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THE BAADE-WESSELINK METHOD AND THE DISTANCES TO RR LYRAE STARS. VII. THE FIELD STARS SW ANDROMEDAE AND DX DELPHINI AND A COMPARISON OF RECENT BAADE-WESSELINK ANALYSES

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ABSTRACT

We present new *BVK* photometry and radial velocities from high-resolution spectroscopy for the metal-rich ([Fe/H] = −0.2) *ab*-type RR Lyrae star SW Andromedae. We utilize these data, along with previously published optical data, to derive the distance and absolute magnitudes of this star using the infrared surface brightness version of the Baade-Wesselink method. We also analyze the metal-rich ([Fe/H] = −0.2) RR Lyrae variable DX Delphini using the same technique and previously published data. We obtain $\langle M_V \rangle = 1.10$ mag, $\langle M_{\rm bol} \rangle = 1.10$ mag, and $\langle M_K \rangle = +0.10$ mag for SW And, and $\langle M_V \rangle = 0.68$ mag, $\langle M_{\rm bol} \rangle = 0.69$ mag, and $\langle M_K \rangle = -0.35$ mag for DX Del. We compare these results, and the values we derived in previous papers in this series, with the results from three other recent applications of the Baade-Wesselink method, and find that *they agree to within* 0.10 mag *when the appropriate phase intervals are considered.* Excluding the highly reddened and highly evolved stars, we derive

$$\langle M_V({\rm RR})\rangle = 0.16[{\rm Fe/H}] + 1.02\ ,$$

$$\langle M_{\rm bol}({\rm RR})\rangle = 0.21[{\rm Fe/H}] + 1.04\ ,$$

$$\langle M_K({\rm RR})\rangle = -2.33 \log P - 0.88\ ,$$

using weighted least-squares fitting for 18 stars.

Subject headings: stars: fundamental parameters — stars: individual (DX Del, SW And) — stars: oscillations — stars: variables: other

1. INTRODUCTION

The Baade-Wesselink method provides a direct means of measuring the distances and absolute magnitudes of pulsating stars such as RR Lyrae variables. In the past, the application of this method has been plagued by difficulties and inconsistencies due, in part, to limitations in the precision of the data, particularly the radial velocities, and the assumptions made in the color-temperature transformations (Gautschy 1987; Jones 1987). The development of instrumentation capable of producing high-precision radial velocity curves, such as the echelle spectrometers of the Center for Astrophysics (CfA) (Latham 1985; Wyatt 1985) and CORAVEL (Baranne, Mayor, & Poncet 1979), and more sophisticated techniques of analysis has led to significantly improved results. In previous papers of this series (Carney & Latham 1984; Jones et al. 1987a, b; Jones 1988; Jones, Carney, & Latham 1988a, b, hereafter Papers I through VI, respectively), we have applied the surface brightness version of the Baade-Wesselink method, using near-simultaneous photometry and spectroscopy, to seven field RR Lyrae stars spanning a wide range of metallicity. In these papers, we have shown that angular diameters derived from optical colors are systematically distorted during the expansion phase of the pulsation cycle due to an apparent flux redistribution during these phases. Use of the $V-K$ color index alleviated this problem and yielded angular diameters from photometry that matched those derived from the radial velocities through most of the pulsation cycle. The superiority of infrared photometry over optical colors in Baade-Wesselink–type analyses of RR Lyrae stars was first demonstrated by Longmore et al. (1985); this United Kingdom group has applied an infrared flux version of the Baade-Wesselink method to six field stars (Fernley et al. 1989; Skillen et al. 1989; Fernley et al. 1990a, b, hereafter UK1 through UK4, respectively). More recently, Liu & Janes (1990a, hereafter LJ90) have analyzed 13 field stars using the surface brightness method and the $V-K$ color index, while Cacciari, Clementini, & Buser (1989a) and Cacciari et al. (1989b, hereafter Cac89a, Cac89b) applied this same method to six field stars. This latter group used optical colors, namely $V-R$ and $V-I$, but restricted the phase interval to avoid the flux redistribution problems associated with the expansion phase. Finally, the surface brightness version has been applied to globular cluster RR Lyrae stars by Liu & Janes (1990b), who analyzed four M4 variables using *VK* data, by Cohen & Gordon (1987), who used *B*, *i* photometry and a restricted phase interval for four RR Lyrae stars in M5, and by Beck (1988, reported in Paper VI), who analyzed variable V8 in M5 using *VK* data. Results for other clusters, including 47 Tuc and M92, will be available soon.

In this paper, we extend our field star work to the very metal-rich RR Lyrae stars SW Andromedae and DX Delphini ($\Delta S = -0.5$ and -0.2, respectively: Butler 1975), utilizing unpublished observational data for the former star and pre-

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viously published observations for the latter. We also examine in detail our results and those of the other groups listed above to resolve discrepancies in some of the derived distances, luminosities, and radii. Finally, we use the results obtained using infrared photometry to derive relationships between the absolute magnitudes and the metallicity, and we also test the dependence of $\langle M_K \rangle$ upon log P and [Fe/H].

2. OBSERVATIONS OF PROGRAM STARS

2.1. *Observations of SW Andromedae*

BVK photometry of SW And was acquired by B. W. C. on the night of 1987 September 26–27 using the SIMULPHOT system on the 1.3 m reflector at Kitt Peak National Observatory. The SIMULPHOT system consists of optical and infrared photometers mounted such that light from an object can be alternated from one photometer to the other using a motor-driven mirror. The Mk 3 optical photometer and the infrared detector "Hermann" were employed. The conditions were fully photometric. The extinction was measured from the observations of four stars, while measurements of 10 infrared standards from Elias et al. (1982) and 18 optical standards from Landolt (1983) permitted transformations of the observed data to the standard *BV* system and to the "CIT" *K* system. Two stars near SW And—SAO 73957 and SAO 73960—were measured to set up local standards for differential photometry. We obtained $V = 8.892$ mag, $B-V = 0.583$ mag for SAO 73957 and $V = 10.109$ mag, $B-V = 0.528$ mag for SAO 73960 from one set of measurements for each star. *BV* photometry of SW And was obtained only near maximum light in order to properly phase the observations with prior data; these observations were transformed directly to the standard system and are presented in Table 1. Infrared observations only were measured for this star from 1987 September 27–October 4; the conditions were fully photometric, except for the night of 1987 September 27–28, where conditions permitted differential photometry only. The *K* photometry of SW And obtained on these nights are shown in Figure 1 and listed in Table 2. Twenty-nine observations each of the two nearby comparison stars were obtained on photometric nights at air masses of less than 1.3; these were averaged to derive $K = 7.569 \pm 0.001$ mag for SAO 73957 and $K = 8.752 \pm 0.002$ mag for SAO 73960. (The errors are internal only and are errors of the mean.)

Radial velocities of SW And were measured at various times between 1980 September and December and 1981 September and December using an echelle spectrograph and photon-counting Reticon on the 1.5 m Tillinghast reflector at the Whipple Observatory on Mount Hopkins, Arizona. One order (45 Å) centered at 5190 Å was recorded at a dispersion of 2.2 Å mm^{-1} and with a resolution of 10 km s^{-1}. Each exposure was cross-correlated against a high signal-to-noise spectrum of the twilight sky (see Latham 1985; Wyatt 1985 for details). The radial velocities are listed in Table 3 and depicted in Figure 2. The errors of the values listed in Table 3 are generally less than 1 km s^{-1} (Paper I).

The phases listed in Tables 1–3 were derived using the period of LJ90, 0.442266 days, and our observed time of maximum light, HJD 2,447,065.768. There may be a question as to the accuracy of these phases, since SW And is reportedly affected by the Blazhko effect of period 36.83 days (Kukarkin et al. 1970). However, according to Szeidl (1976), the Blazhko effect ceased in SW And in 1956, with only periodic variations of the "hump" on the ascending branch of the light curve remaining. More recently, Barnes et al. (1988) observed this star and noted that their photometry showed evidence of the Blazhko effect at the 0.02 mag level. Also, the observations of Cacciari et al. (1987, hereafter Cac87) seemed to indicate a large variation in the light curve of SW And, particularly around minimum light, with a magnitude spread of about 0.2 mag. Since most of their observations were obtained over a time interval of only 3 days, and since neither the observations of McNamara & Feltz (1977) nor of Liu & Janes (1989, hereafter LJ89) showed this variation, it is worthwhile to examine this large-magnitude spread in more detail.

TABLE 1

BV PHOTOMETRY OF SW ANDROMEDAE

HJD (−2447000)	Phase	V	$B-V$
65.7008.........	0.8480	10.098	0.530
65.7025.........	0.8518	10.085	0.529
65.7085.........	0.8655	10.075	0.524
65.7099.........	0.8687	10.069	0.521
65.7165.........	0.8835	10.035	0.499
65.7180.........	0.8868	9.996	0.493
65.7239.........	0.9003	9.888	0.445
65.7254.........	0.9037	9.852	0.437
65.7313.........	0.9170	9.688	0.398
65.7331.........	0.9211	9.630	0.384
65.7396.........	0.9358	9.487	0.325
65.7412.........	0.9394	9.465	0.321
65.7474.........	0.9534	9.395	0.290
65.7489.........	0.9568	9.363	0.278
65.7548.........	0.9703	9.248	0.235
65.7564.........	0.9738	9.215	0.235
65.7626.........	0.9878	9.157	0.207
65.7641.........	0.9913	9.145	0.203
65.7701.........	0.0048	9.146	0.204
65.7717.........	0.0084	9.145	0.202
65.7776.........	0.0218	9.175	0.209
65.7794.........	0.0258	9.175	0.216
65.7852.........	0.0389	9.214	0.225
65.7866.........	0.0421	9.216	0.226
65.7959.........	0.0632	9.271	0.243
65.7972.........	0.0660	9.268	0.252
65.8062.........	0.0863	9.324	0.271
65.8076.........	0.0895	9.321	0.276
65.8125.........	0.1006	9.349	0.284
65.8138.........	0.1035	9.348	0.287

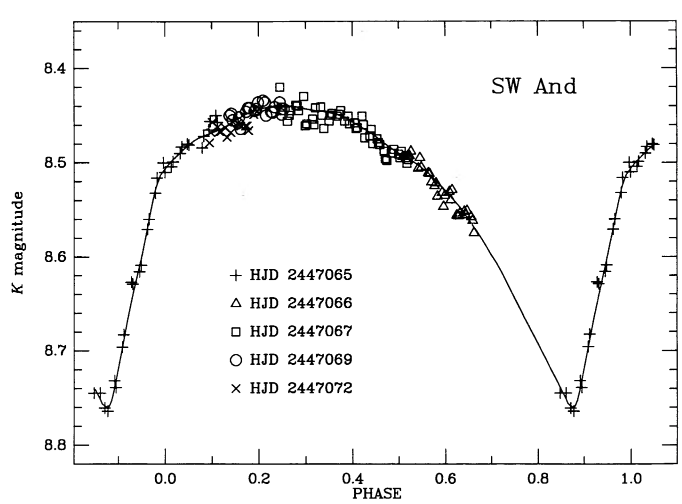

FIG. 1.—*K* magnitudes of SW And plotted vs. phase. The solid line represents the curve fitted to the data.

