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BOUND POPULATIONS AROUND cD GALAXIES AND cD VELOCITY OFFSETS IN CLUSTERS OF GALAXIES

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ABSTRACT

We investigate recent claims in the literature for the existence of a low velocity dispersion population of galaxies in the cores of rich clusters. The suggested population, which has been generally interpreted as gravitationally bound to the potential well of the first-ranked cluster galaxy (often morphologically identifiable as a D/cD galaxy), is taken as evidence in support of the galactic cannibalism model for D/cD galaxy formation. The statistical tests of kinematic evidence which have been used to support this claim are, however, fundamentally flawed. In particular, tests for bound populations which rely on the binning of small numbers of galaxy redshifts may lead to spurious results. We examine five clusters which have been suggested in the literature to possess bound populations of galaxies associated with the D/cD. We develop an improved statistical test, the Indicator test, which compares the number of galaxies with relative velocities (measured with respect to the D/cD) less than a given fraction of the cluster scale ("dispersion") to the number expected for an assumed parent velocity distribution (for convenience taken to be a Gaussian). We find strong supporting kinematic evidence for the existence of a bound population in only one of the five clusters investigated, Klemola 44. Marginal kinematic evidence in support of the bound population hypothesis is found for two clusters, A1991 and A2589. We find no strong supporting kinematic evidence for a bound population in two clusters, A2107 and A2593.

To avoid difficulties with small number statistics, some authors have employed tests for bound populations which rely on a summation of redshift distributions from many different clusters. Detection of a bound population is claimed if the summed distribution cannot be adequately fit with a single Gaussian. We show, however, that the null hypothesis for such a procedure is incorrect. A finite mixture of velocities drawn from clusters with nonidentical dispersions is *not a Gaussian*, but rather a distribution which is generally peakier than Gaussian in the center, lighter than Gaussian in the middle quantiles, and heavier than Gaussian in the outer tails. This realization, along with sample-dependent selection effects, can explain differences in the proposed mixture model parameters between Cowie & Hu and Bothun & Schombert, and can also account for the anomalously large second-component dispersion in the mixture models by both sets of authors. Simulations of the expected pooled samples when draws are made from clusters with a range of dispersions are compared with the observed distributions of Cowie & Hu and Bothun & Schombert. A one-parameter functional form which matches such mixture populations well, at least in the central portion, is the Logistic function. *No strong kinematic evidence is found for the existence of a low-dispersion "bound" population of galaxies in the vicinity of the D/cD for most clusters*, when compared to Logistic fits to the expected pooled samples. We conclude, in agreement with the analysis of Lauer, that the high multiplicity of first-ranked galaxies in clusters is best explained as a result of the central structure of galaxy clusters.

We next consider claims for the existence of significant velocity offsets of a number of D/cD galaxies with respect to the central location in velocity space ("mean") of the remaining cluster galaxies. We point out that, particularly in clusters with small numbers of available redshifts, confidence intervals on central location are often asymmetric. Estimates of (symmetric) confidence intervals on the mean velocity obtained via canonical procedures are potentially misleading and may lead to false claims of significance. We employ a bootstrap resampling technique to obtain realistic confidence intervals on cluster central locations and incorporate observational errors in the measurement of the D/cD velocity in a consistent manner. Of 14 clusters (15 D/cD galaxies) noted in the literature with discrepant D/cD velocities, we confirm only four (A1795, A1809, A2670, and Shapley 8). Three of the four clusters for which we confirm suspected velocity offsets have more than 40 redshifts available, enough to reduce the error in central velocity location of the samples to the point that velocity offsets of the D/cD galaxies on the order of 250–400 km s^{-1} become statistically significant. Three clusters (A85, A2634, and Klemola 44) have only marginally significant D/cD velocity offsets. The remaining seven clusters, which generally have only 10–30 redshifts available, are shown not to exhibit statistically significant D/cD velocity offsets. Many available redshifts per cluster are required before apparently large D/cD velocity offsets can be accepted in most clusters.

Subject headings: galaxies: clustering — galaxies: redshifts

1. INTRODUCTION

The relatively large numbers of measured redshifts for galaxies in clusters which are now becoming available enable searches for important, but often subtle, kinematic clues to the formation and evolution of the morphologically distinct D/cD galaxies which preferentially inhabit the high-density regions of many clusters. Two important effects have drawn considerable recent attention. The first is a search for evidence in the radial velocity distributions of galaxies in the region of the D/cD for the presence of a low-dispersion ("bound") population—galaxies that, according to conventional wisdom, will eventually merge with the D/cD via the process of dynamical friction (Cowie & Hu 1986, hereafter CH; Bothun & Schombert 1988, hereafter BS I; Bower, Ellis, & Efstathiou 1988; Bothun & Schombert 1990, hereafter BS II; Green, Godwin, & Peach 1990, hereafter GGP). Such searches are complicated by the fact that a single cluster velocity sample may be composed of contributions from at least three kinematical populations—the presumed bound population, galaxies which are members of the "normal" core population, and galaxies which may be on highly eccentric orbits focused on the D/cD galaxy (Tonry 1985). Even large samples of redshifts in a given cluster may have difficulty placing useful constraints on such models.

The second, and at first blush, rather perplexing result is that the D/cD galaxies in some clusters appear to exhibit rather large velocity differences relative to the mean velocity of the rest of the cluster galaxies (Hill et al. 1989; Sharples, Ellis, & Gray 1988; BS II; Teague, Carter, & Gray 1990, hereafter TCG; Zabludoff, Huchra, & Geller 1990, hereafter ZHG). This result has been used to call into question models for D/cD formation which imply that these galaxies should be at rest in the cluster potential. Given that this notion goes against the overwhelming weight of evidence presented to date in support of the conventional interpretation (e.g., Beers & Geller 1983, who show that D/cD galaxies are preferentially located on local density maxima in clusters; Lauer 1988, who presents evidence of distorted isophotal structures in D/cD galaxies probably due to present-day mergers; Jones & Forman 1984, who show that X-ray surface brightness maps are accurately centered on the position of the central D/cD galaxy; and Quintana & Lawrie 1982, ZHG, TCG, and others, who have argued that the great majority of D/cD galaxies in clusters occupy a kinematically privileged location at the center of the cluster velocity distribution), we must scrutinize the estimation of statistical errors in cluster mean velocities on which this result is based.

The statistical techniques used thus far to assess the reality of these two effects are, however, rather crude, and may result in spurious claims. One difficulty that must be faced by any investigation is the specification of kinematic parameters for the parent population upon which the presumed bound population of galaxies, or "high-velocity" D/cD, is superposed. As emphasized by the results of Beers, Flynn, & Gebhardt (1990, hereafter BFG), canonical techniques for estimating central locations and scales in velocity space often do not provide an adequate picture of the parent kinematic populations in clusters of galaxies and fail to capture the underlying variability of the estimators in their stated confidence intervals. These issues become particularly important for the present analysis, as the reality or absence of the proposed effects relies on a comparison of a subset of the velocity data to the global kinematic properties of the cluster.

In this paper we present simple statistical techniques for examining kinematic evidence for bound populations and high-velocity D/cDs and compare our results with previous work. In § 2 we discuss a statistical test that determines whether an excess of radial velocities exists within a specified range of the cluster radial velocity distribution by comparing the empirical distribution function (EDF) of observed velocities to the cumulative distribution function of the proposed parent population. This technique has several advantages. First and foremost it obviates the need for binning of small data samples. It can also be easily extended to arbitrary, non-Gaussian, parent populations. We compare the results of this test to previous analyses of five clusters with proposed bound populations: A1991 and A2589, listed by BS II as definite detections, A2107 and A2593, listed by BS II as marginal detections, and Klemola 44, suggested to possess a bound population by GGP, Of these proposed cases, we find clear kinematic evidence for a bound population in only one, the cluster Klemola 44. Marginal kinematic evidence in support of the bound population hypothesis is found for A1991 and A2589. We find no strong supporting kinematic evidence for a bound population in A2107 and A2593.

We discuss the analysis of pooled samples of relative velocities in § 3. Simulations of the expected contribution from clusters with a wide range of dispersions can account for much, if not all, of the excess counts in the central bins of pooled samples, in disagreement with previous interpretations of this effect as due to the presence of a bound population of galaxies in the vicinity of the dominant galaxy.

In § 4 we apply the bootstrap resampling technique to determine confidence intervals on the central locations of cluster velocity distributions. These intervals are then compared to the measured velocity of the D/cD galaxy to assess the reality of any supposed velocity offset. We examine 14 clusters suggested in the literature to exhibit significant D/cD velocity offsets. Of these, only four cases are supported by our analysis: A1795, A1809, A2670, and Shapley 8. Three clusters (A85, A2634, and Klemola 44) have only marginally significant D/cD velocity offsets. The remaining seven clusters, which generally have only 10–30 redshifts available, are shown not to exhibit statistically significant D/cD velocity offsets. Possible implications of our results are discussed in § 5.

2. KINEMATIC DETECTION OF BOUND POPULATIONS IN INDIVIDUAL CLUSTERS

Not all clusters with D/cD galaxies are expected to exhibit evidence for a bound population associated with the dominant galaxy. Indeed, if the galactic cannibalism model for D/cD formation is correct, dynamical friction time-scale calculations suggest that a low-velocity dispersion population of galaxies intimately associated with a D/cD galaxy should be consumed within one quarter to one half of the Hubble time (Merritt 1985; BS II, and references therein). This suggests that detectable bound populations may be relatively rare in present-day clusters. Furthermore, the intrinsic properties of each bound population may differ from cluster to cluster. The kinematic properties of a bound population, for example, may scale with the mass of the parent D/cD, the mass of the parent cluster, the nature of the bound galaxies (whether they possess tight nuclei which might survive even if their outer envelopes are disrupted), and even the difference (if any) in the distribution of dark matter associated with the D/cD galaxy as compared to that of the cluster. It would seem advisable, therefore, to test for the presence of bound populations on a cluster by cluster basis. One possible test is discussed below.

2.1. *Binned Velocity Tests*

If a given cluster has achieved dynamical equilibrium between its galaxies and the underlying mass distribution (whatever its true nature), one might expect to find that the observed radial velocities of its galaxies exhibit a Gaussian distribution (Sarazin 1986). Although this might not be a reasonable expectation for the velocity distribution of the complete set of galaxies within a given cluster, one might hope that the Gaussian approximation might be a useful description at least for the apparently high-density central regions of many clusters. Deviations of radial velocity distributions from Gaussian might be used to identify kinematic subpopulations of galaxies within the cluster core region.

As the redshift data available for most clusters are still relatively sparse, every effort must be expended to employ tests which make efficient use of the data in hand. Ideally, one would also like to employ a test which is " tuned " to the particular deviation of interest. Omnibus tests for non-Gaussian behavior provide some information, but by design respond to a variety of deviations other than that due to a hypothesized bound population. For the present application it should also be kept in mind that we are considering a rather specific alternative hypothesis—that the number of galaxies in the vicinity of the D/cD galaxy with small radial velocity differences (relative to the D/cD) is greater than might be expected had they been chosen at random from the (presumably) Gaussian population of the remaining cluster. Unfortunately, omnibus tests for non-Gaussian behavior respond with equal likelihood to *deficits* in the velocity sample with respect to the Gaussian model as they do to excesses. Furthermore, an excess number of radial velocities in the tails of a given cluster distribution, which surely has nothing to do with the identification of a bound population, also may inflate an omnibus statistic to the point of rejection.

