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THE TOPOLOGY OF LARGE-SCALE STRUCTURE. V. TWO-DIMENSIONAL TOPOLOGY OF SKY MAPS

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ABSTRACT

In a recent series of papers we have developed and tested algorithms for quantitatively measuring the topology of large-scale structure of the universe. In this paper, we apply our two-dimensional algorithm to observed sky maps and numerical simulations. We find that when topology is studied on smoothing scales larger than the correlation length, the topology is approximately in agreement with the random phase formula for the two-dimensional genus-threshold density relation, $G_2(\nu) \propto \nu e^{-\nu^2/2}$. Some samples show small "meatball shifts" similar to those seen in corresponding three-dimensional observational samples and similar to those produced by biasing in cold dark matter simulations. The observational results are thus consistent with the standard model in which the structure in the universe today has grown from small fluctuations caused by random quantum noise in the early universe. In all cases (both observations and simulations), the two-dimensional results are in good agreement with the three-dimensional results.

Subject headings: galaxies: clustering — large-scale structure of the universe

1. INTRODUCTION

Gott, Melott, & Dickinson (1986, hereafter GMD) pointed out that it is possible to quantitatively measure the topology of large-scale structure of the universe, and they suggested an algorithm for doing this. In a recent series of papers we have extended this approach. (See Hamilton, Gott, & Weinberg 1986, hereafter HGW; Weinberg, Gott, & Melott 1987; Melott, Weinberg, & Gott 1988; Gott, Weinberg, & Melott 1987, for detailed discussions.) In the three-dimensional case, Gott et al. (1989) have demonstrated that at scales significantly larger than the correlation length of galaxies, the topology is spongelike, consistent with the random quantum noise scenario while at scales comparable to the correlation length, they find slight meatball shifts which are also observed in biased CDM models.

Melott et al. (1989, hereafter MCHGW) have pointed out that topological information can also be derived from two-dimensional density fields. Gott et al. (1990) have applied this approach to microwave background fluctuations.

In this paper, we apply this technique to observational two-dimensional angular galaxy catalogs and demonstrate that the results are consistent with the standard model, and in good agreement with the three-dimensional results. The UGC and ESO catalogs we analyze here are particularly important because they cover regions (and to similar depths) for which we have three-dimensional data (Gott et al. 1989, and Gott 1991) and thus we can show that the three- and two-dimensional methods give consistent results in these regions. This allows one to use the two-dimensional methods with confidence on other data sets. The two-dimensional techniques for analyzing sky maps developed here can be used in the future on much deeper data sets like the Maddox et al. (1990) two million galaxy sample for which complete reshift data is not yet available, thus allowing one to investigate the topology statistically over very large volumes of space.

In § 2 we briefly describe our methods. In § 3 we present the observational and simulation results. In § 4 we discuss these results further and summarize our conclusions.

2. METHOD

Our principal tool has been the genus-threshold relation. In this section we briefly review relevant results, with emphasis on the two-dimensional techniques.

In three-dimensions, we use a Gaussian convolution to turn a discrete galaxy distribution into a smooth density field. We then study isodensity contour surfaces by measuring the genus of these surfaces as a function of contour threshold densities. For a simulation with periodic boundary conditions or a sample drawn from a large universe, an appropriate definition of the three-dimensional genus is

$$G_3 \equiv (\text{number of holes}) - (\text{number of isolated regions}) , \quad (1)$$

where holes mean holes like a donut has. According to the Gauss-Bonnet theorem the genus as defined in equation (1) is proportional to the integral of the Gaussian curvature $K \equiv (r_1 r_2)^{-1}$ over the contour surfaces

$$G_3 = -\frac{\int K\,dA}{4\pi} , \quad (2)$$

where r_1 and r_2 are the two principal radii of curvature. For a Gaussian random field, the average genus per unit volume is (HGW; see also Adler 1981 and Bardeen, Steinhardt, & Turner 1986)

$$g_3 = \frac{1}{(2\pi)^2}\left(\frac{\langle k^2 \rangle}{3}\right)^{3/2}(1 - \nu^2)e^{-\nu^2/2} , \quad (3)$$

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where v is the threshold density in units of standard deviations from the mean and $\langle k^2 \rangle$ is the square of the wave number k averaged over the smoothed power spectrum. The curve is symmetric in v and its shape is the same for all random phase distributions (note: a symmetric genus curve does not necessarily imply the distribution itself is Gaussian. However, it does indicate that it is a distribution which could have grown from Gaussian initial conditions by gravitational instability).