TABLE 2

K Photometry of SW Andromedae

HJD (-2447000)	PHASE	*K*	HJD (-2447000)	PHASE	*K*	HJD (-2447000)	PHASE	*K*
65.7008	0.8480	8.745	66.9287	0.6244	8.556	67.7465	0.4736	8.498
65.7064	0.8608	8.745	66.9301	0.6276	8.557	67.7507	0.4831	8.490
65.7123	0.8707	8.761	66.9315	0.6307	8.554	67.7521	0.4861	8.490
65.7138	0.8774	8.764	66.9357	0.6403	8.552	67.7533	0.4890	8.486
65.7200	0.8915	8.732	66.9372	0.6436	8.556	67.7578	0.4990	8.491
65.7214	0.8947	8.739	66.9385	0.6465	8.551	67.7591	0.5019	8.495
65.7275	0.9085	8.696	66.9426	0.6559	8.558	67.7603	0.5048	8.488
65.7290	0.9118	8.683	66.9439	0.6589	8.562	67.7644	0.5141	8.495
65.7359	0.9274	8.627	66.9452	0.6618	8.575	67.7657	0.5170	8.501
65.7374	0.9307	8.629	67.6452	0.2446	8.420	67.7670	0.5199	8.497
65.7436	0.9449	8.616	67.6465	0.2475	8.441	69.8091	0.1372	8.450
65.7451	0.9482	8.609	67.6479	0.2506	8.445	69.8105	0.1405	8.448
65.7513	0.9621	8.571	67.6523	0.2607	8.456	69.8119	0.1437	8.455
65.7527	0.9653	8.560	67.6539	0.2642	8.450	69.8167	0.1545	8.458
65.7589	0.9794	8.532	67.6554	0.2676	8.445	69.8181	0.1576	8.452
65.7603	0.9826	8.516	67.6603	0.2787	8.439	69.8195	0.1607	8.464
65.7664	0.9964	8.500	67.6616	0.2816	8.440	69.8248	0.1728	8.446
65.7678	0.9996	8.510	67.6629	0.2845	8.445	69.8264	0.1765	8.442
65.7740	0.0133	8.504	67.6677	0.2954	8.430	69.8280	0.1800	8.442
65.7754	0.0167	8.499	67.6692	0.2987	8.461	69.8328	0.1909	8.445
65.7820	0.0316	8.490	67.6706	0.3019	8.459	69.8341	0.1939	8.443
65.7834	0.0349	8.483	67.6761	0.3144	8.454	69.8360	0.1980	8.436
65.7887	0.0468	8.480	67.6775	0.3175	8.460	69.8403	0.2090	8.434
65.7901	0.0500	8.481	67.6789	0.3207	8.442	69.8423	0.2124	8.436
65.8030	0.0791	8.484	67.6840	0.3322	8.441	69.8438	0.2156	8.450
65.8044	0.0822	8.472	67.6853	0.3351	8.450	69.8484	0.2262	8.446
65.8094	0.0936	8.465	67.6870	0.3389	8.464	69.8499	0.2295	8.447
65.8107	0.0965	8.456	67.6918	0.3500	8.456	69.8515	0.2332	8.441
65.8156	0.1077	8.450	67.6932	0.3531	8.450	69.8565	0.2444	8.436
65.8170	0.1108	8.457	67.6947	0.3565	8.448	69.8581	0.2481	8.449
66.8771	0.5078	8.494	67.6995	0.3672	8.451	69.8599	0.2521	8.443
66.8788	0.5117	8.494	67.7013	0.3714	8.449	72.8860	0.0943	8.479
66.8818	0.5183	8.493	67.7027	0.3746	8.445	72.8873	0.0973	8.468
66.8834	0.5220	8.493	67.7066	0.3832	8.456	72.8888	0.1006	8.457
66.8848	0.5253	8.488	67.7097	0.3903	8.451	72.8936	0.1116	8.462
66.8920	0.5414	8.506	67.7113	0.3939	8.459	72.8951	0.1151	8.467
66.8934	0.5446	8.495	67.7159	0.4044	8.458	72.8965	0.1182	8.467
66.8948	0.5478	8.505	67.7172	0.4073	8.464	72.9011	0.1285	8.463
66.9013	0.5624	8.511	67.7185	0.4103	8.463	72.9024	0.1315	8.473
66.9027	0.5657	8.512	67.7228	0.4199	8.451	72.9060	0.1397	8.468
66.9042	0.5691	8.521	67.7241	0.4230	8.460	72.9073	0.1427	8.456
66.9073	0.5760	8.525	67.7255	0.4260	8.474	72.9127	0.1546	8.458
66.9088	0.5794	8.522	67.7304	0.4371	8.472	72.9141	0.1578	8.460
66.9101	0.5824	8.536	67.7317	0.4401	8.465	72.9154	0.1610	8.461
66.9145	0.5923	8.532	67.7331	0.4430	8.480	72.9201	0.1715	8.461
66.9158	0.5953	8.547	67.7372	0.4525	8.475	72.9215	0.1746	8.461
66.9170	0.5984	8.535	67.7386	0.4556	8.479	72.9229	0.1778	8.466
66.9215	0.6082	8.531	67.7399	0.4586	8.481	72.9278	0.1888	8.449
66.9228	0.6112	8.540	67.7439	0.4677	8.488	72.9292	0.1920	8.443
66.9244	0.6146	8.529	67.7452	0.4706	8.496	72.9307	0.1953	8.446

The *V* data of Table 1 were combined with the *V* observations of LJ89, and a mean light curve was fitted to the combined data set using cubic splines. There was excellent agreement between the two data sets, and the scatter around the mean curve was well under 0.01 mag over the entire phase cycle. The quantity $\Delta V = V(\text{Cac87}) - V(\text{mean light curve})$ was then computed for all of the data points of Cac87, and is plotted against HJD in Figure 3. It can be seen that there are short intervals of large variations in magnitude during each night. This behavior is not characteristic of the Blazhko effect, which is a multiday cycle-to-cycle variation. The most likely explanation of this behavior is possible variability of the comparison star employed by Cac87, SAO 73957, for which they reported $V = 8.980$ mag, $B-V = 0.591$ mag. This star was

TABLE 3

RADIAL VELOCITIES OF SW ANDROMEDAE

HJD (-2444000)	PHASE	v_{rad} ($km\ sec^{-1}$)	HJD (-2444000)	PHASE	v_{rad} ($km\ sec^{-1}$)
509.9469	0.0780	-49.38	590.6148	0.4747	-12.99
509.9603	0.1083	-46.92	590.6286	0.5059	-9.23
509.9735	0.1381	-44.34	591.5832	0.6644	-2.05
509.9868	0.1682	-39.23	592.6819	0.1486	-42.33
510.8916	0.2140	-35.90	592.6886	0.1638	-41.54
510.9368	0.3162	-25.42	592.6961	0.1807	-39.03
510.9594	0.3673	-19.34	592.7015	0.1929	-38.13
513.8068	0.8055	7.45	592.7071	0.2056	-36.96
513.8775	0.9654	-45.41	592.7128	0.2185	-35.13
513.9589	0.1495	-44.06	592.7217	0.2386	-33.28
514.0016	0.2460	-32.42	592.7291	0.2553	-31.35
536.7173	0.6081	-5.01	592.7373	0.2739	-30.25
536.8036	0.8032	5.96	867.9736	0.6058	-7.58
571.5827	0.4416	-14.05	886.6774	0.8966	5.51
571.5938	0.4667	-12.47	886.6867	0.9176	-3.93
571.6029	0.4873	-11.10	886.6950	0.9364	-26.07
571.6184	0.5223	-8.78	886.7044	0.9577	-42.34
571.6329	0.5551	-7.35	886.7127	0.9764	-50.52
571.6441	0.5804	-6.66	886.7239	0.0018	-55.72
571.6548	0.6046	-6.44	886.7321	0.0203	-55.29
571.6662	0.6304	-3.93	886.7404	0.0391	-54.83
571.6764	0.6535	-3.35	886.7490	0.0585	-52.80
571.6869	0.6752	-1.66	886.7578	0.0784	-51.25
571.6958	0.6973	-0.07	887.6953	0.1982	-36.59
571.7055	0.7193	1.55	887.7034	0.2165	-34.93
571.7162	0.7435	2.78	887.7116	0.2350	-33.41
572.5683	0.6701	-2.58	887.7203	0.2546	-31.70
572.5886	0.7160	2.23	887.7287	0.2737	-29.99
572.6046	0.7522	4.23	887.7371	0.2927	-28.33
572.6154	0.7766	4.35	887.7462	0.3133	-26.69
572.6321	0.8144	6.13	887.7547	0.3325	-24.48
572.6468	0.8476	8.02	887.7631	0.3515	-23.18
572.6650	0.8888	5.72	887.7714	0.3702	-21.68
572.6856	0.9353	-24.02	887.7889	0.4098	-18.37
572.6960	0.9589	-42.61	887.7975	0.4292	-16.37
572.7073	0.9844	-50.68	887.8059	0.4482	-15.23
572.7213	0.0161	-53.43	887.8183	0.4763	-13.39
572.7321	0.0405	-52.61	887.8283	0.4989	-11.61
572.7454	0.0706	-49.89	954.6032	0.4824	-14.38
572.7568	0.0963	-47.69	954.6154	0.5093	-11.94
572.7683	0.1223	-46.86	954.6268	0.5398	-10.11
572.7800	0.1488	-41.82	954.6386	0.5625	-8.28
590.5954	0.4309	-15.66	954.6503	0.5889	-7.41
590.6060	0.4548	-13.18	954.6620	0.6154	-7.29

also used as a comparison by LJ89, who obtained $V = 8.923 \pm 0.001$ mag, $B-V = 0.576 \pm 0.001$ mag based on 26 observations. Finally, as noted above, our single measurement of this star on a fully photometric night yielded $V = 8.892$ mag, $B-V = 0.583$ mag. The differences in these observed values are too large to be explained by observational error, but instead seem to indicate some type of variability in SAO 73957. Further evidence of this comes from the infrared photometry of this star: LJ89 obtained $K = 6.984 \pm 0.007$ mag, based on three observations, while we measured a value of 7.569 ± 0.001 mag as reported above. However, a detailed comparison of our observed K values of SAO 73957 and SAO 73960 showed no evidence of variation between the two stars;

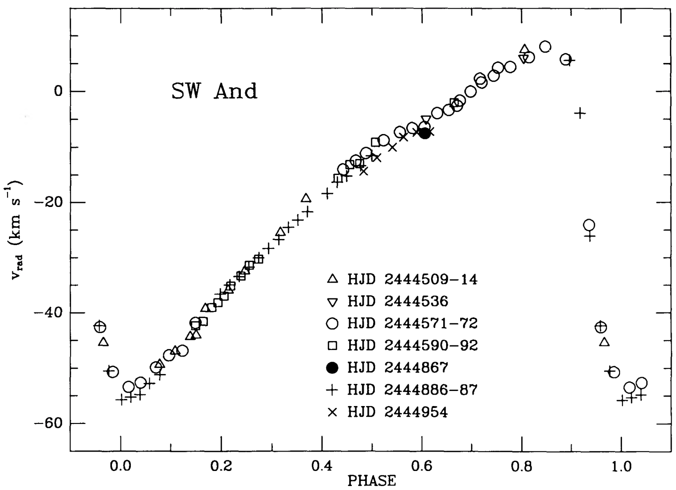

FIG. 2.—Radial velocities vs. phase for SW And

Liu (1988, private communication) also saw no evidence of variability in his 26 optical values, which were obtained on two consecutive nights. This seems to indicate that the variability may be intermittent in nature; this star clearly warrants further investigation. Liu (1988, private communication) also reports that his observations of SW And were reduced using mean extinction coefficients derived using several comparison stars each night, rather than differentially with respect to SAO 73957, which explains why the data of LJ89 do not show the large-magnitude spread.

The problems with SAO 73957 discussed above, and the variability of BD +29°3132 detected in Paper VI, illustrate the danger of relying on only one comparison star to monitor the extinction, particularly if differential photometry is employed.

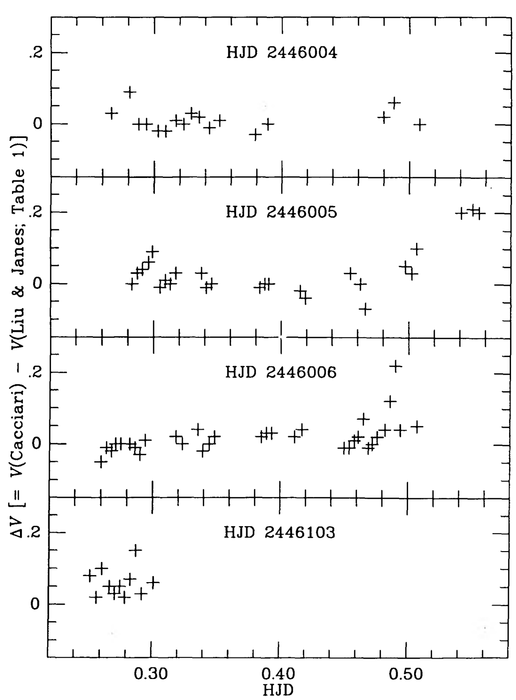

FIG. 3.—Difference in V magnitude, ΔV, vs. time for SW And. Quantity ΔV is defined in the text.

We have chosen to use the phases computed assuming no Blazhko effect for SW And, and also to utilize the V data of LJ89, since their values were obtained in an interval of a few days and show no evidence of variation. Also, they obtained their photometric and spectroscopic data for this star nearly simultaneously, so that their radial velocity curve can be used to check our results.

2.2. *Metallicities and Reddenings*

Both SW And and DX Del were analyzed by Butler (1975), who obtained ΔS values of -0.5 and -0.2 for these stars, respectively. He also employed curve-of-growth analyses to obtain [Fe/H] values of $+0.16$ and -0.11, respectively. We note that the high-dispersion spectra were obtained in phases around maximum light. Thus the curve of growth analyses may have been vulnerable to systematic effects. We therefore adopt [Fe/H] $= -0.2$ for both of these stars based solely on their ΔS values and the ΔS – [Fe/H] calibration of Butler (1975). For the reddenings, we continue to use the reddening scale based on the Walraven $E(V-B)_p$ values obtained from the period-color relations of Lub (1979), using the transformation $E(B-V) = 2.175E(V-B)$ described in Paper VI. For DX Del, an $E(V-B)_p$ value of 0.033 mag (Lub 1979) leads to $E(B-V) = 0.072$ mag. SW And was not observed by Lub (1979); in Paper VI, we obtained reddenings for stars not on Lub's list by using stars of similar periods and metallicities that were in Lub (1979). The only star comparable to SW And in period and metallicity is DX Del. Reddening values between 0.06 and 0.07 mag can be obtained for SW And following the method of Paper VI, depending upon the assumed metallicity difference between the two stars. These values are consistent with the adopted value of LJ90, $E(B-V) = 0.06 \pm 0.01$, which was obtained by averaging the values derived from several sources. We adopt this same value for SW And.