BS II consider an alternative test, the $\sqrt{N}/N$ test, for the presence of a bound population centered on the D/cD velocity. This test demands that the central bin of a cluster velocity histogram for which one claims to exhibit a bound population must rise above the expected frequency for a Gaussian distribution (with parameters estimated from the complete data set) by an amount at least equal to an estimate of the counting statistics for that bin. This technique is somewhat better suited to the problem at hand, but still suffers from (1) arbitrary bin selection, (2) uncertainties in parameter estimation of the parent population (due to the use of nonresistant estimators), and (3) makes no prediction as to how significant a detection might be, just that it (possibly) exists. Below we describe a simple test which avoids some of these difficulties.

2.2. *The Indicator Test*

For the present investigation, we proceed on the assumption that radial velocities of the normal cluster population are Gaussian distributed. Although a variety of analytic functions might be used to model the specified cluster population, it is doubtful that we could meaningfully choose between alternatives to the Gaussian with such small data sets.

We first obtain rest-frame relative velocities from observed redshifts by dividing the observed heliocentric velocity " cz " by $(1 + z_{\rm BI})$, where $z_{\rm BI}$ is an estimate of the central location in redshift space obtained from the resistant biweight estimator $C_{\rm BI}$ (see BFG for details). The scale of these relative velocities is then obtained by the use of the biweight estimator on scale, $S_{\rm BI}$. We next consider the range in relative velocity over which we endeavor to test for an excess number of galaxies. For a comparison to a standard normal we obtain the standard deviates (Z-scores) which correspond to the range of interest

$$Z_{l,u} = \frac{V_{l,u} - C_{\rm BI}}{S_{\rm BI}}\,, \tag{1}$$

where $V_{l,u}$ are the values of relative velocity for the lower and upper endpoints of the range. To test for an excess in velocity space around the velocity of the D/cD the range is given by

$$V_{l,u} = V_{\rm cD} \mp f(S_{\rm BI})\,, \tag{2}$$

where $V_{\rm cD}$ is the relative velocity of the D/cD with respect to the central location of the remaining cluster galaxies and f takes on values between 0.1 and 1.5. The extension of this range to locations anywhere in the distribution is trivially obtained by replacing $V_{\rm cD}$ with the appropriate value. From the Z-scores we evaluate the probability, p, that a data point chosen at random from the parent population is contained within the specified range by evaluating the area under the standard normal curve in that interval.

The Indicator function, $I(V_{l,u})$ is defined to be equal to 1 if a given relative velocity, V_i, lies within the interval defined in equation (2), and equal to 0 otherwise. Clearly, such a function is binomial distributed, hence the probability, P, of finding less than n galaxies within the specified range is given by

$$P = \sum_{x=0}^{n-1} \binom{N}{x} p^x (1-p)^{N-x}\,, \tag{3}$$

where N is the total number of galaxies in the sample. The probability of finding n or more galaxies in the specified range is given by $1 - P$.

2.3. *Examples*

We apply the Indicator test described above to five clusters for which the existence of a bound population has been claimed in the literature—A1991, A2107, A2589, A2593, and Klemola 44. Klemola 44 possesses two D/cD galaxies; thus we test for bound populations around each of these galaxies considered separately. The results of the Indicator test are shown in Figure 1—the velocity data is that given by BS I (A2589), BS II (A1991, A2107, A2593), or GGP (Klemola 44). Galaxies included in the BS I and BS II data sets range up to 400 kpc from the central galaxy ($H_0 = 100$ km s^{-1} Mpc^{-1}). The maximum distance covered in the Klemola 44 data set is 470 kpc. The horizontal axis in Figure 1 is the relative velocity of galaxies normalized by the scale of the cluster ($f = V_i/S_{\rm BI}$); the vertical axis is the number of galaxies found within the specified range relative to the velocity of the D/cD galaxy. As we are looking for the excess number of galaxies at low velocities relative to the D/cD, this galaxy is explicitly not included in the number of galaxies within a given velocity range. The circles in Figure 1 represent the data. The solid line is the predicted number of galaxies within the specified scale fraction assuming draws from a Gaussian parent distribution, given by $N \times p$. The dashed line corresponds to the predicted number required to achieve an excess at the 95% significance level. In Figure 2 the clusters A1991, A2589, and A2593 are reanalyzed with larger samples of redshifts, to be published in a paper in preparation (Beers et al. 1991). The enlarged samples include gal-

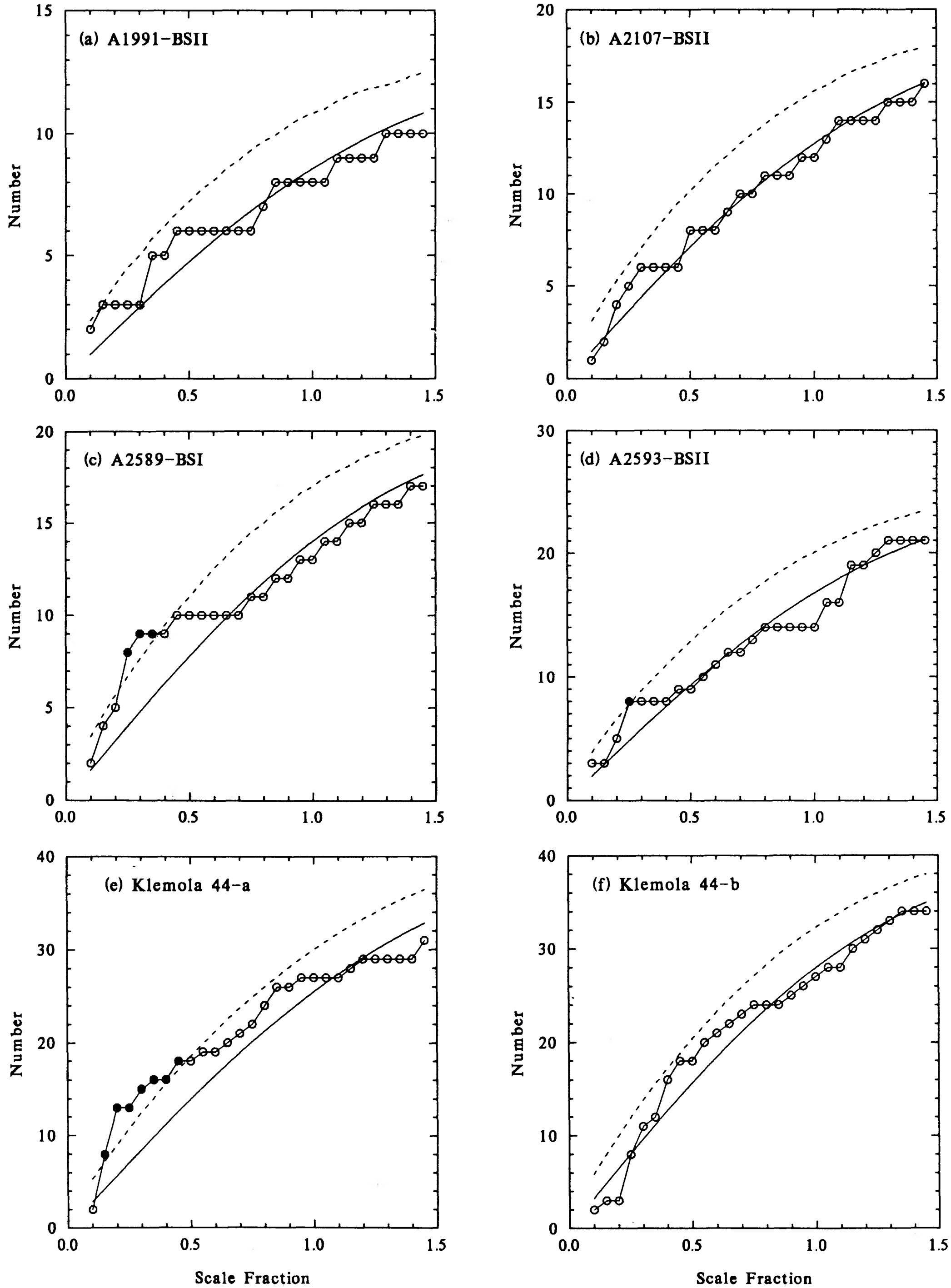

FIG. 1.—Results of the Indicator test. The horizontal axis represents the scale fraction ($f = V_i/S_{BI}$), where the scale estimate for the individual clusters are given in Table 1. The vertical axis is the number of galaxies within the specified range in velocity relative to the D/cD galaxy. Solid line is the expected number of galaxies within a given scale fraction for a Gaussian distribution. Dashed line represents the required number for an observed excess number to be at a significance level of 95%. Circles represent the data; filled circles are significant at the 95% level. (*a*) Data from BS II; (*b*) data from BS II; (*c*) data from BS I; (*d*) data from BS II; (*e*) data from GGP—results relative to IC 5353; (*f*) data from GGP—results relative to IC 5358.

axies located up to 870 kpc from the D/cD (A1991), 600 kpc (A2589), and 500 kpc (A2593).

In Table 1 we summarize the kinematic properties of the individual clusters. Column (1) lists the cluster name. Columns (2) and (3) identify the source of the redshift data and number of galaxies in the sample, respectively (including the D/cD galaxy). Column (4) is the biweight estimator of central location in velocity space (excluding the D/cD) in km s^{-1}. Column (5) is the biweight estimator of scale (also excluding the D/cD) in km s^{-1}. Column (6) lists our assessment of the results of the Indicator test. If, for a given cluster, the EDF at any position within the examined scale fraction rises to exceed the 95% line,

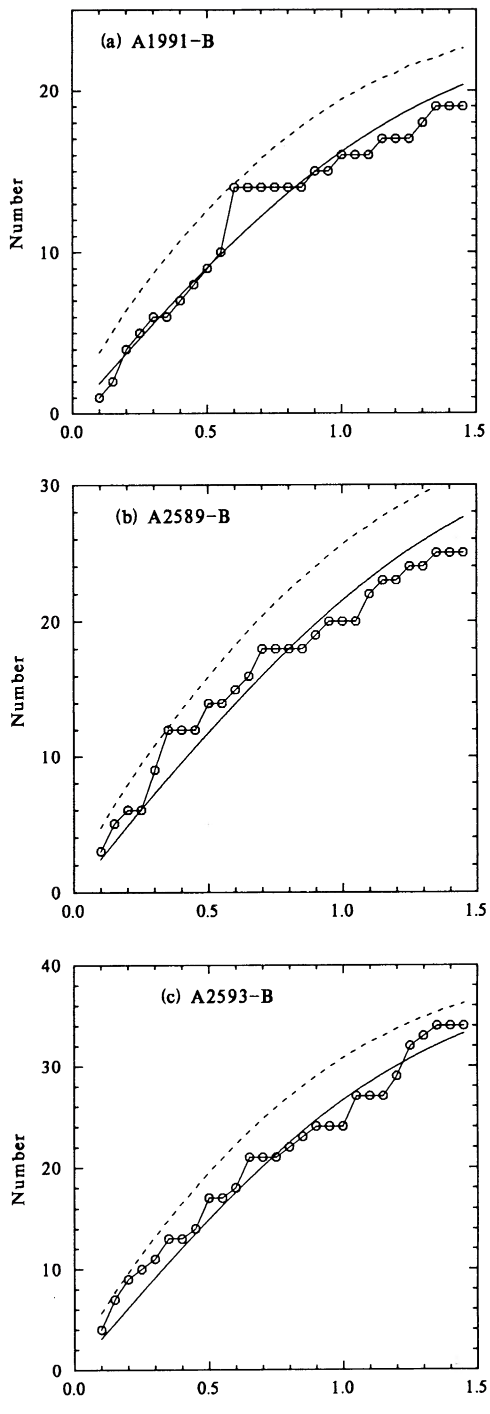

FIG. 2.—Results of the Indicator test for three clusters of BS I and BS II which have larger available samples of redshifts from Beers et al. (1991). Axes and circles represent the same quantities as in Fig. 1.

we list a "Y" marking that cluster as meeting our criteria for existence of a bound population. A cluster for which the EDF rises to meet the 95% line is considered marginal and is labeled with an "M." A cluster for which the EDF fails to reach the 95% line is not considered statistically significant and is marked with an "N." For the purpose of illustration we have carried out three powerful omnibus tests for normality for each cluster velocity distribution—the Cramer von-Mises statistic (W^2), the Watson statistic (U^2), and the Anderson-Darling statistic (A^2) (see D'Agostino 1986 for a detailed description of these tests). The values of these statistics for each data batch and their associated p-values are listed in columns (7)–(13). Below we discuss our analyses of the individual clusters on a case-by-case basis.