A two-dimensional genus is defined as

$$G_2 = (\text{number of isolated high-density regions}) - (\text{number of isolated low-density regions})\,. \quad (4)$$

[The two-dimensional equivalent of eq. (1) would be $G_2 =$ (number of holes) − (number of isolated regions), but since an isolated low-density region is a hole in the high-density region, we have $G_2 =$ (number of isolated low-density regions) − (number of isolated high-density regions), G_2 is thus equal to the three-dimensional genus a two-dimensional slab of finite thickness would have if the high-density regions were solid and the low-density regions were empty space. Then we simply change the sign of G_2 (purely a sign convention) so that eq. (4) is obtained. This sign convention avoids a minus sign in eq. (6) below. See MCHGW for details.]

The genus of isodensity contours in a two-dimensional density distribution can be measured from a two-dimensional analog of the Gauss-Bonnet theorem (Gott et al. 1990)

$$G_2 = \frac{\int C\,ds}{2\pi}\,, \quad (5)$$

where the line integral follows a contour and C is the curvature of the line, negative or positive depending on whether a low- or high-density region is enclosed. Equations (4) and (5) provide a simple way to characterize and measure the topology of two-dimensional distributions, by measuring the turning angles taken by the contour at pixel vertices in a discretized version of the density field in exact analogy with the three-dimensional procedure of GMD.

The genus per unit area of a two-dimensional Gaussian random field is (MCHGW; see also Coles 1988 and Gott et al. 1990 for related two-dimensional topology measures)

$$g_2 = \frac{1}{(2\pi)^{3/2}} \frac{\langle k^2 \rangle}{2} v e^{-v^2/2}\,. \quad (6)$$

In principle when we know the power spectrum $P_3(k)$ of a three-dimensional sample, we can calculate the two-dimensional power spectrum $P_2(k)$ of the two-dimensional density field of a slice constructed from the three-dimensional field. For a Poisson or white noise density distribution,

$$\langle k^2 \rangle = \frac{2}{\lambda_l^2}\,, \quad (7)$$

where λ_l is the two-dimensional smoothing length with a Gaussian window function $W(r) \propto e^{-r^2/\lambda_l^2}$. Simple analytic expressions exist for $P_3(k) \propto k^n$ only in the limit where the smoothing length is much greater than or much less than, the slice half-thickness z_0 (MCHGW). For $\lambda_l \gg z_0$ (thin slice)

$$\langle k^2 \rangle = \begin{cases} \dfrac{n+3}{\lambda_l^2} & (-3 < n < -1) \\ \dfrac{2}{\lambda_l^2} & (n \geq -1)\,. \end{cases} \quad (8)$$

and for $\lambda_l \ll z_0$ (thick slice)

$$\langle k^2 \rangle = \begin{cases} 0 & (-3 < n \leq -2) \\ \dfrac{n+2}{\lambda_l^2} & (n > -2)\,. \end{cases} \quad (9)$$

For sky maps, where we are seeing galaxies in projection, the thick slice approximation is useful.

3. ANALYSIS AND RESULTS

In dealing with sky maps we shall use the techniques developed by Gott et al. (1990) for measuring the topology of microwave background fluctuations. First we smooth the galaxy distribution in the observed samples with a Gaussian smoothing function, $W(r) \propto e^{-r^2/\lambda^2}$, where r is the angular distance on the celestial sphere and λ is the angular smoothing length. The size of pixels used to define the density field is chosen so that $\lambda \geq 2.5$ pixels. In this case, no corrections for finite pixel size need to be applied (HGW). After we find the smoothed density distribution, we use the methods outlined in MCHGW and Gott et al. (1990) to calculate the genus. We then apply the statistical bootstrap procedure used by Gott et al. (1989) in their three-dimensional genus analysis to estimate errors in the measured genus.