2.3. *Characteristics of Program Stars*

Following the procedure of our earlier papers, we smoothed the V magnitudes and v_{rad} values of SW And and DX Del into bins of 0.01 in phase by the use of local cubic splines. For SW And, we used the values of Tables 1 and 3 along with the V data of LJ89, while for DX Del, we utilized the V data of Barnes et al. (1988) and the radial velocities of Burki & Meylan (1986). Since the observations of DX Del were obtained from various sources, we have rephased the data according to the ephemeris of UK2, HJD(max) $= 2439362.614 + 0.47261838E$. A further complication arose for DX Del during the fitting of the v_{rad} data. Due to the fact that there are not many data points for this star, the resulting spline fit had "wiggles" at some phases. We note that the Fourier fit to the data by Burki & Meylan (1986) also had "wiggles" (their Fig. 1*c*). Since we need accurate values of the derivative of this curve to compute the effective gravities, we smoothed these "wiggles" by requiring that the derivative vary continuously.

As before, there was too much scatter in the K values to permit accurate numerical curve-fitting by splines, so we fitted the data of Table 2 for SW And and of UK2 for DX Del with hand-drawn curves. Such a procedure is subjective and may introduce systematic biases into our results. In order to evaluate the effects of this, two of us (R. V. J. and J. S.) fitted the data of DX Del independently. Each fitted curve was analyzed separately; the results are discussed in § 3. One of us (R. V. J.) fitted the data and performed the analysis of DX Del prior to reading UK2, in order not to be influenced by their results. The curve-fitting and analysis of SW And were performed in 1988, prior to the publication of LJ90, and thus were also not influenced by knowledge of the results of others. The fitted K curve of SW And is shown in Figure 1.

TABLE 4

ADOPTED DEREDDENED QUANTITIES FOR SW AND AND DX DEL

Quantity	SW And	DX Del
Period (days)	0.442266	0.47261838
[Fe/H]	−0.2	−0.2
p	1.30	1.36
γ (km s^{-1})	−19.2	−55.3
A_B (mag)	1.27	0.97
$E(B-V)$ (mag)	0.06	0.072
$\langle B\rangle_{int}$	9.852	10.075
$\langle V\rangle_{int}$	9.510	9.714
$\langle K\rangle_{int}$	8.508	8.676
$\langle m_{bol}\rangle_{int}$	9.513	9.724
$\langle B\rangle_{int} - \langle V\rangle_{int}$	0.342	0.361
$\langle V\rangle_{int} - \langle K\rangle_{int}$	1.002	1.038
$(B-V)_{mag}$	0.370	0.381
$(V-K)_{mag}$	1.032	1.056

Table 4 lists the observed dereddened quantities and amplitudes for these two stars. For the photometric data, the mean quantities are intensity averages—averages computed by first converting the data into intensities, calculating the time average, and converting the mean value back into a magnitude—except where noted. For SW And, the optical values were taken directly from LJ90, while for DX Del, the optical quantities were derived using the spline fits to the BV data. The magnitudes and colors of both stars were dereddened using the adopted reddening values and the absorption relaxations of Savage & Mathis (1979). Finally, the value of the systemic velocity, γ, was calculated by integrating the v_{rad} curve over the entire phase cycle, which is equivalent to demanding that $\Delta R(0, 1) = 0$ for phases 0 to 1.

3. ANALYSIS AND RESULTS

As in previous papers of this series, we employed a variation of the surface brightness method first introduced by Wesselink (1969) and further developed by Manduca & Bell (1981) and in Papers I–VI. Values of the photometric angular diameter, θ_{phot}, defined at a given phase ϕ by

$$\theta_{phot}(\phi) = \text{dex}\ \{0.2[S_0 - (m_\lambda + BC_\lambda) - 10 \log T_{eff}]\} \quad (1)$$

were determined from the observed dereddened K magnitudes and $V-K$ color values using the synthetic colors computed from the unpublished static model atmospheres of R. L. Kurucz. As in our previous analyses, we adopted $S_0 = 41.160$ mag (Paper I), applied a 0.240 mag zero-point shift to the synthetic visual bolometric corrections (Paper I), and applied a shift of -0.011 mag to the observed $V-K$ values of SW And to convert them to the Johnson system (Paper II). The color-temperature transformations needed in equation (1) were calculated at the adopted metallicity, -0.2 for both stars, using the values of the effective gravity, g_{eff}, computed at the appropriate phase by

$$g_{eff}(\phi) = g(\phi) + d^2r/dt^2 = GM/R^2 + d(v_{pul})/dt\ , \quad (2)$$

where M is the stellar mass, assumed to be 0.6 $M_\odot$, and v_{pul} is the pulsational velocity (the derived results are insensitive to the assumed metallicity and mass values due to the relative insensitivity of the $V-K$ index to metallicity and gravity effects). We also performed the transformations using the normal gravity, g, as a check on the gravity sensitivity. The radius, $R(\phi)$, was found from

$$R(\phi) = R(\phi_0) + \Delta R(\phi_0, \phi) = R(\phi_0) - \int_{\phi_0}^{\phi} p[v_{\rm rad}(\phi) - \gamma]P\,d\phi\ , \quad (3)$$

where P is the period in seconds, p is the conversion factor between radial and pulsational velocities, and ϕ_0 is an arbitrary reference phase. The spectroscopic angular diameter, $\theta_{\rm spec}$, was obtained from $R(\phi)$ by

$$\theta_{\rm spec}(\phi) = 2R(\phi)/d\ , \quad (4)$$

where d is the distance in the appropriate units.

An appropriate value of p is required to calculate the radii using equation (3). In our earlier papers, we adopted a value of 1.30 for p for CfA velocities, based on the resolution of the CfA spectrometers and the work of Parsons (1972). A p-value of 1.36 has been determined for the CORAVEL system (Burki, Mayor, & Benz 1982). As we showed in Paper V, these p-values are consistent with each other; the CORAVEL velocities appear to require a larger p-value than CfA measurements. LJ90 matched derived pulsational velocity curves to show that the CORAVEL velocities needed a larger p-value than did their velocities. We therefore employed p-values of 1.30 for SW And (CfA velocities) and 1.36 for DX Del (CORAVEL velocities) in equation (3).

In Paper VI, we outlined a procedure of matching the $\theta_{\rm spec}$ variation with that of $\theta_{\rm phot}$ that minimizes the influence of one curve on the other, since there may be systematic differences between the two data sets. We discuss it here in detail. Each angular diameter curve can be divided into three phase intervals: one ranging between ϕ_{m_1} and ϕ_{m_2}, which is roughly centered on maximum radius; an interval from ϕ_{e_1} to ϕ_{e_2}, which occurs during the expansion part of the curve; and an interval between ϕ_{c_1} and ϕ_{c_2}, which covers the contraction part of the cycle. The difference between the $\theta_{\rm spec}$ values of phase ϕ_m, which occurs during the maximum phase interval, and $\phi_{e,c}$, which lie in the expansion or the contraction intervals, can be found from equation (4) to be

$$\Delta\theta_{\rm spec}(\phi_{e,c}, \phi_m) = \theta_{\rm spec}(\phi_m) - \theta_{\rm spec}(\phi_{e,c}) = (2/d)[R(\phi_m) - R(\phi_{e,c})]\ . \quad (5)$$

Summing $\Delta\theta_{\rm spec}$ over all phases in all three phase intervals leads to

$$\text{Sum (spect)} = (2/d)\sum_{j=m_1}^{m_2}\left[\sum_{i=e_1}^{e_2}\Delta R(\phi_i, \phi_j) + \sum_{k=c_1}^{c_2}\Delta R(\phi_k, \phi_j)\right]. \quad (6)$$

This sum essentially measures the amplitude of the $\theta_{\rm spec}$ variation and is dependent upon the distance. An identical sum can also be generated for the photometric angular diameters:

$$\text{Sum (phot)} = \sum_{j=m_1}^{m_2}\left[\sum_{i=e_1}^{e_2}\Delta\theta_{\rm phot}(\phi_i, \phi_j) + \sum_{k=c_1}^{c_2}\Delta\theta_{\rm phot}(\phi_k, \phi_j)\right]. \quad (7)$$

Our procedure is to first estimate a value of $R(\phi_0)$, then compute the radii at other phases using equation (3), then the effective gravities from equation (2), then the $\theta_{\rm phot}$ values and the photometric sums using equations (1) and (7). Values of ΔR are determined from the integration of the radial velocity curve, and sets of spectroscopic sums are calculated using various values of the distance d and equation (6). The photometric and spectroscopic sums are compared to see which distance produces the best match; the value $R(\phi_0)$ is then adjusted, and photometric sums recomputed, until the $\theta_{\rm spec}$ curve derived at the distance of best match overlies that of $\theta_{\rm phot}$. Separate matches are performed for the expansion case (first terms only in eqs. [6] and [7]), the contraction case (second terms only) and the combined case (both terms). In practice, the interval endpoints are defined such that $\phi_{e_2} = \phi_{m_1}$ and $\phi_{c_1} = \phi_{m_2}$, so that the expansion case is equivalent to fitting the phase interval $\phi_{e_1} - \phi_{m_2}$, the contraction case $\phi_{m_1} - \phi_{c_2}$, and the combined case $\phi_{e_1} - \phi_{c_2}$. The sums in microarcseconds for SW And for all three cases are presented in Table 5 to indicate the sensitivity to distance of each case.

TABLE 5

SUMS FROM MATCHING PROCESS FOR SW AND

Type of Sum (Phases)	Expansion (0.05–0.50)	Contraction (0.30–0.65)	Combined (0.05–0.65)
Photometric	1661.8	532.1	2193.9
Spectroscopic $(m-M)_0$:			
8.37	1675.7	557.5	2233.2
8.39	1660.3	552.4	2212.7
8.41	1645.1	547.3	2192.4
8.43	1630.0	542.3	2172.3
8.45	1615.1	537.3	2152.4
8.47	1600.2	532.4	2132.6
$(m-M)_0$ of "match"	8.39	8.47	8.41

Figure 4 presents the angular diameter curves of SW And, while Figure 5 depicts that of DX Del. As before, we restricted the analysis to exclude the phase region from just before minimum radius to maximum optical light (phases 0.8–1.0), due to the probable presence of shock waves, strong accelerations and hence large changes in log $g_{\rm eff}$, possible changes in the conversion factor, p, due to rapid changes in $T_{\rm eff}$, and the probable breakdown of the assumption of quasi-static equilibrium implicit in the use of static model atmospheres (see the relevant discussions in Jones 1987, Cac89b, LJ90, and UK3). In addition, further phase restrictions were required for SW And and DX Del, as they have been for each star that we have analyzed previously. For SW And, there is a large gap in the K curve (Fig. 2) between phases 0.67 and 0.85. Although this region does not include an inflection point in the K photometry, so that an interpolation between the edges of the gap should be reasonably accurate, we have chosen to exclude phases beyond 0.65 in our analysis of this star to avoid any

TABLE 6

RESULTS FOR SW AND AND DX DEL

Star	Quantity	Expansion	Contraction	Combined
SW And	Phase interval	0.05–0.50	0.30–0.65	0.05–0.65
	$(m-M)_0$ (mag)	8.39	8.47	8.41
	$\langle R\rangle/R_\odot$	4.01	4.16	4.06
DX Del	Phase interval	0.00–0.45	0.30–0.75	0.00–0.75
	$(m-M)_0$ (mag)	9.00	9.07	9.03
	$\langle R\rangle/R_\odot$	4.97	5.16	5.07

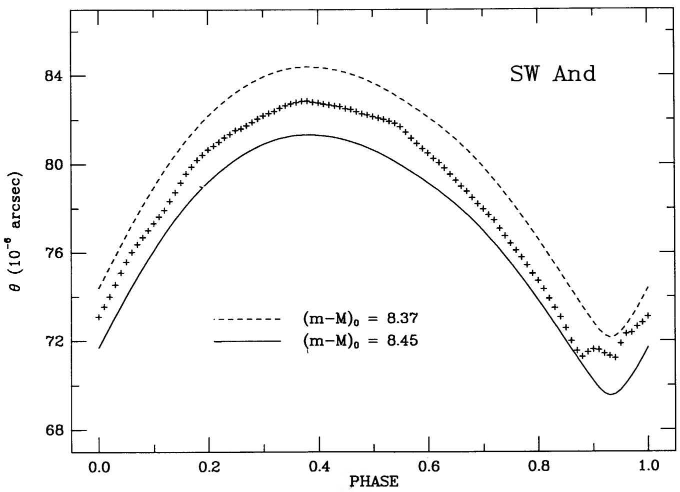

FIG. 4.—Angular diameters vs. phase for SW And. Symbols depict photometric angular diameters obtained from K, $V-K$, while lines represent spectroscopic diameters derived from the radial velocity data at the listed distances.