2.3.1. *A1991*

In Figure 1*a* it is clear that the velocity data from the BS II sample exhibit a slight tendency to lie above the expected line at scale fractions below $f = 0.6$ (319 km s^{-1}), but nowhere is this excess significant. The small number of available galaxy redshifts in this sample ($N = 14$ excluding the D/cD) limit any further speculation. Long plateaus in the EDF, which indicate regions where no additional velocities are found, are the dominant feature of this data batch. Note that the omnibus normality tests of Table 1 show no evidence for significant deviations from Gaussian in the BS II sample. Presumably the BS II claim for a significant deviation from Gaussian is due to inadequacies of the χ^2 test which they employ. In Figure 2*a* we show the results of the Indicator test for a sample of redshifts which is roughly double the sample size of BS II. With the increased sample size ($N = 24$ excluding the D/cD), the EDF within $f = 0.5$ lies very close to the expected line. At $f = 0.6$ (400 km s^{-1}) the EDF does take a sharp leap, from 10 galaxies to 14, followed by no additional galaxies until $f = 0.9$ (600 km s^{-1}), but never rises above the 95% significance line. We consider A1991 to show only marginal kinematic evidence for the existence of a bound population. We note, however, that the omnibus normality tests of Table 1 do reject a Gaussian hypothesis for the extended velocity sample, at critical levels of better than 2%. This result is clearly due to the spike in the EDF at $f = 0.6$.

2.3.2. *A2107*

The $\sqrt{N}/N$ test employed by BS II suggested to them that this cluster has an excess of galaxies in its central 200 km s^{-1} bin, although their χ^2 test could not reject the Gaussian hypothesis. In Figure 1*b*, a slight (but not significant) excess above the expected line is seen within $f = 0.5$ (265 km s^{-1}). Six galaxies are found where four are expected. The omnibus tests cannot reject the hypothesis of draws from a Gaussian.

2.3.3. *A2589*

The BS I sample for A2589 ($N = 21$; Fig. 1*c*) exhibits a significant excess of galaxies at $f = 0.25$ (210 km s^{-1}). Nine galaxies are found where five are expected. Our results for this data batch are similar to that of BS II, who claim the existence of a bound population from their $\sqrt{N}/N$ test and χ^2 test. The omnibus tests of Table 1 also seem to indicate a marginal rejection of the Gaussian hypothesis, with critical levels between 3% and 7%. When the sample is increased to $N = 34$ (Fig. 2*b*), A2589 no longer exhibits a significant excess, although there still exists a marginal excess at $f = 0.35$ (230 km s^{-1}). Twelve galaxies are found where eight are expected. The omnibus tests reject the Gaussian hypothesis at the 1% critical level.

2.3.4. *A2593*

In the BS II sample ($N = 25$; Fig. 2*d*) A2593 shows an excess at $f = 0.25$ (190 km s^{-1}). Eight galaxies are found where five are expected. This excess is formally significant, although the small number of galaxies involved suggests caution in the interpretation of this result. It is important to note that the

TABLE 1

KINEMATIC PARAMETERS, INDICATOR RESULTS, AND EDF TESTS FOR NORMALITY

Cluster (1)	Source (2)	N (3)	C_{BI} (4)	S_{BI} (5)	Indicator Results (6)	W^2 (7)	$p(W^2)$ (8)	U^2 (9)	$p(U^2)$ (10)	A^2 (12)	$p(A^2)$ (13)
A1991	BS II	15	17836	531	M	0.06	0.44	0.05	0.43	0.32	0.54
A1991	B	25	17784	667	M	0.16	0.02	0.15	0.02	1.05	0.01
A2107	BS II	20	12335	529	N	0.06	0.34	0.06	0.38	0.41	0.34
A2589	BS I	22	12470	827	Y	0.13	0.04	0.13	0.03	0.69	0.07
A2589	B	35	12535	660	M	0.20	0.01	0.19	0.01	1.16	0.01
A2593	BS II	26	12534	755	Y	0.04	0.70	0.04	0.68	0.27	0.66
A2593	B	40	12443	728	N	0.06	0.30	0.05	0.42	0.37	0.42
Klemola 44a	GGP	42	8670	924	Y	0.19	0.01	0.18	0.01	0.98	0.01
Klemola 44b	GGP	42	8635	936	N	0.21	0.01	0.20	0.01	1.09	0.01

NOTES.—B: Beers et al. 1991; BS I: Bothun & Schombert 1988; BS II: Bothun & Schombert 1990; GGP: Green et al. 1990.

omnibus tests of Table 1 give no reason to reject the Gaussian hypothesis. When the sample is increased to $N = 39$ (Fig. 2c), we see a similar behavior. The only place where the Indicator test approaches marginal significance is at $f = 0.2$ (150 km s^{-1}). Nine galaxies are found where six are expected. Again, the omnibus tests cannot reject a Gaussian hypothesis.

2.3.5. *Klemola 44*

Klemola 44 has two D/cD galaxies. GGP argue that an excess of low-velocity galaxies exists, centered on a velocity location midway between these two dominant galaxies. We carry out an Indicator test for both D/cD galaxies; the results are shown in Figures 1e and 1f. There is a definite detection of a low-dispersion population around IC 5353 (GGP galaxy no. 1), which has a radial velocity of 8181 km s^{-1}. Within $f = 0.2$ (185 km s^{-1}) 13 galaxies are found where only six are expected. There remains a significant excess out to $f = 0.5$ (460 km s^{-1}), and a detectable, but not significant, excess to $f = 1.0$ (925 km s^{-1}). The Indicator test finds no similar population at the velocity of the other D/cD galaxy (IC 5358; GGP galaxy no. 2), which has a radial velocity of 8573 km s^{-1}. Note that there is an increase in the number of galaxies found within $f = 0.8$ (750 km s^{-1}), where the velocity distance extends to include the low velocity dispersion population surrounding IC 5353. The omnibus tests reject a Gaussian hypothesis for both data batches at the 1% critical level.

3. DETECTION OF BOUND POPULATIONS FROM POOLED SAMPLES

Above we have shown how simple tests can be carried out to evaluate kinematic evidence for the presence of a bound population of galaxies in a number of clusters. It is also clear, however, that our results are limited by the still relatively small numbers of galaxies with measured redshifts in each cluster. This limited amount of redshift information has prompted previous workers to consider the nature of the distribution of radial velocities (measured relative to the D/cD galaxy) for a pooled sample of clusters. Two different sorts of data have been examined in this way. The first involves a study of the relative velocities of the multiple nuclei which are found in roughly 50% of clusters (Hoessel & Schneider 1985). Smith et al. (1985), Tonry (1985), CH, and Lauer (1988) examine the distribution of velocities for galaxies chosen quite close (generally $r \leq 20$ kpc) from the brighter component. BS II examine the distribution of relative velocities for galaxies in clusters projected within 400 kpc of the dominant galaxy. Although these two applications consider two rather different samples, the statistical problems encountered in the analysis are similar.

3.1. *Previous Analysis*

Smith et al. (1985) analyze a sample of 34 relative velocities for multiple nuclei in 30 different clusters. Their analysis indicates that a single Gaussian ($\sigma = 833$ km s^{-1}) adequately describes the distribution of relative velocities in the pooled sample. When the sample is divided into those nuclei within 20 kpc of the central galaxy, and those outside this region, single Gaussians still provide adequate fits to the velocity distributions. The inner sample has a dispersion of 754 km s^{-1}; the outer sample a dispersion of 974 km s^{-1}. Smith et al. conclude that the distribution of relative velocities for multiple nuclei is consistent with random sampling from a parent population with velocity dispersion similar to that of a typical rich cluster.

Tonry (1985) obtained relative velocities for 19 multiple nuclei in 14 clusters, all chosen within 20 kpc of the central galaxy. His analysis suggests that roughly 55% of the galaxies in his sample are on rather eccentric orbits, with apocenters on order 100 kpc from the brightest cluster galaxy. Based on the strongly peaked number density distribution of galaxies in many clusters (Beers & Tonry 1986), which may be a result of a population of galaxies on eccentric orbits focused on the brightest cluster galaxy, Tonry argues that no more than 20% of his sample are consistent with membership in the general cluster population and are seen in projection. The remaining 25% were thought to be galaxies on bound orbits that should eventually merge with the brightest cluster galaxy.

CH contend that the Smith et al. and Tonry velocity samples are too small to constrain such interpretations and call for the need of a control sample farther from the central galaxy to assess the meaning of their results. These authors assembled an extended sample of 75 multiple nuclei (and other cluster galaxies out to 200 kpc from the D/cD), incorporating the lists of Tonry and Smith et al. as well as their own measurements. CH argue, based on a χ^2 test of their pooled sample, that the relative velocities in this extended sample cannot be well fitted by a single Gaussian distribution with dispersion on order 800 km s^{-1}. Rather, they argue for a mixture model with 60% of the galaxies coming from a population with dispersion on order 250 km s^{-1} and 40% from a population with a dispersion of 1400 km s^{-1}. Their analysis indicated that such a mixture model also provides an adequate fit when applied exclusively to the velocity data for the multiple nuclei galaxies within 20 kpc of the dominant galaxy. Although CH express concern over the rather large dispersion of their second component, they cannot explain why it should be so high compared to the global dispersions of the clusters from which their sample is drawn.

Lauer (1988) considers the relative velocities of multiple nuclei from Smith et al. and Tonry which were drawn from clusters of known dispersion. From a plot of these velocities, scaled by their parent cluster dispersions, he concludes that the multiple nuclei are kinematically indistinguishable from other cluster galaxies. As seen below, we concur with such a view.

BS II sum the relative velocities from eight clusters to obtain a sample of 169 galaxies with projected radii less than 400 kpc from the D/cD. These authors argue that this pooled sample is not well fitted by a single Gaussian, but rather requires a mixture model with 20% of the galaxies drawn from a population with a dispersion of 120 km s^{-1} and 80% of the galaxies from a population with a dispersion on order 1200 km s^{-1}. BS II further argue that their mixture model population fractions differ from CH because their search radius is sufficiently large to include significant numbers of the low-dispersion population, which they (following Tonry 1985) take to be galaxies on eccentric orbits with apocenters on order 150 kpc from the D/cD. This result prompts the question as to why the great majority of galaxies found in the cores of their clusters have apparently been drawn from a population with velocity dispersion significantly larger than the global dispersions of 95% of well-studied clusters of galaxies (ZHG).

