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THE BAADE-WESSELINK METHOD AND THE DISTANCES TO RR LYRAE STARS. VIII. COMPARISONS WITH OTHER TECHNIQUES AND IMPLICATIONS FOR GLOBULAR CLUSTER DISTANCES AND AGES

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ABSTRACT

We summarize all techniques that derive M_V or $M_{\rm bol}$ as a function of [Fe/H] for RR Lyrae variables. Baade-Wesselink analyses indicate $\langle M_V(\rm RR)\rangle \propto 0.16$[Fe/H] and $\langle M_{\rm bol}(\rm RR)\rangle \propto 0.21$[Fe/H]. We revise recent results from main-sequence fitting to account for metallicity-dependent errors in $B-V$ from model isochrones, finding $\langle M_V(\rm RR)\rangle \propto 0.12$[Fe/H]. We redefine the equilibrium temperature scale for RR Lyrae stars, a crucial quantity for comparison of theory with observations and the period-shift analyses of Sandage. We readdress the period-shift analyses for RR Lyrae stars in globular clusters and in the field, finding in both cases $\langle M_{\rm bol}(\rm RR)\rangle \propto 0.19$[Fe/H]. All methods now agree that, within the errors, $\langle M_V(\rm RR)\rangle \propto 0.15$[Fe/H] and $\langle M_{\rm bol}(\rm RR)\rangle \propto 0.19$[Fe/H]. There is no need to invoke an anticorrelation between helium and heavy-element abundances. If [O/Fe] = +0.3, the Galaxy's age may be as high as 20 ± 3 Gyr, and the most metal-poor clusters are on average older than the more metal-rich ones. If [O/Fe] ∝ −0.4[Fe/H], the Galaxy's age is about 14 ± 2 Gyr, and the cluster's ages do not correlate with metallicity. Regardless of the oxygen abundance, there is a significant range in ages among the intermediate-metallicity clusters. The spread correlates with horizontal branch morphology. The Oosterhoff classes are very distinct in a plot of RR Lyrae richness versus metallicity, suggesting that the division results from a sudden change in the way the horizontal branch is populated at [Fe/H] ≈ −1.7.

Subject headings: globular clusters: general — stars: fundamental parameters — stars: oscillations — stars: statistics — variables: other

1. INTRODUCTION

RR Lyrae variables have long been recognized as valuable distance indicators because of their ease of recognition and the relatively small dispersion in their intrinsic luminosities. They are found in essentially all old stellar systems, and consequently have been used to derive distances to globular clusters (see, for example, Zinn 1985) and the Galactic center (see the recent work by Oort & Plaut 1975; Walker & Mack 1986; Fernley et al. 1987). They have also been used to estimate the mass distribution in the Galaxy's halo (see Saha 1985; Suntzeff, Kinman, & Kraft 1991, hereafter SKK). RR Lyrae stars may further be used to estimate the distances to more remote stellar systems, including most of the other members of the Local Group (*LMC*: Graham 1977; Nemec, Hesser, & Ugarte 1985; Walker & Mack 1988b; Walker 1989, 1990; *SMC*: Walker & Mack 1988a; *NGC 147*: Saha, Hoessel, & Mossman 1990; *NGC 185*: Saha & Hoessel 1990; *M31*: Pritchet & van den Bergh 1987). See also the review by Pritchet (1989).

Distances are important for more than the spatial distributions of old populations. Perhaps the most crucial role is the estimation of ages. Globular cluster ages are determined most reliably from the comparison of model isochrones to cluster photometry in the region of the main-sequence turnoff. One key set of variables, the chemical composition, is amenable to observation, and model stellar evolution tracks may be computed for the proper abundance mixtures. Were cluster distances known precisely, the cluster ages would follow directly from the luminosity of the cluster main-sequence turnoffs. This has not, however, been the most common age estimation technique. Instead, isochrones' luminosities *and* temperatures have been used in comparison with clusters' color-magnitude diagrams. While this would seem better, matching in two variables rather than one, and while excellent matches have been obtained (see Hesser et al. 1987 and Richer & Fahlman 1987, for example), it has been accomplished by the use of additional observational free parameters and less certain physics. For example, main-sequence solar-mass stars' luminosities are computationally independent of surface boundary conditions and convection, whereas the radii, hence the effective temperatures, are very dependent upon these two uncertain physical problems. Additionally, one must worry about the conversion from effective temperature to color index, most often $B-V$, which itself depends on gravity and metallicity as well as temperature. Thus even were the chemical abundances all well determined, the isochrones involve some choice of α (the ratio of the convective mixing length to, typically, the photospheric pressure scale height) and color-temperature relations, plus either "gray atmosphere" or model atmosphere matches to the interior solutions to provide the surface boundary conditions. Ages derived from isochrone versus color-magnitude diagram matches thus require more and less certain physics than ages derived from turnoff luminosities alone. There is, of course, a benefit: one can see directly the remarkably good agreement between isochrones and color-magnitude diagrams and thus gain some confidence that the results appear to be correct, but we must remember this is accomplished by allowing for small color shifts in the isochrones and distance shifts for the clusters (thereby sacrificing one of the true observables to help match one of the free parameters), and choosing α on the basis of matching portions of the color-magnitude diagram that have lesser or greater sensitivity to

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convection, such as the turnoff and the red giant branch, respectively. How precise are such ages? Heasley & Christian (1986) have graphically shown that good matches between isochrones and color-magnitude diagrams can be obtained for any cluster by varying $\delta(B-V)$ (the isochrone color shift), the distance, and the age together, even ignoring the additional problems introduced by convection theory. In our view, this is not the most desirable procedure. Instead, distances should be determined to high precision, enabling good age estimates directly from the turnoff luminosities alone. *Then* we may test the other aspects of the model isochrones, but without using distance as a free parameter. This point has been made before, of course, most notably by Sandage (1981, 1982, 1990b) and Sandage & Cacciari (1990), as well as by the isochrone builders themselves. In this paper we address the use of RR Lyrae stars to measure cluster distances directly, and then use these results to estimate cluster ages.

Before we discuss the distance scale for RR Lyrae stars, it is worth recalling our astrophysical goals. To establish relative or absolute cluster ages to a precision of 10% using turnoff luminosities requires that relative or absolute distances have errors of less than 10%. The absolute ages set a lower limit to the age of the Galaxy for direct comparison with estimated ages of the universal expansion or the ages of the elements (see Cowan, Thielemann, & Truran 1991). The relative ages bear directly on the formation of the Galaxy, or at least its halo. If the halo evolved rapidly and homogeneously to form the Galactic disk, as summarized in the work of Eggen, Lynden-Bell, & Sandage (1962, hereafter ELS), the total age spread of the globular clusters should be less than about 0.2 Gyr, the free-fall time for the collapse of the proto-Galaxy. This is an age spread too small to be detected using current means. If the analyses by Isobe (1974) and Saio & Yoshii (1979) of the ELS data are correct, the time scale for such evolution might have been considerably longer, perhaps several Gyr, which we might be able to detect. It may be an even more complex situation, for based on their analyses of the metallicities and kinematics of metal-poor field stars, Norris (1986) and Carney, Latham, & Laird (1990) have argued that the halo evolved independently of the Galactic disk and that, as suggested earlier by Searle & Zinn (1978), the halo may have been assembled from a variety of independent fragments over a long period of time. In that case a large (>1 Gyr) age spread might be detectable, and it would not necessarily show a smooth age-metallicity relation. This issue is not settled (see, for example, Sandage & Fouts 1987), but there are already hints from the study of the globular clusters. There are cases where, by comparing a cluster's color-magnitude diagram with other clusters of the same metallicity, an age spread is indicated. These examples include the well-known differences between NGC 288 and NGC 362 (Bolte 1989; Green & Norris 1990; Sarajedini & Demarque 1990) and the clusters Palomar 12 (Gratton & Ortolani 1989; Stetson et al. 1989; VandenBerg, Bolte, & Stetson 1990), and Ruprecht 106 (Buonanno et al. 1990a). VandenBerg et al. (1990), in particular, have utilized such matches between 22 globular clusters in three metallicity regimes to argue that there are increasingly large age spreads as the mean metallicity increases.

The goal of this paper is to reevaluate the relations between absolute magnitudes of RR Lyraes and metallicity. We summarize in the next section the current situation regarding empirical or theoretical derivations of $\langle M_V(RR)\rangle$ versus [Fe/H]. There has not been good agreement between all the methods, and we highlight the problems. In the subsequent sections, we rederive the $\langle M_V(RR)\rangle$ versus [Fe/H] relation for RR Lyrae stars obtained by main-sequence fitting using model isochrones, correcting the previous analyses for metallicity-dependent distance errors. We also discuss the issue of RR Lyrae richness in clusters, noting that it is inappropriate to compare with equal weights a cluster with very few variables to one rich in variables. We then discuss the period-shift analyses presented over the past decade by Sandage and his collaborators. Our analyses differ, however, in that we utilize a more physically correct definition of the equilibrium temperature of RR Lyrae variables, which is the true independent variable of the analysis. We argue that all methods employed now show good agreement. We close by discussing the implications of such a conclusion and suggest future investigations.

2. CURRENT RESULTS

Qualitative evidence supporting a small range in absolute magnitudes for RR Lyrae stars has been offered by Barnes & Hawley (1986) and by Strugnell, Reid, & Murray (1986), based on their statistical parallax analyses. The uncertainties, due to small sample sizes, remain too large for any definitive conclusions. Walker & Mack (1986) argued that the observed range in apparent V-magnitudes of RR Lyrae stars toward Baade's Window ($l = 1°.0$; $b = -3°.9$) is too narrow to be reconciled with an $\langle M_V(\mathrm{RR})\rangle$ versus [Fe/H] slope as steep as that proposed by Sandage (1982). However, Walker & Terndrup (1991) have found the metallicity spread among these variables to be smaller than expected, so that this conclusion can no longer be rejected or supported. Clearly we require more quantitative results, which we summarize below.

2.1. *Baade-Wesselink Analyses*

In our previous paper (Jones et al. 1992 [Paper VII of this series], hereafter JCSL), we summarized the results from four different groups applying the Baade-Wesselink method to field RR Lyrae variables. After eliminating the highly reddened stars, V445 Oph and AR Per, and the two apparently highly evolved stars, DX Del and SS Leo, the following relations, based on 18 stars, were obtained:

$$\langle M_V(\mathrm{RR})\rangle = 0.16(\pm 0.03)[\mathrm{Fe/H}] + 1.02(\pm 0.03)\,, \quad (1)$$

$$\langle M_{\mathrm{bol}}(\mathrm{RR})\rangle = 0.21(\pm 0.03)[\mathrm{Fe/H}] + 1.04(\pm 0.03)\,, \quad (2)$$

$$\langle M_K(\mathrm{RR})\rangle = -2.33(\pm 0.20)\log(P) - 0.88(\pm 0.06)\,. \quad (3)$$

There was no clear evidence for any metallicity sensitivity in the last relation. The uncertainties quoted above reflect only those derived from the *internal* errors. They do not include errors that might arise from systematic effects, such as the zero point of the color index versus temperature relation and the projection factor p that converts measured radial velocities into pulsational velocities. Since the slopes of the above relations do not in principle depend on these systematic factors, we argue that the slopes and their errors given above are realistic, whereas the quoted uncertainties in the zero points of equations (1)–(3) are underestimates. JCSL estimated that these amount to about ±0.15 mag. The agreement of the slope of equation (3), derived for field stars at a variety of distances, with the slopes of apparent K-magnitudes within individual globular clusters determined by Longmore et al. (1990, hereafter UK90) is especially encouraging. UK90 commented that pulsation theory predicts a slope of −2.22, if there is no dependence of mass on metallicity. They noted further that their study of the variables in the cluster ω Cen (NGC 5139) showed a slope of -2.28 ± 0.07 with no evidence for any metallicity sensitivity, and a weighted result from a total of eight clusters of -2.23 ± 0.05. If we eliminate the two clusters with five or fewer variables, leaving six clusters each with 20 or more variables, the slope is -2.31 ± 0.06. All these results confirm our result, and imply that our relative distances are of good precision. Additional cluster data are highly desirable, of course.

The zero points of equations (1)–(3) must, as noted, have somewhat lower precision, but they, too, may be tested directly, using two different methods. First, Strugnell et al. (1986) used a statistical parallax analysis to obtain $\langle M_V(\mathrm{RR})\rangle = +0.75 \pm 0.2$ mag at a mean metallicity of $\langle[\mathrm{Fe/H}]\rangle = -1.35$. Equation (1) predicts $\langle M_V(\mathrm{RR})\rangle = +0.80$ mag at that metallicity. In a similar analysis, Barnes & Hawley (1986) derived $\langle M_V(\mathrm{RR})\rangle = +0.80 \pm 0.14$ mag at $\langle[\mathrm{Fe/H}]\rangle = -0.98$ (once their result is converted to the same reddening scale used by Strugnell et al. and in the Baade-Wesselink analyses). Equation (1) predicts $\langle M_V(\mathrm{RR})\rangle = +0.86$ mag. Second, there is one halo dwarf, HD 103095, with a trigonometric parallax precise enough so that its M_V value is determined to better than 0.07 mag (1 σ). HD 103095 also happens to be a single, cool (hence unevolved) star with essentially the same metallicity ([Fe/H] $= -1.44$; Carney et al. 1987) as the globular cluster M5 (NGC 5904), [Fe/H] $= -1.4$ (Zinn 1985) or -1.47 (Gratton & Sneden 1991). The excellent color-magnitude diagram of Richer & Fahlman (1987) may then be used to derive the cluster's distance from main-sequence fitting, and the $\langle M_V(\mathrm{RR})\rangle$ for its RR Lyrae stars may be obtained directly. The result is $\langle M_V(\mathrm{RR})\rangle = +0.86 \pm 0.12$ mag (Jones, Carney, & Latham 1988 [Paper VI in this series], hereafter JCL). This compares, again, very well with equation (1), which predicts $\langle M_V(\mathrm{RR})\rangle = +0.80$ mag.

Do the results for the field stars apply to variables within globular clusters? This is a major complication, which we address in detail later. The basic problem is that, in general, we expect to discover most field stars in their longest-lived evolutionary stage. Hence field RR Lyrae stars are, on average, at or near their zero-age horizontal-branch (ZAHB) location. Selection of globular clusters for study is not usually driven by such a consideration, however. Thus comparisons are often made between clusters like M3 (NGC 5272), which is very rich in RR Lyrae stars and so is likely to be representative of the field RR Lyrae stars at that metallicity, with clusters like 47 Tuc (NGC 104), which in spite of its large mass has only one confirmed RR Lyrae variable, V9 (Storm et al. 1992) and so may not be representative of high-metallicity RR Lyrae stars. To date, Baade-Wesselink analyses have been done for RR Lyrae stars in two globular clusters, and results for more clusters will be available shortly. (We will be reporting on two variables in M5, two in M92, and one in 47 Tuc.) Cohen & Gordon (1987) studied four RR Lyrae stars in M5, using *Bi* photometry. For reasons discussed in JCSL, we prefer to avoid all Baade-Wesselink analyses obtained that rely on short-wavelength (i.e., blue) magnitudes. Cohen is continuing her work on these variables and several in M92 using infrared photometry. Beck (1988, reported by JCL) analyzed V8 in M5, which is a cluster rich in RR Lyrae stars. She obtained $M_V = +0.86 \pm 0.16$ mag, in excellent agreement with equation (1) and the main-sequence fitting result. Liu & Janes (1990b) analyzed V2, V15, V32, and V33 in the intermediate-metallicity and RR Lyrae-rich cluster M4 (NGC 6121). They also found excellent agreement with equation (1), and especially impressive agreement with equation (3) on a star-by-star basis. (They are now working on variables in M15 and ω Cen.) These cluster results and the data that went into equations (1), (2), and (3) are plotted in Figures 1, 2, and 3. Solid dots are from JCL and JCSL, solid squares are from Liu & Janes (1990a), and the solid triangles are the two stars in common. Circles and open squares are the results for cluster variables. It appears that the field stars and the near ZAHB cluster variables are drawn from the same population, and that the Baade-Wesselink results for the field and cluster variables stars have good relative and absolute precision.