possible systematic effects such interpolations might produce. Also, SW And, like most of our other stars, shows an indication of a "bump" on its K curve at about phase 0.0. As discussed in Paper VI, the size of this "bump" seems to be correlated with the amplitude of the optical light curve, such that it is strongest for the large-amplitude stars RS Boo and TW Her and essentially nonexistent in the small-amplitude stars DX Del and VY Ser. This feature occurs in a phase region where the temperature undergoes a large variation in *ab*-type stars, and its prominence seems to correlate with the blue amplitude, which is an indicator of the temperature variation. Since there is no "bump" in the radius variation at this phase, this feature, unlike the rest of the K curve, is probably primarily produced by changes in temperature, not radius. In our previous analyses, we excluded the phase region 0.0–0.1 in the *ab*-type stars due to this effect, except for RS Boo, whose "bump" so dominated its K curve that the entire expansion phase region had to be excluded (Paper VI), and for VY Ser, where no phases needed to be excluded due to the absence of the "bump." The "bump" is weaker in SW And than in all of the other *ab*-type stars (except DX Del and VY Ser), so only the

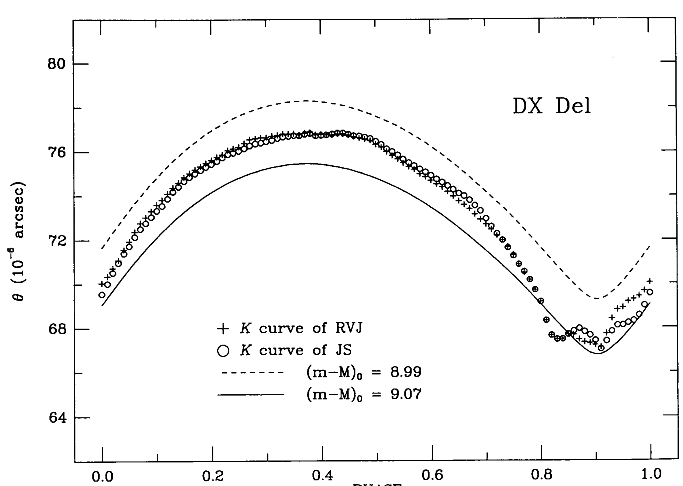

FIG. 5.—Same as Fig. 4 for DX Del. Different symbols depict $\theta_{\rm phot}$ values from the different K curves.

phase interval 0.00–0.05 was excluded. For DX Del, like VY Ser, no phases needed to be excluded due to this effect. Finally, the photometric angular diameter curves of DX Del show a pronounced "dip" near minimum radius; this "dip" may be associated with the large "hump" seen on the ascending branch of its optical light curve at about this phase (see Fig. 2 of Barnes et al. 1988). The phases beyond 0.75 were therefore excluded.

Table 6 presents the $(m-M)_0$ and $\langle R \rangle$ values derived for both stars in each of the three phase cases. The values of $\langle R \rangle$, the time-averaged radius, can be easily found from the reference value $R(\phi_0)$ using equation (3),

$$\langle R \rangle = \langle R(\phi_0) + \Delta R(\phi_0, \phi) \rangle$$
$$= R(\phi_0) - pP\left\langle \int_{\phi_0}^{\phi} [v_{\rm rad}(\phi) - \gamma] d\phi \right\rangle , \qquad (8)$$

where the average is taken over the entire pulsation cycle (that is, over all phase intervals, ϕ) and p is assumed to be constant. As before, we adopt the values obtained using the combined phase case for each star (0.05–0.65 for SW And, 0.00–0.75 for DX Del), since this covers the largest reliable phase interval. The other two cases, using more limited phase coverage, are needed to check the asymmetry of the angular diameters. As can be seen in Table 6, the distance moduli derived in the three phase intervals agree with each other within 0.10 mag for each star, indicating a lack of asymmetry in the θ-curves determined using the $V-K$ index. The angular diameters obtained from optical colors, on the other hand, show very large differences between the values derived from the expansion phase and those derived from the contraction phase due to the flux redistribution problem (Paper IV).

It should also be noted that the results listed for DX Del in Table 6 were obtained by combining the values from the two $\theta_{\rm phot}$ curves shown in Figure 5; these curves were derived from the two fitted K curves discussed in § 2.3. The values obtained in the combined 0.00–0.75 interval from these curves were in excellent agreement, differing only by 0.03 mag in the resultant distance modulus, which indicates that the uncertainty arising from our fits to the K data is probably on the order of 0.05 mag or less for our stars. Also, SW And was reanalyzed using the radial velocity curve of LJ89 to test for effects arising from the possible Blazhko effect in this star and the nonsimultaneity of our photometric and spectroscopic data. The derived $(m-M)_0$ agreed with that listed in Table 6 for the combined case to within 0.02 mag, indicating that the uncertainty due to the Blazhko effect, if present, is not very large for this star. Finally, the analysis was repeated for both stars using log g values instead of log $g_{\rm eff}$ in the color-temperature transformations. The derived distance moduli differed only on the order of 0.001 mag, confirming the gravity insensitivity of the $V-K$ results in these phase regions (essentially constant gravity).

We therefore adopt values of $(m-M)_0$ of 8.41 and 9.03 mag and $\langle R \rangle$ values of 4.06 and 5.07 $R_\odot$ for SW And and DX Del, respectively. Using these distance moduli and the intensity-averaged magnitudes of Table 4 yields $\langle M_V \rangle = \langle M_{\rm bol} \rangle = 1.10$ mag, $\langle M_K \rangle = +0.10$ mag for SW And and $\langle M_V \rangle = 0.68$ mag, $\langle M_{\rm bol} \rangle = 0.69$ mag, and $\langle M_K \rangle = -0.35$ mag for DX Del. Following the discussions in Paper VI, the final estimated uncertainty in each of these magnitudes is on the order of 0.15 mag.

4. COMPARISON WITH OTHER BAADE-WESSELINK RESULTS

We have established in our earlier papers that the use of optical color indices to derive $T_{\rm eff}$ values leads to distorted $\theta_{\rm phot}$

curves due to a flux redistribution during the expansion phase of the pulsation cycle. This problem could lead to systematic errors in Baade-Wesselink analyses which employ optical colors, particularly if phase shifts are utilized to "correct" the phasing mismatch between the photometric and spectroscopic angular diameters presumably arising from data acquired nonsimultaneously. As first demonstrated in Longmore et al. (1985), and also in UK1–UK4 and Papers II–VI, use of infrared fluxes or colors yield results which are essentially free of the flux redistribution problem. Recent work by our group, LJ90, and UK1–UK4 all employ infrared photometry, while that of Cac89a, b uses restricted phase intervals to avoid the expansion phase. It is of interest to see how the results of all four groups compare. Since different groups have adopted different reddenings for some of the stars in common, we compare only the derived $(m - M)_0$ and $\langle R \rangle$ values, which are relatively insensitive to the adopted reddening.

4.1. *The Surface Brightness Method of Cacciari et al.*

Six stars—the *ab*-type variables SW And, SW Dra, SS For, RV Phe, and V440 Sgr and the *c*-type star YZ Cap—were analyzed by Cac89a, b using the surface brightness method, the unpublished Kurucz models, and the Cousins $V-R$ and $V-I$ colors. We have independently analyzed two of their stars, SW And (this paper) and SW Dra (Paper III, revised in Paper VI). For SW And, Cac89b restricted the phase region of analysis to 0.40–0.70 to avoid the distorted $\theta_{\rm phot}$ region and obtained distances of 501 and 546 pc, corresponding to $(m - M)_0$ values of 8.50 and 8.69 mag, from the $V-R$ and $V-I$ colors, respectively. They combined these values to derive a distance of 523 pc, equivalent to a modulus of 8.60 mag, and a mean radius $\langle R \rangle$ of 4.25 $R_{\odot}$. Our adopted values from Table 6 are $(m - M)_0 = 8.41$ mag and $\langle R \rangle = 4.06\ R_{\odot}$. Using the results only from the contraction case (phases 0.30–0.65) yields $(m - M)_0 = 8.47$ mag and $\langle R \rangle = 4.16\ R_{\odot}$. Both sets of values agree with those from Cac89b within the uncertainties of 0.20 mag in the values of Cac89b and 0.15 mag in our values; there is particularly good agreement between the values from the $V-R$ color of Cac89b and the contraction phase of our analysis, and also between the values of Cac89b and LJ90 discussed below. (As noted in § 2.1, there is a problem with the optical photometry of SW And from Cac87, but it is not clear what effect, if any, this has on their results.) For SW Dra, Cac89b utilized a restricted phase interval of 0.50–0.80 to derive distances of 834 and 846 pc, corresponding to moduli of 9.61 and 9.64 mag, from $V-R$ and $V-I$, respectively. Their combined results yielded a distance of 840 pc, equivalent to a distance modulus of 9.62 mag, and an $\langle R \rangle$ value of 4.90 $R_{\odot}$. These values compare extremely well with our adopted values from Paper VI, $(m - M)_0 = 9.61$ mag and $\langle R \rangle = 4.89\ R_{\odot}$. This agreement seems to indicate that $V-R$ and $V-I$ colors can yield results consistent with those from $V-K$ provided that both are used and the phase intervals are restricted to avoid the expansion phase and also secondary "bumps," which presumably arise from shock waves (Gillet & Crowe 1988), during the contraction phase. Because of the more restricted phase interval, the uncertainties are generally larger, but the observations may be more easily acquired, particularly for cluster variables.

4.2. *The Surface Brightness Method of Liu and Janes*

LJ90 analyzed 13 stars using the surface brightness method, $V-K$ photometry, and the same unpublished Kurucz models that were employed by Cac89a, b and by our group. We have two stars in common with them, SW And and UU Vir (Paper VI). For SW And, LJ90 derived a distance of 511 pc, which results in a modulus of 8.54 mag, and a mean radius of 4.36 $R_{\odot}$, while we obtained 8.41 mag and 4.06 $R_{\odot}$. For UU Vir, their derived distance was 813 pc (erroneously listed as 913 pc), or $(m - M)_0 = 9.55$ mag, and $\langle R \rangle = 4.35\ R_{\odot}$. We obtained 9.67 mag, 4.60 $R_{\odot}$. The differences between their derived distance moduli and ours are thus $+0.13$ mag and -0.12 mag for SW And and UU Vir, respectively, which lie within the relative uncertainties of these values. The differences also suggest that there is no zero point shift between the two sets of results. However, we note that they used $S_0 = 42.19$ mag in equation (1), while we employed a value of 42.16 mag. This 0.03 mag difference propagates directly through the analysis, so that their derived distance moduli need to be increased by 0.03 mag, or ours need to be decreased by this amount, to bring the two sets of results onto the same surface brightness scale. This zero point shift means that the differences between their derived distance moduli and ours are then $+0.16$ mag and -0.09 mag for SW And and UU Vir, respectively. LJ90 used a phase region of 0.30–0.80 in their analysis; utilizing our contraction case results only (listed in Table 6) for SW And decreases the difference to ± 0.10 mag for this star. (The contraction case results for UU Vir are essentially the same as the combined case.) To summarize, the difference between our distance moduli and theirs is on the order of 0.10 mag, which is within the relative uncertainties of these values, and no zero-point difference other than in S_0 is apparent.

4.3. *The Infrared Flux Method of Fernley et al.*

4.3.1. *General Comparisons*

Fernley and collaborators (UK1–UK4) have studied six stars—the *ab*-type stars X Ari, DX Del, SS Leo, V445 Oph, and VY Ser and the *c*-type variable DH Peg—using a version of the Baade-Wesselink method based on the infrared flux method of Blackwell & Shallis (1977) to estimate temperatures. They derived $\langle M_V \rangle$ values of 0.47, 0.62, and 0.63 mag for X Ari, DH Peg, and VY Ser, respectively, compared to our values 0.74, 0.89, and 0.90 mag (Paper VI). At first glance, there appears to be a zero-point shift of 0.27 mag between the two sets of values. The situation becomes more complicated when DX Del is included; they derived $\langle M_V \rangle = 0.83$ mag (UK2), compared to our value of 0.68 mag. Comparing the derived distance moduli instead of M_V to minimize the effects due to differences in adopted reddening values shows that the differences between their adopted distances and ours are $+0.18$, -0.16, $+0.27$, and $+0.24$ mag for X Ari, DX Del, DH Peg, and VY Ser, respectively. These differences are significantly larger than those between our results and those of Cac89a, b and LJ90, so we examine below six possible causes: temperature scales; magnitudes employed; the conversion of radial velocities into pulsational velocities; number of data points; smoothing techniques; and phase intervals employed.

The differences are probably not due to the temperature scales. Both sets of results were obtained using temperatures derived from the use of Kurucz model atmospheres—the models of Kurucz (1979) in UK1–UK4, and the unpublished models in Papers I–VI. (The models employed for VY Ser in UK3 are unclear; UK3 states that fluxes from the MARCS program of Gustafsson et al. 1975 were used, but further state that the fluxes were tabulated in Table 10 of UK1. The fluxes in that table are labeled as being from Kurucz 1979, however.) The two sets of Kurucz models, the published ones used by the UK group, and the unpublished ones we have used, differ mainly in the treatment of convection. These differences are minor. UK1 have shown that the temperatures they derived for X Ari using the infrared flux method are essentially the same as we obtained from synthetic $V-K$ color indices.