3.2. *The Functional Form of Pooled Cluster Samples*

The above analyses (except that of Lauer 1988) have attempted, by various methods, to resolve a proposed single-dispersion velocity distribution, of on order 800–1000 km s^{-1}, which the authors have argued is "typical" of the velocity dispersions of the clusters from which their samples are drawn, into (at least) a two-component mixture. These attempts rely on the assumption that draws of relative velocities from a pooled sample of clusters, each of which possess a one-component (Gaussian) velocity distribution, should, if not for the presence of a bound, low-dispersion population (or some other complicating effect), result in a sample which is itself well fitted by a single Gaussian. Our concern with such an interpretation is the following. It can be shown that *a finite mixture of nondegenerate (i.e., nonidentical dispersion) Gaussians is not itself a Gaussian.* A succinct proof of the above statement can be found in Titterington, Smith, & Makov (1985).

The importance of the above realization to the present analysis cannot be overemphasized. For example, the Smith et al. sample includes draws from clusters with a range in velocity dispersion from 500 to 1200 km s^{-1}. The Tonry clusters exhibit a similar range of dispersions, as does the CH sample. The BS II sample includes clusters with dispersions as low as 500 km s^{-1} to as high as 1500 km s^{-1}. ZHG have shown that D/cD galaxies are found in clusters exhibiting a wide range of velocity dispersions, similar to the range of dispersion in clusters without D/cD galaxies. An analysis of their reported dispersions for 19 cD clusters shows that the distribution has a central location $C_{\rm BI} = 758$ km s^{-1} and scale $S_{\rm BI} = 240$ km s^{-1}. For the purpose of the present analysis, therefore, there really is no "typical" velocity dispersion for clusters of galaxies. Careful consideration must be given to the expected contribution of velocities from each cluster to the combined sample—when clusters of different dispersions are pooled the contribution of galaxies chosen from the "normal" cluster population of low-dispersion clusters may *dominate* the expected signal from the presumed "bound" populations. The net result is that the mixture model parameters obtained by different authors are set, in reality, not by differences in the bound populations which they seek to study, but by differences in the velocity dispersions of the parent clusters which make up each authors' sample.

We illustrate our concern by example. In Figure 3*a* we show frequency distributions for three Gaussians of identical location (and total area), with dispersions set equal to 518 km s^{-1} (one scale length below that of the ZHG central dispersion for cD clusters), 758 km s^{-1} (equal to the ZHG central dispersion), and 998 km s^{-1} (one scale length above the ZHG central dispersion). The normalized sum of these three distributions is shown as the dotted line in Figure 3*b*. The solid line is the Gaussian distribution with dispersion set equal to 758 km s^{-1}. Clearly, the pooled distribution is not Gaussian. The central region is enhanced from the contribution of the low-dispersion population, the middle quantiles are depleted due to a lack of galaxies from the low- and high-dispersion populations, and the tails are composed almost entirely of galaxies selected from the high-dispersion population. Figure 3*c* is a difference plot (scaled to the reference Gaussian) between these two functional forms ([(pooled − reference)/reference] × 100%). From this figure one more clearly sees the expected distortions. The maximum frequency difference in the central region is small for this case, but the trend is obvious.

One might argue that the above problem could be easily avoided by scaling each galaxy's relative velocity by the dispersion of the cluster from whence it came. However, there are at least three reasons why scaling relative velocities is not a preferable solution at present. First, it obviously requires knowledge of the cluster dispersion to which one seeks to scale. Only 70% of the clusters in the CH sample have known dispersions, and many of these are based on fewer than 10 galaxies per cluster. Scale estimates based on such a limited amount of data have rather large errors of determination; thus scaling prior to pooling the data would add a considerable amount of noise. There does not yet exist an adequate amount of kinematic data to pare the sample down to only include those galaxies from clusters with well-measured global dispersions. Finally, as argued in § 2, it is not clear that one should expect the kinematic properties of a bound population to be proportional to the global dispersion of the cluster. It may well be the case that a bound population of galaxies has a "universal" dispersion on order that of the stars in the D/cD galaxy (200–300 km s^{-1}). Pooled samples of scaled relative velocities would result in loss of this information. Clusters with low global dispersions (on order 500 km s^{-1}) would be required to have bound populations with dispersions on order 50–100 km s^{-1} to contribute a signal as significant as that for bound populations with 200–300 km s^{-1} dispersion in moderate- to high-dispersion clusters (750–1250 km s^{-1}). If a "universal" bound population dispersion does exist, the signal contributed by clusters of varying global dispersions would have rather different shapes after scaling. Low global dispersion clusters would exhibit a signal due to the presence of a bound population which is spread out over half of the fractional scale; high global dispersion clusters would exhibit a bound population signal which is confined to low fractional scales. Thus, even after scaling, interpretation of the data would still require consideration of the mixture of global dispersions. For these reasons we prefer to analyze pooled samples of relative velocities without resorting to scaling.

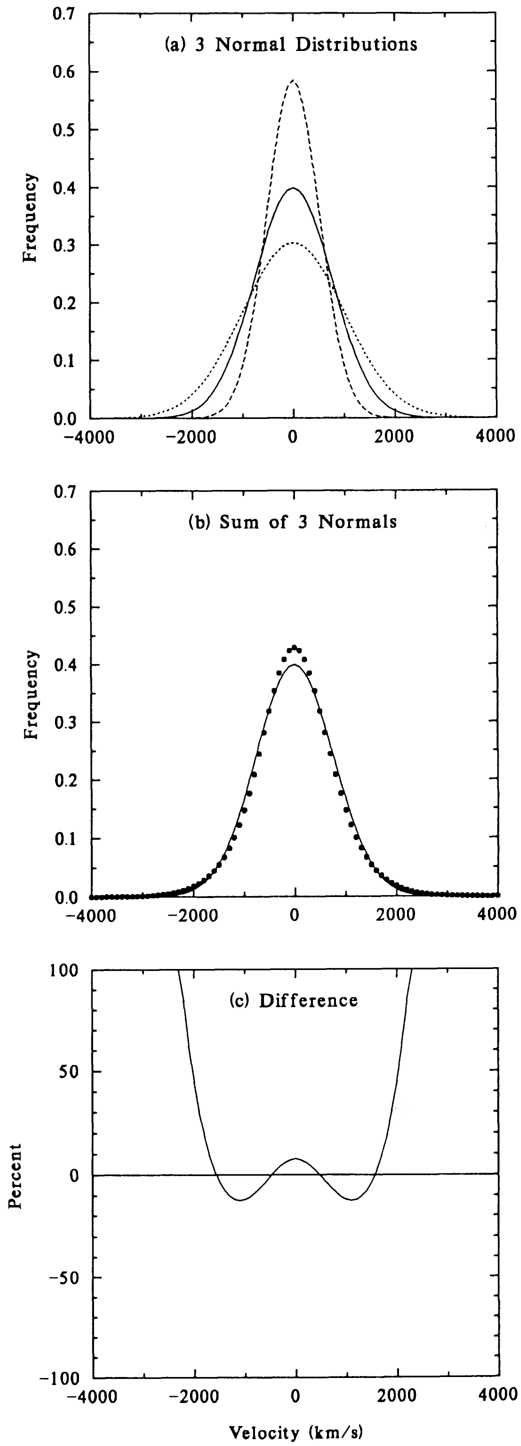

FIG. 3.—(*a*) Frequency distribution for three Gaussians of equal area with parameters $\sigma = 518$ km s^{-1} (*large dashed line*), $\sigma = 758$ km s^{-1} (*solid line*), and $\sigma = 998$ km s^{-1} (*small dashed line*), respectively. (*b*) Frequency distribution of the Gaussian with $\sigma = 758$ km s^{-1} (*solid line*) and the normalized summed distribution of the three Gaussians in (*a*) (*filled circles*). (*c*) Percent difference of the summed distribution from the Gaussian distribution.

Fortunately, a realistic picture of the expected distribution of relative velocities from pooled clusters is easily constructed. We first obtain a set of dispersions by drawing 10,000 times from a normal population with parameters $\mu = 758$ km s^{-1}, $\sigma = 240$ km s^{-1}. From each selected Gaussian, we draw 20 velocities at random. The summed population frequency is shown as the solid line in Figure 4*a*. The open circles represent the best Gaussian fit obtained from a maximum likelihood (ML) fit to the entire list of 200,000 velocities ($\sigma = 797$ km s^{-1}). A nonlinear least-squares (NLS) fit of the binned frequency distribution to a Gaussian is shown by the filled circles ($\sigma = 708$ km s^{-1}). In Figure 4*b* we plot the frequency difference between these two fits and our simulation. It is obvious that a ML Gaussian fit is rather poor. Perhaps this result could have been anticipated, as the ML fit is expected to be dominated by the contribution of high-velocity galaxies in the tails of the pooled distribution. Note the large excess (on order 20%), relative to the ML Gaussian fit in the middle of the distribution, as well as the large deficits (on order 15%) at ± 1200 km s^{-1}. The NLS fit (which puts equal weight on all of the bins) is a considerable improvement in the middle of the distribution, but cannot match the tails outside 1200 km s^{-1}. It is no surprise that the omnibus normality tests discussed above are able to reject a Gaussian hypothesis for the simulated pooled sample at critical values less than 0.001.

It is tempting to ask whether a Double Gaussian fit, with parameters close to those found by CH and BS II, could provide an adequate description of our simulated data. We obtain a NLS fit of the simulated frequency distribution to the model,

$$G'(\alpha, \sigma_1, \sigma_2) = \alpha G(\sigma_1) + (1 - \alpha) G(\sigma_2) , \qquad (4)$$

where σ_1 and σ_2 represent the dispersions of the two components. The parameter α is the fraction of the mixture from the first component; $1 - \alpha$ the contribution attributed to the second component. Figure 4*c* is a plot of the simulated data and the resulting Double Gaussian fit, indicated by the filled circles. We obtain $\alpha = 0.20$, $\sigma_1 = 365$ km s^{-1}, and $\sigma_2 = 843$ km s^{-1}. Figure 4*d* shows the corresponding difference plot. Note that the Double Gaussian fit, with three free parameters, matches the central part of the simulated distribution very well, but cannot fit the tails outside 2000 km s^{-1}.

It would be useful to identify a simple parametric model which captures the nature of the expected velocity distribution when samples from many clusters are pooled in the manner described above. One family of functions that suggests itself is the Logistic, which is peakier than Gaussian in the center and somewhat heavier than Gaussian in the tails. The Logistic function has density

$$L(a, b) = \frac{b^{-1} e^{-(x-a)/b}}{[1 + e^{-(x-a)/b}]^2} , \qquad (5)$$

where x-values are here taken to be the relative velocities, the parameter a serves the role of the central location, and b is a scale parameter. As our simulated data is symmetric, the a-parameter is taken to be identically zero. A NLS fit of equation (5) to our simulated frequency distribution yields a scale $b = 438$ km s^{-1}. This fit is shown in Figure 4*e* as the filled circles. The corresponding difference plot is shown in Figure 4*f*. As is seen in the Figure, the Logistic does an excellent job of mimicking the simulated data, capturing the central peak as well as the heavier-than-Gaussian tails. Such a fit may prove useful for future examination of this problem.