3.1. *UGC Catalog*

The UGC sample (Nilson 1973) covers the declination range north of $\delta = -2^\circ\!.5$ and includes galaxies with angular diameters $D \geq 1'$. We use an angular smoothing length of 5° and a corresponding pixel size of 2°. We then select 6449 galaxies with $b > 29^\circ$, $\delta > -2^\circ\!.5$ and $D \geq 1'$. The active survey region area A is 2.52 steradians. The mean angular separation between galaxies is $d \approx 1^\circ\!.13$. The angular correlation length θ_0, the place where $w(\theta_0) = 1$, is $\theta_0 \approx 0^\circ\!.32$ (Lahav, Nemiroff, & Prian 1990, hereafter LNP). The mean depth of the survey is about $D_* \approx 70h^{-1}$ Mpc (Hudson 1990). The three-dimensional correlation length for galaxies is $r_0 \approx 6h^{-1}$ Mpc [$\xi(r_0) = 1$], with a corresponding angular scale $\theta_* = r_0/D_* = 4^\circ\!.9$. The quantity θ_* is always greater than θ_0 because projections add a random background which lowers the amplitude of the density fluctuations and thus suppresses the $w(\theta_*)$ below unity. For a model with random phase initial conditions we expect approximately random phase results today in three-dimensions providing we smooth on scales larger than r_0 (Gott et al. 1989), because we are looking at fluctuations that are still basically in the linear regime. Thus in a two-dimensional study we might expect to obtain approximately random phase results providing we choose $\lambda > \max(d, \theta_*) = 4^\circ\!.9$. Thus we have adopted $\lambda = 5^\circ$ to satisfy these constraints and yet provide the maximum number of resolution elements in the map. Also $\lambda \sim 5^\circ$ corresponds roughly to a physical scale of $\sim 6h^{-1}$ Mpc which is approximately the smoothing length we used in our three-dimensional analysis (Gott et al. 1989) so that we may easily compare the two-dimensional and the three-dimensional results.

Figure 1 shows the median (*thick solid line*), 16% low (*dashed line*), and 16% high (*thin solid line*) density contours by area fraction, the galaxies are plotted in a Lambert azimuthal equal area projection centered on the galactic pole. The density contours trace the structures very well. The 16% high contour traces the most prominent features—the Virgo cluster, and background Coma supercluster and the Ursa Major cluster. The median contour nicely encloses the supergalactic plane.

FIG. 1.—Contour plot for UGC catalog ($\delta > -2°.5$, $b > 29°$). The galaxies are plotted in a Lambert azimuthal equal area projection centered on the galactic pole. The thick solid line is the median density contour ($\nu = 0$), the thin solid line is the 16% high-density contour ($\nu = 1$), the dashed line is the 16% low density contour ($\nu = -1$) by area fraction. *All the contour curves in this paper will be plotted in similar fashion.* The smoothing length is $\lambda = 5°$.

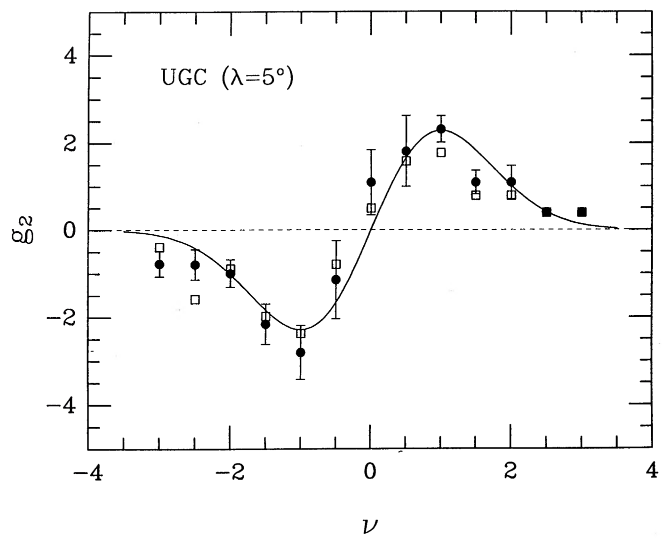

FIG. 2.—Genus curve for the UGC catalog ($\delta > -2°.5$, $b > 29°$) plotted with the best-fit random phase theoretical curve $g_2(\nu) \propto \nu e^{-\nu^2/2}$. The open squares are obtained from simply taking the raw data and assuming exactly average density in the universe beyond the observation survey region. The mean values (shown as filled circles), and the error bars are computed from the statistical bootstrap technique. In 15 statistical bootstrap runs we take the observed galaxies and throw them back in a Poisson fashion at their original positions (cf. Gott et al. 1989), and in the region exterior to the sample we put a Poisson distribution of galaxies with the average density. Not only do the bootstrap runs allow us to compute error bars due to sampling, but the means are less subject to boundary effects resulting from picking a single uniform external density. *All the genus curves in this paper will be plotted in similar fashion unless explicitly stated;* $\lambda = 5°$.

We compute the raw genus curve by simply taking the raw data and assuming exactly average density beyond the observation region. Next we use statistical bootstrap technique which was discussed extensively in Gott et al. (1989) to calculate the mean genus values and corresponding errors. As can be seen in Figure 2. The genus curve shown is approximately random phase but shows a small shift to the left (a meatball shift)—note that the median density contour ($\nu = 0$) has a genus significantly above zero. Having the genus curve shifted slightly to the left (meatball shift) indicates a slight prominence of isolated clusters over isolated voids and in this sample is provided by the prominence of the large Virgo cluster, Coma supercluster complex.

