL FIELDS

We have seen that an upper limit can be derived for the strength of the azimuthal field at the surface. We would also like to have an upper limit on $B_{r,*}$. Unfortunately, we cannot use the momentum equation of the usual (one-dimensional) magnetic rotator model (e.g., Weber & Davis 1967; Friend & MacGregor 1984) to obtain this limit. In Paper I, we suggested that an approximate value could be obtained by using MHD stability conditions (e.g., Tayler 1980), which require that one component of the magnetic field should not be very much larger than another.

We pointed out in Paper I that the wind parameters give us Nerney's upper bound in the form

$$B_{r,*} \leq (\dot{M}V_\infty)^{3/2}/MRV_{\rm rot}, \tag{13}$$

where V_∞ is the terminal speed of the wind. If an upper limit involving interior constraints is required as an independent check, then we can combine equation (12) with a result for B_r/B_ϕ at the base of the wind zone, which states that (e.g., Friend & MacGregor 1984)

$$B_\phi = \frac{B_r\Omega r(1 - r^2/r_{\rm A}^2)}{v_{\rm A}(1 - v_r r^2/v_{\rm A} r_{\rm A}^2)}, \tag{14}$$

where v_r is the radial speed of the wind at distance r from the rotation axis of the star, $r_{\rm A}$ is the distance of the Alfvén point, and $v_r = v_{\rm A}$ when $r = r_{\rm A}$. If $r = r_0$ at the base of the wind zone and if we have $v_{r0} r_0^2 \ll v_{\rm A} r_{\rm A}^2$, as is found in magnetic rotator models, we get

$$\left|\frac{B_{\phi,0}}{B_{r,0}}\right| = \frac{V_{\rm rot}(1 - r_0/r_{\rm A}^2)}{v_{\rm A}} . \quad (15)$$

We might expect that equation (15), which is valid at the base of the wind zone, to also give, approximately, the ratio $B_{r,*}/B_{\phi,*}$, at the stellar "surface" (optical depth = 1), provided that there is no sharp change in B_r or B_ϕ between the surface and the base of the wind zone. Recent work (e.g., Karpen, Antiochos, & DeVore 1990) suggests that there will be no current sheets in this region, which could cause such a sharp change in the magnetic field strength across this region. Also, to avoid having to include the effects of other forces, such as the line radiation force (e.g., MacGregor & Friend 1987), we shall consider only those cases where the base of the wind zone is sufficiently close to the photosphere. Then, we might take the right side of equation (15) to be an approximation for $B_{r,*}/B_{\phi,*}$. This can then be combined with condition (12) to give an upper limit on $B_{r,*}$. Such an upper limit can be used only as a check, after models have been computed to give values of $r_{\rm A}$ and $v_{\rm A}$. However, in typical hot stars with winds, the strengths of radial magnetic fields required to explain FMR winds, or nonthermal emission, are usually much less than the theoretical upper limit derived in Paper I.

An important difference between the Weber-Davis magnetic rotator models for normal stars and those for hot stars (e.g., MacGregor & Friend 1987) is that in the former, the ratio $B_{\phi,0}/B_{r,0}$ must be very large, whereas, in the latter, this ratio is of order 10. An interesting question is whether the Weber-Davis models may, over a long period of time, be subject to the MHD instabilities discussed by Tayler (1980).

6. LOWER BOUND DUE TO STRITTMATTER AND NORRIS

As an explanation of the difference between the magnetic nature of Ap-type and "normal" main-sequence A-type stars, Strittmatter & Norris (1971) argued that rotationally driven meridional circulation would drag a stellar magnetic field beneath the surface unless the Lorentz *force* due to the magnetic field is stronger than the centrifugal *force* due to rotation. This requirement is different from the one used by us in Paper I. We should note, from equations (6) and (7) above, that, when applied to the usual magnetic rotator models, the Strittmatter-Norris condition (hereinafter SNC) gives a lower bound on $B_{\phi,*}$ and not on $B_{r,*}$. An extra condition would be required to extend it to the latter. Strittmatter & Norris have assumed that $n = 2$ and $B_{r,*} \approx B_{\phi,*}$, as we did in Paper I.