Use of differing bandpasses do make a difference, however. In the extreme case, as we have shown (Paper IV), color indices utilizing blue magnitudes err seriously during the expansion phases. On a less significant level, UK1–UK4 derived their results using both the H and the K magnitudes, while we employed K only. An inspection of the temperature curves of UK1–UK4 reveals that the temperatures derived from the H magnitudes at minimum temperature are systematically larger than those obtained using the K magnitudes. Since the temperature values derived from various colors should be in good agreement at minimum temperature (Paper IV), this difference is perhaps due to systematic errors in the model atmosphere surface fluxes at either H or K. One way to test this is to utilize color-color diagrams of stars with known temperature, gravity, and metallicity values, as we did in Papers III and IV with optical colors. Until this is done, it is not certain which set of temperatures, if not both, are affected. We note that the H filter falls at that minimum of the H^- continuous opacity, so fluxes in this wavelength band may be more sensitive to gravity effects and hence less certain than those from K. Further, the continuum fluxes at the H band do not form at the same physical depth as the V and K fluxes. We consider this to be a serious potential problem in an expanding/contracting photosphere since the emergent flux may then originate in different atmospheric levels during different phases, leading to systematic errors. We therefore compare our results only with their K results. The differences in the derived $(m-M)_0$ values then become +0.10, −0.18, +0.18, and +0.21 mag for the four stars.

Another difference between the two sets of analyses is in the adopted conversion factor, p, between radial and pulsational velocities. The UK group adopted a p value of 1.33 for all of these stars, employing the same radial velocity curves as we did, while we utilized a value of 1.30 for the velocities from CfA and 1.36 for CORAVEL values. Repeating our analysis of the four stars, using a p value of 1.33 instead of our adopted values, reduces the differences between their K results and ours to +0.05, −0.13, +0.14, and +0.16 mag for X Ari, DX Del, DH Peg, and VY Ser, respectively.

The difference between the adopted values for X Ari, +0.05 mag when the above systematic effects are taken into account, appears to be well within the relative uncertainties of the two sets of analyses, indicating good agreement. The situation is more complicated, however, for if we restrict the comparisons to $\phi = 0.1$–0.7, our derived distance modulus is 8.39 mag, using a p value of 1.33 and the data of our Paper II (as corrected in our Paper VI), while the UK analysis of *the same data with the same p value* led to 8.54 mag. This 0.15 mag difference is larger than we have encountered in our comparisons with the results of LJ90. In the case of X Ari, the uncertainties should be larger, however, due to the smaller-than-average number of available K magnitudes. Another example is DH Peg, where our initial analysis similarly involved relatively few K magnitudes. Since DH Peg is a low-amplitude c-type variable, such a small sample can have a profound effect in principle. As an illustration, UK4 used our radial velocities and K data (Paper V) and obtained a distance modulus of 8.58 mag (using $\phi = 0.3$–0.8, which corresponds to $\phi = 0.1$–0.6 for the ephemeris given in Paper VI). UK4 derived a distance modulus of 8.21 mag, on the other hand, when they replaced our 23 K data points with their 32 K data points. Does this mean any given Baade-Wesselink analysis has a random uncertainty of about 0.2 mag? No. DH Peg is a c-type variable, so the effects of small samples are most pronounced. Further, the UK numerical fitting technique is very sensitive to such small data sets (compared to, for example, our use of smoothed curves—see below). For example, when UK4 doubled the phase coverage ($\phi = 0.0$–1.0) to double the number of points, we would have expected the difference in the distance modulus derived from the Paper V and UK4 K data sets to drop from 0.37 mag to perhaps 0.26 mag, if $\sqrt{N}$ statistics are valid. In fact, UK4 found a difference of only 0.13 mag. It is thus not simply a matter of data points, but the method of analysis. A detailed comparison of the results from UK4 and UK3 shows that when the number of H or K magnitudes in each data set exceeds 50, their numerical fitting technique yields differences in distance moduli smaller than 0.1 mag in all cases, so their technique is robust for such large samples.

A criticism of our previous results (and, by implication, those of LJ90) is our use of smoothed K light curves, and that such use might be responsible for differences between distance moduli we have obtained compared to those of the UK group. We disagree. *Both* smoothing and the numerical fitting employed by the UK group reduce the random uncertainties of the derived values, although at the risk of introducing systematic effects. Systematic effects between differing data sets, as in the photometric zero points, may be taken into account prior to analyses. It is true that biases introduced by smoothed curves are harder to ascertain, as in our K data fits. We argue the effects are minor: one need only look at our fits (e.g., Fig. 1). They are clearly as good as the data warrant. Further, consider again DH Peg, where the errors in the fitting have the greatest effects due to the low amplitude. As we have noted, UK4 derived distance moduli that differed by 0.37 mag when two (small) K data sets were employed. Their analysis using *all* 55 K data points in the phase interval $\phi = 0.3$–0.8 yielded a distance modulus of 8.38 mag for $p = 1.33$. When our (Paper V) hand-smoothed curve is used *in this phase interval and based upon only half as many data points*, and if we change our p value from 1.30 to 1.33, we find a distance modulus of 8.44 mag, in excellent agreement with UK4. It is only when UK4 use *their* numerical fitting technique *and* small samples that large uncertainties in the distance moduli arise.

To test this further, and especially to examine the importance of the phase intervals employed, we have studied our results for X Ari DX Del, and VY Ser but restricted the formal phase intervals for fitting to those employed by UK1–UK4. We emphasize that no changes have been made to our data except the ϕ values used in the equations discussed in § 3. Table 7 presents the derived values for the three ab-type stars in our adopted phase intervals (0.1–0.7 for X Ari, 0.0–0.75 for DX Del, and 0.0–0.4 for VY Ser) and also in the two phase intervals from UK1–UK4, the "constant gravity" and "full cycle" phase intervals, that contained the most data points in their analyses and also included phases on both sides of maximum radius. (Note that due to a typographical error, the adopted value of $\langle R \rangle$ for X Ari was incorrectly listed in Table 16 of Paper VI. It should have been 5.34 $R_\odot$, as given in Table 7.) The values from the combined K data sets of UK1–UK4 are also listed in Table 7. As can be seen, the distances we obtained using $p = 1.33$ agree with the corresponding UK values to within 0.10 mag for every selection of the phase interval, indicating that the two sets of analyses agree very well with each other when systematic effects, such as reddening, p value, magnitude employed, *and selection of phase intervals*, are taken into account. Of equal importance in Table 7 is the fact that our values show the same *trends with phase* as do those of the UK group. An examination of the results of UK1–UK4 shows that, with rare exceptions, the values derived using the H filter follow the same trends with phase as do those from K. This indicates that the differences between the distance moduli derived from different phase intervals arise from systematic effects. It can be seen, by examining the angular diameter curves of LJ90, Papers III–VI, and Figures 4–6 of this paper that the $\theta_{\rm phot}$ values tend to systematically deviate from the $\theta_{\rm spec}$ curves in the phase region from just before minimum radius through maximum optical light. It is probable that there are shock waves passing through the atmosphere during this phase region (Preston & Paczyński 1964; Gillet & Crowe 1988), which may lead to non–LTE effects and a breakdown of the assumption of quasi-static equilibrium required in the application of static model atmospheres to these dynamic atmospheres, as noted already. Since the UK results show the same trends as do our results, and since both sets of analyses rely upon static model atmospheres, both are vulnerable to these dynamical effects. An examination of Figure 6*a*, which illustrates the angular diameter curves of X Ari used in this analysis and in that of Paper VI (these curves were generated following the corrections applied to the data discussed in Paper VI and therefore supercede those of Paper II), and of Figure 6*b*, which replicates the angular diameters of DH Peg from Paper V, reveals that systematic deviations in $\theta_{\rm phot}$ also may be present in the phase region around 0.7–0.8, which may contain secondary shock waves (Gillet & Crowe 1988). It is clear that these phase regions should probably be excluded from Baade-Wesselink analyses until dynamic models are developed that can accurately produce $\theta_{\rm phot}$ values throughout the pulsation cycle.

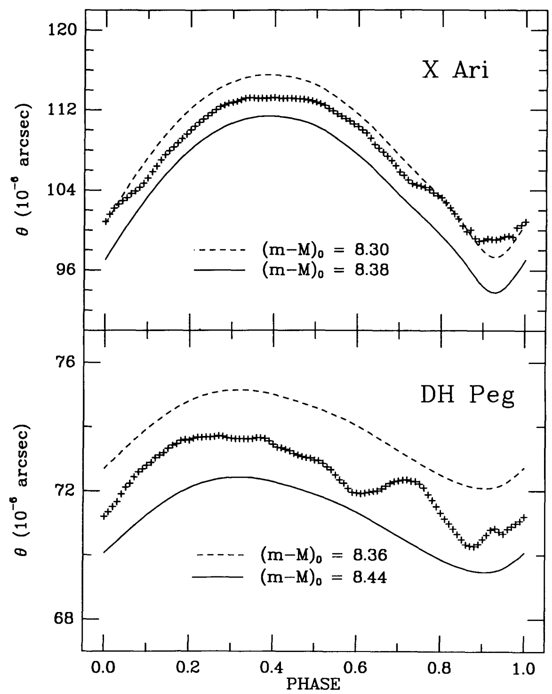

FIG. 6.—Same as Fig. 4, for (*a*) X Ari and (*b*) DH Peg

TABLE 7

RESULTS FROM DIFFERENT PHASE INTERVALS

Star	Group	p	Quantity	Adopted Interval	Constant Gravity	Full Cycle
X Ari	JCSL	1.30	$\Delta\phi$	0.10–0.70	0.10–0.85	0.00–1.00
			$(m - M)_0$ (mag)	8.34	8.44	8.56
			$\langle R\rangle/R_\odot$	5.34	5.59	5.93
	JCSL	1.33	...			
			$(m - M)_0$ (mag)	8.39	8.50	8.61
			$\langle R\rangle/R_\odot$	5.46	5.75	6.09
	UK	1.33	...			
			$(m - M)_0$ (mag)	...	8.41	8.54
			$\langle R\rangle/R_\odot$	...	5.70	6.09
DX Del	JCSL	1.36	$\Delta\phi$	0.00–0.75	0.10–0.10	0.00–1.00
			$(m - M)_0$ (mag)	9.03	9.00	8.91
			$\langle R\rangle/R_\odot$	5.07	4.99	4.76
	JCSL	1.33	...			
			$(m - M)_0$ (mag)	8.98	8.96	8.86
			$\langle R\rangle/R_\odot$	4.96	4.89	4.66
	UK	1.33	...			
			$(m - M)_0$ (mag)	...	8.95	8.77
			$\langle R\rangle/R_\odot$	...	5.03	4.62
VY Ser	JCSL	1.30	$\Delta\phi$	0.00–0.40	0.05–0.85	0.00–1.00
			$(m - M)_0$ (mag)	9.17	9.48	9.39
			$\langle R\rangle/R_\odot$	5.26	6.14	5.86
	JCSL	1.33	...			
			$(m - M)_0$ (mag)	9.22	9.53	9.43
			$\langle R\rangle/R_\odot$	5.38	6.27	5.99
	UK	1.33	...			
			$(m - M)_0$ (mag)	...	9.49	9.42
			$\langle R\rangle/R_\odot$	...	6.42	6.20

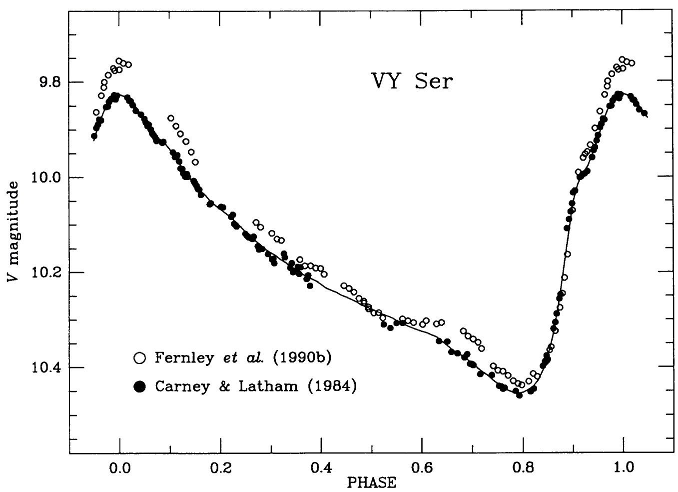

FIG. 7.—V magnitude vs. phase for VY Ser, obtained from the listed sources. Solid line represents the spline fit to the data of Paper I.

Thus we conclude that the published differences in distance moduli, radii, and absolute magnitudes obtained by the UK group and by us do not differ significantly, once the reddenings, projection factors, and magnitudes employed are taken into account. The differences are largely due to differing phase intervals employed. We argue that any phase interval where there are odd structures such as "bumps" in the radial velocity or visible/infrared light curves must be avoided. Such structures are likely caused by shocks and other nonthermal phenomena, and for which the existing model atmospheres cannot be expected to provide useful temperatures. This policy has been followed by ourselves, by LJ90, and by Cac89a, b.