The UGC sample covers approximately the same region as the three-dimensional CfA sample (Huchra et al. 1983). Gott et al. (1989) found that the genus curve for the CfA sample is approximately random phase with a slight meatball shift. Ryden et al. (1989) also arrived at the same result using the one-dimensional level-crossing statistics. Thus the three-, two-, and one-dimensional methods applied to the same region give consistent results.

3.2. *ESO Catalog*

ESO sample (Lauberts 1982) covers the sky with $\delta < -17°.5$. 6650 galaxies are selected with $b < -29°$, $\delta < -17°.5$ and $D \geq 1'$. The active survey region area A is 1.79 steradians. The mean angular separation between galaxies is $d = 0°.93$ and the angular correlation length is $\theta_0 = 0°.23$ (LNP). The mean depth of the ESO sample is $D_* \approx 80h^{-1}$ Mpc (Hudson 1990). The three-dimensional correlation length for galaxies is $r_0 \approx 6h^{-1}$ Mpc. So $\theta_* = r_0/D_* \approx 4°.3$. Thus for the standard model we might expect to find approximately random phase results provided that $\lambda > \max(d, \theta_*) = 4°.3$. The difference between the values of d, θ_0, and D_* for the UGC and ESO samples is in accord with claims that the ESO sample is about 15% deeper (e.g., Lahav, Rowan-Robinson, & Lynden-Bell 1988; Lynden-Bell, Lahav, & Burstein 1989). The ESO sample is deeper than the UGC sample because it was done on a higher sensitivity film (so the ESO angular diameter limit of 1′ is thus not equivalent to the UGC angular diameter limit of 1′), and therefore it has a different selection function. In Figure 3 we show the median, 16% low and 16% high-density contours. It is obvious that the density distribution of the ESO catalog is smoother than the UGC catalog. The corresponding genus curve shown in Figure 4 is approximately random phase—note the median density contour ($\nu = 0$) has a genus that is consistent with zero.

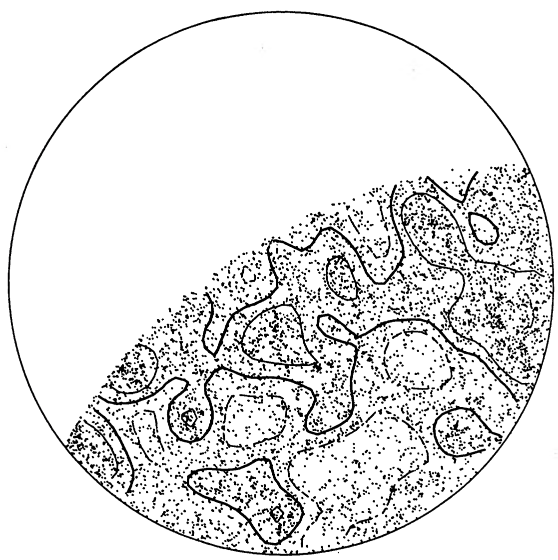

FIG. 3.—Contour plot for the ESO catalog ($\delta < -17°.5$, $b < -29°$); $\lambda = 5°$

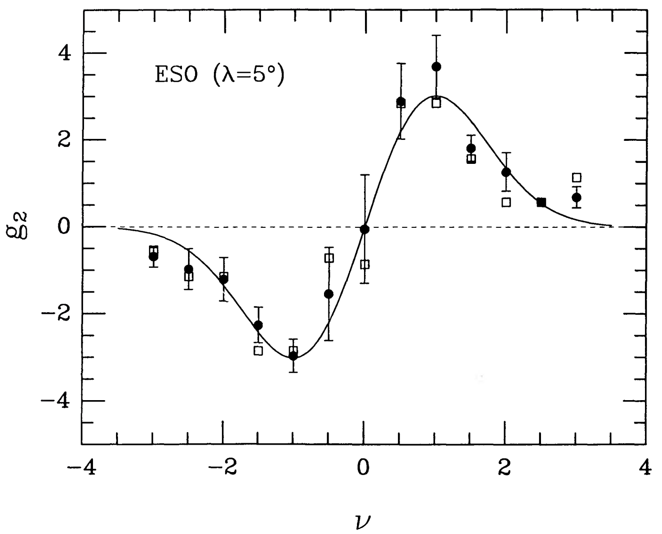

FIG. 4.—Genus curve for the ESO catalog ($\delta < -17^\circ\!.5$, $b < -29°$); $\lambda = 5°$