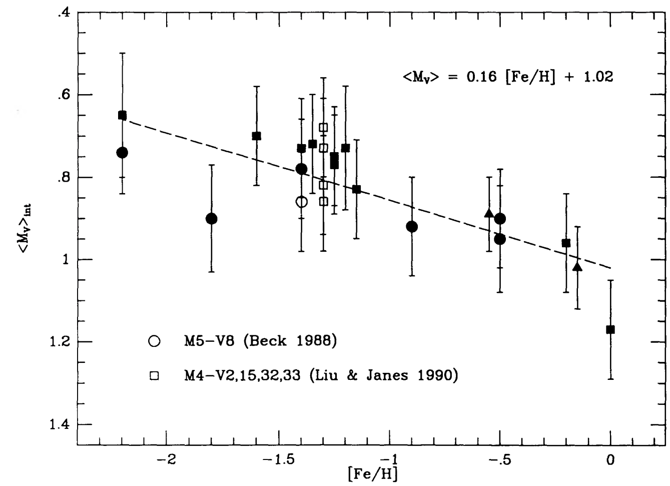

FIG. 1.—Baade-Wesselink M_V vs. [Fe/H] results for field and cluster RR Lyrae stars. Filled circles are from JCL; filled squares are from Liu & Janes (1990a); and filled triangles are the two stars in common.

There is one additional complication we must keep in mind. The results for the field stars obtained using the Baade-Wesselink method apply to stars that are on average slightly (albeit not significantly; see JCSL) evolved. But some of the other methods of analysis discussed below refer to unevolved, zero-age horizontal-branch (ZAHB) stars. There is a slight difference between mean and ZAHB absolute magnitudes. Sandage (1990a) has studied the vertical structure of the hori-

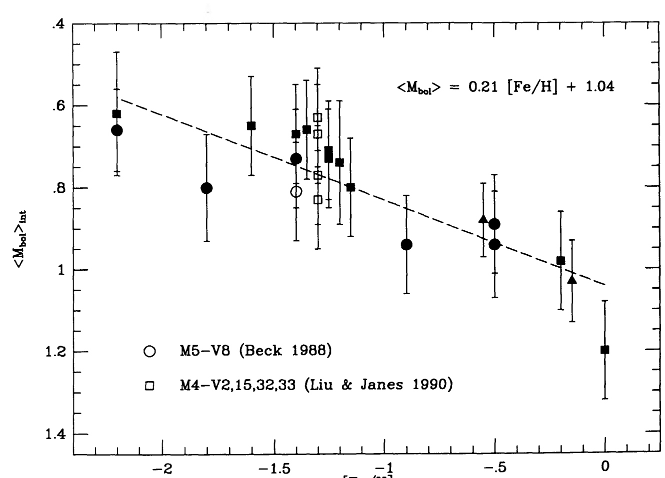

FIG. 2.—Same as Fig. 1, but for M_{bol} vs. [Fe/H]

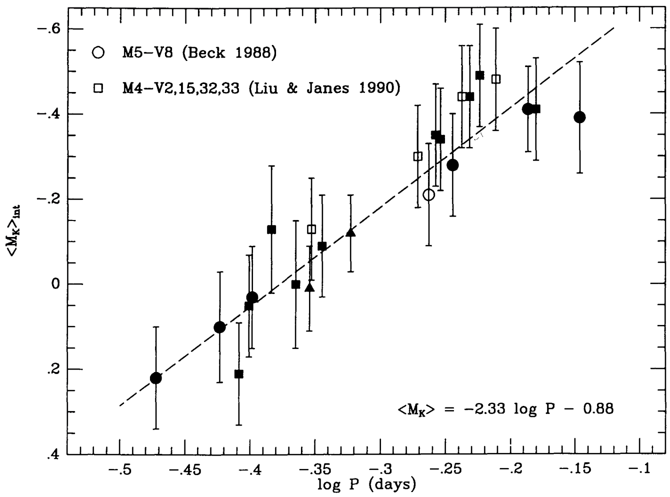

FIG. 3.—Same as Fig. 1, but for M_K vs. log P

zontal branch in considerable detail, finding, as predicted by theory, that metal-poor clusters are narrower in their absolute/apparent magnitude distribution than are metal-rich clusters. We wish to correct for this effect insofar as it is possible. In Table 13 of Sandage (1990a), data are presented for nine globular clusters, showing the distribution with respect to an adopted ZAHB. We have computed the difference in V-magnitude between the ZAHB and the median RR Lyrae variable. The results are plotted as a function of metallicity in Figure 4. (We give zero weight to the result for M4, since Sandage's ZAHB levels differ by 0.3 mag from that obtained by Buonanno, Corsi, & Fusi Pecci 1989, hereafter BCF. This difference probably reflects problems with differential reddening and absorption in this cluster.) A linear fit to the results for the eight clusters yields

$$\langle V \rangle = V_{\rm ZAHB} - 0.05[{\rm Fe/H}] - 0.20 . \quad (4)$$

Thus the slopes of equations (1) and (2) should be increased by 0.05 if a comparison is to be made with ZAHB predictions.

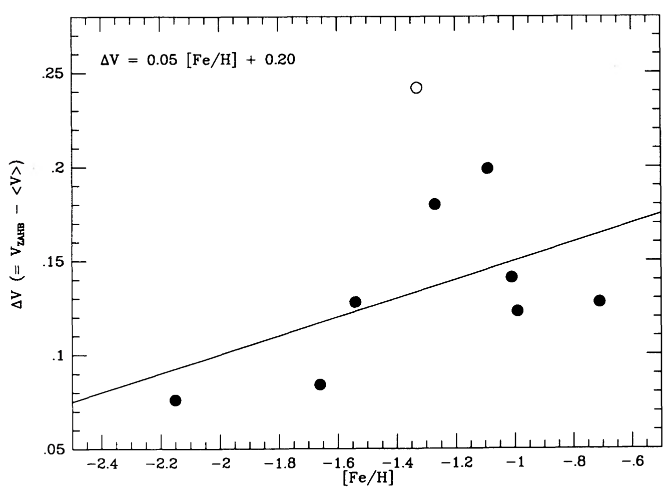

FIG. 4.—Difference between median RR Lyrae magnitude and the level of the ZAHB as a function of metallicity.

2.2. *Horizontal-Branch Theory*

Lee, Demarque, & Zinn (1990, hereafter LDZ) have published results of synthetic horizontal-branch models. Their models represent an improvement over previous work, in that they have a more realistic chemical composition and they follow the stars' evolutionary tracks well away from the ZAHB, so their results apply to the mean, not the ZAHB level. We discuss some of their results below, but we note here that their models predict

$$\langle M_V({\rm RR}) \rangle = +0.17[{\rm Fe/H}] + 0.82 , \quad (5)$$

and

$$\langle M_{\rm bol}({\rm RR}) \rangle = +0.20[{\rm Fe/H}] + 0.81 , \quad (6)$$

for $Y = 0.23$ during the main-sequence stage of evolution. The slopes of equations (5) and (6) agree very well with those of equations (1) and (2), but it is worth noting that the zero points differ by about 0.2 mag. This is outside the apparent range of error of equations (1) and (2), but given the internal and systematic errors in both the observations and the model calculations, the agreement is encouraging. As LDZ noted, a revision in the helium core masses toward lower values would bring their models into agreement with the Baade-Wesselink results. Lee (1990) has in fact redone the calculations and found that by lowering Y to 0.20 (0.22 on the horizontal branch),

$$\langle M_V({\rm RR}) \rangle = +0.19[{\rm Fe/H}] + 0.97 , \quad (7)$$

and

$$\langle M_{\rm bol}({\rm RR}) \rangle = +0.224[{\rm Fe/H}] + 1.00 , \quad (8)$$

when the clusters with very blue horizontal branches are excluded. Agreement with equations (1) and (2) is excellent in both slope and zero point, although the main-sequence helium abundance is somewhat lower than expected (Steigman et al. 1989).

2.3. *Red Giant Branch Theory*

Two uses have been made recently of model evolutionary tracks for red giant stars in order to determine relative distances to globular clusters with differing metallicities and thereby derive the slope (but not the zero point!) of the $\langle M_V({\rm RR}) \rangle$ versus [Fe/H] relation. In the first case, the prediction of the stellar luminosity at the time of core helium ignition is used in comparison with the observed apparent bolometric magnitude of the brightest red giant in a cluster of known chemical composition. This yields relative distance estimates for several clusters, so that $\langle V({\rm RR}) \rangle$ may be converted to $\langle M_V({\rm RR}) \rangle$. Alternatively, a relation between $\langle M_V({\rm RR}) \rangle$ and [Fe/H] may be tested by comparing the derived luminosity of the red giant branch tip with the model predictions. This technique was used originally with optical photometry, but the advent of infrared photometry made the comparison with the models much easier, since the bulk of the luminosity of red giants is emitted at wavelengths longward of 1 μm. Frogel, Persson, & Cohen (1981) began such work, and noted that the brightest star in each of 16 clusters agreed with the red giant evolution models of Sweigart & Gross (1978) to within 0.09 mag. We note that Frogel et al. assumed $M_V({\rm HB}) = +0.8$ mag for the most metal-rich clusters and $+0.6$ mag for the most metal-poor, similar to the results of equation (1).

VandenBerg & Durrell (1990) utilized the theoretical prediction that all clusters with similar metallicities would have similar M_{bol} (1st) (the bolometric magnitude of the brightest cluster member). Optical color-magnitude diagrams were used to shift clusters of equal metallicities to equal distances, then determine whether the main-sequence turnoff luminosities were the same. They found that all very metal-poor clusters seemed to have the same ages, whereas the most metal-rich clusters studied seemed to show larger scatter. The models were then used to infer no discernible mean age difference between the metal-poor and metal-rich clusters.

Da Costa & Armandroff (1990) have revisited this problem, pointing out the difficulties in unambiguously identifying the brightest cluster member, and the continuing problems of stellar effective temperature and bolometric corrections. The results from LDZ were used to estimate cluster distances to determine a (horizontal-branch theory) estimate of M_{bol} (1st) for each of eight clusters. The resultant prediction is that M_{bol} (1st) is propoportional to -0.20[Fe/H]. The "red giant branch RGB) theory" of Sweigart & Gross (1978) resulted in a slope of -0.23. The agreement between the two relative theoretical distance sets is quite good, although it should be noted that the two differ by 0.12 mag in their zero points. The conclusion is that the RGB theory supports the slope for $\langle M_V(\mathrm{RR})\rangle$ versus [Fe/H] obtained by LDZ, and that the derived $\langle M_V(\mathrm{RR})\rangle$–[Fe/H] slope is close to 0.17. It is not, however, clear from the text whether the derived slope refers to the main RR Lyrae level or to the ZAHB.

Recently, red giant evolution theory has been applied to the luminosity functions of red giant stars in globular clusters. A "bump" in the luminosity function is caused by the pause in a star's evolution as its hydrogen-burning shell passes through the discontinuity left behind by the retreating convective envelope. The predicted luminosity of this "bump" may be compared with observations of the giant branch luminosity functions. Relative distances may then be used to derive relative luminosities of the RR Lyrae stars. Based on results from 13 clusters (three of which were binned into one data point), Fusi Pecci et al. (1990) found that dM_V/d[Fe/H] probably lies between 0.15 and 0.20. The slope in this case refers to the ZAHB, so comparison with equation (1) translates into a slope of 0.10–0.15.

2.4. *Main-Sequence Fitting*

As noted already, and discussed at length by BCF, the "oberver's route to distances" that relies on trigonometric parallaxes of field stars does not yet have enough high-precision data to derive adequate relative cluster distances, and an absolute distance may be obtained only for those clusters whose abundances match those of the halo dwarf HD 103095. (See Laird, Carney, & Latham 1988 for a summary of halo dwarf parallax data.) Instead, we must rely upon the "theoretician's route," in which one establishes the relative cluster distances by comparing main-sequence photometry and model isochrones. Recent work has resulted in rather different slopes for the M_V–[Fe/H] relation for HB stars. BCF found that their homogeneous sample of 19 globular clusters and the VandenBerg & Bell (1985, hereafter VB85) model isochrones were consistent with a slope of 0.37, but with a large uncertainty (± 0.14). King, Demarque, & Green (1988, hereafter KDG) used a slightly different set of clusters, a different set of model isochrones (the Revised Yale Isochrones; Green, Demarque, & King 1987), and a slightly different method of analysis, and obtained a slope of 0.20. The discordant results have been revisited by Buonanno et al. (1990b, hereafter BCCF). The primary cause of the difference was found to be the reliance by BCF solely on relatively unevolved main-sequence stars, and the use by KDG of turnoff and subgiant stars. In our opinion, the former is the safer approach, relying upon a part of the color-magnitude diagram that is insensitive to age to derive the parameters that result in an age estimate. The new analysis by BCCF averaged over the two different sets of clusters and the two sets of isochrones, but relied only on these relatively unevolved stars, and concluded that

$$M_V = +0.39[\mathrm{Fe/H}] + 1.32 . \quad (9)$$

This result actually refers to the ZAHB level, since BCF were careful to determine these magnitudes in the clusters they studied. This steep slope, which has an "informal" uncertainty of $+0.14$, is much steeper than that obtained using any of the previously discussed methods. As discussed by BCCF, it does not appear to be sensitive to errors in either the helium or CNO abundances. We discuss this method in § 3 below, noting that a subtle but significant error is present in the predicted isochrone colors and that the error is a function of metallicity.

2.5. *The Period-Shift Analyses*

In an extensive series of papers, Sandage and his collaborators (Sandage, Katem, & Sandage 1981; Sandage 1981, 1982, 1990a, b; Sandage & Cacciari 1990) have pioneered the use of pulsation theory to derive relative luminosities of RR Lyrae variables in globular clusters and in a sample of field RR Lyrae stars studied by Lub (1977, 1987). The basis for the relative distances comes from equations (2) and (3) of van Albada & Baker (1971, hereafter vAB71):

$$\log(P_0) = -1.772 - 0.68 \log\left(\frac{M}{M_\odot}\right) + 0.84 \log\left(\frac{L}{L_\odot}\right) + 3.48 \log\left(\frac{6500}{T_{\mathrm{eff}}}\right), \quad (10)$$

$$\log\left(\frac{P_0}{P_1}\right) = 0.095 - 0.032 \log\left(\frac{M}{M_\odot}\right) + 0.014 \log\left(\frac{L}{L_\odot}\right) + 0.09 \log\left(\frac{6500}{T_{\mathrm{eff}}}\right). \quad (11)$$

Here P_0 is the fundamental period and P_1 is the first-overtone period. Ignoring for the moment possible differences in mass, the essence of the period-shift analysis is to adopt a fiducial set of equal-composition RR Lyrae stars, such as those in the intermediate-metallicity cluster M3, then compare variables in other clusters or in the field to those in M3 at equal temperatures. Differences in log P should then reflect differences in log L. From such analyses, Sandage and his collaborators have derived relations between $\langle M_V(\mathrm{RR})\rangle$ and [Fe/H], using clusters and field stars of widely differing metallicities. Additional information regarding the masses of RR Lyrae stars as a function of metallicity may then be included to derive the final relationship.

The observables that relate most directly to the stellar temperature are the heart of the issue. In the early work, Sandage et al. (1981) showed that M3 and the metal-poor cluster M15 (NGC 7078) followed different period-amplitude and period–rise time relations. Insofar as the blue amplitude, $A(B)$, or the rise time (from minimum to maximum light), $\Delta\phi_{\mathrm{rise}}$, is directly related to temperature, such data may be used to infer differences in luminosities of the RR Lyrae stars in the two clusters, hence differences in their relative distances, hence differences in their main-sequence turnoff luminosities, and hence differences in their ages. Use was also made of mean $B - V$ colors of the variables, corrected for both reddening and crowding effects within the clusters. To convert from such a color index into a temperature, one must decide whether to utilize the magnitude-averaged, $(B - V)_{\rm mag}$, or the intensity-averaged, $\langle B \rangle - \langle V \rangle$, color indices. Sandage et al. (1981) discussed this at length, opting for the latter. The conversion from color index and metallicity into effective temperature was accomplished using synthetic flux spectra computed by R. A. Bell (quoted by Butler, Dickens, & Epps 1978). In the recent work, Sandage (1990a, b) used this same color index–metallicity–temperature calibration, but returned to the use of $(B - V)_{\rm mag}$. The differences in the final results are minor, and he concluded from an analysis of cluster of variables and field variables from Lub (1977) that the slope of the $\langle M_V({\rm RR}) \rangle$–[Fe/H] relation is 0.37, consistent with the main-sequence fitting results of BCF. He did point out (Sandage 1990b) that the true relation remains uncertain, given the wide variation in the slopes determined from the different methods, and that an important datum is the mass–metallicity relation for the RR Lyrae stars, statements with which we agree fully.