3.3. *Comparison with Real Data*

The comments above do not rule out the possibility of detecting multiple kinematic populations in clusters of galaxies

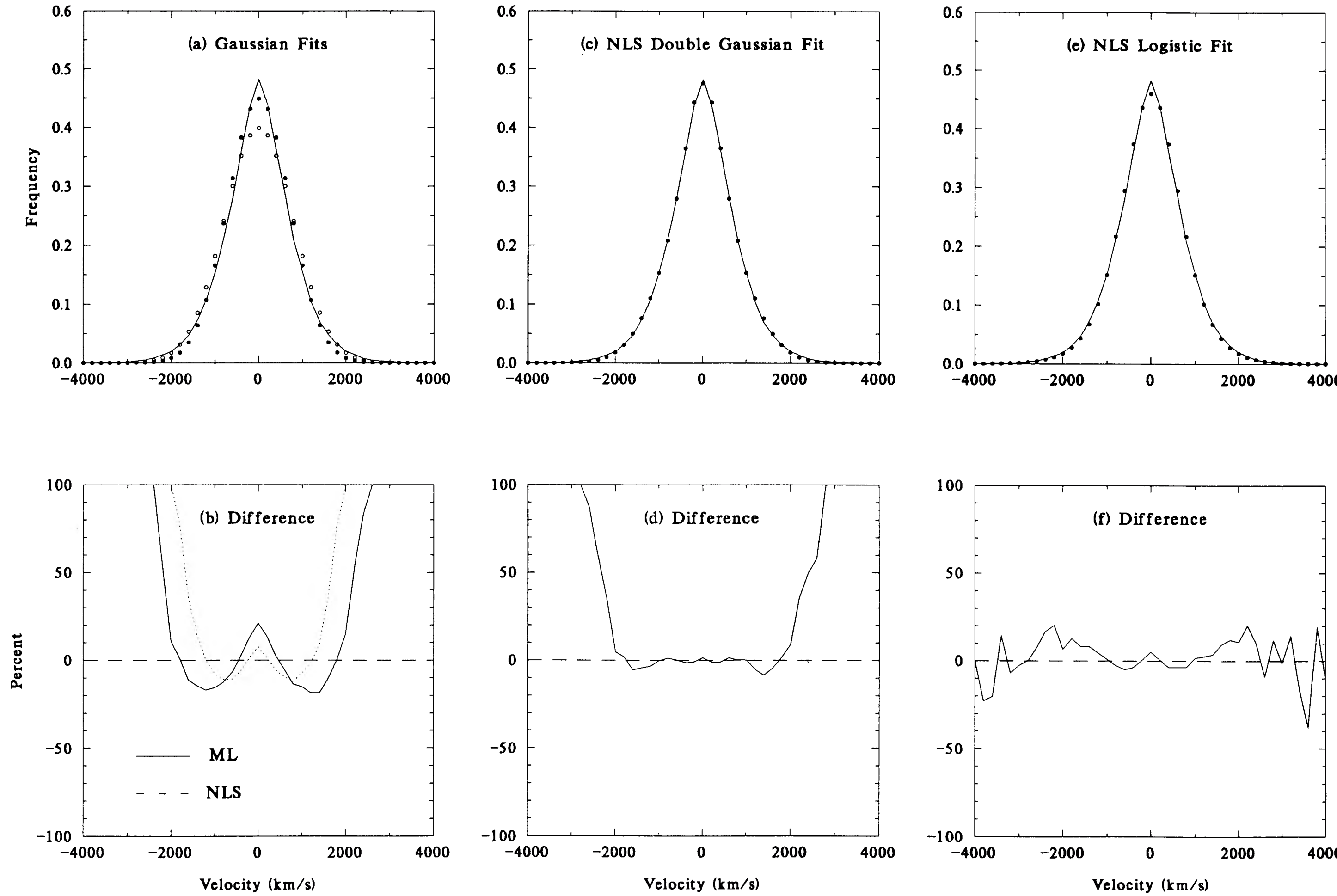

FIG. 4.—Simulation results based on random draws of 20 galaxies each from 10,000 clusters with dispersions drawn from the distribution of cluster dispersions described by ZHG. (*a*) Frequency distribution of the simulated data. A ML Gaussian fit is indicated by the open circles; a NLS Gaussian fit is indicated by the filled circles. (*b*) Percent difference of the simulated data with respect to the ML Gaussian fit (*solid line*), and the NLS Gaussian fit (*dashed line*). (*c*) Frequency distribution of the simulated data. A NLS fit to a Double Gaussian model is indicated by the filled circles. (*d*) Percent difference of the simulated data with respect to the NLS Double Gaussian fit. (*e*) Frequency distribution of the simulated data. A NLS fit to a Logistic model is indicated by the filled circles. (*f*) Percent difference of the simulated data with respect to the NLS Logistic fit.

by use of pooled samples, but merely serve to underscore the need to consider the expected distribution of velocities which result from a mix of clusters with different dispersions. Below we consider whether any significant excess of galaxies with low velocities exists in the pooled samples of CH and BS II when compared to simulations of the expected cluster contributions.

3.3.1. *The Double Root Residual Plot*

A comparison of real data (rather than simulated data) with a specific model requires a more sophisticated display than a simple difference plot. We choose to employ a plot of the so-called Double Root Residuals (hereafter referred to as DRRs). The DRR plot is a variation of a hanging histogram, which itself is just a (nonnormalized) difference plot between binned data and a proposed fitting function. Unfortunately, a hanging histogram gives equal weight to residuals from bins near the extrema of a distribution (which are usually poorly populated) as to residuals from bins near the middle of a distribution (which are usually well populated, and hence subject to greater $\sqrt{N}/N$ variations). To compensate for this effect, instead of using the actual value of counts in each bin to obtain a difference plot, the DRR technique takes advantage of the variance stabilizing properties of a square-root transformation. A refinement to the canonical DRR plot, which avoids some problems with small data sets (Velleman & Hoaglin 1981), is given below. The DRR in our calculations is defined as:

$$\mathrm{DRR} = \sqrt{2 + 4(\mathrm{observed})} - \sqrt{1 + 4(\mathrm{fitted})} \quad \text{if observed} \geq 1 \; , \tag{6a}$$

$$\mathrm{DRR} = 1 - \sqrt{1 + 4(\mathrm{fitted})} \quad \text{if observed} = 0 \; . \tag{6b}$$

The additive constants under the square roots in equation (6a) are not the same because for low bin frequencies the fitted count can take on arbitrarily small values, whereas the observed count is constrained to integer values. There are several clear advantages of a DRR plot over a simple difference plot. First, it graphically emphasizes *where* in a distribution lack of fit between data and model exists—the square-root transformation puts the residuals throughout the fit on an equal footing. Second, if the model is an adequate fit to the data, then the DRRs are roughly equivalent to normal deviates. Thus a DRR with numerical value $\geq \pm 2$ is significant at the 95% ($2\ \sigma$) level, whereas DRRs with absolute magnitude less than one indicate a reasonable agreement between data and model.

3.3.2. *The Cowie and Hu Sample*

In Figure 5*a* we plot the extended sample of CH ($N = 75$), with bins of 200 km s^{-1}. The solid line is the ML fit of a single Gaussian to the unbinned data ($\mu = -201$ km s^{-1}, $\sigma = 981$ km s^{-1}). Note that there exists a zero point offset of the fit due to an asymmetry in the distribution of relative radial velocities, a feature which would clearly be lost if one chose to fold the data (by plotting absolute relative velocities) prior to fitting. The dashed line is the Logistic model described above. We emphasize that we have not fitted the CH data to this model, but have simply scaled the Logistic fit to our simulations to the size of the CH sample. In Figure 5*b* we show the DRR plot for the Gaussian fit. The fit is rather poor, as noted by the CH analysis. The maximum DRR in the middle of the plot allows a rejection of the ML Gaussian at the 5% critical level. Also note that the deficits at ± 1000 km s^{-1} are rather large, indeed they are as significant as the middle peak. The omnibus normality tests support this impression: they all reject a single Gaussian model at better than the 1% critical level. In Figure 5*c* we show a DRR plot for the Logistic model obtained from the simulations described above. Note that the deviation of the central peak is now considerably less, on order 1.5, though it still might be argued that such a residual is larger than might be expected if the Logistic model represents an adequate description of the CH data.

Ideally, we would like to compare the CH data to simulations based on draws from the actual cluster dispersions which contribute to their data set. Unfortunately, as noted above, only roughly 70% of their clusters have available dispersions (Struble & Rood 1987; ZHG). It is instructive, nevertheless, to consider the distribution of velocity dispersions for their clusters which do have measurements. For the 30 clusters in their sample with known dispersions, five (17%) have dispersions less than 500 km s^{-1}, 21 (70%) have dispersions between 500 and 1000 km s^{-1}, and four (13%) have dispersions greater than 1000 km s^{-1}. However, it must be kept in mind that of the 59 galaxies in the CH sample drawn from clusters of known dispersion, six (10%) are from the low-dispersion group, 26 (44%) are from the medium-dispersion group, and 27 (46%) are from the high-dispersion group. As a result of the enhanced contribution of galaxies from high-dispersion clusters, the CH distribution might therefore be expected to be heavy-tailed relative to our simulation (which has 16% of its sample drawn from clusters with dispersions less than 500 km s^{-1}, 68% drawn from clusters between 500 and 1000 km s^{-1}, and 16% drawn from clusters above 1000 km s^{-1}). Some evidence for this effect is seen in the DRR plot in Figure 5*c*. Further speculation is limited by our ignorance of the global dispersions from which the remaining 16 CH galaxies are drawn.

CH also consider the distribution of relative velocities for 47 galaxies chosen exclusively with projected distances $r \leq 20$ kpc from the dominant galaxy. They find that the Double Gaussian mixture model which fits the extended sample also provides a reasonable fit to the central data. In Figure 6*a* we plot the CH central data along with the ML Gaussian fit to the extended data set, and the Logistic fit to the simulations described above. The DRR plot of the inner CH data with respect to the ML Gaussian shown in Figure 6*b* shows a clear excess in the middle. The omnibus normality tests again reject such a model at high levels of confidence. The DRR plot of the data with respect to the Logistic model shown in Figure 6*c* shows markedly better agreement, particularly in the central bins.

In summary, we believe that the CH data can be adequately explained by a simple assemblage of relative velocities drawn from a wide range of cluster dispersions. There is no compelling reason, at least given the data in hand, and considering our ignorance of the global dispersions for 30% of the CH clusters, to require existence of a population of satellite galaxies of low dispersion (on order 250 km s^{-1}) which "swarm" around the central dominant galaxy.