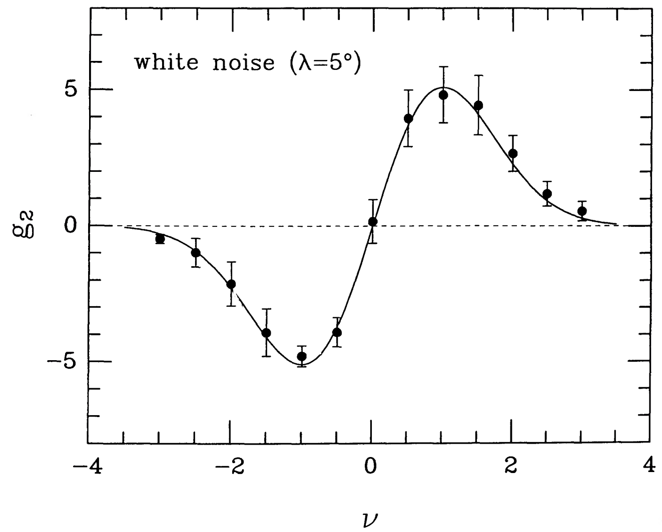

FIG. 5.—Genus curve for the white noise simulation plotted with best-fit random phase theoretical curve (*solid line*). The mean values are shown as filled circles; $\lambda = 5°$.

Thus the ESO survey region appears to be more nearly random phase than the UGC region. A three-dimensional topology study (cf. Gott 1991) of the Southern Sky Redshift Survey (SSRS), out to a distance of $50h^{-1}$ Mpc, drawn from the ESO survey, shows a rather accurately random phase three-dimensional genus curve, with a smoothing length of $\lambda = 6.5h^{-1}$ Mpc. This smoothing length corresponds well to our effective smoothing length of $\lambda = 7h^{-1}$ Mpc when we use $\lambda = 5°$ in our $80h^{-1}$ Mpc deep sample. (For further discussion of the three-dimensional topology of the SSRS sample at other smoothing lengths, some of which show slight meatball shifts, see Park et al. 1991.) Thus again the three- and two-dimensional results are in good agreement.

The UGC sample shows a meatball shift because of the prominence of the Virgo cluster and Coma supercluster in the sample. Note that the "Great Attractor" region (see, e.g., Fig. 8 of Lynden-Bell et al. 1988) is not included in our ESO sample since we restricted the sample to $b < -29°$. Gott et al. (1990) showed that for three-dimensional samples of this general size the biased CDM model simulations produced three-dimensional genus curves which were either random phase or showed slight meatball shifts. An analysis by Park & Gott (1991) and Park (1990b) shows that the slight meatball shifts seen in the biased CDM model at the present epoch are largely due to a nonlinear interaction of statistical biasing in the galaxy distribution with gravitational evolution. In the next section we will compare our two-dimensional results with simulations.

3.3. *Simulations*

In order to understand the topology of observed distribution of galaxies shown by our two-dimensional genus curves, we make comparisons with simulated skies with similar selection effects. We randomly choose an observer's location and his "North Pole," and make a redshift catalog of "galaxies." In these surveys we use a selection function from the Southern Sky Redshift Survey (SSRS), whose diameter limit is $1'\!.26$ (Park et al. 1991), scaled so as to match the 1′ diameter limit of the ESO sample we are simulating, i.e., the selection function $\rho_s(r)$ as defined in Gott et al. (1989) is drawn from the known selection function of SSRS $\rho_{s,\mathrm{SSRS}}(r)$ by the equation $\rho_s(r) = \rho_{s,\mathrm{SSRS}}(r/1.26)$. Furthermore, the selection function is truncated at the distance of $307.2h^{-1}$ Mpc which is the size of the simulation cubes. We use a sky coverage corresponding to that of the UGC catalog to give us a larger active survey region.

First we consider a toy model where galaxies are distributed randomly in three-dimensional space. For a Poisson field smoothed with a Gaussian window function the expected genus per steradian is, from equations (6) and (7)

$$g_2 = \frac{1}{(2\pi)^{3/2}\lambda^2}\,\nu e^{-\nu^2/2}\,, \qquad (10)$$

which gives $g_2 = 5.1$ at $\nu = 1$ with a smoothing length $\lambda = 5°$. This theoretical value is in excellent agreement with the genus per steradian at $\nu = 1$ of the best-fit random phase genus curve shown as a solid curve in Figure 5. The measured genus curve is also very close to the random phase curve. This justifies our