In § 4 below, we revisit the period-shift problem, using cluster and field variables, but with one major modification. We define the temperature scale rather differently, and then investigate the consequences. As we discuss below, the temperature scale we adopt results in significant changes in the period-shift analysis.

Sandage's (1990b) results for the cluster variables refer to the ZAHB, whereas the field stars must obviously refer to the mean magnitude level.

2.6. M_K *versus* log P

UK90 used the periods and $\langle K \rangle$ magnitudes for RR Lyrae stars in eight globular clusters to derive relative distances from a variant of equation (3). They were then able to easily derive relative $\langle M_V({\rm RR}) \rangle$ values. They found a slope of 0.32 for the $\langle M_V({\rm RR}) \rangle$–[Fe/H] relation, unless, as they and Liu & Janes (1990a, b) argued, the ratio of visual to selective absorption, $R = A_V/E(B - V)$, was equal to 3.8 for M4 and NGC 6171, both of which are near the Sco-Oph cloud complex, in which case they found that the slope would drop to 0.16. An unpublished renalysis (R. Dixon, 1991, private communication) done using realistic errors in both $\langle M_V({\rm RR}) \rangle$ and [Fe/H] has revealed a slope of 0.19 ± 0.12. Although this method has great promise, we do not yet attach great significance to their results, for three reasons. First, there is the ambiguity of the absorption corrections for the two clusters that wholly define the metal-rich end of the relation. Second, if we substitute the Zinn (1985) metallicity scale in place of the BCF scale used by UK90, the original slope of 0.32 drops to 0.23. The revised slope is less affected. Finally, there is the matter of cluster richness, mentioned earlier and discussed more fully in the next section. More work with this method is needed, especially for unobscured RR Lyrae–rich clusters that are metal-poor (such as NGC 4590) and metal-rich (such as NGC 6723).

3. MAIN-SEQUENCE FITTING

As mentioned already, BCF utilized the model isochrones of VB85 to derive the relative distances to 19 globular clusters, using NGC 7099 as the fiducial cluster. The true distance of NGC 7099 is uncertain, so all the cluster distances are relative, not absolute. However, as they discussed thoroughly, the results are not definitive, owing to the lack of high-precision parallax data for metal-poor dwarfs, and to lingering uncertainties in the $B - V$ color indices in the isochrones. We cannot solve the first problem, but we discuss the latter herein in detail. We point out also that comparisons of cluster and field star results cannot be done on the basis that one cluster equals one field star, since clusters cover a wide range in richness in RR Lyrae stars.

3.1. *Isochrone Color Indices*

As VB85 point out, there is some concern about the zero points of the predicted $B - V$ color indices. The problem lies in part with the model atmospheres used to compute the emergent flux (Kurucz 1979), which predict too blue a $B - V$ value for the Sun. While a simple zero-point shift may be used, it is not clear that it is appropriate at all metallicities. This is because the model atmospheres probably include too little line opacity at the shorter wavelengths to match the Sun well, but, as the metallicity decreases, the effect of this missing opacity diminishes. A possible result of this effect may be the frequent requirement of additional color shifts for the isochrones when they are compared with high-precision globular cluster color-magnitude diagrams (e.g., Hesser et al. 1987; Richer & Fahlman 1987).

We test here the possibility of a metallicity-dependent color error in the VB85 isochrones. We approach this problem by identifying a set of field stars whose metallicities overlap those of the globular clusters studied by BCF. We estimate $T_{\rm eff}$ for each star using a metallicity-insensitive parameter. The basis for our temperature determinations is the slope of the Paschen continuum. Carney (1983) reported results of emergent flux calculations obtained using Kurucz's code ATLAS6, and the comparison with 100 Å resolution spectrophotometry. The Paschen continuum slope is insensitive to either metallicity or convection, but the small effects of metallicity were taken into account on a star-by-star basis. The results of such temperature determinations were then compared with existing broad-band photometry, and it was found that the color index least affected by metallicity and gravity and most sensitive to temperature for F, G, and early K stars is $V - K$. We thus use here either the spectrophotometric data or $V - K$ photometry for each star to estimate its temperature. With $T_{\rm eff}$ known, then if the metallicities of these stars are also known, we may use the VB85 isochrones directly to *predict* a $B - V$ color index, which may be compared to the *observed* values. We use as our data set the recent survey of over 900 proper-motion stars carried out by Carney, Latham, & Laird. Laird et al. (1988) have published photometry, temperatures, and metallicities with good precision for over 700 field stars, most of which are dwarfs. They have, in addition, completed a study of an additional 500 stars, selected mostly from the similar program undertaken by Sandage & Fouts (1987). We have taken a subset of the full sample of over 1450 proper-motion stars. The stars used here were chosen to be only those with metallicities derived from low signal-to-noise but high-resolution echelle spectra, following the methods discussed by Carney et al. (1987). The estimated precision is about 0.13 dex per star. We have used only those stars for which the reddening is known to be small, $E(B - V) \leq 0.05$ mag. We have eliminated all stars that are double-lined, evolved, or have any uncertainties in their photometry, proper motions, or velocities. Since the VB85 isochrones have [Fe/H] ≤ -0.45, we similarly restrict our sample of field stars to these lower metallicities. We have as a result a sample of 355 stars. Since BCF restricted their main-sequence fitting to stars with $M_V \geq +5.0$ and $M_V \geq +5.5$ when possible, we consider similar limits on our sample. (This also helps to eliminate evolved stars. In the full sample, it is estimated that the subgiant/giant contamination in the proper-motion sample is about 10% [Laird et al. 1988], but most of these stars are the hotter ones, where the differences in absolute magnitude between main-sequence and subgiant stars at a fixed temperature are not too great. Thus, in a magnitude-limited survey, the hotter, evolved, more luminous subgiant stars are not too much more distant than main-sequence stars of the same color indices. Hence subgiants are found among stars of bluer color indices, although in diminished numbers relative to main-sequence stars, in a proper-motion–limited sample. For main-sequence stars with $M_V \geq 5.0$, however, the evolved stars at comparable color indices are giants and very much brighter, and hence in general much more distant. For these stars to be included in a proper-motion catalog thus requires extreme values of the tangential velocity. Hence restriction to such faint main-sequence stars helps eliminate the evolved stars very effectively. This is important, since $B-V$ is weakly dependent on gravity.) We find that the difference between the $B-V$ value *predicted* by the VB85 isochrones for these stars using the spectroscopic metallicity and the metallicity-insensitive $T_{\rm eff}$ value and that *observed* for each star is clearly a function of metallicity. We did the analyses for both helium abundances available in the VB85 isochrones, $Y = 0.2$ and 0.3, and for samples restricted to $M_V \geq +5.0$ and $+5.5$, and $[m/H] \leq -0.45$. We show a typical result in Figure 5, where we plot the difference, $\Delta(B-V) = (B-V)_{\rm predicted} - (B-V)_{\rm observed}$, against [m/H] for $Y = 0.2$ and $M_V \geq +5.0$, including the 235 data points and the linear least-squares fit

$$\Delta(B - V) = -0.034[m/H] - 0.055 . \quad (12)$$

The uncertainties are $+0.003$ in the slope and $+0.004$ in the intercept, and the correlation coefficient is -0.56. The rms scatter is 0.029 mag, consistent with the combined errors in the derived temperatures and metallicities and in the BV photometry. If we adopt, following BCF, $Y = 0.23$, and average the results from the two M_V limits, we find that the slope of the relation is, again, about -0.034 mag dex^{-1}. The sense of this result is consistent with increasing missing line opacity in the blue spectral region as the metallicity increases.

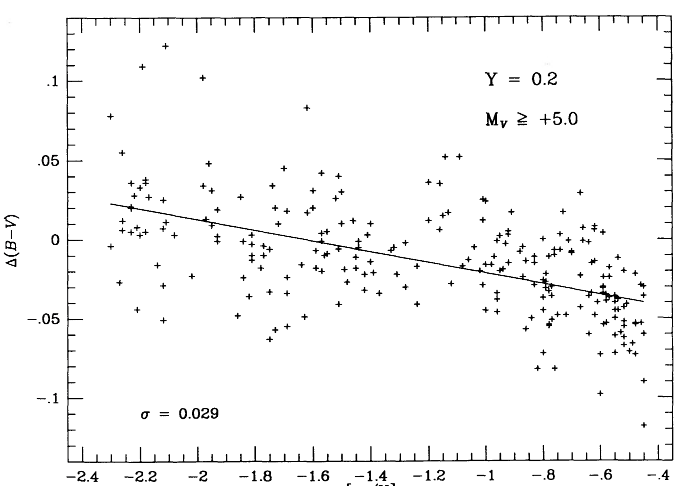

FIG. 5.—$\Delta(B-V)$ vs. [Fe/H] for $Y = 0.2$, $M_V \geq +5.0$

How does this affect the derived relative distances? Consider the BCF methodology, which uses the isochrones to determine the relative distances of clusters compared with the metal-poor cluster NGC 7099 ([Fe/H] = −2.13). They found that using the VB85 isochrones, a metal-rich cluster such as 47 Tuc ([Fe/H] = 0.71) has a horizontal branch intrinsically fainter than that of NGC 7099 by 0.47 mag. Using all 19 clusters resulted in a steep $\langle M_V({\rm RR})\rangle$–[Fe/H] relation, specifically a slope of about 0.37, the more metal-rich variables being much fainter than the metal-poor ones. All else being equal, a metal-rich isochrone of unevolved stars is brighter than a metal-poor isochrone at equal $B-V$ color indices. The metallicity-dependent color error makes the metal-rich model isochrones even bluer than they should be. Thus the correction for this effect, which moves the metal-rich isochrones more to the red than the metal-poor ones, seems to make the entire metal-rich main sequence *even brighter at a fixed color index*. Thus the correct inferred absolute magnitudes for the RR Lyrae stars in metal-rich clusters become brighter than estimated by BCF, and the slope of the $\langle M_V({\rm RR})\rangle$–[Fe/H] relation becomes shallower. Since the slope of the M_V–$(B-V)$ main sequence is quite steep (>5 in the relevant cases), an error of over $5 \times 0.034 \times (2.13 - 0.71)$, or 0.24 mag, results. Thus, 47 Tuc's horizontal branch is more than 0.24 mag brighter with respect to that of NGC 7099 than claimed by BCF. Correction for this systematic color effect in the VB85 isochrones reduces the slope of the $\langle M_V({\rm RR})\rangle$–[Fe/H] relation by a significant amount. We therefore repeat the BCF analyses in Table 1, where we give for each of the 19 clusters the metallicity adopted by BCF, their derived M_V(HB) (but with an uncertain zero-point offset for all clusters, recall), our estimate of the error in $B-V$ in the VB85 isochrones for such a metallicity compared with that of NGC 7099, the slope of the unevolved main-sequence M_V–$(B-V)$ relation at this metallicity, using the VB85 isochrones and $Y = 0.23$, the effect this has on the

TABLE 1

CORRECTIONS TO MAIN-SEQUENCE FITTING

Cluster (1)	[Fe/H] (2)	M_V[a] (3)	$\Delta(B-V)$ (4)	S (5)	ΔM_V (6)	$M_{V,{\rm rev}}$[a] (7)
NGC 104	−0.71	0.89	0.048	5.60	0.27	0.62
NGC 288	−1.40	0.74	0.025	5.22	0.13	0.61
NGC 362	−1.27	0.85	0.029	5.25	0.15	0.70
NGC 2808	−1.37	0.77	0.026	5.24	0.14	0.63
NGC 4590	−2.09	0.48	0.001	5.14	0.01	0.47
NGC 5139	−1.59	0.65	0.018	5.21	0.10	0.55
NGC 5272	−1.66	0.71	0.016	5.20	0.08	0.63
Pal 5	−1.27	0.76	0.029	5.25	0.15	0.61
NGC 5904	−1.40	0.79	0.034	5.31	0.18	0.61
NGC 6121	−1.28	0.77	0.029	5.25	0.15	0.62
NGC 6171	−0.85	1.02	0.044	5.50	0.24	0.78
NGC 6205	−1.65	0.71	0.016	5.20	0.08	0.63
NGC 6341	−2.24	0.47	−0.004	5.13	−0.02	0.49
NGC 6397	−1.91	0.47	0.007	5.16	0.04	0.43
NGC 6752	−1.54	0.57	0.020	5.22	0.10	0.47
NGC 6809	−1.82	0.50	0.011	5.18	0.05	0.45
NGC 7078	−2.15	0.46	−0.001	5.14	0.00	0.46
NGC 7099	−2.13	0.42	...	5.24	...	0.42
NGC 7492	−1.34	0.74	0.042	5.24	0.14	0.60

[a] Derived *assuming* $M_V = +0.42$ for NGC 7099.

derived horizontal-branch absolute magnitudes, ΔM_V, and, in the last column, the corrected horizontal-branch absolute magnitudes. (A reminder: The zero-point offset is uncertain here.) A linear least-squares fit reveals a slope of 0.16 for all 19 clusters, and 0.15 if we exclude the highly reddened (and uncertain absorption-to-reddening ratio) clusters M4 and NGC 6171. The BCF results indicated slopes of 0.34 and 0.37, respectively. The slopes refer to the ZAHB level (BCF), so to transform to the mean horizontal-branch levels, we must decrease the slope by about 0.05, so that $\Delta\langle M_V(\mathrm{RR})\rangle \propto 0.11[\mathrm{Fe/H}]$. The correction for the metallicity-dependent errors in the $B-V$ color indices in the VB85 isochrones thus brings the slope in the $\langle M_V(\mathrm{RR})\rangle$–[Fe/H] relation down from its high value to one even shallower than that predicted by equation (1).

3.2. *Cluster Richness Effects*

There is an additional subtle effect often overlooked in comparisons between results from field and cluster RR Lyrae stars: the role of RR Lyrae richness in the latter. In general, a sample of field RR Lyrae stars is an unbiased sample of such objects. However, the selection of which globular cluster to study is not usually made without such a bias, but rather is made on the basis of a cluster's proximity, or low reddening, or one of many other variables. Put simply, the question is: If the globular clusters in the sample we have studied were dissolved, would their RR Lyrae stars be representative of the field RR Lyrae stars? Even excluding the absence of short-period metal-rich RR Lyrae stars in globular clusters which are common in the field, one also asks: Should a cluster such as M13 (NGC 6205), which has very few RR Lyrae stars, be given equal weight with M3, which has very many, even though the two clusters' metallicities are essentially identical? In our opinion, any result from a study of globular cluster RR Lyrae stars should be weighted by each cluster's RR Lyrae richness before a comparison may be reasonably made with unbiased samples of field stars. In this section we therefore evaluate an RR Lyrae richness index, then reevaluate the cluster main-sequence fitting results.

We confine our analyses to those Oosterhoff clusters discussed by Sandage (1990b), since one of the continuing problems in the study of RR Lyrae stars is the cause of the Oosterhoff dichotomy. We see two simple methods by which we may define an RR Lyrae richness index. First, there is a parameter which measures the number of RR Lyrae variables, V, compared with the number of blue horizontal-branch stars, B, and the number of red horizontal-branch stars, R. Lee, Demarque, & Zinn (1992) have summarized the results of this ratio, $V/(B+V+R)$, for many clusters which have color-magnitude diagrams deep enough to sample fully the horizontal-branch distribution, and for clusters where field contamination has been either estimated directly or claimed to be negligible. To their data we have added results for NGC 5824 (Cannon, Sagar, & Hawkins 1990) and NGC 6229 (Carney, Fulton, & Trammell 1991). We have also used the extensive results of Woolley (1966) for ω Cen to estimate the value for the most interesting cluster. Table 2 gives the results for 31 clusters which have at least three RR*ab* stars with known periods so that $\langle P(\mathrm{RR}ab)\rangle$ may be computed and an Oosterhoff class assigned to each cluster. The first richness index, $V/(B+V+R)$, is given in column (2), and its (Poisson statistics only) error is given in column (3). One fascinating result that highlights the importance of any richness index is that for ω Cen. This cluster has the second largest total number of RR Lyrae stars and so is often taken to be representative of the Galactic halo, but in fact its $V(B+V+R)$ value is quite low, 0.093 ± 0.006, compared with, say M3, with 0.420 ± 0.051, in spite of the two clusters having very similar mean metallicities. Were the field RR Lyrae stars formed out of equal masses of ω Cen–like and M3–like clusters, over 80% of the field would be from a cluster like M3 rather than one like ω Cen.