4.3.2. *Special Cases: VY Ser and SS Leo*

Figure 7 illustrates the variation of the V magnitude with phase for VY Ser, using the data of Paper I (with phases corrected as described in Paper VI) and of UK3. The apparent zero point difference of 0.04 mag between the two data sets (UK3) has not been taken into account. It is clear from this figure that the amplitude of variation is different in the two data sets; that of Paper I has an amplitude of about 0.63 mag, while the data of UK3 varies with an amplitude of 0.68 mag in V and 0.88 mag in B. Lub (1977) lists a V amplitude of 0.292 (in log 10 units) for this star; converting this value to a magnitude yields 0.73 mag. The B amplitude of Lub's data is 0.98 mag (Sandage 1990b). Finally, an examination of Table 2 and Figure 2 of Varsavsky (1960) reveals an estimated V amplitude of 0.57 mag and a B amplitude of 0.73 mag (incorrectly listed as 0.76 mag in Table 12 of Paper VI) for VY Ser. This star thus apparently possesses a rather sizeable variation in amplitude. Since there is an indication that the V amplitude of *ab*-type RR Lyrae variables is correlated with the radial velocity amplitude (Paper VI; Liu 1991), the radial velocity curve may also vary. RS Boo, for example, a star that possesses the Blazhko effect, has been shown to have a variable v_{rad} curve as well (Paper VI). The variability of these curves could introduce systematic errors in Baade-Wesselink analyses which employ non-simultaneous photometry and spectroscopy of VY Ser. A further complication is revealed in Figure 11*c* of UK3; most of the θ_{phot} values lie below the fitted curve for phases less than 0.20 and above the curve for phases between 0.6 and 0.9. At first glance, this behavior suggests the possibility of a phase mismatch between θ_{phot} and θ_{spec}. An examination of Figure 12 of Paper VI also shows that the θ_{phot} values are larger during the contraction phase than the corresponding θ_{spec} values. Since the source of the asymmetric θ_{phot} curve may introduce systematic errors in the derived results, this is worth a closer look.

Figure 8 depicts the K magnitudes versus phase for VY Ser using the data of Paper VI and UK3. The latter group applied a shift of 0.02 mag to their data set to bring it into agreement with that of Paper VI; we have applied a further shift of 0.01 mag to their data so that the maximum and minimum values of the two data sets are in agreement. As can be seen in Figure 8, the two data sets appear to be significantly different in the phase region 0.4–0.8, reaching a difference of about 0.03 mag near phase 0.6. We note that the data from Paper VI in this phase region were obtained mainly on the night of 1987 March 27–28, when the conditions permitted differential photometry only. We note further that the conditions on that night required the aperture of the photometer to be opened from 15″ to 22″ at about phase 0.4. It is possible that a second star may have partially entered the beam during the observations, which would account for the brighter K values in this phase interval. On the other hand, the following night (1987 March 28–29) was completely photometric, and the aperture diameter was again set at 15″. The K magnitudes obtained during this night just after phase 0.7 also appear to be brighter and seem to follow the same trend as the data from phases 0.6–0.7. Thus, it is possible that VY Ser itself possesses a variable K curve in this phase region. To test the effects of the different K curves, we fitted a curve to the adjusted K data of UK3 only in this phase region. The new fitted curve and the original curve are both shown in Figure 8. The angular diameter curves derived from these two infrared curves are depicted in Figure 9; we note that the θ_{phot} curve from the original fitted K curve is the same as that in Figure 12 of Paper VI. The new θ_{phot} curve appears to be more symmetric than the old one. We performed matches using the illustrated θ curves, a p value of 1.30, and two phase intervals to compare the derived values from the two curves with each other. We present the derived $(m - M)_0$ in Table 8, including the results for the expansion and contraction cases (§ 3) as well as the combined phase intervals. As can be seen, the results from expansion and contraction cases in

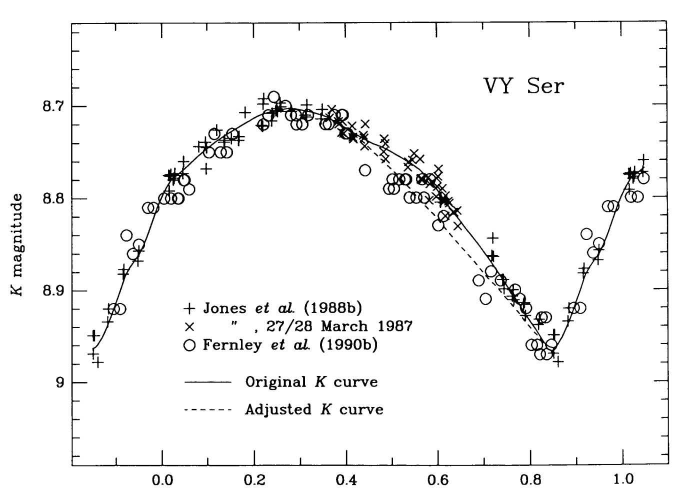

FIG. 8.—K magnitude vs. phase for VY Ser from the listed sources. Lines represent the curves fit to the data.

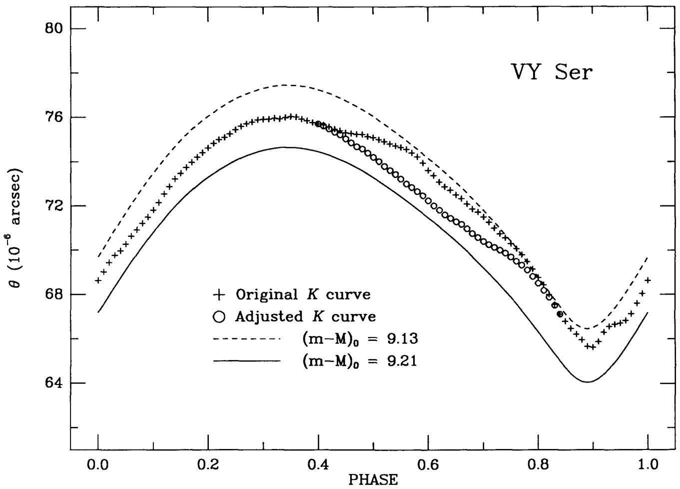

FIG. 9.—Same as Fig. 5 for VY Ser

each phase interval agree to within 0.05 mag for the adjusted curve, compared with the large difference obtained using the original curve. Since the results from UK3 agreed well with those obtained from the original K curve (Table 7), it is probable that their results derived using the K photometry of Paper VI may have been significantly affected by the systematic deviations in that data set. Since the distances derived from the adjusted K curve agree with our adopted value from Paper VI (listed also in Table 7) to within 0.05 mag, we continue to use our previously adopted values for this star, but factor in an additional uncertainty of 0.05 mag in the distance modulus to cover possible effects introduced by the differences between the K curves.

We have also performed a preliminary analysis for SS Leo, utilizing the data presented in UK3. Unfortunately, we experienced serious problems. First, the V data from the BV photometry of 1987 and the VRI photometry of 1988 appeared to be slightly different in form; the V curve from the BV photometry appeared to have a larger amplitude, by about 0.05 mag, and also possessed a more pronounced "bump" at about phase 0.72. Second, we were unable to match the θ_{phot} and θ_{spec} curves using the effective gravities to perform the color-temperature transformation because the derived radii were so large that g_{eff} values near maximum radius became less than zero for assumed stellar masses of less than 0.8 $M_\odot$. Matching performed using log g instead of log g_{eff} to derive θ_{phot} resulted in $(m-M)_0 = 11.02$ mag, $\langle M_V \rangle = 0.00$ mag, $\langle M_{bol} \rangle = -0.05$ mag, $\langle M_K \rangle = -1.10$ mag, and $\langle R \rangle = 7.32\ R_\odot$ for this star. We obtained the same results using the log g_{eff} values if we assumed a stellar mass of greater than 0.8 $M_\odot$. On the other hand, UK3 derived $\langle M_V \rangle = 0.29$ mag, $\langle M_K \rangle = -0.79$ mag, and $\langle R \rangle = 6.53\ R_\odot$ using four phase intervals and both the H and K magnitudes. We performed our analysis, using the K magnitude data only, over the same phase intervals and using the same adopted p value and reddening as they did; unfortunately, our work yielded distance moduli at least 0.2 mag larger than theirs in all cases. Even accounting for the possible use of a different set of model atmospheres by UK3, the difference in the $(m-M)_0$ values was still more than 0.15 mag in most cases. We find this to be perplexing, given the good agreement (within 0.10 mag in the same phase intervals) between our results and theirs for X Ari, DX Del, DH Peg, and VY Ser. We note that *if* the log g_{eff} values represent the appropriate gravity for this star, and *if* the g_{eff} values are near or below zero near maximum radius, then the use of static model atmospheres cannot give accurate results for this star because the static models cannot physically have gravities less than zero. In that case, neither our results, derived using log g, nor those of UK3, obtained using a constant log g of 2.7, may be valid. Clearly, this star warrants further investigation.

TABLE 8

VY SER DISTANCE MODULI FROM DIFFERENT K PHOTOMETRY

Total Phase Interval	Subinterval	Prior K Data	Adjusted K Data
0.05–0.85............	0.05–0.40	9.13	9.13
	0.30–0.85	9.58	9.18
	0.05–0.85	9.48	9.17
0.00–1.00............	0.00–0.40	9.17	9.17
	0.30–1.00	9.44	9.22
	0.00–1.00	9.39	9.21

In conclusion, a comparison of our results with those of LJ90, Cac89a, b, and UK1–UK4 shows that our results agree with theirs within 0.10 mag, except for SS Leo, when the proper phase intervals are compared and known systematic effects are taken into account. This good agreement between results derived by several groups using various techniques and photometric colors indicates that the Baade-Wesselink method of producing results with a *relative* uncertainty on the order of 0.10 mag or less, *provided* that the phase intervals are chosen to avoid the region of minimum radius to maximum optical light, and that infrared photometry, or optical colors with a very restricted phase interval, are used to compute temperatures and photometric angular diameters. In the case of optical colors, employment of more than one index, preferably red ones such as $V-R$, $V-I$, or $R-I$, is recommended due to small size of the acceptable phase region.

5. THE ABSOLUTE MAGNITUDE RELATIONS

5.1. *Adopted Values of the Program Stars*

The relationships between $\langle M_{bol}(RR) \rangle$ or $\langle M_V(RR) \rangle$ and [Fe/H] have important implications in determining the Galactic and Local Group distance scales, and the relative distances and ages of globular clusters (Sandage, Katem, & Sandage 1981; Sandage 1981; 1982; 1990a, b; Sandage & Cacciari 1990). We have therefore utilized the results from the most recent Baade-Wesselink work on field stars (this paper; Papers II–VI; LJ90; UK1–UK4; Cac89a, b) to rederive these relationships, and also to investigate the dependence of $\langle M_K(RR) \rangle$ on log P (Longmore, Fernley, & Jameson 1986; Paper VI; LJ90; Longmore et al. 1990). We have elected to use only the results obtained using the K magnitude and $V-K$ color for consistency, since there are many more such results and it is unclear what systematic effects might arise if we employ different magnitudes (e.g., H) or color indices (e.g., $V-R$). This restriction also excludes the results of Cac89a, b, based on optical color indices; we compare the resulting relationships with their values as a consistency check. Finally, we exclude the two highly reddened [$E(B-V) > 0.20$ mag] stars V445 Oph and AR Per, since the uncertainty in the adopted reddening values for each star is at least 0.05 mag, leading to an absolute magnitude uncertainty of 0.15 mag or more. This leaves us with a sample of 20 stars.

Before deriving the relationships, we first remove systematic differences between the data sets. As discussed in § 4.2, there is a 0.03 mag difference in the adopted value of S_0 between our analysis and that of LJ90. One way to remove this difference is by increasing the distances of LJ90 by 0.03 mag, which brightens the derived absolute magnitudes of their Table 8 by 0.03 mag. Alternatively, we could adjust our values to their scale by increasing our magnitude values by 0.03 mag. This would lead to relationships that have the same slopes as in the first case, but with zero points larger by 0.03 mag. We choose to adjust the LJ90 values, but include the zero point difference in the determination of the uncertainties. For X Ari, DX Del, and DH Peg, the values of UK1, UK2, and UK4 were derived by employing a p value of 1.33; we have chosen instead to adopt p values of 1.30 for CfA velocities (X Ari and DH Peg) and 1.36 for CORAVEL values (DX Del). Also, their derived values for these stars need to be adjusted to exclude the phase regions which contain systematic deviations in the $\theta_{\rm phot}$ values. For DX Del, we use their results derived in the 0.1–0.8 phase interval. UK2 obtained a distance modulus of 8.95 mag and a mean radius of 5.03 $R_\odot$ in this interval using $p = 1.33$. Inspection of the results of Table 7 shows that the difference between the values derived using $p = 1.36$ and those from a p value of 1.33 are 0.04 mag, 0.10 $R_\odot$ in this phase interval. Applying these adjustments to the UK2 values yields $(m - M)_0 = 8.99$ mag, $\langle R \rangle = 5.13$ $R_\odot$. Their adopted values for this star were $\langle M_V \rangle = \langle M_{\rm bol} \rangle = 0.83$ mag, $\langle M_K \rangle = -0.19$ mag at a distance corresponding to a modulus of 8.87 mag. Adjusting these values to a distance of 8.99 mag results in a value of 0.71 mag for the optical magnitudes and -0.31 for $\langle M_K \rangle$. For DH Peg, we consider their results from the combined K data set and their phase interval 0.3–0.8, $(m - M)_0 = 8.38$ mag and $\langle R \rangle = 3.87$ $R_\odot$. Adjusting these values to a p factor of 1.30 involves applying shifts of -0.04 mag and -0.08 $R_\odot$ to their values, yielding a distance modulus of 8.34 mag and an $\langle R \rangle$ value of 3.79 $R_\odot$. UK4 adopted values of 0.62 mag for $\langle M_V \rangle$, 0.65 mag for $\langle M_{\rm bol} \rangle$, and -0.08 mag for $\langle M_K \rangle$ at a distance modulus of 8.67 mag. Adjusting these magnitudes to the new modulus of 8.34 yields $\langle M_V \rangle = 0.95$ mag, $\langle M_{\rm bol} \rangle = 0.98$ mag, and $\langle M_K \rangle = +0.25$ mag for this star.