FIG. 6.—Contour plot for a biased CDM simulation; $\lambda = 5°$

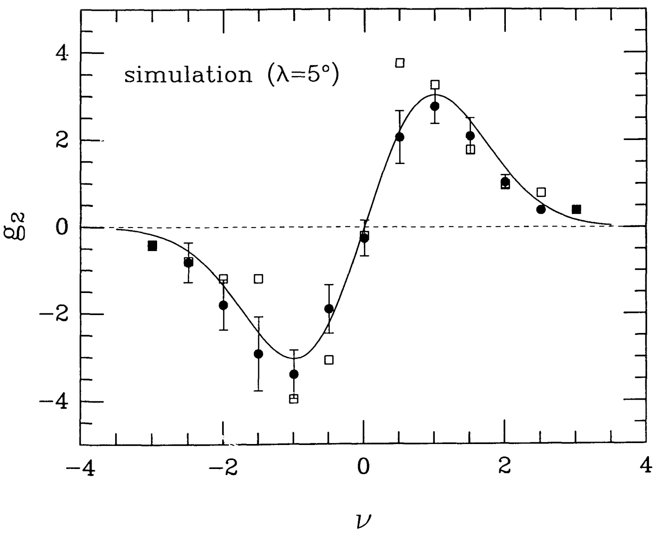

FIG. 7a

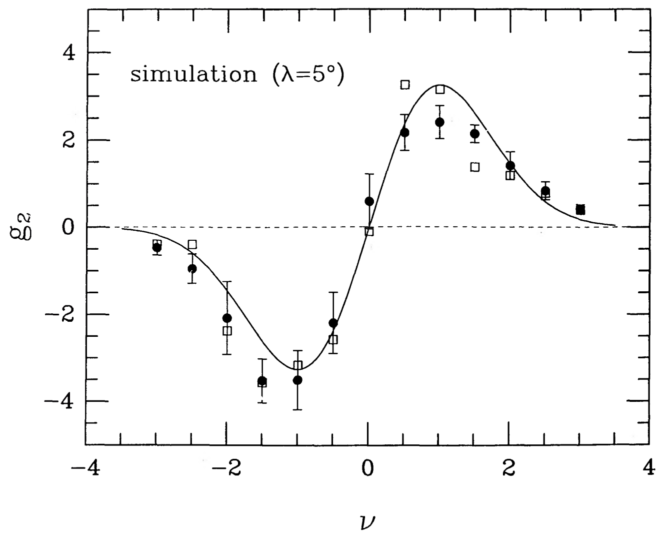

FIG. 7b

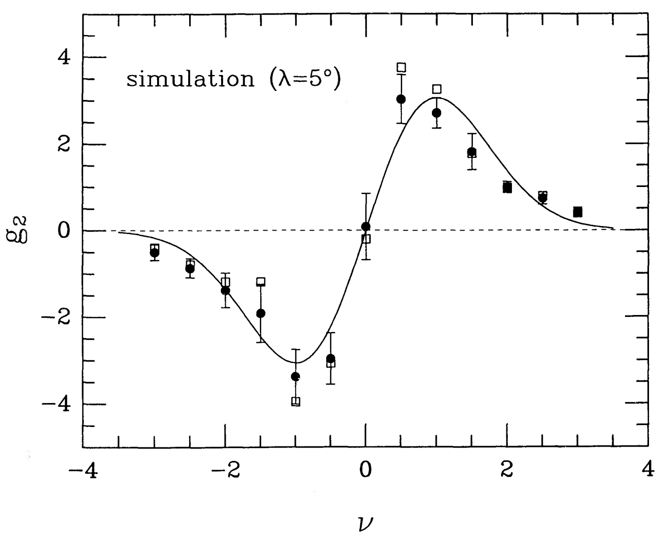

FIG. 7c

FIG. 7.—(a)–(c) Genus curves for three biased CDM simulations; $\lambda = 5°$. Fig. 7a shows the genus curve for the sample shown in Fig. 6.

choice of the smoothing length $\lambda > \max(d, \theta_0, \theta_*)$, i.e., the smoothed density field is recovered without discreteness effects.

We next study simulated skies of "galaxies" which are gravitationally evolved from an initially random Gaussian distribution. We locate an observer randomly in each of three $\Omega = 1$ CDM simulations of Park (1990a) and observe the biased "galaxy" particles as described in Park (1990a). The biased particles are allocated as sites of galaxy formation in the beginning of the simulations and evolved with massive CDM particles. This standard biasing scheme has been extensively studied in Park (1991). One of the simulated skies mimicking the ESO catalog in depth but with the larger sky coverage of the UGC catalog is shown in Figure 6. The catalogs from the simulations are treated in the same way as the observations: 15 bootstrap replacement runs are made to estimate the uncertainty in the genus. Figure 7 shows the results for the three catalogs. The genus curves are all approximately random phase but one of the three (Fig. 7b) shows a slight meatball shift. So again the simulations and the observations are consistent—we have two observational samples: one is random phase and one shows a slight meatball shift; we have three simulations: two are random phase and one shows a slight meatball shift. Gott et al. (1989) found that in the three-dimensional case, for observational samples of this size CDM models were either random phase or showed small meatball shifts. So again the two- and three-dimensional results are in good agreement.