A second possible index is the "specific frequency," or number of RR Lyrae stars per unit luminosity. This is a measure formulated by Kukarkin (1973), who chose to normalize with respect to $M_V = -7.5$ mag, which is near the peak of the globular cluster luminosity function. Pritchet & van den Bergh (1987) have defined a similar specific frequency, ν_{RR}, which is the number of variables per $M_V = -10$ luminosity. It is obviously not important which definition we take, and we choose to follow Pritchet & van den Bergh, since one of the interesting questions yet to be studied thoroughly is the possible differences in RR Lyrae populations in other galaxies versus those in our own Galaxy. In columns (4) and (5) of Table 2 we give ν_{RR} and its estimated error, again assuming $\sqrt{N}$ statistics. Column (6) gives the *total* (RR*ab* + RR*c*) number of RR Lyrae stars in each cluster, taken from SKK, and column (7) gives the cluster's absolute magnitude, obtained from available apparent magnitudes and reddenings and distances estimated using equation (1).

These two richness indicators, $V/(B+V+R)$ and ν_{RR}, are reasonably well correlated (see Fig. 6), and it is not clear which is preferable. We have therefore chosen to combine them. We first, however, must normalize them, which we do by dividing the value for each index for each cluster by the maximum value for each index observed in this sample. Thus each value of $V/(B+V+R)$ is divided by 0.420 ± 0.051, and each ν_{RR} value is divided by 1136 ± 159. The final richness index, or weighting factor, W_{RR}, is then formed by the weighted average of these two normalized indices. This value, and its error, are given in

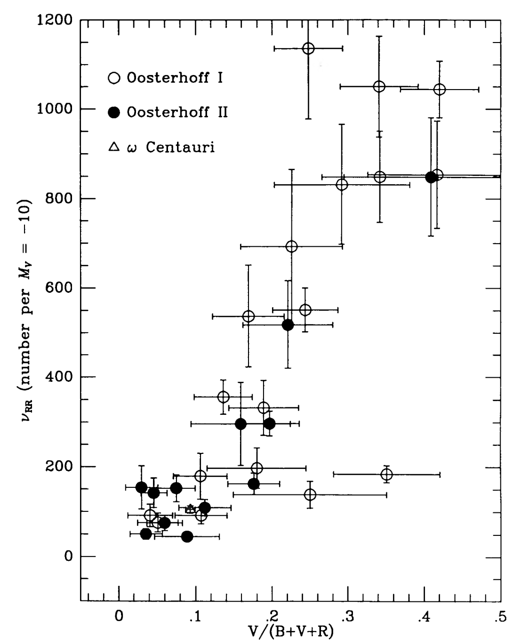

FIG. 6.—Richness of RR Lyrae stars along the horizontal branch, $V/(B+V+R)$, vs. the frequency of RR Lyrae stars per unit $M_V = -10$ luminosity, ν_{RR}.

TABLE 2

CLUSTER RR LYRAE RICHNESS VALUES

Cluster (1)	V/(B+V+R) (2)	σ (3)	ν_{RR} (4)	σ (5)	N (6)	M_V (7)	[Fe/H] (8)	W_{RR} (9)	σ (10)	$<P_{ab}>$ (11)	Oo (12)
NGC 362	0.051	0.026	76	21	13	-8.07	-1.27	0.073	0.020	0.542	I
NGC 1261	0.180	0.065	197	45	19	-7.46	-1.29	0.193	0.045	0.563	I
NGC 1851	0.107	0.034	92	19	22	-8.44	-1.33	0.091	0.020	0.573	I
NGC 2419	0.112	0.034	109	18	37	-8.82	-2.10	0.106	0.020	0.650	II
NGC 3201	0.341	0.051	1050	113	85	-7.27	-1.56	0.865	0.113	0.558	I
NGC 4147	0.226	0.067	692	173	16	-5.91	-1.80	0.573	0.123	0.525	I
NGC 4590 (M68)	0.409	0.114	848	132	41	-6.71	-2.09	0.796	0.138	0.625	II
NGC 4833	0.046	0.017	142	33	18	-7.75	-1.86	0.119	0.027	0.684	II
NGC 5024 (M53)	0.176	0.034	162	24	44	-8.58	-2.04	0.167	0.028	0.633	II
NGC 5053	0.159	0.065	296	93	10	-6.32	-2.58	0.289	0.079	0.672	II
NGC 5139 (ω Cen)	0.093	0.006	105	8	152	-10.40	-1.59	0.118	0.013	0.653	II
NGC 5272 (M3)	0.420	0.051	1044	64	260	-8.49	-1.66	0.951	0.109	0.551	I
NGC 5466	0.221	0.059	518	98	28	-6.83	-2.22	0.479	0.088	0.637	II
NGC 5824	0.089	0.042	45	8	27	-9.44	-1.87	0.041	0.009	0.624	II
NGC 5904 (M5)	0.244	0.043	551	49	123	-8.37	-1.40	0.514	0.068	0.547	I
NGC 6121 (M4)	0.248	0.045	1136	159	51	-6.63	-1.28	0.712	0.108	0.538	I
NGC 6171 (M107)	0.169	0.047	537	114	22	-6.53	-0.99	0.438	0.086	0.527	I
NGC 6229	0.250	0.101	138	30	20	-7.90	-1.40	0.129	0.032	0.527	I
NGC 6266 (M62)	0.136	0.038	356	38	87	-8.47	-1.29	0.316	0.048	0.544	I
NGC 6341 (M92)	0.075	0.024	152	30	25	-8.04	-2.24	0.144	0.029	0.626	II
NGC 6402 (M14)	0.351	0.070	184	19	86	-9.17	-1.39	0.177	0.028	0.564	I
NGC 6626 (M28)	0.041	0.029	92	25	13	-7.87	-1.44	0.083	0.024	0.565	I
NGC 6656 (M22)	0.060	0.023	75	17	18	-8.44	-1.75	0.074	0.017	0.651	II
NGC 6712	0.106	0.035	179	51	12	-7.06	-1.01	0.181	0.044	0.557	I
NGC 6723	0.189	0.046	332	61	29	-7.35	-1.09	0.330	0.059	0.540	I
NGC 6934	0.417	0.091	853	120	50	-6.92	-1.54	0.815	0.128	0.545	I
NGC 6981 (M72)	0.292	0.089	831	134	38	-6.65	-1.54	0.720	0.129	0.552	I
NGC 7006	0.342	0.076	848	102	68	-7.26	-1.59	0.767	0.115	0.567	I
NGC 7078 (M15)	0.197	0.039	297	28	112	-8.94	-2.15	0.291	0.041	0.640	II
NGC 7089	0.036	0.021	51	12	17	-8.79	-1.62	0.048	0.012	0.636	II
NGC 7099	0.030	0.021	154	48	10	-7.03	-2.13	0.106	0.034	0.698	II

columns (9) and (10), followed by the mean periods of the RR*ab* variables in each cluster (taken from Table 1 of Sandage 1990b) and the Oosterhoff class assignment.

The BCF main-sequence fitting results, corrected for the VB85 isochrones' color errors, may now be reanalyzed. First we add, in Table 3, the richness index data, defined as above, for the seven BCF clusters not included in Table 2. These clusters generally lack RR Lyrae stars, and thus cannot be assigned to an Oosterhoff class, and they consequently carry little weight in the cluster versus field variables comparisons. When all the W_{RR} values are used in addition to the M_V results from Table 1, the new slopes for the M_V–[Fe/H] relations are

TABLE 3

ADDITIONAL CLUSTER RR LYRAE RICHNESS DATA

Cluster (1)	$V/(B+V+R)$ (2)	σ (3)	ν_{RR} (4)	σ (5)	N (6)	M_V (7)	[Fe/H] (8)	W_{RR} (9)	σ (10)
NGC 104 (47 Tuc)	0.013	0.013	2	2	1	−8.92	−0.71	0.003	0.002
NGC 288	0.010	0.010	26	26	1	−6.44	−1.40	0.024	0.017
NGC 2808	0.036	0.026	4	3	2	−9.17	−1.37	0.004	0.003
Pal 5	0.200	0.110	257	128	4	−5.48	−1.47	0.266	0.108
NGC 6205 (M13)	0.027	0.016	22	10	5	−8.35	−1.65	0.023	0.009
NGC 6809 (M55)	0.029	0.017	106	33	10	−7.43	−1.82	0.084	0.026
NGC 7492	0.034	0.034	138	138	1	−4.65	−1.51	0.093	0.068

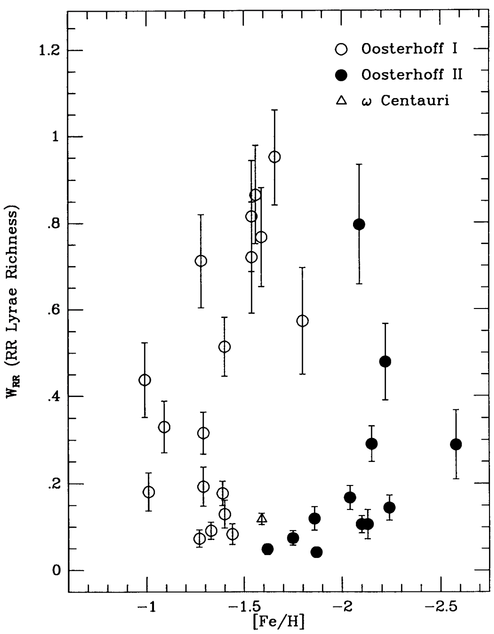

FIG. 7.—Combined RR Lyrae richness, W_{RR}, vs. metallicity for a sample of Oosterhoff I and II globular clusters.

0.156 and 0.187, respectively, excluding and including both NGC 6121 and NGC 6171. The correlation coefficients are 0.86 and 0.79, respectively. We adopt an average value of 0.17, decreasing to 0.12, as before, to correct the ZAHB results to the mean horizontal-branch level.

The results contained in Table 2 merit additional attention. In Figure 7 we show the combined weighting factor, W_{RR}, plotted against metallicity, with separate symbols for the Oosterhoff I and II cluster (and for the unusual cluster ω Cen). It appears that the transition from Oosterhoff I clusters into Oosterhoff II clusters is not smooth. The former increase in richness of RR Lyrae stars as the metallicity decreases, maintaining mean fundamental periods of around 0.55 days. There is then a sudden transition in the vicinity of [Fe/H] $= -1.6$ to -1.8. In the overlap regions, the Oosterhoff I clusters are rich in RR Lyrae stars, whereas the Oosterhoff II clusters are very deficient. In our opinion, this suggests a dramatic difference in the evolutionary status of the variables in these two types of clusters. Most of the lifetime of a horizontal-branch star is spent near the ZAHB. Thus clusters very rich in RR Lyrae stars probably have most of their RR Lyrae stars in or near the ZAHB. Later horizontal-branch evolution is rapid, so the RR Lyrae stars in clusters with low richness indices are probably few in number because they are highly evolved. Thus we suggest that the very low frequency of RR Lyrae stars in the more metal-rich of the Oosterhoff II clusters is due to their being predominantly evolved stars. This would also explain why their periods are longer than those of their Oosterhoff I counterparts. Note also that as metallicity declines further, the RR Lyrae richness increases again in the Oosterhoff II clusters, suggesting a shift back toward the red (or perhaps simply less blue) side of the horizontal branch. Lee (1990) and Sandage (1990a) have discussed this point at length as well, on the basis of the observed horizontal-branch colors/morphologies at different metallicities. Model evolution tracks, such as those of LDZ, account for this behavior only in part.

4. PERIOD-SHIFT ANALYSES

4.1. *Introduction*

Probably the most compelling argument in favor of the steeper absolute magnitude versus metallicity relations for RR Lyrae stars is the period-shift analysis discussed by Sandage (1990a, b and references therein). The apparently unavoidable consequence of the steep slope, however, is that the helium mass fraction is inversely correlated with metallicity. This surprising result has spurred much theoretical work on pulsation theory and horizontal-branch evolution, but, as Sandage has noted in his reviews of the problem, the consequence is apparently unavoidable. This anticorrelation is difficult to accept, for three reasons. First, it cannot be understood on the basis of simple chemical evolution. On the contrary, normal stellar evolution and ejection of material into the interstellar medium should enrich it in both helium and heavier elements. Second, the anticorrelation conflicts with the limited data we have that may be used to estimate helium abundances in globular clusters. Caputo, Martinez-Roger, & Paez (1987) have summarized the recent work on the so-called *R* method, in which numbers of horizontal-branch stars are compared with the numbers of first-ascent red giant branch stars, $R = N(\mathrm{HB})/N(\mathrm{RGB})$. The data for nine clusters spanning [Fe/H] $= -1.3$ to -2.2, and the slope of the relation predicted by the period shift (but with an arbitrary zero point), are shown in Figure 8. No anticorrelation between Y and [Fe/H] is apparent, much less one as steep as predicted by the period shifts. Finally, studies of metal-poor extragalactic H II regions indicate that the helium mass fraction increases as the heavy-element abundances rise (see Steigman, Gallagher, & Schramm 1989).

The implied helium-metallicity anticorrelation may be a false clue, relying as it does upon theoretical models to rule out alternative explanations. However, we are uncomfortable with the period-shift analyses done to done, for two reasons, each of which is significant. First, Sandage's analyses have been done using either an intensity-averaged or a magnitude-averaged $B-V$ color index to infer an average temperature, using the color-temperature transformations quoted by Butler et al. (1978). All the recent Baade-Wesselink analyses done to date on RR Lyrae variables have shown that the angular diameters

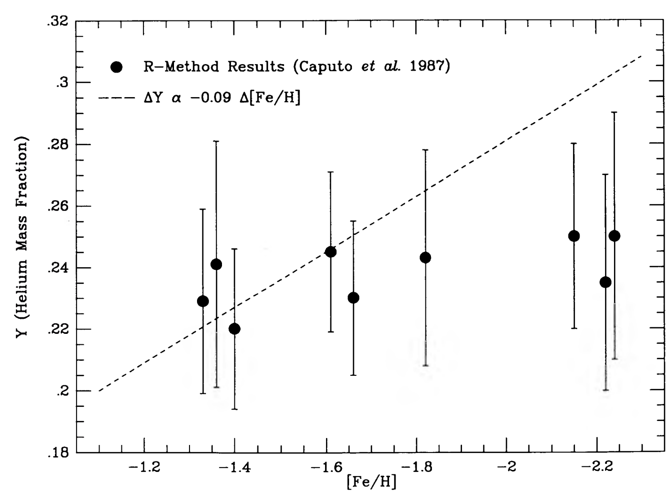

FIG. 8.—Helium mass fractions implied by the "*R* method" results of Caputo et al. (1987) compared with the slope of the Y-[Z] relation implied from Sandage's (1990b) period-shift analyses (*dashed line*).

derived from such blue photometry do not match those derived from the radial velocities. Jones (1988) argued that the problem arises from excess short-wavelength flux emitted during the rise from minimum to maximum radius. This problem occurs for *both* $B-V$ and $b-y$ color indices. (Indeed, this caused the failure of our first attempt at a Baade-Wesselink analysis: Carney & Latham 1984.) But whatever the cause, such results imply that any type of average $B-V$ color over the full pulsation cycle leads to erroneous temperatures. An alternative color index is required, one less sensitive to such effects. As pointed out first by Jameson, Fernley, & Longmore (1987), infrared magnitude and colors are much more suitable to such an analysis, and have been utilized successfully by JCL, JCSL, Liu & Janes (1990a, b), and the UK group (UK90 and references therein).

However, even were we to replace the mean $B-V$ color index by a mean $V-K$ index, we would still be introducing an error. A comparison with horizontal-branch evolution tracks and with pulsation theory requires the use of a star's equilibrium temperature, T_{eq}, which is not the same as its mean temperature, $\langle T_{eff} \rangle$. The equilibrium temperature is defined by the equilibrium luminosity, L_{eq}, and the equilibrium radius, R_{eq}, so that $T_{eq} = (L_{eq}/4\pi\sigma R_{eq}^2)^{1/4}$. An average or mean temperature is not the same, but is in fact $\langle T \rangle = \langle L/4\pi\sigma R^2)^{1/4} \rangle$. It is thus not mathematically the parameter we seek, even when $L_{eq} = \langle L \rangle$ and $R_{eq} = \langle R \rangle$. Fortunately, the Baade-Wesselink method itself produces naturally both the mean luminosity, $\langle L \rangle$, based on an intensity-weighted average over bolometric magnitudes, and a mean radius, $\langle R \rangle$, based on an average over the pulsational velocities. True equilibrium temperatures may thus be defined readily.