When the condition (SNC) that the Lorentz force be larger than the centrifugal force at the surface is applied, α and β in equation (7) must satisfy

$$\beta^2 > \alpha^2 , \quad (16)$$

or, equivalently,

$$B_{\phi,*}^2 > 4\pi\rho_* V_{\rm rot}^2/(n - 1) . \quad (17)$$

Another interpretation of SNC is that $(n - 1)^{1/2}$ times the azimuthal Alfvén speed at the surface must be larger than the equatorial rotation speed, i.e.,

$$(n - 1)^{1/2} V_{{\rm A},\phi,*} > V_{\rm rot} . \quad (18)$$

In contrast, the condition that we have used (in Paper I) is that the radial Alfvén speed near the surface must be larger than the speed of large-scale meridional currents, which directly gives a lower bound on $B_{r,*}$. Since the speed of circulation currents is significantly slower than equatorial rotation speed, our lower bound is much smaller than theirs.

In stars with moderate rotation speeds, the values of $B_{\phi,*}$ required to satisfy SNC will be very large. In fact, conditions (8) and (16) require that α^2 must be less than or equal to 0.5. Also, when α^2 gets close to 0.5, $B_{\phi,*}$ will be very close to the upper limit allowed by the condition for hydrostatic balance. Some numerical values in support of this are given in the following section. When this lower bound is applied to the extension of equation (15) to the stellar surface, we should find that, as shown by MacGregor & Friend (1987), $B_{r,*}$ is almost an order of magnitude larger than $B_{\phi,*}$. Though, from the point of view of mechanical equilibrium, such a large $B_{r,*}$ may not be excluded, computations in Table 1 of Paper I show that the values of $B_{r,*}$ required by SNC will be very much larger than the upper limit allowed by Nerney's formula given in equation (13). Thus, SNC cannot be relevant for FMR stars, and it appears that only slow rotators, such as the Ap stars, in which $B_{r,*}$ is of order 1000 G, will be able to satisfy their condition. We conclude that SNC is not the appropriate condition for determining lower bounds on the strengths of surface magnetic fields in magnetic rotator wind models.

7. APPLICATIONS

Bieging et al. (1989) have estimated surface magnetic fields of early-type stars using the theory of White (1985), and their observations of nonthermal emission. According to White's theory, the field primarily determines the turnover frequency, ν_t, at which the spectrum changes from a $\nu^{0.5}$ power—that is, held at low frequencies—to a flat spectrum. Although it is difficult to estimate ν_t, the values of the surface field derived by Bieging et al. provide a useful test for our upper bound on the average surface radial field. Bieging et al. note that, qualitatively, only the very luminous stars are detected as having nonthermal emission. The lack of nonthermal emission from lower luminosity and cooler stars may now be understood as a result of low photospheric speeds associated with their winds that have a smaller mass-loss rate, *or* a larger radius. Here, we consider two stars with similar mass-loss rates ($\sim 1 \times 10^{-5}$ $M_\odot$ per yr). The smaller star 9 Sgr O4 V shows evidence of nonthermal emission, while the larger star ζ^1 Sco B1 Ia does not.