4. DISCUSSION

In addition to evaluating whether or not the topology is random phase, we can also use our two-dimensional genus curves coupled with the thick slice approximation to estimate the effective index of the power spectrum $n_{\rm eff}$.

From equations (6) and (9), we find that genus amplitude per steradian

$$A = \frac{1}{(2\pi)^{3/2}} \frac{\langle k^2 \rangle}{2} = \frac{1}{(2\pi)^{3/2}} \frac{(n_{\rm eff} + 2)}{2\lambda^2} . \tag{11}$$

Therefore

$$n_{\rm eff} = 2A\lambda^2(2\pi)^{3/2} - 2 . \tag{12}$$

We find the amplitude A by using best least-squares fitting curve ($g_2 = A\nu e^{-\nu^2/2}$). Then we calculate the effective power index by using equation (12).

For the white noise sample, we find $n_{\rm eff} = 0.08$ which is in good agreement with the theoretical value ($n = 0$).

For the UGC and ESO catalogs, we find $n_{\rm eff} = -1.1$ and $n_{\rm eff} = -0.81$, respectively. Most of the uncertainty in measuring $n_{\rm eff}$ comes from the difference in the two samples. Thus, the observed power index at this smoothing scale is roughly $n_{\rm eff} \approx -0.96 \pm 0.15$. For the three evolved CDM simulations, shown in Figures 7a–7c matched in depth to the ESO survey, we find $n_{\rm eff} = -0.80$, -0.72, and -0.80, respectively, in good agreement with the value of the ESO observational sample. Since the ESO sample is somewhat deeper than the UGC sample, with the $\lambda = 5°$ smoothing length, we are looking at somewhat larger scales (approximately $7h^{-1}$ Mpc instead of $6h^{-1}$ Mpc), and with a CDM power spectrum $n_{\rm eff}$ slowly increases with increasing scale. From our three-dimensional theoretical formulae, for the CDM model ($\Omega = 1$, $h = 0.5$) we find $n_{\rm eff} = -1.02$ at a smoothing scale of $6h^{-1}$ Mpc increasing to $n_{\rm eff} = -0.92$ at a smoothing scale of $7h^{-1}$ Mpc. These values are quite consistent with the results obtained from the UGC and ESO catalogs, respectively.

In conclusion, we have applied our two-dimensional topology measuring algorithm to two important catalogs of galaxies and to numerical simulations. We find that in general the topology is approximately in agreement with the random phase formula $G_2(\nu) \propto \nu e^{-\nu^2/2}$. This is consistent with the standard model where structure forms out of initial quantum fluctuations.

When the topology is studied on scale $\lambda = 5°$, we find the UGC catalog has a small meatball shift, which agrees with the three-dimensional genus curve for the three-dimensional CfA sample which covers the same area with similar effective smoothing length (Gott et al. 1989). The ESO sample is more nearly random phase, which also agrees with the three-dimensional result for the SSRS which covers the same area (cf. Gott 1991) with $\lambda = 6.5h^{-1}$ Mpc corresponding well to the effective $\lambda = 7h^{-1}$ Mpc smoothing length in our $\lambda = 5°$ ESO sample. Thus in each sample the results obtained in two-dimensions are consistent to those from three-dimensional studies in the same regions. The fact that we find distributions that are either random phase or show slight meatball shifts is just what we find from biased CDM simulations. Using the amplitudes of the genus curves we can estimate the effective index of the power spectrum at the smoothing length scale. We find $n_{\rm eff} \approx -1$ for the observations (cf. Gott & Rees 1975), again quite close to that produced by biased CDM simulations. As in the three-dimensional case the overall success of the standard biased CDM model is good.

The biased standard CDM model with $\Omega = 1$, a gravitational instability model, has shown a distribution of galaxies as complex as presented by the universe itself. CDM models with $\Omega = 0.4$ are similar (cf. Gott 1982) but have more power on large scales in better agreement with the Maddox et al. (1990) angular covariance function data, and produce great walls that are as spectacular if not more so than the $\Omega = 1$ model (Park 1990a). The fact that the standard CDM model produces the observed large scale features well is remarkable since it has relatively low large-scale power. This is encouraging not only for itself but for the entire class of gravitational instability models [including, for example, texture seeded models (Park, Spergel, & Turok 1991) and extended inflation models (La, Steinhardt, & Bertschinger 1989)], a class of models which produce no gross distortions in the cosmic microwave background spectrum in agreement with the recent *COBE* FIRAS result (Mather et al. 1990). In the future, deeper observational data on the large-scale structure of the universe and extensive statistical tests of numerical simulations should enable us to narrow our choices.