A question we must ask first is whether $\langle L \rangle$ and $\langle R \rangle$ are in fact the equilibrium values, L_{eq} and R_{eq}. In the former case, the answer is unambiguously yes, since the mean luminosity is not affected by the pulsation itself, being the result of processes deep in the star's interior. In the latter case, we are asking in essence whether the mean radius of the oscillator is the same as the value were the oscillation to cease. We cannot answer this question empirically, but can offer some circumstantial evidence in favor of the mean being close to the equilibrium value, at least when we consider the very asymmetrical radial velocity curves. We ask: Is the mean radial velocity measured for RR Lyrae stars the same value that would result if the stars were not pulsating? If it is, we can more readily believe that the mean radii and equilibrium radii are very nearly the same. We address the radial velocity question by using the results of Liu & Janes (1990b). The mean radial velocity for all four RR Lyrae stars in M4 they analyzed yields $\langle v_{rad} \rangle = +70.15 \pm 1.35$ km s^{-1}, where the error given is the error of the mean. If the mean radial velocities of these pulsating stars are truly representative of their systemic (i.e., nonpulsating) velocities, the mean for the four stars should equal the mean radial velocity of the cluster itself. Peterson (1985) and Peterson & Latham (1986) have measured radial velocities for nine blue horizontal-branch and 19 red giant branch stars. The weighted mean radial velocity is $+70.14 \pm 0.13$ km s^{-1}, where, again, the error is the error of the mean. Given the small internal velocity dispersion of M4, we consider that the agreement is excellent, and that mean radial velocities of RR Lyraes are in fact very close to the equilibrium values.

On the theoretical side, A. Cox and J. Guzik have graciously computed a mean radius from one of their nonlinear models of RR Lyrae stars. They selected a star with $T_{eff} = 7000$ K, $M = 0.65\ M_{\odot}$, and a luminosity of 1.950×10^{35} ergs s^{-1} ($M_{bol} \approx +0.52$ mag). The equilibrium radius, before the star was allowed to begin pulsating, was $R_{eq} = 3.376 \times 10^{11}$ cm. After turning on the pulsation and letting it stabilize, then integrating through 343 time steps over one pulsation cycle, they found $\langle R \rangle = 3.397 \times 10^{11}$ cm. In this particular case, R_{eq} and $\langle R \rangle$ agree to within 0.6%.

4.2. *The Equilibrium Temperature Scale*

In Table 4A we collect the data for the 15 RR*ab* stars used in JCSL to derive the relations between [Fe/H] and $\langle M_V(\text{RR}) \rangle$ and $\langle M_{bol}(\text{RR}) \rangle$ and between $\langle M_K(\text{RR}) \rangle$ and log *P*. We exclude here the RR*c* variables. Since any T_{eq} relation should apply to all RR*ab* variables and not only those near the ZAHB, we include the two highly evolved field variables DX Del and SS Leo, discussed also by JCSL. We did not adopt a final absolute magnitude for SS Leo in our previous paper (JCSL), owing to difficulties of the analysis presumably associated with the star's high luminosity. However, the M_{bol} value derived ignoring the acceleration terms, $\langle M_{bol} \rangle = -0.05$, and the radius, $\langle R \rangle = 7.32\ R_{\odot}$, yield $T_{eq} = 6440$ K. Fernley et al. (1990) derived quite different magnitudes and radii ($\langle M_{bol} \rangle = +0.24$, $\langle R \rangle = 6.57\ R_{\odot}$), but the resultant T_{eq} value is very similar to ours, 6359 K. (Note that we have used our bolometric correction to their derived $\langle M_V \rangle$ result to derive $\langle M_{bol} \rangle$.) We therefore adopt $T_{eq} = 6400$ K for this star, but consider the luminosity and radius to be indeterminate. We give in columns (2)–(6) the metallicity, period, blue amplitude $A(B)$, magnitude- and intensity-averaged $B-V$ color indices, $(B-V)_{mag}$, and $\langle B \rangle - \langle V \rangle$. In columns (7)–(9) we give the bolometric magnitudes, M_{bol}, the mean radii (in solar units), and the resultant equilibrium temperatures, T_{eq}.

We seek a relation between T_{eq} and some minimal set of parameters. The most likely variable is the mean $B-V$ color index. Although we have noted that neither the magnitude-averaged, $(B-V)_{mag}$, nor the intensity-averaged, $\langle B \rangle - \langle V \rangle$, color index gives a correct measure for the RR Lyrae stars, because of excess short-wavelength emission during the rise from minimum to maximum radius, there should still be *some* relation between the equilibrium temperatures and either of these average color indices. Of course, since $B-V$ is affected by line blanketing, we expect that a metallicity term is required. Are these the only terms? We consider it likely that at least one additional term is required, for two reasons. First, $B-V$ is somewhat sensitive to gravity, as the synthetic color indices we have computed for our Baade-Wesselink analyses show. Second, the difference between the observed mean $B-V$ color index and what it would be without the excess short-wavelength emission is probably affected by the shape of the light curve, or at least the amplitude of the pulsation. In the limit, a nonpulsating star's $B-V$ value does give a correct measure of its true color index, after all. We have two variables that we may employ to assess the gravity effect and light-curve effect: the pulsation period and the pulsation amplitude. Thus we seek relations between T_{eq} and $(B-V)_{mag}$, [Fe/H], and either log *P* or $A(B)$. Finally, since $B-V$ does have some obvious problems, we seek at least one other relation between T_{eq} and some of the observables. The obvious choice is to use log *P*, $A(B)$, and [Fe/H]. The use together of $A(B)$ and log P should account for temperature and gravity effects. We therefore proceed to use the Θ_{eq} ($= 5040/T_{eq}$) values for the field RR Lyrae stars and derive relations involving the above variables

TABLE 4

FIELD RR LYRAE STAR TEMPERATURES AND MASSES

Star (1)	[Fe/H] (2)	$\log P_0$ (3)	$A(B)$ (4)	$(B-V)_{mag}$ (5)	$<B>-<V>$ (6)	M_{bol} (7)	$R/R_{\odot}$ (8)	T_{eq} (9)	$M/M_{\odot}$ (10)
(a) *RR*ab Variables									
SW And	-0.15	-0.35432	1.27	0.370	0.342	1.030	4.210	6622	0.515
X Ari	-2.20	-0.18631	1.26	0.328	0.296	0.660	5.330	6409	0.525
RS Boo	-0.50	-0.42327	1.65	0.307	0.250	0.940	3.980	6953	0.561
RR Cet	-1.25	-0.25725	1.21	0.336	0.311	0.710	5.020	6528	0.574
DX Del	-0.20	-0.32549	0.97	0.381	0.361	0.700	5.100	6524	0.753
SU Dra	-1.60	-0.18018	1.26	0.334	0.306	0.650	5.150	6535	0.471
SW Dra	-1.40	-0.24438	1.22	0.340	0.310	0.730	4.890	6584	0.514
RX Eri	-1.40	-0.23118	1.14	0.363	0.341	0.670	5.300	6412	0.603
RR Gem	-0.20	-0.40087	1.62	0.318	0.270	0.980	4.050	6830	0.545
TW Her	-0.50	-0.39837	1.69	0.313	0.253	0.890	4.160	6880	0.576
RR Leo	-1.15	-0.34449	1.64	0.279	0.229	0.800	4.350	6869	0.536
SS Leo	-1.51	-0.20319	1.49	0.339	0.295	.....	.....	6400	.....
TT Lyn	-1.35	-0.22371	0.92	0.368	0.353	0.660	5.400	6367	0.616
AV Peg	0.00	-0.40852	1.41	0.349	0.314	1.200	3.800	6703	0.479
VY Ser	-1.80	-0.14624	0.88	0.376	0.361	0.800	5.260	6247	0.446
TU UMa	-1.25	-0.25365	1.22	0.340	0.314	0.730	5.000	6511	0.562
UU Vir	-0.55	-0.32275	1.50	0.325	0.282	0.875	4.475	6657	0.538
(b) *RR*c Variables									
TV Boo	-2.20	-0.3836	.....	.....	.....	0.620	4.33	7176	0.601
DH Peg	-0.90	-0.4723	0.64	0.197	0.187	0.940	3.76	7154	0.573
T Sex	-1.20	-0.3649	0.53	0.201	0.196	0.740	4.05	7218	0.478

using nonlinear least-squares methods. We find the following relations:

$$\Theta_{eq} = 0.611(B-V)_{mag} - 0.023[Fe/H] + 0.5355 , \quad (13)$$

$$\Theta_{eq} = 0.691(B-V)_{mag} + 0.010A(B) - 0.024[Fe/H] + 0.4937 , \quad (14)$$

$$\Theta_{eq} = 0.434(B-V)_{mag} + 0.141 \log P - 0.006[Fe/H] + 0.6531 , \quad (15)$$

$$\Theta_{eq} = 0.261 \log P - 0.028A(B) + 0.013[Fe/H] + 0.8910 . \quad (16)$$

The rms scatter in each of these relations is very small: 57, 48, 41, and 54 K, respectively.

4.3. *Period Shifts for Cluster Variables*

With the above relations, it is a straightforward matter to repeat the analyses by Sandage. But our first question must be: *Which of the above equations is most physically correct?* The concern is that the above relations have been defined mostly using unevolved stars (see JCSL), yet we seek to apply the results to some evolved cluster variables. We argue that the best test is to focus on a cluster with a large W_{RR} value, with well-studied variables, and test to see whether the differences in the luminosities *predicted by the period-shift analysis* match the *observed* differences in luminosities. The obvious cluster for study is M3, using the data from Table 5 of Sandage (1990a). Using each of the above equations, we constructed a log P versus log T_{eq} diagram. We sought a modest number of stars that adequately define the lower, shorter period, presumably unevolved, boundary of the distribution. We found that V19, V72, and V83 do this well in all four cases. We then derived a linear fiducial relation between log P and log T_{eq} using these three stars. Then for all the other variables in the cluster, we computed $\Delta(\log P)$, the difference between the observed period and the period for an unevolved star at the same equilibrium temperature. Using equation (10), we then derived the predicted luminosity shift, Δm_{bol}. (We assumed equal masses for all stars.) We then computed the difference in each star's V-magnitude and the means of the three fiducial variables, $\langle V \rangle = 15.69$. *If* we have reasonable temperature calibrations, *and if* the vAB71 relations are valid, we expect to see a very close relationship between Δm_{bol} and ΔV. (Relevant bolometric corrections were also studied but found to make no difference. We have omitted them here for the sake of brevity.) The results are shown in Figures 9–12. Use of equation (13) (Fig. 9) produces enormous scatter with only a very small degree of correlation between the two magnitude differences. Equation (14) leads to similar scatter (Fig. 10), but equation (15) produces less scatter (Fig. 11). The best correlation follows from equation (16), as shown in Figure 12. On the basis of the appearances of Figures 9–12, we claim that equation (16) gives the best results in the derivations of equilibrium temperatures for RR Lyrae stars, an important result for the period-shift analyses we carry

TABLE 5

CLUSTER PERIOD SHIFTS

		NUMBER OF RR LYRAE	EQUATIONS (16)–(18)			EQUATIONS (15)–(17)		
CLUSTER (1)	[Fe/H] (2)	STARS (3)	Δ(log *P*) (4)	σ_μ (5)	σ (6)	Δ(log *P*) (7)	σ_μ (8)	σ (9)
NGC 3201..........	−1.56	42	0.002	0.002	0.010	−0.023	0.002	0.016
NGC 5904..........	−1.40	7	−0.010	0.006	0.016	−0.027	0.014	0.037
NGC 6121..........	−1.28	15	−0.015	0.004	0.014	−0.015	0.014	0.055
NGC 6171..........	−0.99	14	−0.027	0.004	0.016	−0.057	0.008	0.031
NGC 6341..........	−2.24	6	0.040	0.005	0.011	0.004	0.012	0.029
NGC 6712..........	−1.01	7	−0.018	0.005	0.013	−0.050	0.029	0.076
NGC 6723..........	−1.09	14	−0.013	0.005	0.020	−0.028	0.012	0.046
NGC 6981..........	−1.54	16	−0.013	0.003	0.010	−0.032	0.006	0.025
NGC 7078..........	−2.15	24	0.030	0.003	0.013	0.062	0.004	0.021

out below, and for comparisons of model horizontal-branch evolution with observed RR Lyrae stars. We presume that equations (13)–(15) are more sensitive to evolutionary state than is equation (16). However, we retain one of the temperature calibrations that uses $B-V$, equation (15), in the period-shift analyses.

Our first step in comparing the period shifts from one cluster with those from another is to select the reference locus. For the same reasons given above, and following Sandage, we use the M3 variables as our fiducial points. We use equations (15) and (16) to obtain the two log *P* versus log $T_{\rm eq}$ planes. We compute linear least-squares fits to the results, using first log $T_{\rm eq}$, then log *P*, as the independent variable. For the final relations, we adopt the bisector of the two fits (following Isobe et al. 1990), as shown in Figures 13 and 14. These bisectors, which may be seen to fit the data well, are described by

$$\log P = -4.264 \log T_{\rm eq} + 15.9999 , \quad (17)$$

$$\log P = -4.135 \log T_{\rm eq} + 15.5223 , \quad (18)$$

where in the first case we have used equation (15) and in the second case equation (16). These fits were obtained from all but two of the M3 variables, the very long period, low-amplitude stars I-42 and I-100 having been excluded.

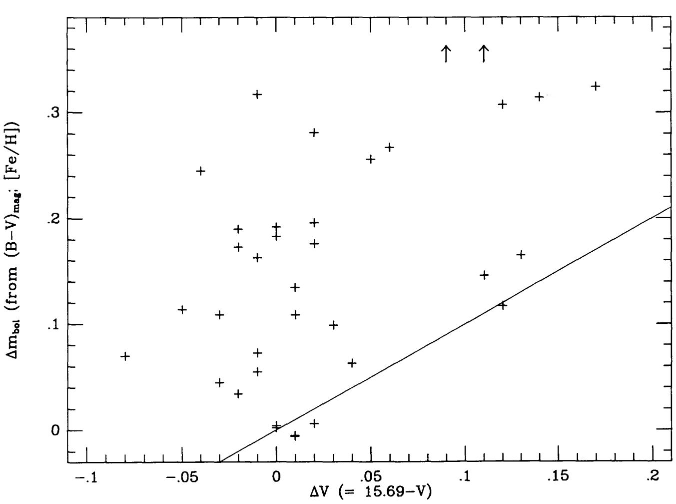

FIG. 9.—Predicted difference in $M_{\rm bol}$ from vAB71, plotted against the observed differences in *V*-magnitudes. Variables 19, 72, and 83 defined the "unevolved" sequence, and the temperatures were obtained from eq. (13).

With these fits, we compare the variables in other clusters with those in M3 to derive cluster-to-cluster period shifts. We proceed by assuming that the slopes of the two log *P* versus log $T_{\rm eq}$ relations apply to all clusters, then compute the mean zero-point offsets. We use the same cluster data given by Sandage (1990a), augmented by new results for M5 and M92 (NGC 6341; Storm, Carney, & Beck 1991; Storm, Carney, & Trammell 1992). The results are given in Table 5. In columns (1)–(3) we give the cluster names, the metallicities, and the number of RR*ab* variables used in the analyses. In columns (4)–(6) we give the results using equations (16) and (18), including the scatter derived for each star (σ) and the error of the mean (σ_μ). In columns (7)–(9) we give similar results obtained from the color-based relations, equations (15) and (17). We note that the period shifts have been computed using the *means* rather than a "lower three" set of stars that might (or might not) represent in each cluster the unevolved ZAHB. We do this partly because we must use such means when we study the field RR Lyrae stars (see below) and because, in the case of at least some of our clusters, either the sample sizes are too small to identify the unevolved stars unambiguously or the clusters are themselves so poor in RR Lyrae stars (i.e., small $W_{\rm RR}$ values) that it is not clear whether *any* of the variables are unevolved.