Estimates of the various bounds on the radial component surface magnetic field, for different values of the rotation parameter α, are shown in Figures 1 and 2, respectively, for the B supergiant ζ^1 Sco, and for the main-sequence O-type star 9 Sgr. Figures 3 and 4 show bounds on the azimuthal component of the field. Table 1 gives the values of the different parameters for these two stars. In Figures 1 and 2, curves B and C have been drawn for constant values of $\dot{M}$ and V_∞. This would correspond to the situation in which the mass-loss rate and terminal speed of the wind were known from observations, or specified for the star, and we were attempting to establish limits on the unknown rotation speed and magnetic field strength. In Figure 2, for 9 Sgr, several possible values of $\dot{M}$ are considered in order to determine the minimum $\dot{M}$ that would explain the estimated radial magnetic field of 1.8 G. The effects of changing the values of $\dot{M}$ and V_∞, on the different bounds, are discussed below. Note that it would be possible to draw similar figures when $\dot{M}$ and V_∞ are taken to be functions of α and formulas for their variation with α are known. However,

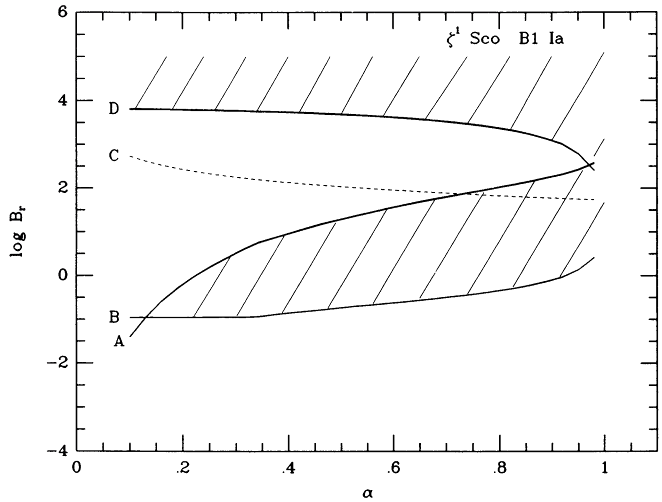

FIG. 1.—Bounds on the radial component, B_r, of the surface magnetic field vs. rotation rate, given by $\alpha = \Omega/\Omega_c$ for the star ζ^1 Sco, with stellar parameters shown in Table 1. The cross-hatched areas represent forbidden field regions. Curve A gives the lower bound on a strong B_r, determined by Eddington-Vogt currents. Curve B gives the upper bound on a weak B_r that may exist in a hot star with outward photospheric velocities associated with a wind. Curves A and B have been obtained by taking the outer boundary, S_d, of the radiative diffusion zone to be at optical depth 25. Curve C gives values of B_r determined by Nerney as capable of explaining the mass-loss rate and terminal wind speed via the FMR wind. Curve D represents the upper limit on B_r consistent with the conditions of hydrostatic support.

we do not expect that there would be any significant qualitative change in the figures.

In all these models, we find that the surface magnetic field of a rotating star may have a radial component whose strength should fall into one of two distinct intervals, which we refer to as "weak" and "strong." In Figures 1 and 2, a radial field with strength less than, or equal to, the values on curve B would belong to the "weak" category. A "strong" field required for a star to be an FMR will lie above curve A, which represents the lower bound (derived in Paper I) for this regime. A surface magnetic field with average strength lying between values on curve B and on curve A would not have a radial field projecting out from the major portion of the stellar surface. For a very slow rotator with value of α less than that at which curves A and B intersect, there will be no distinction between the weak and strong regimes, and the only relevant bound would be the upper bound imposed by the condition of hydrostatic equilibrium. The very slow rotators thus form a separate class of objects.