The situation is more difficult for X Ari, since UK1 did not derive values from the combined K data set using a phase interval which was free of the systematic effects, as discussed in § 4.3.1. Therefore, the appropriate adjusted values must be determined indirectly. Examining Table 7, we note that the difference between their values and ours derived with $p = 1.33$ are -0.09 mag, $-0.05/R_\odot$ for the 0.10–0.85 phase interval and -0.07 mag, 0.00 $R_\odot$ for the full cycle case. Averaging these values leads to values of -0.08 mag, -0.03 $R_\odot$ as the differences between their results from the combined data set and ours for equal p factors and phase intervals. Applying these differences to our adopted values in the phase interval 0.1–0.7, with $p = 1.30$, yields $(m - M)_0 = 8.26$ mag, $\langle R \rangle = 5.31$ $R_\odot$ as our estimate of their adjusted values. They adopted $\langle M_V \rangle = 0.47$ mag, $\langle M_{\rm bol} \rangle = 0.41$ mag, and $\langle M_K \rangle = -0.63$ mag at a distance modulus of 8.52 mag. Adjusting these magnitudes to the new modulus results in values of 0.73 mag for $\langle M_V \rangle$, 0.67 mag for $\langle M_{\rm bol} \rangle$, and -0.37 mag for $\langle M_K \rangle$. Finally, due to their use of optical data sets with different amplitudes, K data sets that differ systematically at some phases (§ 4.3.2), and possibly due to a different set of model atmospheres, which may result in systematic differences of unknown size, we do not attempt to use their results for VY Ser. We rely instead on our previous (Paper VI) results.

Table 9 presents the adopted values for the 20 program stars from Paper VI, LJ90, UK3, and this paper, corrected as discussed above. Stars analyzed by more than one group have average values given. We list the adopted values of UK3 for SS Leo along with our own preliminary results; these values are too different to be simply averaged together. The metallicities listed are those of Paper VI and LJ90—the values for SW And and UU Vir are the averages of the values from both sources—with two exceptions. First, the metallicity of DH Peg has been

TABLE 9

Adopted Values of Program Stars

Star	[Fe/H]	log P_0	A_B	$\langle M_V \rangle$	$\langle M_{\rm bol} \rangle$	$\langle M_K \rangle$	σ	$\langle R \rangle / R_\odot$
				RR*ab*				
Sw And	−0.15	−0.35432	1.27	1.020	+1.030	+0.010	0.10	4.210
X Ari	−2.20	−0.18631	1.26	0.735	+0.660	−0.410	0.10	5.330
RS Boo	−0.50	−0.42327	1.65	0.950	+0.940	+0.100	0.13	3.980
RR Cet	−1.25	−0.25725	1.21	0.750	+0.710	−0.350	0.12	5.020
DX Del	−0.20	−0.32549	0.97	0.695	+0.700	−0.330	0.10	5.100
SU Dra	−1.60	−0.18018	1.26	0.700	+0.650	−0.410	0.12	5.150
SW Dra	−1.40	−0.24438	1.22	0.780	+0.730	−0.280	0.12	4.890
RX Eri	−1.40	−0.23118	1.14	0.730	+0.670	−0.440	0.12	5.300
RR Gem	−0.20	−0.40087	1.62	0.960	+0.980	+0.050	0.12	4.050
TW Her	−0.50	−0.39837	1.69	0.900	+0.890	+0.030	0.12	4.160
RR Leo	−1.15	−0.34449	1.64	0.830	+0.800	−0.090	0.12	4.350
SS Leo	−1.51	−0.20319	1.49	0.000	−0.050	−1.100	...	7.320
				0.290	...	−0.790	...	6.530
TT Lyn	−1.35	−0.22371	0.92	0.720	+0.660	−0.490	0.12	5.400
AV Peg	0.00	−0.40852	1.41	1.170	+1.200	+0.210	0.12	3.800
VY Ser	−1.80	−0.14624	0.88	0.900	+0.800	−0.390	0.13	5.260
TU UMa	−1.25	−0.25365	1.22	0.770	+0.730	−0.340	0.12	5.000
UU Vir	−0.55	−0.32275	1.50	0.885	+0.875	−0.115	0.09	4.475
				RR*c*				
TV Boo	−2.20	−0.3836	...	0.650	+0.620	−0.130	0.15	4.330
DH Peg	−0.90	−0.4723	0.64	0.920	+0.940	+0.220	0.12	3.760
T Sex	−1.20	−0.3649	0.53	0.730	+0.740	+0.000	0.15	4.050

changed from the value listed in Paper VI, -0.80, which was based on the value of Butler (1975), to -0.90, which was found by averaging the values of Butler (1975) and Kemper (1982). The value for SW Dra has also been significantly changed. In Papers III and VI, we adopted [Fe/H] = -0.8 for this star, based on the ΔS value of 3.8 listed in Butler (1975). A preliminary abundance analysis of this star by Jones (1987) indicated that this star may be as metal-poor as -1.5, based on a single echelle spectrogram obtained with the Kitt Peak 4 m telescope. More recently, Smith (1990) reanalyzed this star and determined a ΔS value of 7.1, corresponding to a metallicity of -1.37 using the relation of Butler (1975). We have therefore adopted a new metallicity of -1.40. Due to the insensitivity of the $V-K$ index to metallicity, our derived $(m-M)_0$ and $\langle R \rangle$ values, and hence $\langle M_V \rangle$ and $\langle M_K \rangle$, do not change. However, due to the metallicity dependence of the bolometric corrections, $\langle M_{\rm bol} \rangle$ is 0.02 mag brighter at the new metallicity.

Also listed in Table 9 are the relative uncertainties, σ, of the absolute magnitudes of each star. These values were obtained by noting that the estimates for the random uncertainty in the derived magnitudes of each *ab*-type star ranges from 0.08 mag to 0.15 mag (Table 16 of Paper VI; Table 9 of LJ90; Table 14 of UK3), and that the estimated uncertainties for *c*-type variables are generally higher than the values for RR*ab* stars. We have therefore adopted an uncertainty value of 0.12 mag for RR*ab* stars and 0.15 mag for RR*c* stars that were analyzed by only one group. For UU Vir, the estimated internal uncertainty is smaller, 0.09 mag, since the values listed in Table 9 are the averages of results derived independently using different data sets. For SW And, X Ari, DX Del, and DH Peg, the Table 9 values are also averages of results derived independently, but with some or all of the subsets of the data (for example, radial velocities) being used by more than one group. Slightly higher uncertainties of 0.10 mag for the three RR*ab* stars and 0.12 mag for DH Peg are adopted for these "semi-independent" values. For RS Boo and VY Ser, an extra uncertainty of 0.05 mag has been factored in for each star due to the Blazhko effect in RS Boo (Paper VI) and the problems with the photometry for VY Ser (§ 4.3.2). This 0.05 mag has been added in quadrature to the 0.12 mag values for single-valued RR*ab* stars to yield 0.13 mag. No uncertainty value was estimated for SS Leo due to the large differences in the derived values.

Finally, the log P values listed in Table 9 are that of the fundamental mode of oscillation, log P_0. For the first overtone RR*c* stars TV Boo, DH Peg, and T Sex, their observed first-overtone periods, log P_1, had to be "fundamentalized" to log P_0. This was accomplished by utilizing equations (2) and (3) of van Albada & Baker (1971),

$$\log P_0 = -1.772 - 0.68 \log (M/M_\odot) + 0.84 \log (L/L_\odot) + 3.48 \log (6500/T_{\rm eff}) \quad (9)$$

$$\log (P_0/P_1) = 0.095 - 0.032 \log (M/M_\odot) + 0.014 \log (L/L_\odot) + 0.09 \log (6500/T_{\rm eff}) , \quad (10)$$

to derive

$$\log P_0 = 1.027 \log P_1 + 0.145 - 0.015 \log (M/M_\odot) - 0.0079 \log (L/L_\odot) . \quad (11)$$

Values of log P_0 were computed for the three RR*c* variables using this equation, the derived $\langle M_{\rm bol} \rangle$ values of each star, and an assumed mass of 0.6 $M_\odot$. We note that the resulting "fundamentalized" periods are only weakly dependent upon the adopted mass and luminosity; changing the mass by 0.1 $M_\odot$ or the log L by 0.1 (equivalent to a change in $\langle M_{\rm bol} \rangle$ of 0.25 mag) changes the computed log P_0 by at most 0.001.

5.2. *Evolutionary Status of the Program Stars*

In Paper VI, we estimated absolute magnitudes for seven field RR Lyrae stars. More recently, the UK group (UK1–UK4) and Cacciari et al. (1989a, b) have done the same for six stars each, some of which are in common with our work, as we have noted. Results for absolute magnitude versus metallicity relations from any one of these small samples are, of course, vulnerable to serious distortion due to just one abnormal object. The evolutionary models (see Lee, Demarque, & Zinn 1990, for example) predict that as a horizontal branch star evolves, it becomes brighter, and that the brightest stages are also the shortest lived. Thus a metal-poor (-rich) star that began core helium burning on the blue (red) side of the instability strip might evolve into the instability strip, becoming an RR Lyrae variable, albeit only briefly. Such a star would be significantly brighter than stars near the ZAHB and in the instability strip. We seek to avoid inclusion of such stars in our final results so that we may determine more reliable distances to ensembles of RR Lyrae stars in globular clusters or in other galaxies, where we can be more certain about stars' evolutionary states since RR Lyrae–rich populations presumably involve near–ZAHB stars. There are two procedures at our disposal.

First, we may rely upon the results of the Baade-Wesselink analyses themselves, identifying as evolved stars those few that are significantly brighter than the mean relations defined by the full sample. SS Leo and DX Del are abnormally bright, compared to stars of the same metallicity, and therefore candidates for highly evolved status. While we believe the Baade-Wesselink results of this paper are especially good in star-to-star absolute magnitude comparisons, and thus capable of identifying correctly the rare, abnormally luminous RR Lyrae stars, a second method, and one that is independent of models and reddening, is also available for RR*ab* stars.

Sandage (1981) relied upon the blue amplitude, A_B, to estimate the temperatures of RR Lyrae stars. Differences in the periods of such stars at fixed A_B (temperature) then implied differences in luminosities, according to the van Albada & Baker (1971) theory discussed above. Here A_B has the virtue, as does log P, of being reddening-independent. Of course, the transformation of A_B into temperature is a problem, one bound up with a variety of effects which we will discuss in the following paper, but at a fixed metallicity, it is likely that *relative* A_B values are reliable indicators of *relative* temperatures. We therefore can in principle identify evolved *field* RR Lyrae stars by comparing them in an A_B versus log P plot with numerous, presumably only slightly evolved, *cluster* variables of similar metallicity. We begin at the lowest metallicities, those with [Fe/H] ≤ -2.0. We use for comparison the RR Lyrae stars in M15 (NGC 7078), with data taken from Table 6 of Sandage (1990a). In Figure 10 we show these data, along with a point representing X Ari. With [Fe/H] = -2.2 for this star, compared to -2.15 for M15 (Zinn 1985), this is an appropriate comparison. We see that X Ari appears to be similar to typical M15 variables, and thus is unlikely to be significantly evolved.

In Figure 11 we show similar data for the RR Lyrae–rich cluster M3 (NGC 5272), whose metallicity is [Fe/H] = -1.6

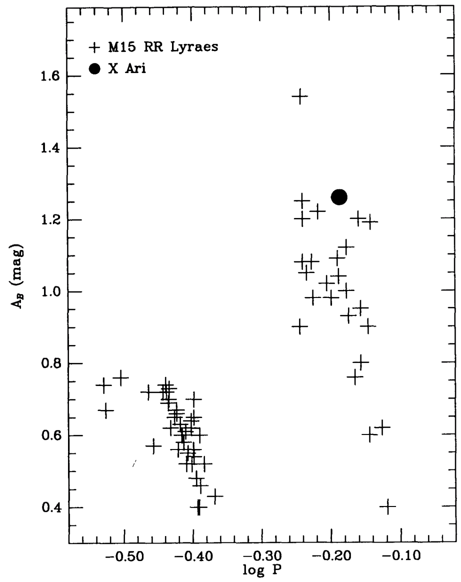

FIG. 10.—*B* amplitude vs. log *P* for stars with [Fe/H] ≤ −2.0 as listed

(Zinn 1985). The data are from Table 5 of Sandage (1990a). We plot in this figure the points for the field RR*ab* variables with [Fe/H] = −1.15 to −1.8. We include also the only RR*ab* variable, V8, in M13 (NGC 6205), whose metallicity is very similar to that of M3. In this case we expect such a rare star to be in a short-lived, presumably highly evolved and luminous, state. It certainly has a much longer period than M3 variables of similar blue amplitude, hence temperature. This confirms our assumption that it is overly luminous. Among the field stars, three are seen to deviate also, although not by as great a degree as M13-V8: SS Leo, SU Dra, and VY Ser. Given that more metal-poor clusters generally have longer periods at equal A_B compared to more metal-rich clusters (compare Figs. 10 and 11, for example), the deviation of VY Ser from the mean locus of the more metal-rich M3 variables is not unexpected. (As we have noted also, its amplitude may be variable, so its exact location in Fig. 11 is uncertain.) SU Dra and SS Leo are, however, apparently evolved, and since SS Leo is more metal-rich than M3, its deviation to longer periods suggests an even greater degree of evolution and greater luminosity than average. We conclude that any results from SU Dra should be treated with some caution, but that SS Leo should probably be excluded from any fits to mean absolute magnitude versus metallicity. This confirms our earlier analyses that this star is unusually bright and of greater-than-average radius. Note further that RR Cet, SW Dra, and TU UMa fall in essentially the same place in Figure 11, and that according to Table 9, they have very similar metallicities. Thus they should have very similar M_{bol} values, and in fact Table 9 shows the Baade-Wesselink analyses yield agreement to within ±0.02 mag.