Before presenting the final results for the period shifts as a function of metallicity, we draw attention to the σ-values. Note that, on average, the scatter per star when the (log *P*, *A*(*B*),

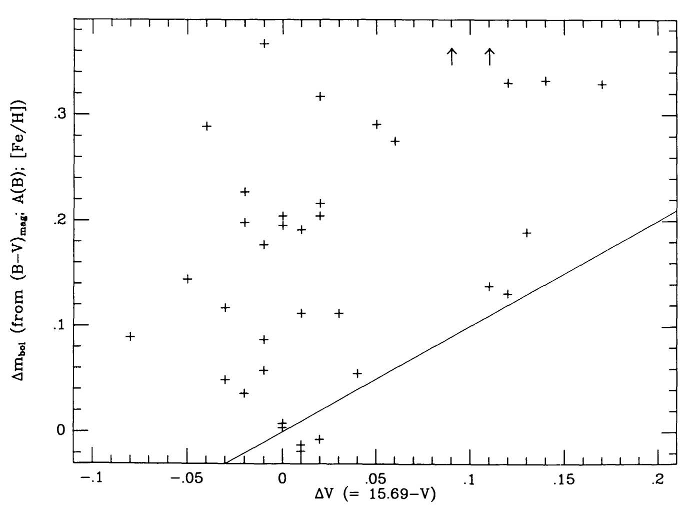

FIG. 10.—Same as Fig. 9, except that eq. (14) was employed

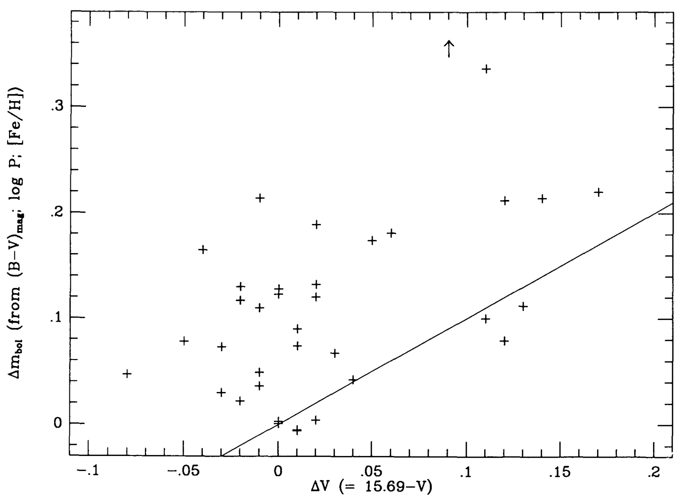

FIG. 11.—Same as Fig. 9, except that eq. (15) was employed

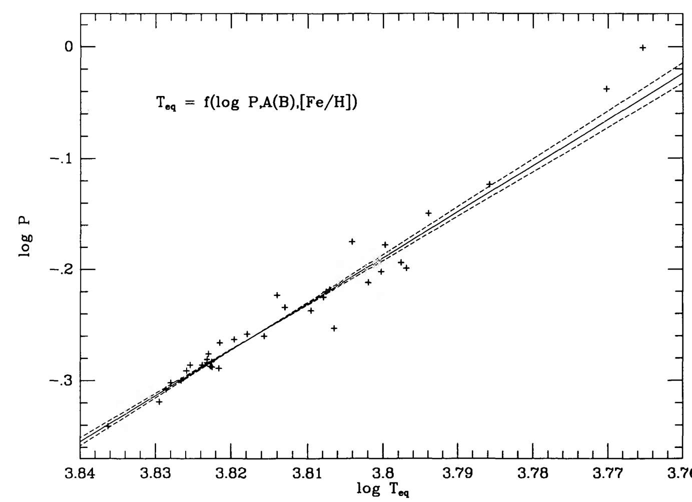

FIG. 13.—The (log P, log T)-plane for RR Lyrae stars in M3 derived from eq. (17).

[Fe/H]) variables are used is only 0.014 in log P, whereas when the $((B-V)_{mag}$, log P, [Fe/H]) variables are used, it rises to 0.037. Again, this suggests that the former set of variables leads to more realistic results. Why is $(B-V)_{mag}$ a poorer primary temperature indicator than log P? As noted, it is presumably due to evolutionary state. In particular, it may be partly due to the gravity sensitivity of $B-V$, which is much greater relative to its temperature sensitivity than for other color indices. For example, if we define a "figure of merit" of a color index (CI) to be $[d(\mathrm{CI})/d \log T]/[d(\mathrm{CI})/d \log g]$, the unpublished synthetic color indices of Kurucz suggest a value of about 90 for $B-V$ and $V-R$, about 140 for $V-I$ and $V-K$, and 190 for $R-I$ in the temperature and gravity domain of RR Lyrae stars. It may also be due to the fact that the flux at B and at V does not form at the same physical depth during the pulsation cycle (Jones 1988). We further suspect it is because the systematic error in temperatures implied by $B-V$ versus phase has a more complex behavior than may be modeled by using only log P as the secondary variable.

To obtain final results for the period shifts for the clusters' variables, we do a weighted, linear least-squares fit to the results in Table 5. Using the (log P, $A(B)$, [Fe/H]) variables, we find $\Delta(\log P) \propto -0.048(\pm 0.003)$[Fe/H] (see Fig. 15), whereas using $((B-V)_{mag}$, log P, [Fe/H]) results in $\Delta(\log P) \propto -0.107(\pm 0.006)$[Fe/H] (see Fig. 16). These are *very* different results, well outside each others' error bars. Note in particular that while in the second case the formal error bar is small, exclusion of either M15 or M92 alone would change the results dramatically. The solution is therefore not robust, while that obtained using the (log P, $A(B)$, [Fe/H]) variables is. While we believe the shallow slope is a superior result, both for reasons given above and because of the smaller scatter in Figure 15 compared with Figure 16, we cannot be certain that this is so. Thus we choose to present our final result for the clusters by taking a weighted average of the two results: $\Delta(\log P) \propto -0.060 \pm 0.003$[Fe/H]. To translate the period shift into a luminosity shift using equation (10), we must assume a relation between mass and metallicity. We chose for the moment to assume there is none, and defer the complete discussion until later. In this case, we find $\Delta M_{bol} \propto 0.18$[Fe/H]. We discuss below how we might be able to reconcile these two period-shift results.

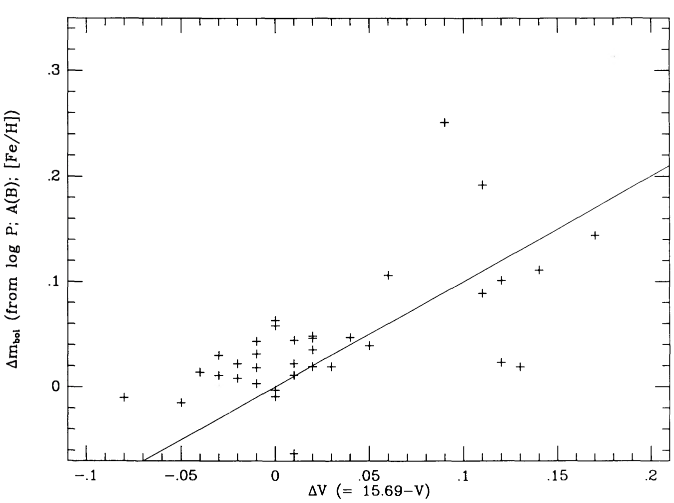

FIG. 12.—Same as Fig. 9, except that eq. (16) was employed

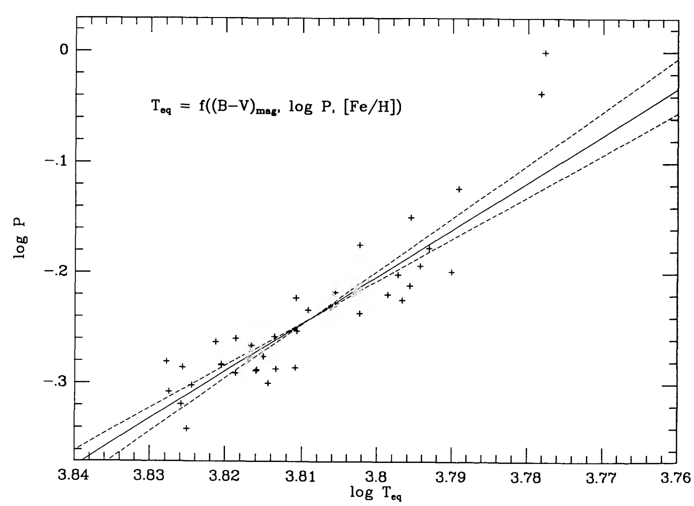

FIG. 14.—Same as Fig. 13, from eq. (18)

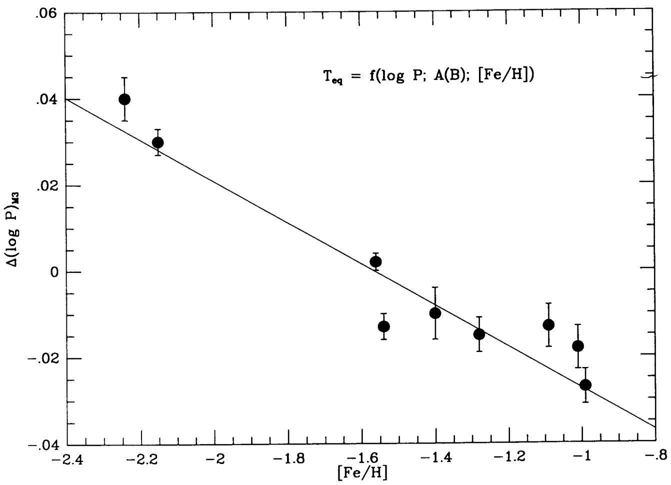

FIG. 15.—Δ (log P) vs. [Fe/H] for cluster variables obtained using eqs. (16) and (18).

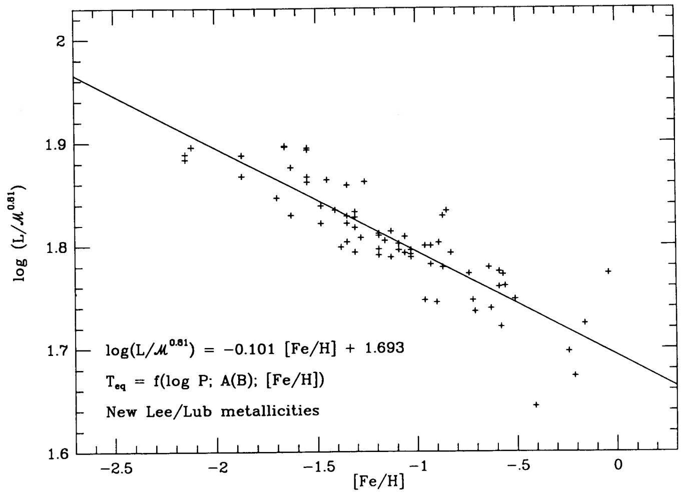

FIG. 17.—Lub's (1977) field star period shifts obtained using eqs. (16) and (18).

4.4. *Period Shifts for Field Variables*

Sandage (1990b) analyzed the field RR Lyrae sample of Lub (1977), and discovered that Δ(log P) versus [Fe/H] relation implied a steep M_{bol} versus [Fe/H] relation. We confirm this result. We have used equations (15) plus (17), and (16) plus (18), and both metallicity scales: that used by Sandage (1990b) and the revised set discussed by Lee (1990). If we continue to assume that mass is not correlated with metallicity, we find $\Delta M_{bol} \propto 0.25$[Fe/H]. It makes no significant difference which metallicity scale or temperature scale is used.

We argue that these results from Lub's very thorough work do not, however, bear directly on the problem at hand, simply because his sample is biased. The stars studied by Lub were selected originally to sample as fully as possible *both* metallicity and period. This introduced an immediate bias toward evolved stars, because at a fixed metallicity and temperature, a longer period star has a lower gravity and a greater luminosity, and thus is probably highly evolved. The results are plotted in Figure 17, but we prefer to seek another sample without such a bias toward longer period, overly luminous variables.

There is a new sample which does not suffer from such selection effects. The Lick group has been searching for fainter RR Lyrae variables in various selected directions for many years, and SKK have recently summarized their results. They have graciously shared their data with us, and we have used them to restudy the period shift versus metallicity relation. The available data include metallicity estimates from ΔS measures, periods, and blue amplitudes. There are 59 stars whose data are considered "good," and 82 whose data are considered to be of lower quality. We have used equations (16) and (18) to derive period shifts. We considered three cases: Only the "good sample," only the "poor sample," and the combined sample. The results were not distinguishably different, so we report here the results for only the combined sample. Figure 18 shows the data, with different symbols to distinguish the two data subsets. We performed linear least-squares fits using both Δ(log P) and [Fe/H] as the independent variables (since both contain errors). The equations are

$$\Delta(\log P) = -0.046(\pm 0.004)[\mathrm{Fe/H}] - 0.075(\pm 0.006) \quad (19)$$

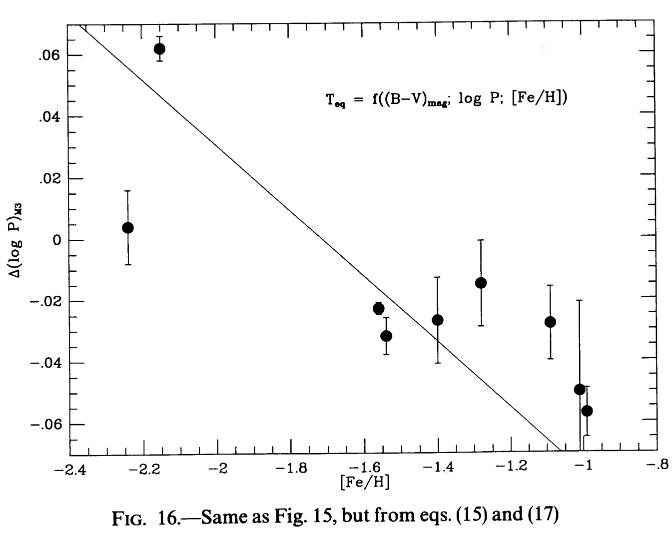

FIG. 16.—Same as Fig. 15, but from eqs. (15) and (17)

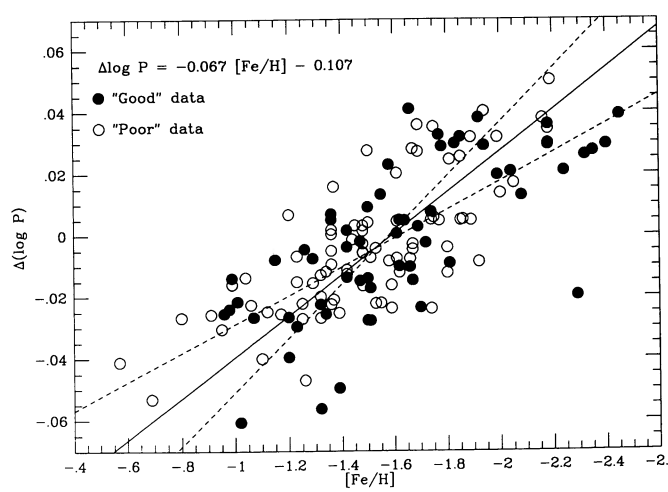

FIG. 18.—Lick sample of RR Lyrae period shifts obtained using eqs. (16) and (18).

when [Fe/H] is the independent variable, and

$$\Delta(\log P) = -0.087(\pm 0.008)[\mathrm{Fe/H}] - 0.138(\pm 0.005) \quad (20)$$

when $\Delta(\log P)$ is the independent variable and the equation is transformed back into the desired form. Following the recommendations of Isobe et al. (1990), the most physically correct fit to the data is defined in this case by the bisector of the above two relations, so we claim that the unbiased Lick survey of field RR Lyrae stars results in

$$\Delta(\log P) = -0.067(\pm 0.005)[\mathrm{Fe/H}] - 0.107(\pm 0.007)\,. \quad (21)$$

Note the excellent agreement with the results obtained from the cluster variables.

Again assuming for the moment that there is no dependence of mass on metallicity for RR Lyrae stars, the van Albada & Baker (1971) equation used by Sandage predicts

$$M_{\mathrm{bol}} \propto 0.20[\mathrm{Fe/H}]\,. \quad (22)$$

This result applies, of course, to the mean horizontal-branch level. This is very similar to the result from the globular clusters' period shifts, the main-sequence fitting result reported here, and the Baade-Wesselink results, plus the other techniques discussed in § 2. The only remaining question, then, is the relation between mass and metallicity.