We shall now consider some numerical values for the weak-field upper bound. In the case of ζ^1 Sco, observations give $V \sin i = 45$ km s^{-1}, which corresponds to $\alpha = 0.198/\sin i$, where i denotes the angle of inclination of the rotation axis to the line of sight. We find that the upper bound on a weak field would be of order 0.1 G, when α is less than 0.4 and $\dot{M} = 1 \times 10^{-5}\ M_\odot$ yr^{-1}. This is consistent with the observations of Bieging et al. (1989), for which the radial magnetic field at the surface of this star must have an upper limit of 0.4 G. In Figure 2, for the star 9 Sgr, we have drawn curves B corresponding to three different values of $\dot{M}$. This star has $V \sin i = 128$ km s^{-1}, giving $\alpha = 0.155/\sin i$. The radial magnetic field at the surface for this star is estimated, by Bieging et al., to be ~ 1.8 G. If this field is to be in the "weak" domain, with $\alpha = 0.3$, say, the mass-loss rate $\dot{M}$ should be larger than $2.2 \times 10^{-5}\ M_\odot$ yr^{-1}. We note that a mass-loss rate that is significantly smaller than this value would be too low, at any reasonable value of α, to account for the estimated radial field of 1.8 G. The value of $2.2 \times 10^{-5}\ M_\odot$ yr^{-1} is larger than the $1.3 \times 10^{-5}\ M_\odot$ yr^{-1} determined from the empirical mass-loss rate of Garmany & Conti (1989). However, given the uncertainties in both $\dot{M}$ and the field derived by Bieging et al., the expression that we have established for an upper bound on the strength of the average surface radial field, in terms of the mass-loss rate, is encouraging.

Also, referring to the discussion in § 2, we may infer that the magnetic field of order 1 G estimated by Chen & White (1991) for the OB supergiants they considered would belong to the "weak" domain for that star.

In § 3, we discussed the approximation of $B_{r,*}$ by $B_{r,d}$, the latter being the value computed at the outer boundary, S_d, of the radiative diffusion zone. The numerical values presented above for $B_{r,*}$ have been obtained by taking S_d at optical depth 25, as have been the values along curves A and B in Figures 1 and 2. Computations were carried out with S_d taken at different optical depths ranging from a value of 10 to a value of 110 in order to estimate the possible error in the approximation for $B_{r,*}$, corresponding to an error in the location chosen for S_d. For both the stars discussed above, for values of α less than 0.7, the value of $B_{r,d}$ decreased only by a factor of 1.5 when τ increased from 25 to 110. When τ decreased from 25 to 10, $B_{r,d}$ increased by a factor of 1.5. Thus, the estimated upper bound for $B_{r,*}$ could be in error by a factor of between 0.7 and 1.5 due to an uncertainty in the location of S_d.

We shall not go into a discussion of numerical values for the "strong" domain, relevant for FMR stars, because this was done in Paper I. However, we would like to point out that the allowed field strengths in the "strong" region are consistent with the observationally inferred values lying in the range from 300 to 10,000 G (Drake et al. 1987; Stewart et al. 1989; Andre et al. 1988).

Curve C represents Nerney's upper bound on the radial component, $B_{r,*}$, at the surface. It depends on the terminal speed, V_∞, of the wind and the mass-loss rate, $\dot{M}$, and is derived

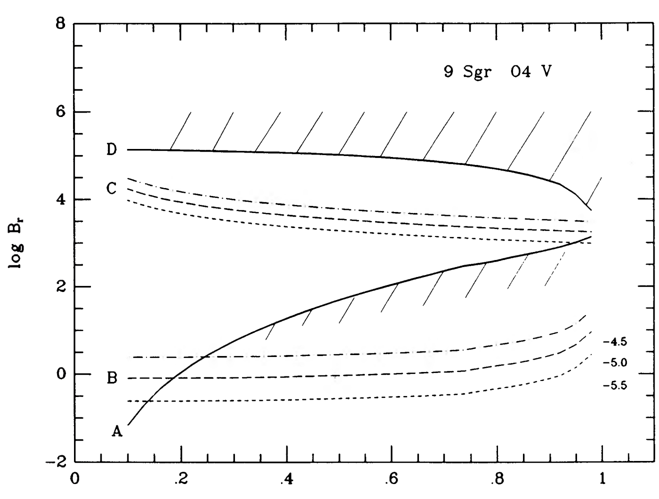

FIG. 2.—Bounds on B_r vs. α for the star 9 Sgr, for three different values of the stellar mass rate 3×10^{-5}, 1×10^{-5}, and $3 \times 10^{-6}\ M_\odot$ yr^{-1}, whose logarithms are shown against the B curves corresponding to them. The position of curve C also depends on the mass-loss rate. S_d has been taken to be at optical depth 25.