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REFERENCES

Adler, R. J. 1981, The Geometry of Random Fields (New York: Wiley)
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Bardeen, J. M., Steinhardt, P. J., & Turner, M. S. 1983, Phys. Rev. D, 28, 679
Coles, P. 1988, MNRAS, 234, 509
Coles, P., & Barrow, J. D. 1987, MNRAS, 228, 407
Coles, P., & Plionis, M. 1991, MNRAS, 250, 75
Gott, J. R. 1982, Nature, 295, 304
Gott, J. R. 1991, in Large-Scale Structures and Peculiar Motions in the Universe, ed. D. W. Latham & L. A. N. da Costa (San Francisco: Astronomical Society of the Pacific), 173
Gott, J. R., Melott, A. L., & Dickinson, M. 1986, ApJ, 306, 341 (GMD)
Gott, J. R., et al. 1989, ApJ, 340, 625
Gott, J. R., Park, C., Juszkiewicz, R., Bies, W. E., Bennett, D. P., Bouchet, F. R., & Stebbins, A. 1990, ApJ, 352, 1
Gott, J. R., & Rees, M. J. 1975, A&A, 45, 365
Gott, J. R., Weinberg, D. H., & Melott, A. L. 1987, ApJ, 319, 1
Hamilton, A. J. S., & Gott, J. R. 1988, ApJ, 331, 64
Hamilton, A. J. S., Gott, J. R., & Weinberg, D. H. 1986, ApJ, 309, 1 (HGW)
Huchra, J., Davis, M., Lantham, D., & Tonry, J. 1983, ApJS, 52, 89
Hudson, M. J. 1990, private communication
La, L., Steinhardt, P. J., & Bertschinger, E. 1989, Phys. Letters B, 231, 231
Lahav, O., Nemiroff, R. J., & Prian, T. 1990, ApJ, 350, 119 (LNP)
Lahav, O., Rowan-Robinson, M., & Lynden-Bell, D. 1988, MNRAS, 234, 677
Lauberts, A. 1982, The ESO-Uppsala Survey of the ESO(B) Atlas (Munich: ESO)
Lynden-Bell, D., Faber, S. M., Burstein, D., Davies, R. L., Dressler, A., Terlevich, R. J., & Wegner, G. 1988, ApJ, 326, 19
Lynden-Bell, D., Lahav, O., & Burstein, D. 1989, MNRAS, 241, 325
Nilson, P. 1973, Uppsala General Catalogue of Galaxies (Uppsala Astron. Obs. Ann., vol. 6)
Maddox, S. J., Efstathiou, G., Sutherland, W. J., & Loveday, J. 1990, MNRAS, 242, 43P
Mather, J. C., et al. 1990, ApJ, 354, L37
Melott, A. L., Cohen, A. P., Hamilton, A. J. S., Bott, J. R., & Weinberg, D. H. 1989, ApJ, 345, 618 (MCHGW)
Melott, A. L., & Fry, J. N. 1986, ApJ, 305, 1
Melott, A. L., Weinberg, D. H., & Gott, J. R. 1988, ApJ, 328, 50
Park, C. 1990a, MNRAS, 242, 59P
———. 1990b, Ph.D. thesis, Princeton Univ.
———. 1991, MNRAS, 251, 167
Park, C., & Gott, J. R. 1991, in preparation
Park, C., Gott, J. R., Melott, A. L., & da Costa, L. N. 1991, in preparation
Park, C., Spergel, D. N., & Turok, N. 1991, ApJ, 372, L53
Peebles, P. J. E. 1980, The Large Scale Structure of the Universe (Princeton: Princeton Univ. Press)
———. 1982, ApJ, 263, L1
Ryden, B. S. 1988, ApJ, 333, L41
Ryden, B. S., Melott, A. L., Craig, D. A., Gott, J. R., Weinberg, D. H., Scherrer, R. J., Bhavsar, S. P., & Miller, J. M. 1989, ApJ, 640, 647
Smoot, G., & Lubin, P. 1979, ApJ, 234, L83
Weinberg, D. H., Gott, J. R., & Melott, A. L. 1987, ApJ, 321, 2
Weinberg, D. H. 1988, PASP, 100, 1373

Note added in proof.—After completing our paper we received a preprint from Coles & Plionis (1991) of a two-dimensional topology analysis of the Lick galaxy catalogue (Shane-Wirtanen galaxy counts) which comes to similar conclusions to those presented here.