3.3.3. *The Bothun and Schombert Sample*

Unlike the CH sample, where we are forced to rely on a simulation of draws from clusters without complete knowledge of the distribution of their dispersions, the sample of BS II is amenable to a much more direct investigation. Of the eight clusters discussed by these authors, three have global dispersions of 550 km s^{-1} or less, three have dispersions between 650 and 850 km s^{-1}, and two have dispersions greater than 1200 km s^{-1}. The number of galaxies from low-, medium-, and high-

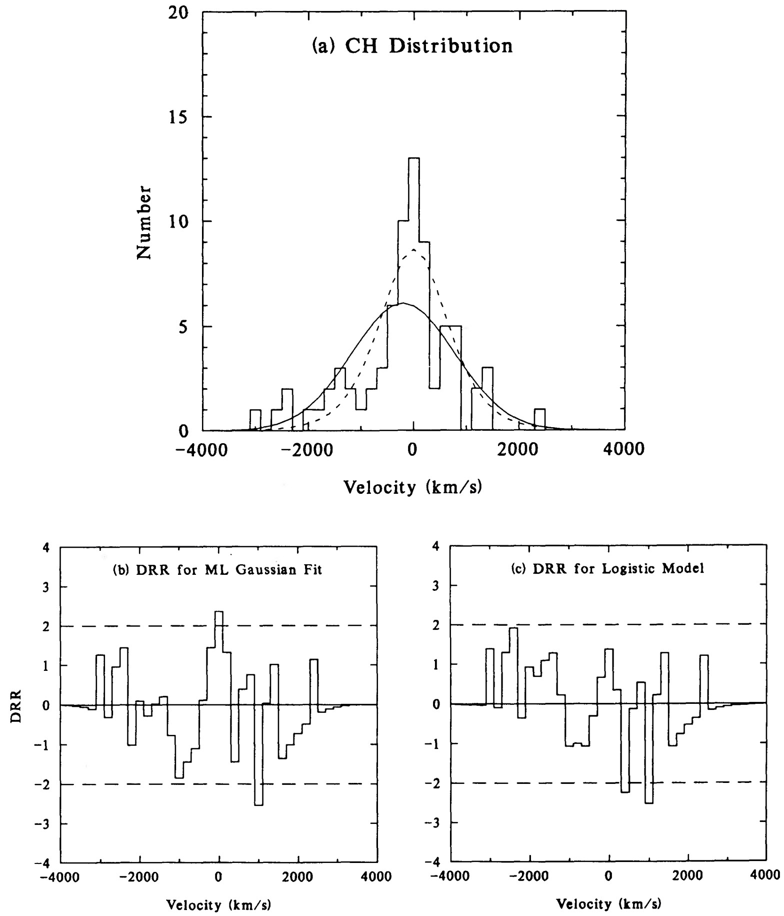

FIG. 5

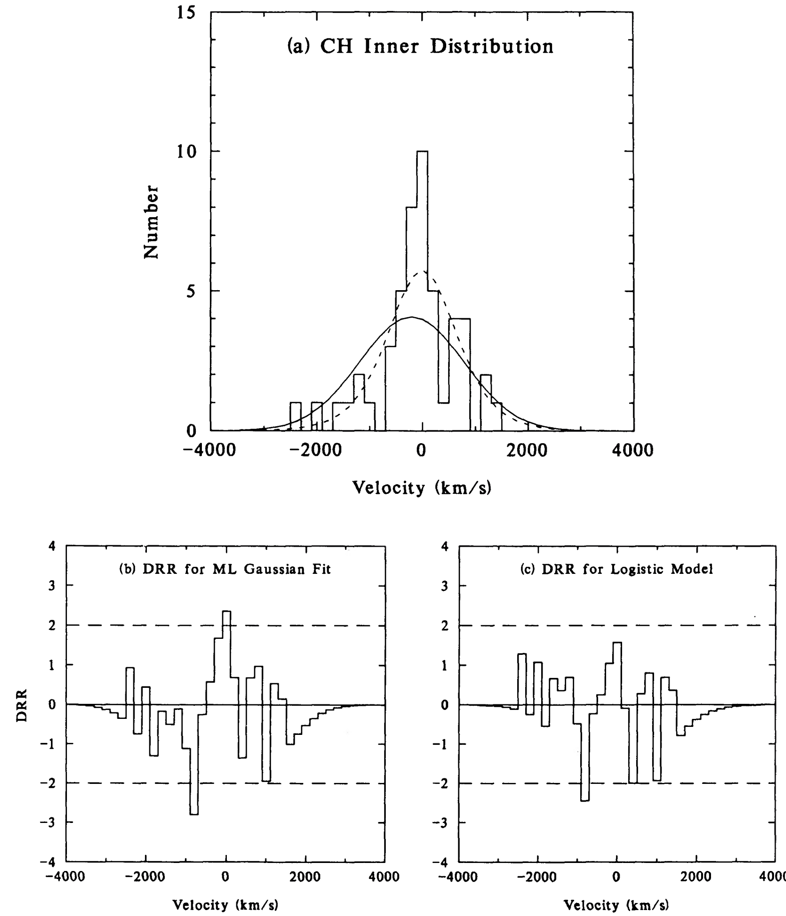

FIG. 6

FIG. 5.—Histogram of the CH extended sample (75 galaxies), with bins of 200 km s^{-1} centered on 0 km s^{-1}. Solid line is the ML Gaussian fit. Dashed line is the Logistic model from the simulation normalized to the size of the data batch. (*b*) Double Root Residual (DRR) plot for the ML Gaussian fit. The dashed lines at ± 2 indicate an approximate 95% significance level. (*c*) DRR plot for the normalized Logistic model. Dashed lines at ± 2 indicate an approximate 95% significance level.

FIG. 6.—Histogram of the CH inner sample ($r < 20$ kpc; 47 galaxies). Solid line is the ML Gaussian fit. Dashed line is the Logistic model from the simulations normalized to the size of the data batch. (*b*) DRR plot for the ML Gaussian fit. Dashed lines at ± 2 indicate an approximate 95% significance level. (*c*) DRR plot for the normalized Logistic model. Dashed lines at ± 2 indicate an approximate 95% significance level.

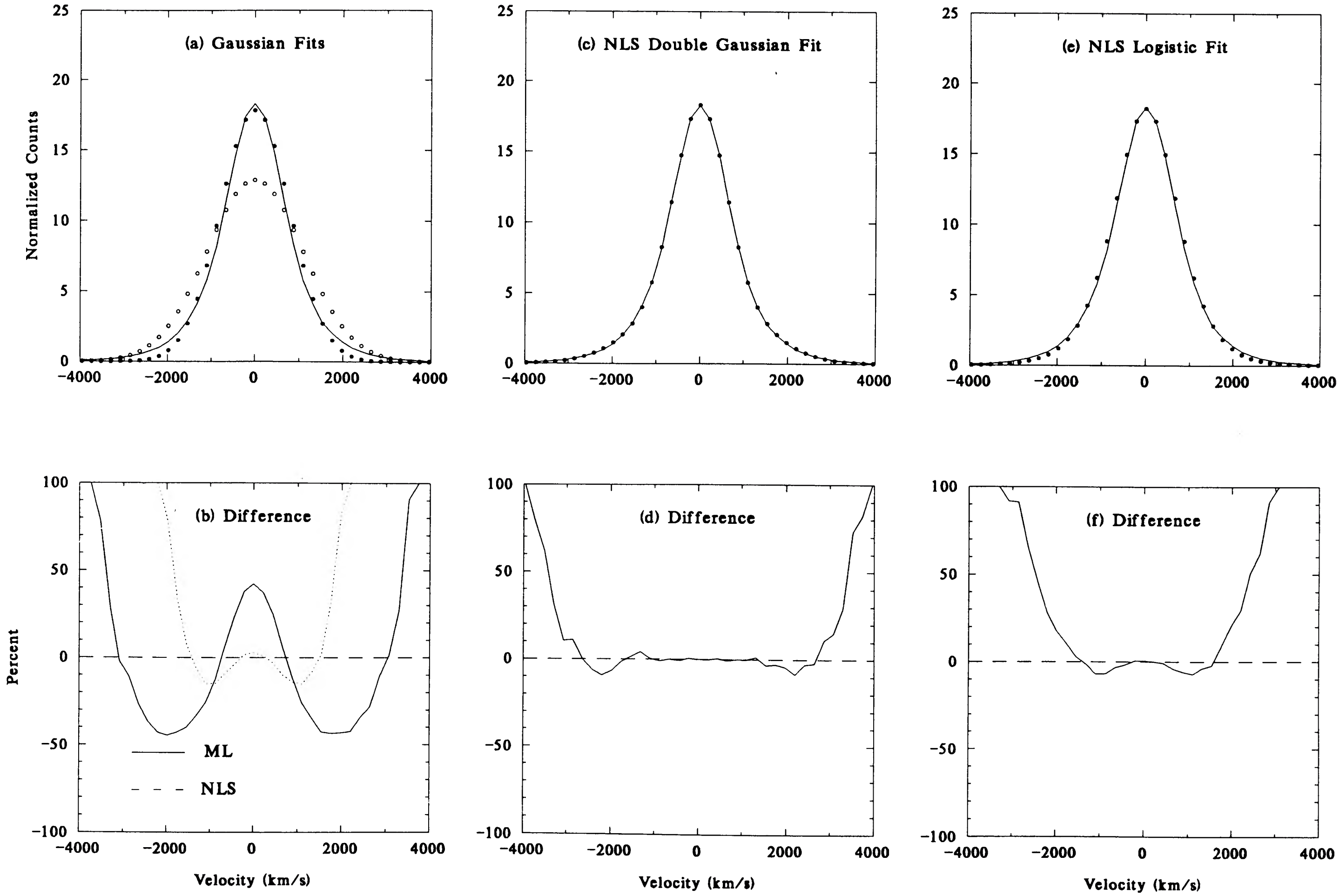

FIG. 7.—Simulation results based on random draws from eight clusters with dispersions set to those values specified by BS II, and in proportion to the number of galaxies in each of the BS II clusters. (*a*) Frequency distribution of the simulated data. A ML Gaussian fit is indicated by the open circles; a NLS Gaussian fit is indicated by the filled circles. (*b*) Percent difference of the simulated data with respect to the ML Gaussian fit (*solid line*) and the NLS Gaussian fit (*dashed line*). (*c*) Frequency distribution of the simulated data. A NLS fit to a Double Gaussian model is indicated by the filled circles. (*d*) Percent difference of the simulated data with respect to the NLS Double Gaussian fit. (*e*) Frequency distribution of the simulated data. A NLS fit to a Logistic model is indicated by the filled circles. (*f*) Percent difference of the simulated data with respect to the NLS Logistic fit.

dispersion clusters are roughly equally divided: 45 (26%), 73 (43%), and 52 (31%), respectively. We simulate the expected velocity distribution for the BS II sample by randomly drawing velocities from Gaussian distributions with dispersions set equal to those given by BS II, and in the same relative proportion obtained from the number of galaxies in each cluster listed in Table 4 of BS II. We repeat this experiment 1000 times to beat down the sampling noise. The solid line in Figure 7*a* shows the results of our simulation. The open circles are the ML Gaussian fit ($\sigma = 959$ km s^{-1}) to the full simulated data, while the filled circles are the NLS Gaussian fit to the binned frequency distribution ($\sigma = 793$ km s^{-1}). The difference plot shown in Figure 7*b* clearly exhibits a strong mismatch between the ML Gaussian fit and the simulated data. Although the difference plot for the NLS Gaussian fit is a dramatic improvement over the ML Gaussian fit, the NLS Gaussian still fails to capture the behavior of the near and outer tails of our simulations. Again, the omnibus normality tests discussed above are able to reject a Gaussian hypothesis for the simulated pooled sample at critical values less than 0.001. Figure 7*c* is a plot of the simulated BS II data and the resulting NLS Double Gaussian fit, indicated by the filled circles. We obtain $\alpha = 0.52$, $\sigma_1 = 578$ km s^{-1}, and $\sigma_2 = 1214$ km s^{-1}. Figure 7*d* shows the corresponding difference plot. In Figure 7*e* the filled circles represent the NLS fit to a Logistic function ($b = 486$ km s^{-1}). Note that this Logistic fit is similar, though not identical, to that obtained above. This result is expected, as the mix of input cluster dispersions is different. In fact, the BS II clusters represent rather more of the low-dispersion population, less of the middle-dispersion population, and more of the high-dispersion population than was the case for the previous simulation. Figure 7*f* is the difference plot for the simulation and the NLS Logistic fit. As can be seen, the Logistic model provides an excellent fit in the central region, though it still fails in the outer tails.