4.5. *RR Lyrae Masses*

An assumption made in the above analyses is that the masses of the RR Lyrae variables do not depend on chemical composition. For two very different reasons this is not easy to support. First, globular clusters of equal age but different metallicities have turnoff masses that differ. For example, the VB85 isochrones predict that for $Y = 0.20$, and for an age of 16 Gyr, clusters with $Z = 0.0001([\mathrm{m/H}] = -2.23)$ have a turnoff mass of 0.81 $M_{\odot}$, whereas clusters with $Z = 0.001([\mathrm{m/H}] = -1.23)$ have a turnoff mass of 0.825 $M_{\odot}$. Thus, if mass loss on the red giant branch is not a function of metallicity (which seems unlikely), low-metallicity RR Lyrae stars should be slightly less massive than high-metallicity ones. The effect under these assumptions (metallicity-independent mass loss and equal ages) would be small on our derived absolute magnitude versus [Fe/H] slopes, however, and would tend in fact to diminish further the slope of the $\langle M_V(\mathrm{RR})\rangle$–[Fe/H] relation. Second, there are RR Lyrae stars, the RR*d* subtype, which pulsate in two modes simultaneously. Pulsation theory, through the medium of the Petersen diagram, wherein the period ratio P_1/P_0 is plotted against P_0, may be used to estimate masses for these stars. To date, results are available for RR*d* stars in the Oosterhoff I clusters IC 4499 and M3, and the Oosterhoff II clusters M68 (NGC 4590) and M15 (see Cox, Hodson, & Clancy 1983; Clement et al. 1986; Clement 1990), as well as three field stars, AQ Leo and two stars from the Lick survey, VIII 10 and VIII 58 (Clement, Kinman, & Suntzeff 1991). Considering the clusters alone, it appears that the metal-poor Oosterhoff II cluster variables have masses near 0.65 $M_{\odot}$, whereas the more metal-rich Oosterhoff I cluster variables have masses near 0.55 $M_{\odot}$. The result from the field stars (see Clement et al. 1991) is more confusing, since all three stars have essentially the same metallicity, but the derived masses are 0.66, 0.58, and 0.49 $M_{\odot}$. The possible problems with interpreting the Petersen diagram have been summarized by LDZ, but ignoring them for the moment, and also ignoring the results from the field variables, the results from the cluster variables would imply that all period-shift analyses should have the slopes of the $\langle M_{\mathrm{bol}}(\mathrm{RR})\rangle$ and $\langle M_V(\mathrm{RR})\rangle$ versus [Fe/H] relations steepened by about 0.09.

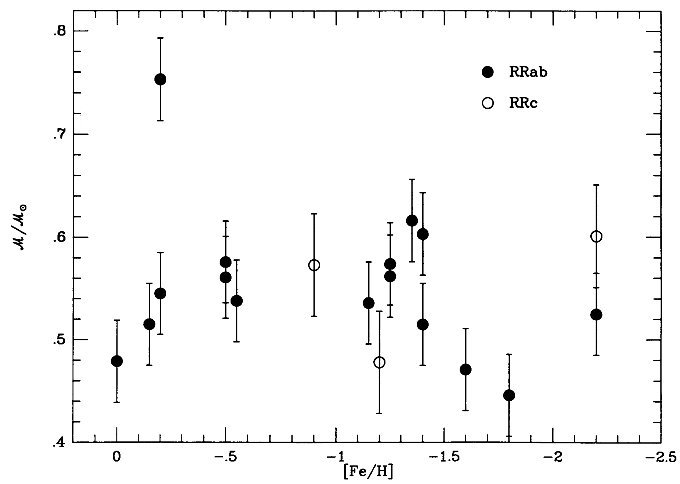

FIG. 19.—Masses of RR Lyrae stars vs. [Fe/H] obtained from the vAB71 models (eqs. [10] and [11]).

Our own results from the Baade-Wesselink method may be used to estimate masses from equations (2) and (3) from vAB71 (eqs. [10] and [11]), following our earlier work (Paper VI) and that of Liu & Janes (1990a). Using the T_{eq}, $\langle L\rangle$, and period data in Tables 4A and 4B, we compute masses and give them in the final columns of the tables. They are plotted as a function of metallicity in Figure 19. If this version of pulsation theory is correct, as well as our results from the Baade-Wesselink analyses, there is no trend with metallicity, and $\langle M\rangle = 0.54$ $M_{\odot}$. We agree with Clement et al. (1991) that there may be a rather large range in the masses of field RR Lyrae stars at a given metallicity. Note that no mass is given for SS Leo, since its luminosity and radius are too uncertain (JCSL). The mass of DX Del is higher than normal, but this is consistent with it having evolved (and brightened) from the more massive, redward side of the instability strip (see Sweigart 1987).

In this case, the slopes of the derived $\langle M_{\mathrm{bol}}(\mathrm{RR})\rangle$–[Fe/H] and $\langle M_V(\mathrm{RR})\rangle$–[Fe/H] relations need no correction. On the other hand, if the results from the double-mode RR Lyrae stars in the four globular clusters are correct, the slope of the M_{bol}–[Fe/H] relation is about 0.26.

Which set of masses should we prefer? *For consistency*, we argue that the vAB71 results are preferable. What we seek, after all, is consistency between the several different methods that yield the $\langle M_V(\mathrm{RR})\rangle$–[Fe/H] slope. The temperature scale used to address the period shifts was taken from the Baade-Wesselink results. The period-shift definitions were taken from the vAB71 formulae. Thus the relevant masses for a consistency check should be those taken from the Baade-Wesselink results and the vAB71 formulae. In this case there is no dependence of mass upon metallicity, and the results for the clusters and for the Lick sample of field variables requires no corrections.

There is one intriguing additional possibility. The vAB71 equations, as applied to our Baade-Wesselink results, suggest that there *is not* a mass-metallicity relation for *field* RR Lyrae stars, which is consistent with the (limited) Petersen diagram results for three field RR*d* variables. Therefore, the period-shift analysis of the SKK field RR Lyrae stars requires no correction for mass versus metallicity effects, and $\langle M_{\mathrm{bol}}(\mathrm{RR})\rangle \propto$

0.20[Fe/H]. Consider now the *cluster* variables, where the (also limited) cluster Petersen diagram results indicate that there *is* a mass-metallicity relation for cluster variables. (Why the cluster and field RR Lyrae stars should differ is beyond the scope of this paper, but would plausibly be related to a difference in age spreads.) The period-shift analysis of the *cluster* variables using *only* $T_{eq} = f(\log P, A(B), [Fe/H])$, which is what we used for the *field* variables, would then require correction for the mass-metallicity effect, in which case we find $\langle M_{bol}(RR) \rangle \propto 0.23[Fe/H]$. This is a possible means of reconciling all the period-shift analyses and mass-metallicity results, but since it comes at the expense of invoking serious differences between field and cluster RR Lyrae stars, we are reluctant to attach great significance to it yet. It will be interesting to see what masses are derived from the vAB71 formulae and the Baade-Wesselink analyses of RR Lyrae stars in the metal-poor, Oosterhoff II clusters M15 and M92.

We note further that at some point the models used to construct the Petersen diagrams should be studied to derive formulae equivalent to equations (10) and (11). Then the Baade-Wesselink and period-shift analyses could be compared in a self-consistent manner.

5. SUMMARY AND IMPLICATIONS

We summarize in Table 6 the results for the slopes of the three absolute magnitude relations derived from the Baade-Wesselink method and the others discussed in this paper. In cases where only an $\langle M_V(RR) \rangle$ or only an $\langle M_{bol}(RR) \rangle$ result is available, we have indicated in parentheses the likely value of the missing one, assuming that the $\langle M_V(RR) \rangle$ versus [Fe/H] and $\langle M_{bol}(RR) \rangle$ versus [Fe/H] relations differ by 0.04 in slope. (This represents a compromise between the Baade-Wesselink results and the LDZ predictions.) We have omitted the M_K versus log P results from UK90 for $\langle M_V(RR) \rangle$ versus [Fe/H] and $\langle M_{bol}(RR) \rangle$ versus [Fe/H] for reasons already discussed. We prefer to tabulate only results for the *mean* RR Lyrae luminosities versus [Fe/H], so in those cases where the analyses prefer to the ZAHB, we have used equation (4) to adjust the one into the other. Such adjustments are indicated with brackets in Table 6. It is clear that with the correction for metallicity-dependent errors in $B-V$ in the VB85 isochrones used for main-sequence fitting, and with the proper selection of temperature scale and field star samples for the period-shift analyses, all the results are in good agreement, and that the slopes are shallower than claimed by BCF, BCCF, and Sandage (1990b).

Before we address the implications for globular cluster ages, we must adopt a final calibration of the $\langle M_V(RR) \rangle$–[Fe/H] relation. Perhaps the major problem is that most of the methods summarized in this paper and in Table 6 address only the slope of the relation and not the zero point. In our opinion, the optimal merger of the results in Table 6 is to compute a mean slope, giving equal weights to the results from the main-sequence fitting, the Baade-Wesselink analyses, the cluster period-shift analysis, the period-shift analysis of the Lick RR Lyrae stars, and the LDZ theoretical prediction. We give half-weight to the results from the red giant branch tips (Da Costa & Armandroff 1990) and red giant branch bump analyses (Fusi Pecci et al. 1990). The resultant slope is 0.15 ± 0.01. To assess the zero points, we rely upon the two statistical parallax analyses and the main-sequence fit of HD 103095 to M5. Using the weights and mean metallicities discussed previously, we find that the zero point should be 1.01 ± 0.08. The final relation is then

$$\langle M_V(RR) \rangle = 0.15(\pm 0.01)[Fe/H] + 1.01(\pm 0.08) . \quad (23)$$

This is the equation we employ to derive cluster ages, and which we recommend for future distance and age estimates. Note that it refers to mean luminosities, not ZAHB levels.

5.1. *Cluster Ages*

We proceed by using the measured magnitude differences between the horizontal branch and the main-sequence turnoff (BCF). We add to their data recent results for five other clusters: NGC 1261 (Bolte & Marleau 1989), Ruprecht 106 (Buonanno et al. 1990a), NGC 6218 (Sato, Richer, & Fahlman 1989), NGC 6254 (Hurley, Richer, & Fahlman 1989), and Palomar 12 (Stetson et al. 1989). Equation (23) is the basis for the distance estimates, and hence the conversion from apparent to absolute main-sequence turnoff magnitudes. However, BCF tabulated the ZAHB levels for globular clusters, which is very reasonable given the wide range in color distributions along the horizontal branch. We must therefore use equation (4) to convert their values back into the mean magnitude levels for correct use of equation (23).

To derive ages from M_V(TO) requires knowledge of the bolometric corrections and the chemical compositions. For the former, we adopt the observed values from Carney (1983). In particular, we adopt a bolometric correction of -0.21 mag for all clusters' turnoffs. For the latter, we must consider three different chemistries. First, there is the assumed helium mass fraction, Y. There are no direct measures of the helium abundances in unevolved halo stars. Following the discussion by Caputo et al. (1987) and Steigman et al. (1989), we adopt $Y = 0.23$. Next, we need a measure of the heavy-element abundances, usually denoted by [Fe/H]. This is a difficult subject, as noted in our discussion of the UK90 results. We

TABLE 6

REVISED M_V AND M_{bol} VERSUS [Fe/H] RESULTS

Method	M_V/[Fe/H]	M_{bol}/[Fe/H]	References
Main-sequence fits	[0.12]	[(0.16)]	BCF; this paper
RGB tips	(0.17)	0.21	Da Costa & Armandroff 1990
RGB "bumps"	[0.10–0.15]	(0.14–0.19)	Fusi Pecci et al. 1990
Baade-Wesselink	0.16	0.21	JCSL
Period shifts:			
Clusters	(0.14)	0.18	Sandage 1990b; this paper
Field	(0.16)	0.20	This paper
Theory	0.17	0.20	LDZ

NOTE.—See text for explanation of parentheses and brackets.

adopt the metallicity scale tabulated by Zinn (1985). In the case of Ruprecht 106, we adopt the new metallicity based on $(B-V)_{0,g}$ from Da Costa, Armandroff, & Norris (1991). Finally, we must adopt the abundances of the lighter but more abundant elements such as carbon, nitrogen, and oxygen. The abundances of the first two elements seem to be present in solar proportions compared with iron, at least down to [Fe/H] $= -2.0$, according to the several studies summarized by Wheeler, Sneden, & Truran (1989). The key element, however, is oxygen, which by itself accounts for roughly half of the atoms heavier than helium within a newly formed main-sequence star. The results summarized by Wheeler et al. (1989) suggest [O/Fe] $= +0.3$ for [Fe/H] ≤ -0.5. However, Abia & Rebolo (1989) have argued that the [O/Fe] ratio rises steadily as [Fe/H] decreases. If we fit their [O/Fe] results for [Fe/H] ≥ -2.5, we find

$$[\mathrm{O/Fe}] = -0.42[\mathrm{Fe/H}] + 0.22 . \quad (24)$$

What happens to equation (23) if the field stars obey equation (24) while the clusters have [O/Fe] $= +0.3$, as summarized by Langer (1991)? The changes are minor. Only two of the methods of Table 6 rely on field stars: the Baade-Wesselink analysis and the period-shift analysis of field stars. To alter them so that they may be applied to cluster variables, the $\langle M_V(\mathrm{RR})\rangle$ versus [Fe/H] slope must be decreased by about 0.04 mag dex^{-1}, based on model calculations by Rood (1984) and Sweigart, Renzini, & Tornambè (1987). The cumulative effect reduces the slope of equation (23) from 0.15 to 0.13. The zero point is even less affected. The absolute magnitudes of the field RR Lyrae stars must be brightened by about 0.04 mag before being applied to cluster variables, while that for HD 103095 must be dimmed by about 0.08 mag before being applied to M5. The zero point thus remains the same as before, 1.01 mag. The formal uncertainty is unchanged, but the true uncertainty probably increases. (This follows because if the field and cluster [O/Fe] ratios are equal at equal [Fe/H], the difference between the statistical parallax and main-sequence fits for the zero point is only 0.13 ± 0.17 mag [errors adding in quadrature], whereas if the field and cluster [O/Fe] values differ at equal [Fe/H], the two sets of zero points differ by 0.24 ± 0.17 mag.) The resultant minor change in slope and lack of change in the zero point of equation (23) means that the cluster distances change little if the clusters and field stars have different [O/Fe] value at equal [Fe/H]. However, the derived ages change significantly. We therefore compute ages for both cases: [O/Fe] $= +0.3$ and [O/Fe] values appropriate to the Abia & Rebolo (1989) results.

We have a choice of isochrones, although it does not make a major difference, since they all predict similar luminosity behaviors. We avoid reliance upon temperature, the variable in which the several sets of isochrones differ significantly (see Straniero & Chieffi 1991). Because of finer grid spacing, we choose to use the Revised Yale Isochrones of Green et al. (1987). We must be careful in our choice of Z, however. These isochrones, like most others, were computed using a solar mixture, so that the abundances of all heavy elements scale directly as [Fe/H]. This is clearly inappropriate for halo stars. We choose to recompute "logarithmic effective heavy-element mass fractions" $[Z_{\mathrm{eff}}]$. In the first case we assume that all the "α-rich" species (Ne, Mg, Ca, Si, S) and oxygen are enhanced by 0.3 dex. We use the Cameron (1982) solar system abundances, according to which $Z_{\odot} = 0.0188$. In the second case we similarly accept [α/Fe] $= +0.3$, but adopt equation (24) to determine the oxygen abundances. Note that we *assume* that $[Z_{\mathrm{eff}}]$ may be compared directly with the log Z values of the isochrones. This is almost certainly not true in detail. Insofar as the effect of metallicity is primarily on the mean molecular weight, which affects the central density and temperature, it is a good assumption. Insofar as the effect of the metallicity is primarily one of opacity, it is not a good assumption. Nevertheless, we have no other choices at present. (Straniero & Chieffi 1991 have stated use of such effective metallicities is in fact a good approximation.) We give the final results for ages in Table 7. We give for each cluster the [Fe/H] value, the derived bolometric magnitude of the turnoff and its error, then the ages derived from the two assumptions regarding [O/Fe]. The errors we have assigned are based *only* on those from the measurement of the horizontal-branch and turnoff points in the color-magnitude diagrams. There are, of course, additional errors due to the uncertainty in the zero point in equation (23) and to the uncertainties in the metallicities. These errors increase the true uncertainty in the ages by at least 10%. However, we are especially interested here in the possibility of an age spread, and the above problems are ones of absolute, not relative, ages.