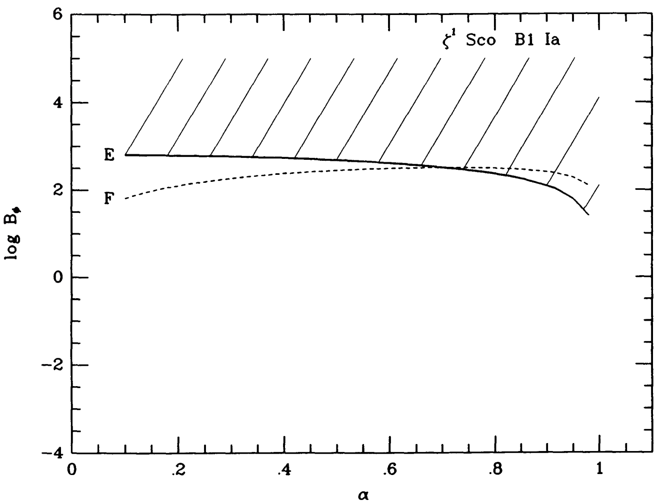

FIG. 3.—Bounds on B_ϕ vs. α for the star ζ^1 Sco. Curve E shows the upper bound on the azimuthal component B_ϕ of the surface magnetic field, determined by the condition of hydrostatic equilibrium. Curve F shows the Strittmatter-Norris lower bound on B_ϕ determined by the condition that the Lorenz force should equal the centrifugal force at the equator.

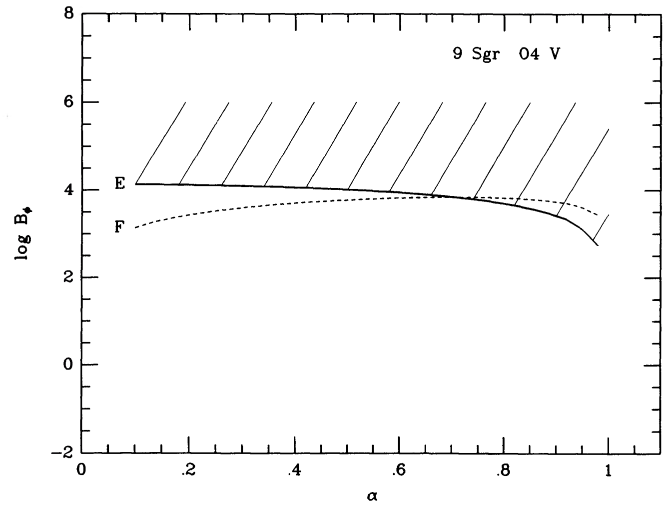

FIG. 4.—Bounds on B_ϕ vs. α for the star 9 Sgr. The curves are as described in Fig. 3.

using the assumption that these observationally estimated wind parameters have values appropriate for the FMR model. If the parametric values of a star place it at a point above curve C, then it would be necessary to reconsider the wind model being used.

Curve D shows the largest possible values of $B_{r,*}$ that would be consistent with hydrostatic equilibrium just below the photosphere and an FMR wind in a hot star. We find that the strength of the radial component of the surface magnetic field in a star with an FMR wind should lie between the values found on curves A and D. Curve D shows the highest level to which curve C can rise, i.e., curve D is an upper limit for curve C. If it is hypothesized that a star conforms to the FMR wind model, then $B_{r,*}$ should not be assigned a value larger than that on curve C. We did not construct detailed models for the wind region to obtain exact values for curve D. It would have been necessary to find the Alfvén radius, r_A, and the radial speed, v_A, of the wind at the Alfvén point, for each model. We assumed that the ratio $B_{\phi,*}/B_{r,*}$ in equation (15) was approximately equal to 0.1 so as to be consistent with MacGregor & Friend (1987). In the figures presented here, curve D gives only a qualitative picture of the upper limit it represents, in relation to the other bounds on $B_{r,*}$.