By counting the binned data in Figure 4 of BS II, we verify that these authors chose to *include* the velocity of the dominant galaxy in the discussion of their grouped sample. Thus the central few bins of the folded distribution discussed in BS II include an additional eight counts (since there were eight clusters in their sample) as compared to a treatment like that in Smith et al. (1985), Tonry (1985), CH, or Lauer (1988). The histogram in Figure 8*a* is the BS II data, here shown excluding the dominant galaxies. We have binned the results using the same bin size (220 km s^{-1} for an unfolded distribution) as used by BS II. The solid line is the ML fit to a single Gaussian ($\mu = 33$ km s^{-1}, $\sigma = 1098$ km s^{-1}); the dashed line is the result of our simulation. In Figure 8*b* we show the DRR plot for the ML Gaussian fit. Note that, compared to a Gaussian model, there does appear to be a significant excess of counts in the

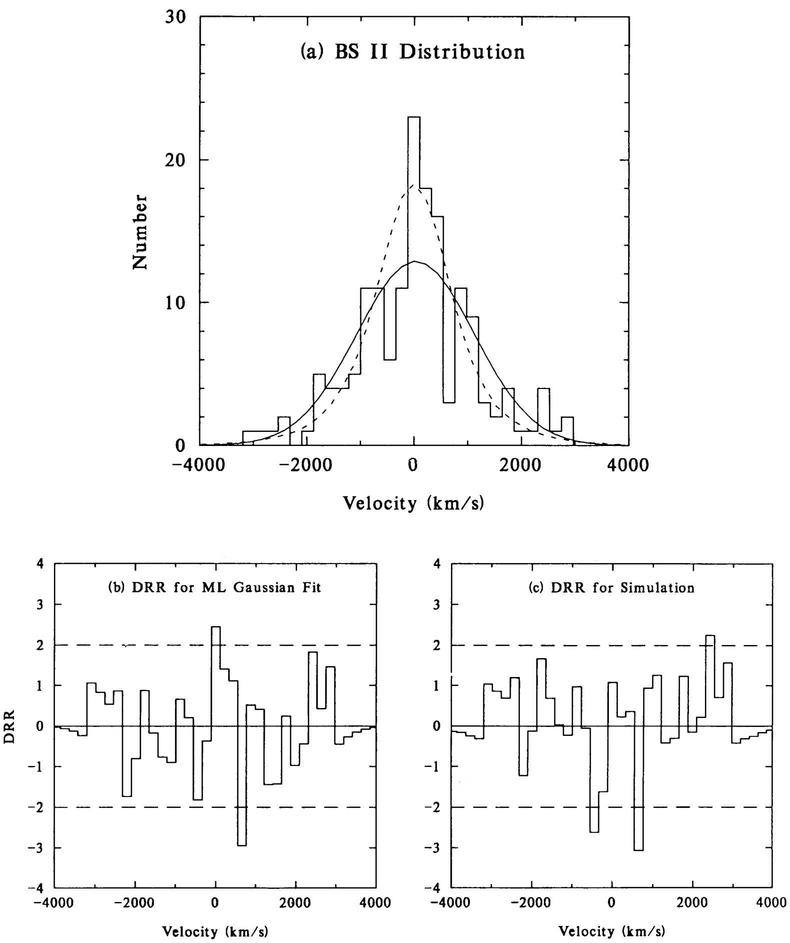

FIG. 8.—Histogram of BS II sample (161 galaxies), with bins of widths 220 km s^{-1} centered on 0 km s^{-1}. Solid line is the ML Gaussian fit. Dashed line is the Logistic model from the simulation normalized to the size of the data batch. (*b*) DRR plot for the ML Gaussian fit. Dashed lines at ± 2 indicate an approximate 95% significance level. (*c*) DRR plot for the normalized Logistic model. Dashed lines at ± 2 indicate an approximate 95% significance level.

central bin, as well as apparently real deficits of counts at a relative velocities of ± 600 km s^{-1}. The omnibus tests support this interpretation, rejecting a Gaussian model at critical levels less than 0.02. In Figure 8c the data are compared to the Logistic distribution obtained from our simulations. The significant excess in the central bins goes away—DRRs in the central bins are consistent with the noise in the rest of the distribution. The deficits which exist on both sides of the center (which may be due to bin placement) become the most significant aspects of the plot. Clearly, it is difficult to argue the case that an excess of low-velocity galaxies exists in the BS II sample based on the pooled distribution alone.

4. TESTS FOR "HIGH-VELOCITY" D/cD GALAXIES

One of the first intriguing results to emerge from the larger redshift surveys of individual clusters is the realization that, in some clusters, the D/cD galaxies exhibit statistically significant offsets in velocity space when compared to the mean velocity of the remaining cluster galaxies. While establishing this effect should be a rather straightforward exercise in statistical analysis, small but important differences in the techniques employed in the literature to date lead to quite different results. For example, while most authors employ canonical methodology (as described by Danese, De Zotti, & di Tullio 1980, hereafter DDd) to determine confidence intervals on their estimate of the cluster mean velocity, BFG argue that such procedures may misrepresent the true confidence intervals, particularly for small- to medium-size samples. In addition, one should consistently incorporate errors in the measurement of the D/cD velocity, an obvious consideration that nevertheless has been ignored by some authors. Finally, it is important to carry out calculations based on rest-frame velocities, rather than the "velocity" cz. For a cluster with mean $cz = 15{,}000$ km s^{-1}, the 5% $(1 + z)$ correction which results is a 10% correction in the confidence interval on location. At $cz = 30{,}000$ km s^{-1}, the correction to the location confidence interval is 20%. We also draw attention to the fact that while the formal statistical significance of an observed D/cD velocity offset can be driven to arbitrarily high values by obtaining large enough numbers of redshifts to shrink the confidence interval on location in velocity, the physical significance of an observed offset is only realized by a comparison with the scale of the parent cluster.

A number of studies have discussed the question of D/cD velocity offsets. Below we reconsider the reality of this effect in 14 clusters which have been suggested in the literature to exhibit statistically significant offsets. An application of the above considerations shows that of these 14, only five meet our criteria for statistical significance. The largest reliable offsets are on order 50% of the cluster scales.

4.1. *Classical Symmetric Tests*

Claims for statistically significant offsets of D/cD velocities reported in the literature have made use of classically determined symmetric confidence intervals on the cluster mean velocity, generally after "cleaning" the cluster velocity distribution following the 3 σ–clipping procedure advocated by Yahil & Vidal (1977), or other data-based clipping procedures (e.g., the χ^2-clip of Chapman, Huchra, & Geller 1988, or the "ad-hoc" gapped-clip of ZHG). A few examples serve to illustrate our concern over such procedures. ZHG apply the DDd technique to derive an rms error on the cluster mean velocity for A478 of ± 274 km s^{-1}. The corresponding 90% confidence interval on the cluster mean velocity is ± 537 km s^{-1}. Our procedure, described below, results in a 90% interval on central location of $(-717, +563)$ km s^{-1}. Note that our interval is asymmetric, reflecting the asymmetry of the velocity histogram for A478 based on only 13 velocities. The velocity offset (in the sense $V_{\rm cD}$ *minus* $V_{\rm cluster}$) of the cD galaxy in A478 reported by ZHG is -651 km s^{-1}, which appears to be highly significant. We obtain a similar velocity offset, $\Delta V_{\rm cD} = -611$ km s^{-1}. However, our velocity offset is now contained within the 90% confidence interval on location, and hence no longer appears significant. BS II obtain a velocity offset of 275 km s^{-1} for the cD galaxy in the cluster A2244 (based on data from Schombert et al. 1989) and claim significance because this value exceeds their estimate of 50 km s^{-1} for their velocity errors of individual galaxy redshifts by a factor of 3. Clearly, this procedure is inappropriate, as errors in individual galaxy redshifts are not the same as a determination of the error in central location of the cluster velocity. Indeed, our procedure indicates that an appropriate 90% interval for the central location in velocity is $(-726, +475)$ km s^{-1}. The velocity offset we obtain, -305 km s^{-1}, is well contained within this interval, and hence also cannot be claimed as significant. We note that this velocity offset would not even be significant if one were to employ the classical 90% interval, which is ± 425 km s^{-1}. Similar results are obtained for two of the three other clusters which BS II suggest to exhibit significant offsets.

4.2. *The Bootstrap Test*

Clearly, we advocate a more rigorous statistical procedure be followed if reports of large velocity offsets of D/cD galaxies in clusters are to be accepted. The prescription we suggest is the following: (1) Specify a window in redshift space within which to consider the data—this should be done a priori without consideration of the observed distribution. A useful specification is to accept for consideration all redshifts within a window ± 4000 km s^{-1} of the velocity of the dominant galaxy—for the present application chosen to be the brightest D/cD galaxy in the cluster. (2) Obtain an estimate of the central location in redshift space using the resistant biweight estimator $C_{\rm BI}$. (3) Obtain rest-frame relative velocities for all cluster galaxies with respect to this central location. (4) Obtain the 90% confidence interval on the central location in velocity space, $IC_{\rm BC_{BI}^*}$, via a bootstrap procedure (see BFG for details). (5) Compare the observed offset of the D/cD galaxy and its 90% measurement error with the above interval. (6) If the velocity offset of the D/cD is contained within the 90% interval on central location in velocity space, it is not significant; if the velocity offset is outside the 90% location interval, it should be reported as significant; if the 90% measurement error on the D/cD overlaps the 90% interval on location, but the actual observed offset is outside the location interval, it should be reported as marginally significant.

A similar procedure should be followed to assess physical significance, with comparison to the parent cluster scale. We suggest the following prescription: (1) After obtaining rest-frame velocities as described above, obtain the biweight estimator on scale, $S_{\rm BI}$. (2) Obtain the normalized offset relative to scale,

$$Z = \frac{V_{\rm cD} - C_{\rm BI}}{S_{\rm BI}} . \tag{7}$$

(3) Obtain a bootstrap estimate of the 90% range in the normalized offset Z. The measurement error of the D/cD velocity can easily be incorporated in this scheme by sampling from a Gaussian distribution with standard deviation set to the reported 68% error as part of the bootstrap. Just how large a normalized offset should be before one ascribes physical significance to the result is a matter of opinion. Nevertheless, it is clearly an interesting number for comparing offsets in one cluster to the next, and might be usefully compared to results of N-body calculations which attempt to simulate the formation and evolution of cD galaxies in clusters (Malumuth & Richstone 1984).

4.3. *Examples*

As an initial application of the procedures described above, we investigate 14 clusters where various authors have claimed significant D/cD offsets. Our results are summarized in Table 2. Column (1) lists the cluster studied. Column (2) lists the source of the redshift data. Column (3) lists the number of galaxies including the D/cD galaxy (in the subsequent calculations the dominant galaxy is not included). Columns (4) and (5) list the 3 σ–clipped estimates of location and the 90% interval on the mean velocity in km s^{-1}, respectively, calculated following the DDd procedure. Column (6) lists the biweight estimate of location in km s^{-1}. Column (7) lists the 90% bootstrap interval on central location in km s^{-1}. Column (8) lists the biweight estimate of the scale in km s^{-1}. Column (9) is the offset in velocity of the D/cD galaxy with respect to central location and its 90% measurement error in km s^{-1}. Column (10) indicates whether the offset is considered statistically significant (Y), not significant (N), or marginally significant (M). Columns (11) and (12) are the normalized offsets, Z, and the associated 90% bootstrap interval on this quantity.