We see from Table 7 (col. [6]) and Figure 20 that if [O/Fe] $= +0.3$, there is an apparent age-metallicity relation, and that the oldest clusters, which define the age of the Galaxy, have ages of close to 20 Gyr. The extent of the age-metallicity relations, about 5 Gyr, agrees very well with that found by Sarajedini & King (1989, hereafter SK), who basically followed the same procedure but used the LDZ relation between $\langle M_V(\mathrm{RR})\rangle$ and [Fe/H], scaled solar abundances, and somewhat different color-magnitude diagram data for some clusters. With the remaining uncertainties in the distance scale's zero point and the lingering uncertainties in the clusters' [Fe/H] values (see Peterson, Kurucz, & Carney 1990), the Galactic age uncertainty must be at least 3 Gyr. For comparison with the universal expansion rates, an age of 17 Gyr or greater is consistent with $H \leq 60$ km s^{-1} Mpc^{-1} for $\Omega = 0$, and $H \leq 40$ km s^{-1} Mpc^{-1} for $\Omega = 1$.

If we accept the oxygen abundances of equation (24), there is no obvious age-metallicity relation (col. [9] and Fig. 21). A weighted mean age is 14.3 Gyr, although, again, the uncer-

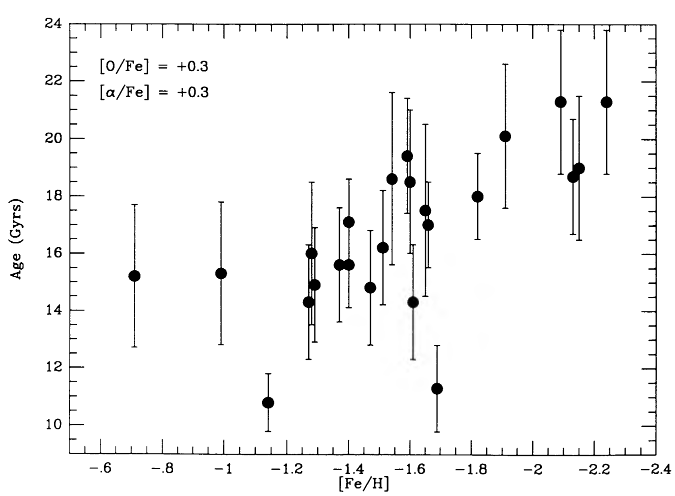

FIG. 20.—Ages of the globular clusters studied by BCF assuming [α/Fe] = [O/Fe] = +0.3.

TABLE 7

CLUSTER AGES DERIVED FROM M_{bol}(TO)

Cluster (1)	[Fe/H] (2)	M_{bol}(TO) (3)	σ (4)	$[Z_{eff}]$ (5)	t_9 (6)	σ (7)	$[Z_{eff}]$ (8)	t_9 (9)	σ (10)
NGC 104	-0.71	4.50	0.18	-0.49	15.2	2.5	-0.35	14.2	3.0
NGC 288	-1.40	4.32	0.12	-1.17	17.1	1.5	-0.83	14.7	1.5
NGC 362	-1.27	4.17	0.14	-1.04	14.3	2.0	-0.74	12.5	2.0
NGC 1261	-1.29	4.21	0.15	-1.06	14.9	2.0	-0.76	13.0	1.5
NGC 2808	-1.37	4.23	0.14	-1.14	15.6	2.0	-0.80	13.5	2.0
Ruprecht 106	-1.69	3.70	0.12	-1.46	11.3	1.0	-1.03	9.5	1.0
NGC 4590	-2.09	3.97	0.12	-1.86	16.4	2.0	-1.27	13.2	2.0
NGC 5139	-1.59	4.36	0.12	-1.36	19.4	2.0	-0.95	16.3	2.0
NGC 5272	-1.66	4.18	0.09	-1.43	17.0	1.5	-1.00	14.2	1.0
Palomar 5	-1.47	4.11	0.14	-1.24	14.8	2.0	-0.87	12.7	1.5
NGC 5904	-1.40	4.21	0.11	-1.17	15.6	1.5	-0.83	13.5	1.5
NGC 6121	-1.28	4.30	0.17	-1.05	16.0	2.5	-0.75	13.9	2.0
NGC 6171	-0.99	4.40	0.18	-0.77	15.3	2.5	-0.55	13.8	2.5
NGC 6205	-1.65	4.22	0.21	-1.42	17.5	3.0	-0.99	14.6	3.0
NGC 6218	-1.61	4.01	0.15	-1.38	14.3	2.0	-0.97	12.2	2.0
NGC 6254	-1.60	4.31	0.15	-1.37	18.5	2.5	-0.96	15.6	2.0
NGC 6341	-2.24	4.20	0.12	-2.01	21.3	2.5	-1.37	16.8	1.5
NGC 6397	-1.91	4.26	0.14	-1.68	20.1	2.5	-1.16	16.2	2.0
NGC 6752	-1.54	4.34	0.17	-1.31	18.6	3.0	-0.92	15.6	2.5
NGC 6809	-1.82	4.18	0.10	-1.59	18.0	1.5	-1.11	15.0	1.5
NGC 7078	-2.15	4.11	0.16	-1.92	19.0	2.5	-1.31	15.3	2.0
NGC 7099	-2.13	4.10	0.14	-1.90	18.7	2.0	-1.30	15.1	2.0
Palomar 12	-1.14	3.90	0.10	-0.91	10.8	1.0	-0.65	9.6	1.0
NGC 7492	-1.51	4.31	0.14	-1.27	16.2	2.0	-0.90	14.5	2.0

tainty must be at least 2 Gyr. A lower limit of 12 Gyr is consistent with H $\leq$ 80 km s^{-1} Mpc^{-1} for $\Omega = 0$ and $H \leq 55$ km s^{-1} Mpc^{-1} for $\Omega = 1$. Note that our limits on the Hubble constant are derived assuming an "instantaneous" formation for the globular cluster. A delay of 1 or more Gyr from the beginning of the universal expansion reduces the Hubble constant limits further still.

What about the age spreads at fixed metallicity? Taken at face value, the [O/Fe] = +0.3 result indicates that the greatest age spread is for intermediate-metallicity clusters. The more metal-rich and more metal-poor clusters do not seem to show such a large age spread, although the sample sizes are smaller.

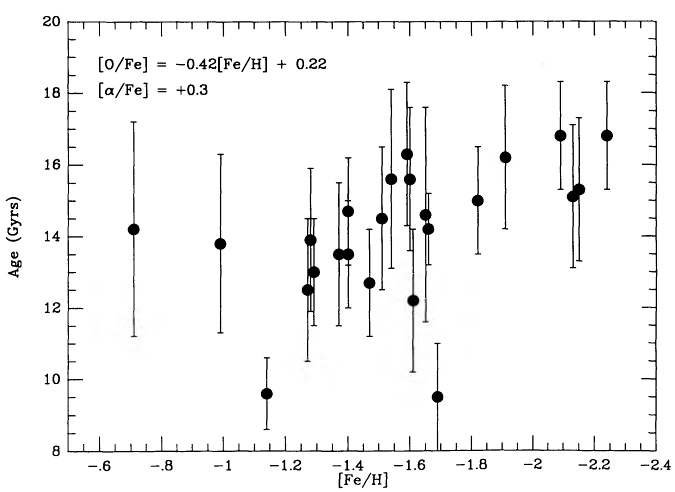

FIG. 21.—Same as Fig. 20, but with [O/Fe] = −0.42[Fe/H] + 0.22

Further, the largest difference in mean age is for the metal-poor clusters compared to the rest. This has two (speculative) impliciations. First, if the Galactic halo evolved homogeneously, it did so slowly at first, so that the metallicity reached [Fe/H] = −1.6 only after several Gyr had passed. After that, the metallicity increased so rapidly we cannot discern an age-metallicity relation. It is worth recalling at this point that the total *number* of heavy-element atoms, which are the products of the nucleosynthesis, doubles in going from [Fe/H] = −1.6 to −1.3, and again by −0.9. The lack of an age-metallicity relation then suggests that the star formation rate and hence supernova rate were very low at first, then began to speed up rapidly. In the case where [O/Fe] $\propto$ −0.42[Fe/H], the clusters in general have similar ages, in agreement with the rapid, homogeneous collapse model of ELS. However, we must recall that the age difference persists between NGC 288 and NGC 362, as well as for the younger-than-average ages for Palomar 12 and Ruprecht 106, regardless of the [O/Fe] ratio. This implies that the evolution of the halo was not *both* homogeneous and rapid.

Our derived age difference (Table 7) between M68 and the other metal-poor clusters conflicts with the conclusions of VandenBerg et al. (1990), who found that such clusters show an excellent match between their turnoff to red giant branch loci inthe color-magnitude diagrams. Hence they have argued that all very metal-poor clusters have ages that are not distinguishably different at the 0.5 Gyr level. What seems to be the problem is that while the less evolved stars match well, the gap between the main-sequence turnoff luminosity and that of the horizontal branch differ from M68 and M92, and by a large amount. BCF found the gap to be 3.39 ± 0.12 mag for M68, but 3.65 ± 0.12 mag for M92. Thus, given nearly identical

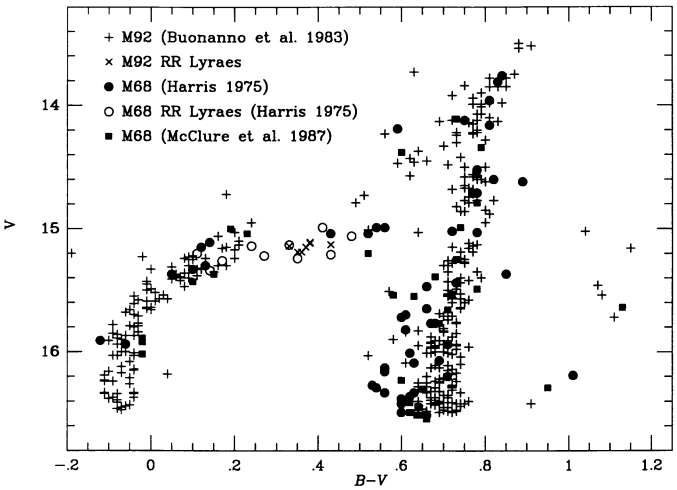

FIG. 22.—Horizontal branches of M68 and M92, after shifting the former by −0.43 in V and −0.033 in $B-V$, following VandenBerg et al. (1990).

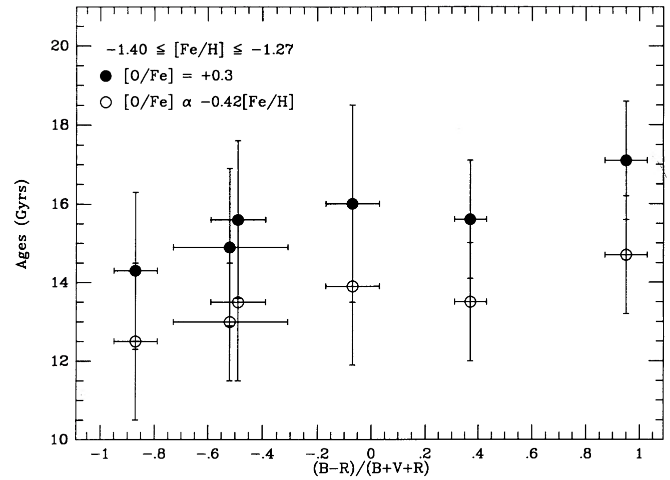

FIG. 23.—Horizontal-branch color of five intermediate-metallicity clusters vs. ages derived for two assumptions about the [O/Fe] ratio.

compositions, hence very similar $\langle M_V(\mathrm{RR})\rangle$, Table 7 shows much different turnoff luminosities, hence much different ages. How do we reconcile these results? We argue that the gaps measured for the two clusters by BCF (and by SK) are incorrect. We take the horizontal-branch color-magnitude diagram data for M68 from Harris (1975) and McClure et al. (1987), and for M92 from Buonanno et al. (1983). The M68 RR Lyrae data are from Harris (1975), and those for M92 from our own work. If we adopt the shift found by VandenBerg et al. (1990) to match the M68 and M92 main sequences and lower giant branches, $\Delta V = -0.43$ mag, and $\Delta(B-V) = -0.033$ mag, we find the results of Figure 22. The shifted M68 horizontal branch agrees very well with that of M92. Indeed, the mean visual magnitudes of the RR Lyrae stars in M92 are 15.15 ± 0.01 mag, and, after the shift, 15.18 ± 0.03 mag for M68. The clusters must therefore really have very similar gaps between the horizontal branch and the turnoff, and thus the same age. (Note: We have plotted M68 in Figs. 20 and 21 so that it has essentially the same age as M92, not the age derived from the BCF data given in Table 7.)

Is there a way in which we might be able to discern smaller age variations? We recall that Searle & Zinn (1978) suggested that the horizontal branch-branch color distribution may be caused by age as well as metallicity. SK studied this problem in detail and concluded that with the limits of their assumptions, for clusters with $-1.75 \leq$ [Fe/H] ≤ -1.25, horizontal-branch morphology does correlate with age. Since differences in horizontal-branch morphology are easier to detect than differences in turnoff luminosities, it is worth looking again at clusters with very similar metallicities to see whether the ages we have derived correlate with horizontal-branch color. If they do, we have a sign that the smaller age differences may be discernible. Since the horizontal branch seems to be undergoing a fundamental change in the vicinity of [Fe/H] ≈ -1.7, we restrict our comparisons to clusters that fall only on the metal-rich side of the Oosterhof transition, and only within narrow limits of [Fe/H]. We thus select the six clusters in Table 7 with $-1.40 \leq$ [Fe/H] ≤ -1.27. We plot these clusters' ages in Figure 23 against their horizontal-branch colors, as measured by $(B-R)/(B+V+R)$ (data taken from Lee, Demarque, & Zinn 1992). We must be especially careful with NGC 2808, whose distribution of stars across the horizontal branch is clearly bimodal (Harris 1974). Nonetheless, we do see an interesting trend: red horizontal branches correlate, albeit weakly, with younger ages. This hints, although not compellingly given the large error bars, that the NGC 288 versus NGC 362 age difference may not be unique, and that an age increase of 2–3 Gyr at these metallicities shifts the horizontal-branch morphology from red to blue. This result is significant in view of the redder-than-average horizontal branches of the most distant globular clusters (see Searle & Zinn 1978; Carney et al. 1991), suggesting that the outermost globular clusters might be considerably younger than the inner halo clusters, as Searle & Zinn suggested.

Thus, at the moment most studies agree, within the limits of the chemical composition uncertainties, that there is little evidence for an age spread among the most metal-poor clusters. Of course, this is at some level a systematic effect, since once star formation begins, whether in the Galaxy or in one of the larger hypothetical proto-Galactic fragments, all the remaining gas in or near the proto-Galaxy will become enriched in metals, and it takes few atoms to increase the mean metallicity from primordial levels to $[Z] = -2$. Only those early stars that formed at about the same time, prior to the first significant burst of star formation in or near the proto-Galaxy, will be metal-poor. On the other hand, there appear to be large age differences among intermediate-metallicity clusters. Hence not all these globular clusters formed at the same time. Until the [O/Fe] abundances are known more precisely, we cannot resolve whether or not there is a systematic variation in ages (i.e., an age-metallicity relation) and thereby learn the degree of mixing of supernova ejecta into the various corners of the young Galaxy.

What remains to be done? Clearly before we can establish or refute an age-metallicity relation, or the absolute ages of the most metal-poor clusters, we must better determine the abundances. Certainly the oxygen-to-iron abundances must be measured to higher precision than has been done heretofore, but we remind the reader that the iron abundances themselves are also important and remain somewhat uncertain (Peterson et al. 1990). It is also clear that we must continue to obtain the best-quality color-magnitude diagrams possible, from the tip of the red giant branch to magnitudes well below the main-sequence turnoff. Data for the M68 RR Lyrae stars would be especially useful. Finally, we should strive to reduce any model dependences on the distance scale Trigonometric and statistical parallaxes of metal-poor main-sequence stars are especially needed, *and* with levels of precision that reduce distance uncertainties to well below 10%. Perhaps convergent point analyses as attempted initially by Eggen & Sandage (1959; see also JCL) would prove useful, as well as dynamical parallaxes for globular clusters from proper motion and radial velocity dispersions (see Cudworth & Peterson 1988). We would also like to see more Baade-Wesselink analyses for RR Lyrae stars in globular clusters to try to resolve the possible differences between field and cluster variables. The good agreement between the variables in M4 and M5 and in the field is encouraging, but we look forward to the coming results for the metal-rich cluster 47 Tuc, the variable metallicity cluster ω Cen, and the metal-poor clusters M15 and M92.

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