For the more metal-rich stars we are handicapped by the lack of any suitable comparison clusters. Metal-rich globular clusters simply do not have large numbers of RR Lyrae stars, unlike the field ("Preston's Conundrum"—see Kraft 1972; Preston 1959). Here we must rely solely upon field variables, and in Figure 12 we plot all stars with [Fe/H] ≥ −0.6 from Paper VI, UK3, and LJ90, including the highly reddened variables AR Per and V445 Oph, and supplemented with two metal-rich field stars (FW Lup and RW TrA) from the Table 3 of Sandage (1990b). Solid figures represent the more metal-poor variables, while open figures represent the more metal-rich ones. There are no obvious deviants, at least when one notes that the outlier UU Vir is also the most metal-poor star. It is possible that the more metal-rich star DX Del deviates, as Sandage (1990b) has noted already based on his period shift analysis. Lacking clear guidance from Figure 12, we assume our Baade-Wesselink analyses are correct and that the star is anomalously bright (compare with and RR Gem), presumably due to its advanced evolutionary state.

5.3. *The Final Absolute Magnitude Relations*

We exclude the highly evolved stars DX Del and SS Leo from the derivation of the magnitude relations. Utilizing the values of the remaining 18 stars listed in Table 9, we derive the following relationship using least squares fitting weighted by

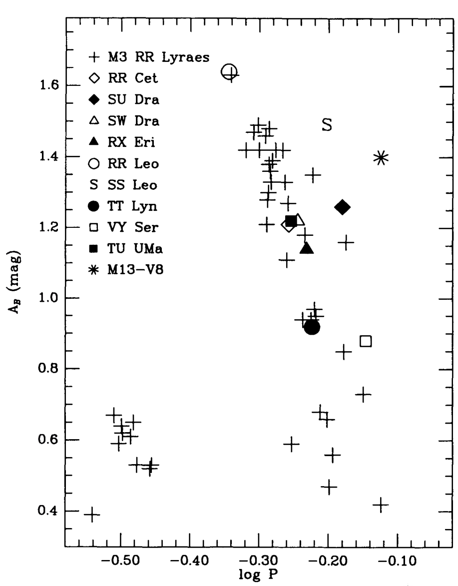

FIG. 11.—Same as Fig. 10 for stars with −1.1 > [Fe/H] ≥ −1.8

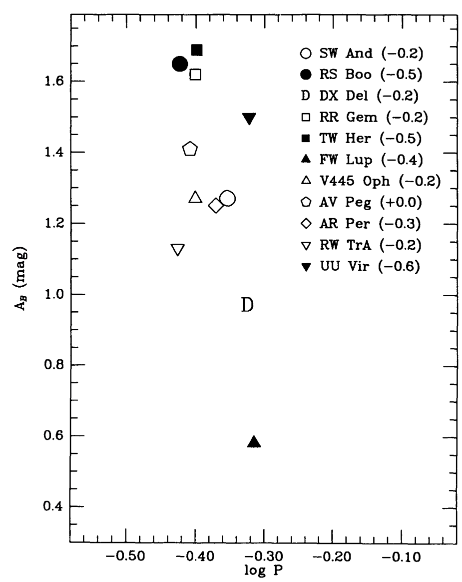

FIG. 12.—Same as Fig. 10 for stars with [Fe/H] ≥ −0.6

the σ-values:

$$\langle M_V(\mathrm{RR})\rangle = 0.16(\pm 0.03)[\mathrm{Fe/H}] + 1.02(\pm 0.03)\,, \quad (12)$$

$$\langle M_{\mathrm{bol}}(\mathrm{RR})\rangle = 0.21(\pm 0.03)[\mathrm{Fe/H}] + 1.04(\pm 0.03)\,, \quad (13)$$

$$\langle M_K(\mathrm{RR})\rangle = -2.33(\pm 0.20)\log P - 0.88(\pm 0.06)\,. \quad (14)$$

The quoted uncertainties are internal error estimates only. The rms values of these three equations are 0.07 mag each, and the correlation coefficients, r, have the values 0.84, 0.89, and -0.95, respectively. We note that equation (12) is in good agreement with the M_V–[Fe/H] relations of Liu & Janes (1991) and Sandage & Cacciari (1990), who obtained slopes of 0.20 ± 0.06 and 0.19 (no error bar), respectively. This good agreement is not surprising, since all three relations were derived using data sets consisting primarily of values from LJ90 and Paper VI. Both Liu & Janes (1991) and Sandage & Cacciari (1990) also included the restricted phase optical results of Cac89a, b, the highly reddened stars AR Per and V445 Oph (incorrectly listed as V455 Oph by both groups), and preliminary values from the UK group in their analyses, while we employed the more recent results of UK1–UK4 and restricted the data set to infrared results from low to moderately reddened stars only. We also removed known systematic differences between the values from the various sources; an inspection of Table 1 of Liu & Janes (1991) shows that they applied a zero point shift of +0.03 mag to the absolute magnitudes of Paper VI to account for the difference in adopted S_0 values discussed in § 4.1 (hence the zero points of their equations need to be decreased by 0.03 mag, or ours increased by 0.03 mag, for both sets to be on the same surface brightness scale), but otherwise, neither of the other two considered the systematic differences. It has been our intention in this paper to remove such systematics so that the true errors of the method may be better understood, and that different results obtained by different groups for the same star may be understood as differences in zero points or phase coverage, and not problems inherent with the Baade-Wesselink method itself.

Longmore et al. (1990) has suggested that the $\langle M_K(\mathrm{RR})\rangle$–log P relation may have a metallicity term. Performing a weighted multivariable least-squares fit yields

$$\langle M_K(\mathrm{RR})\rangle = -2.03(\pm 0.27)\log P + 0.06(\pm 0.04)[\mathrm{Fe/H}] - 0.72(\pm 0.11)\,, \quad (15)$$

with an rms value of 0.07 mag and $r^2 = 0.91$. The uncertainties in the coefficients of equation (15) are larger than the corresponding values in equation (14), and the uncertainty in the metallicity term relative to the coefficient is particularly large, suggesting that the inclusion of a metallicity term may yield an overparameterized system. On the other hand, the value of the metallicity coefficient, 0.06, is the same as in equation (7) of Longmore et al. (1990), suggesting that the metallicity dependence, though relatively weak, may be real.

As noted earlier, the zero points in the above equations would all increase by 0.03 mag if we were to adopt the value of S_0 of LJ90. The quoted errors in the zero points do not include such systematic uncertainties. In Paper V we derived an estimate of 0.13 mag as the net uncertainty due to the combined estimated uncertainties of the color-temperature transformation and the conversion factor, p, which are the main sources of systematic errors. This value is comparable to the estimated value from LJ90 and UK3, 0.12 mag. Additional sources of systematic uncertainties not covered in Paper V are that of the constant S_0 in equation (1), the photometric zero points, and possible systematic effects in the results derived using the K magnitude as compared to those from other magnitudes. For the uncertainty in the S_0 value, we adopt the difference between our value and that of LJ90, 0.03 mag. For the photometry, which includes the calibration of both the optical and the infrared data, we would expect the uncertainty to be less than 0.02 mag, but note that the V values for the comparison stars of VY Ser differed by 0.04 mag between Paper I and UK3. We adopt a conservative value of 0.03 mag for this uncertainty. Finally, a detailed comparison of the results of UK1–UK4 reveals that the distance moduli derived using the H magnitude are, on average, 0.07 mag larger than those obtained using K. We conservatively adopt an uncertainty of 0.05 mag for this effect. Combining these three systematic uncertainties in quadrature with the previous value, 0.13 mag, yields an estimated systematic uncertainty of 0.15 mag, which, when added in quadrature with the internal uncertainties of the above zero points, results in estimates of the true zero point uncertainty of 0.15, 0.15, 0.16, and 0.18 mag for equations (12)–(15), respectively.

We will discuss in detail the implications of these magnitude relations and those derived using other techniques in the following paper (Carney, Storm, & Jones 1992, Paper VIII). For now, we compare the results of Cac89a, b with equations (12) and (13). Cac89a, b analyzed six variables varying in metallicity from -0.2 to -1.5. For SW And, with $[\mathrm{Fe/H}] = -0.2$, Cac89b derived $\langle M_V\rangle = \langle M_{\mathrm{bol}}\rangle = 0.88$ mag; equations (12) and (13) predict 0.99 and 1.00 mag for $\langle M_V\rangle$ and $\langle M_{\mathrm{bol}}\rangle$, respectively. These results agree within the errors, but the Cac89b results may have been affected by the photometric problems discussed in § 2.1. For YZ Cap, with $[\mathrm{Fe/H}] = -1.3$, Cac89a obtained $\langle M_V\rangle = \langle M_{\mathrm{bol}}\rangle = +0.80$ mag. The predicted values are 0.81 and 0.77 mag for $\langle M_V\rangle$ and $\langle M_{\mathrm{bol}}\rangle$, in excellent agreement. SW Dra (listed by Cac89b at a metallicity of -0.8), and V440 Sgr both have $[\mathrm{Fe/H}] = -1.4$. $\langle M_V\rangle$ values of 0.86 and 0.72 mag, respectively, were derived by Cac89a, b for these two stars, yielding an average of 0.79 mag, while the corresponding bolometric magnitudes they derived were 0.81 and 0.69 mag, giving an average of 0.75 mag. Equations (12) and (13) predict values of 0.80 and 0.75 mag, again in excellent agreement. Finally, there are two stars with $[\mathrm{Fe/H}] = -1.5$, SS For and RV Phe. Their derived absolute magnitudes are 0.73, 0.77 mag for $\langle M_V\rangle$, and 0.67, 0.69 mag for $\langle M_{\mathrm{bol}}\rangle$, yielding averages of 0.75 and 0.68 mag. These values also compare favorably with the predicted values, 0.78 and 0.73 mag. As can be seen, the results from Cac89a, b, with the exception of SW And, agree with equations (12)–(13) to within 0.10 mag for the individual stars, well within the relative uncertainties. This agreement indicates that the use of restricted phase optical colors such as $V-R$ and $V-I$ can yield results consistent with those from $V-K$ provided that more than one color is utilized for each star and that the phases are restricted to exclude the expansion phase as well as any secondary "bumps" present in the contraction phase, as was done in Cac89a, b.

As noted in Paper VI, LJ90, and Longmore et al. (1990), a relationship between $\langle M_K\rangle$ and log P could be very useful in obtaining distances to globular clusters, and perhaps to nearby galaxies, due to the relative insensitivity of $\langle M_K\rangle$ to reddening effects. A possible complication in the universality of equation (14) is the fact that the two highly evolved stars, DX Del and SS Leo, apparently do not follow this relation. Using the periods listed in Table 9 for these two stars, equation (14) predicts $\langle M_K \rangle$ values of -0.12 and -0.41 mag for DX Del and SS Leo, respectively. The derived value for DX Del, -0.33 mag, is more than 0.2 mag brighter than the predicted value, while for SS Leo, the derived values listed in Table 9 are 0.38 mag and 0.69 mag brighter than that predicted from equation (14). The use of equation (15) yields $M_K = -0.07$ and -0.40 mag for DX Del and SS Leo, which are even further from the observed values. These results suggest that the "universal" $\langle M_K \rangle$–log P relations may not apply to highly evolved stars, so that these relations probably should not be applied to clusters, such as M13, which contain few, and probably highly evolved, variables. We note that none of the above sources claimed the theoretical or observed relations would apply necessarily to highly evolved variables.

In conclusion, the most recent applications of the Baade-Wesselink method yield results which agree with others, to within 0.10 mag in most cases, when systematic effects, such as the conversion factor, p, are taken into account and when the phase intervals are restricted to exclude regions containing shock waves and probable non–LTE effects. We have used the results from three groups to derive relationships between the intensity-averaged absolute magnitudes and the metallicity, and also between the infrared magnitude $\langle M_K \rangle$ and log P. The implications of these relations will be examined further in Paper VIII.

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