We should note that, when $\dot{M}$ and V_∞ are known for a star and the rotation speed is not known, the point of intersection of curve C with curve A or curve C with curve D, corresponding to the observed $\dot{M}$ and V_∞, gives the maximum possible value of α that would be consistent with an FMR model for that star. For example, if we consider a star with the same parameters as those of ζ^1 Sco, shown in Table 1, we deduce from Figure 1 that the maximum possible α in an FMR model for that star would be approximately 0.74, which is the abscissa of the point where curves C and A intersect.

For a given star, whose mass, luminosity, and radius are known, the position of curve B is influenced by the value of the mass-loss rate $\dot{M}$. For increasing values of $\dot{M}$ the position of curve B moves vertically upward, as seen in Figure 2. For example, if $\dot{M} = 3 \times 10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$, then at $\alpha = 0.3$, the upper bound of the weak field in the star 9 Sgr would be 3 G. Also, an increase in the value of either $\dot{M}$, or of V_∞, will cause an upward shift in the position of curve C.

In Figures 3 and 4, curve E shows the upper limit on $B_{\phi,*}$, imposed by the condition of hydrostatic equilibrium. Curve F gives the values of the lower bound on $B_{\phi,*}$ determined by the SNC. As pointed out in § 6 above, this condition requires that α^2 must be less than 0.5, i.e., curve F terminates at $\alpha = 0.71$, on curve E. For values of α close to 0.7, there is hardly any space between curves E and F. Also, if $B_{\phi,*}$ is large enough to satisfy SNC then it is easy to show that the corresponding $B_{r,*}$ required to fit the FMR model would have to be much larger than the upper bound imposed by Nerney's formula. We shall illustrate this using a numerical example. Consider an FMR model with stellar parameters the same as those of 9 Sgr. When $\alpha = 0.62$, SNC requires that $B_{\phi,*}$ should be larger than 7000 G, which is very close the maximum of 8800 G permitted by hydrostatic equilibrium. This SNC lower bound together with equation (15) and the results of MacGregor & Friend (1987), for hot stars, would require that $B_{r,*}$ should be $\sim 7 \times 10^5$ G for this star to emit an FMR wind. As will be seen from Figure 2, such a value would be very much larger than the upper limit allowed by Nerney's formula (curve C), i.e., it is clear that if $B_{\phi,*}$ satisfies SNC then $B_{r,*}$ cannot satisfy the upper bound imposed by Nerney's formula. On the contrary, the condition used by us in Paper I requires that $B_{r,*}$ be larger than only 150 G to appear above the surface.

As a final application, we consider the case of the Wolf-Rayet star, CV Ser ($L = 3 \times 10^5\ L_\odot$, $M = 13\ M_\odot$, $R = 3\ R_\odot$, $\dot{M} = 2 \times 10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$, and $V_\infty = 2000\ \mathrm{km\ yr}^{-1}$). If this star satisfies the FMR wind model, then $B_{r,*}$ must belong to the strong interval, with a lower bound of ~ 250 (at $\alpha = 0.6$)–1000 (at $\alpha = 0.8$) G. On the other hand, if it is argued that CV Ser has moderate or fast rotation, but the wind is not magnetically driven, then its average surface field must belong to the weak region with an upper bound of ~ 25 G. Of course, if it was a very slow rotator, with α less than 0.2, it could have a radial magnetic field of any strength, subject to an upper limit of order 10^5 G.

In all the applications above, we derived numerical estimates that are valid at the photosphere. We did not construct detailed models for the atmospheric region just above the photosphere, to determine the height to which these bounds may be extended. However, following the work of MacGregor & Friend (1987), we expect that these values may be extended to a height of a few tenths of 1% of the stellar radius, above the photosphere. Our primary interest has been in establishing constraints appropriate for the inner boundary of the wind zone.

8. CONCLUSIONS

We find that, in stars with moderate or rapid rotation, if a radial component of the magnetic field is present at the surface, its strength, averaged over the surface, must belong to one of two distinct intervals—the strong-field regime or the weak-field regime. In such stars, $B_{r,*}$ cannot lie between the upper bound for a "weak" field and the lower bound for a "strong" field. The strengths of radial fields that have been invoked to explain FMR winds are consistent with the bounds derived by us for the strong-field regime. Also, this "strong" interval would be relevant for any rotating early-type star that does not have a wind. In the examples given in the preceding section, to compute numerical values of the lower bound of the strong interval, we have chosen to use the second order result, given by Maheswaran (1968), for the speed of meridional circulation in the diffusion zone of the radiative envelope. However, the formula for the lower bound, derived in Paper I, will accept any appropriate choice for the speed of meridional motions in the envelope of the star.

The allowed interval for weaker fields would be appropriate for hot stars with radiatively driven winds and appreciable mass loss. The upper bound for this weak field regime depends on the mass-loss rate and the rotation rate, and the numerical values computed for the particular stars that we have considered in this paper are consistent with field strengths estimated from radio and X-ray observations. For stars with larger mass-loss rates, this upper bound could be of order 10 G. It should be noted that this weak-field upper bound is applicable to the average surface radial field. However, the strength of the radial field near the poles could be an order of magnitude larger than this upper bound for the average surface radial field (Maheswaran 1969, 1989). This is because large-scale meridional circulation will cause the concentration of radial field lines in a relatively small region around the poles. Thus, when modeling, it would be important to distinguish between phenomena that may be influenced by the polar fields from those that are influenced by the general, or equatorial, field.

We have considered the condition used by Strittmatter & Norris (1971) to determine whether a surface radial field could survive the effects of subsurface motions driven by rotation. In the case of an FMR model star, this condition yields a lower limit on the azimuthal component, $B_{\phi,*}$, of the magnetic field, and additional conditions of the wind model are required to determine a lower limit on $B_{r,*}$. It is found that the SNC would not allow the rotation α to exceed the upper limit of 0.71, and for rotation speeds near this limit, the azimuthal component of the field must have a strength close to the maximum permitted by the condition of hydrostatic equilibrium. Further, for such large values of $B_{\phi,*}$, the relationship between $B_{r,*}$ and $B_{\phi,*}$ that has to be satisfied, in a hot star with an FMR wind (e.g., MacGregor & Friend 1987) would require $B_{r,*}$ to be much larger than the upper limit derived by Nerney (1980). For these reasons, it is suggested that the SNC for a lower limit on surface fields in rotating stars is not valid for FMR models. The condition used by us, in Paper I, is different from the SNC. The strengths of the surface radial fields that are invoked to explain FMR winds in hot stars are able to satisfy the lower bound derived in Paper I and also satisfy the Nerney condition.

In the case of the weak fields, the relationship that we have established between the mass-loss rate and the average strength of a weak surface radial field may be used not only to estimate upper bounds on the field when the mass-loss rate is known, but also to estimate minimum mass-loss rates when the strength of the weak radial field can be determined.

Stellar wind theories, involving rotation and magnetic fields, have had a large measure of success in explaining observed phenomena by treating the wind zone as a separate region, while using suitable boundary conditions at the base of the wind zone. One of the objectives of the present paper, and of Paper I, has been to determine what constraints should be imposed on such boundary conditions as a result of the conditions that prevail in the interior. We have determined the upper limits imposed on the rotation rate and on the magnetic field strength, by the requirement of hydrostatic equilibrium in the surface region. We have also been able to establish results for bounds on the strengths of "weak" or "strong" radial components of the surface magnetic field. These results will directly apply to any example in which the photosphere is taken as the base of the wind zone, or when the base of the wind zone, above the photosphere, is sufficiently small.

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