Figure 9 is a graphical presentation of the bootstrap confidence intervals on velocity central location, the offset in velocity space of each D/cD galaxy, and its measurement error. For the clusters A85 and A1991, we have used the data from Beers et al. (1991). The filled circles represent the central location in velocity for the cluster (excluding the D/cD galaxy). The large vertical bars indicate the bootstrap confidence interval on this quantity. The open circles represent the relative velocity of each D/cD galaxy; the small vertical bars indicate its 90% measurement error. In this figure we also indicate the normalized velocity offsets of each D/cD galaxy, $Z_{\rm cD}$, along with its bootstrapped measurement errors (which explicitly includes the error in determination of central location and scale). The zero point is indicated by a vertical dashed line. The mean absolute Z-score for clusters with marginal or significant offsets is 0.45.

The importance of using a rigorous approach, particularly for small data sets, is demonstrated by our results in Table 2. There are five clusters which have more than 40 galaxies with measured redshifts. Our results confirm that a statistically marginal or significant velocity offset exists in four of these five clusters. For the nine clusters which have less than 40 available redshifts we confirm the existence of a statistically marginal or significant offset in only three clusters. The difference between our results and those of previous authors is due to the larger confidence intervals about the cluster central location in velocity space which we assign from our bootstrap procedure, as well as from our reluctance to trim the data in a subjective manner.

5. DISCUSSION

According to the conventional cannibalism models, dynamical friction time scales for the accretion of low-velocity galaxies trapped in the potential well of a D/cD are sufficiently short that one might expect presently detectable bound populations to be relatively rare. Merritt (1985) outlines arguments for an early, rapid formation of D/cD galaxies, during the collapse and virialization of its parent cluster (or subcluster). In this view as well, one does not expect many clusters to exhibit observable bound populations around the D/cD.

Because the kinematic properties of a bound population may depend in detail on the physical nature and dynamical history of the particular cluster, we have argued that one should ideally test for their presence on an individual basis. With the ever-increasing numbers of available redshifts per

TABLE 2

CENTRAL LOCATIONS, CONFIDENCE INTERVALS, AND D/cD VELOCITY OFFSETS

Cluster (1)	Source (2)	N (3)	$C_{3\sigma}$ (4)	$IC_{3\sigma,3\sigma}$ (5)	$C_{\rm BI}$ (6)	$IC_{\rm BC_{BI}}$, (7)	$S_{\rm BI}$ (8)	$\Delta V_{\rm cD}$ (9)	Significance (10)	Z (11)	Z: $IC_{\rm BC_{BI}}$, (12)
A85	ZHG	15	15825	±795	15734	−1320,836	1706	950 ± 79	Y	0.56	−0.03, 1.43
A85	B	19	16087	±645	16076	−861,613	1646	624 ± 79	M	0.38	−0.01, 1.01
A151	ZHG	13	15207	±992	16265	−2043,189	365	−778 ± 99	N	−2.13	−4.70, 0.58
A478	ZHG	13	26487	±481	26448	−756,562	976	−611 ± 165	N	−0.63	−1.33, 0.44
A1795	ZHG	46	18632	±198	18610	−209,209	817	368 ± 132	Y	0.45	0.13, 0.83
A1809	ZHG	13	23707	±147	23749	−145,242	473	−286 ± 76	Y	−0.61	−1.31, 0.06
A1991	BS II	15	17883	±255	17836	−223,309	531	−169 ± 82	N	−0.32	−0.93, 0.29
A1991	B	25	17820	±208	17784	−222,239	667	−120 ± 76	N	−0.18	−0.58, 0.18
A2067	ZHG	12	22458	±432	22425	−424,597	825	−338 ± 89	N	−0.41	−1.16, 0.23
A2199	ZHG	69	9189	±167	9161	−182,188	881	182 ± 24	N	0.21	−0.02, 0.42
A2244	S	27	29008	±425	28959	−726,475	1341	−305 ± 82	N	−0.23	−0.60, 0.36
A2271	BS I	10	17035	±298	17039	−299,498	497	−297 ± 82	N	−0.60	−1.83, 0.12
A2634	BS I	25	9383	±250	9454	−248,189	703	−343 ± 82	M	−0.49	−0.96, −0.04
A2670	ZHG	223	22849	±101	22843	−104,115	935	408 ± 165	Y	0.44	0.22, 0.66
Shapley 8	TCG	80	14289	±171	14310	−179,197	969	−261 ± 34	Y	−0.27	−0.49, −0.07
Klemola 44a	GGP	42	8782	±227	8670	−334,330	924	−475 ± 165[a]	M	−0.51	−0.89, −0.10
Klemola 44b	GGP	42	8772	±228	8635	−353,336	936	−61 ± 165[b]	N	−0.06	−0.46, 0.48

NOTES.—B: Beers et al. 1991; BS I: Bothun & Schombert 1988; BS II: Bothun & Schombert 1990; GGP: Green et al. 1990; S: Schombert et al. 1989; ZHG: Zabludoff et al. 1990, and references therein.

[a] IC 5353.

[b] IC 5358.

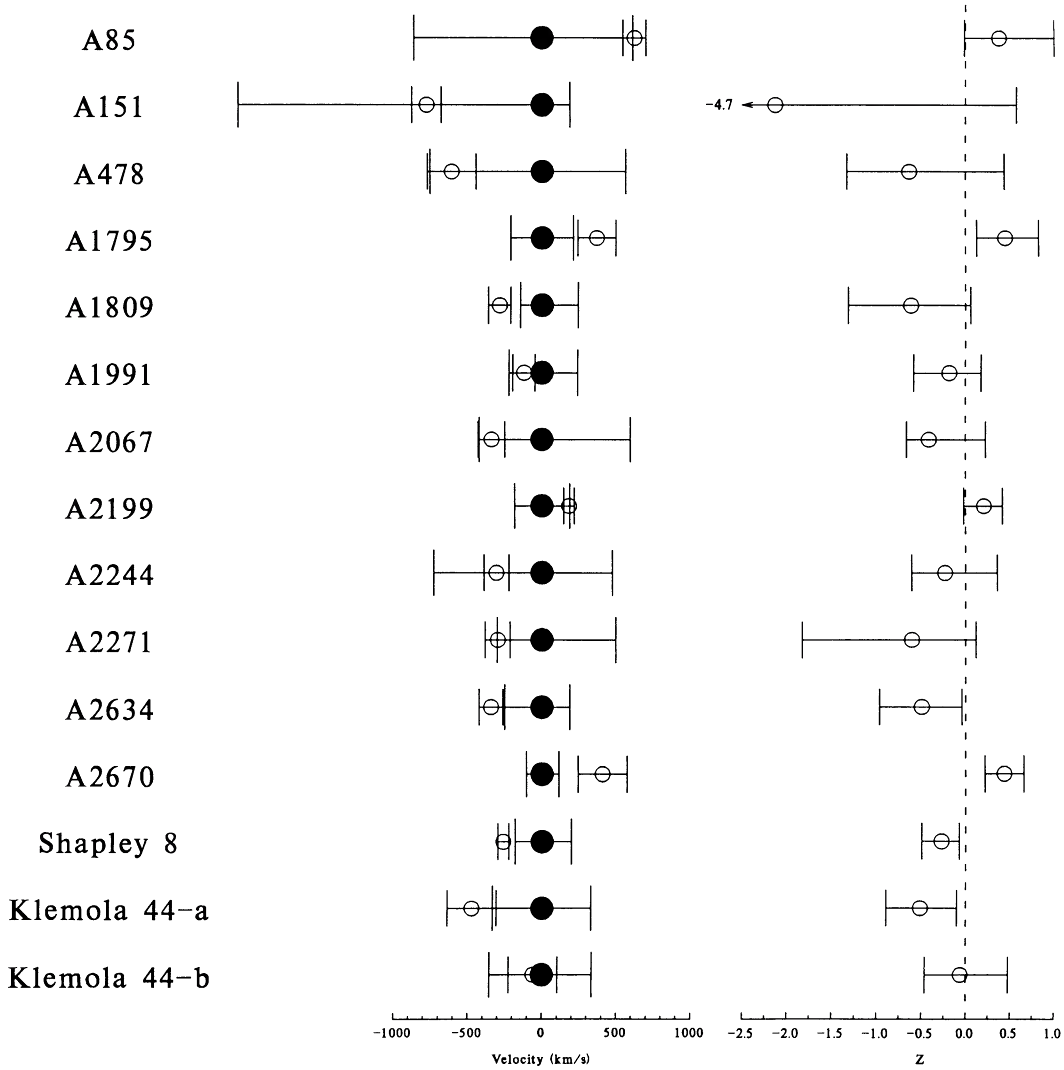

FIG. 9.—(*left*) 90% confidence intervals on location in velocity space for cluster galaxies (excluding the D/cD) and the 90% measurement error of the D/cD velocity. Filled circles are all centered on zero and represent the biweight location estimate. Large vertical bars are the limits of the 90% confidence interval on location. Open circles represent the relative velocity of the D/cD galaxy. Small vertical bars are the limits of the 90% measurement error. (*right*) Velocity offsets of the D/cD galaxy scaled by the dispersion of the cluster. Vertical bars are the 90% bootstrap confidence intervals on this value. Dashed vertical line is the zero point.

cluster, this opportunity may soon become a reality. For now, we are constrained to employ tests which make use of the limited data in hand. We have developed one such test, the Indicator test, which avoids the use of subjective binning of small data sets. Though this test may be conservative, it is efficient and not subject to some of the problems associated with the binning of small data sets. From an application of the Indicator test to five clusters where previous work has suggested the existence of bound populations, we find strong supporting kinematic evidence for only one cluster, Klemola 44. As pointed out by GGP, eight galaxies in the vicinity of IC 5353 have a dispersion of 140 km s^{-1}, as compared to the global cluster dispersion of 924 km s^{-1}. Other proposed clusters with bound populations, A1991, A2107, A2589, and A2593, are shown by the Indicator test to exhibit little, or only marginal, kinematic evidence in support of this hypothesis.

We should emphasize that we have only carried out a detailed investigation of the *kinematic* evidence for individual clusters which have previously been identified as likely bound population candidates. The fact that we find convincing kinematic evidence for the existence of a bound population in only one of the five suggested candidate clusters, based on the available redshift samples, suggests that much more work is required before the existence of bound populations in many clusters can be assumed to be correct.

In BS I and BS II, the authors discuss the nature of the surface brightness profiles in clusters with suspected bound populations. The working hypothesis is that galaxies in the near vicinity of the D/cD (and presumably members of a bound population) should suffer from enhanced tidal truncation due to interaction with the mass distribution associated with the D/cD. Two of the five galaxies suspected to belong to the bound population associated with A2589 are shown by BS I to exhibit tidally truncated profiles. BS II find that some 82% of the low velocity galaxies (those with $|v_{\rm rel}| < 300$ km s^{-1}) in three of the clusters they investigate (A1991, A2107, and A2593) exhibit tidally truncated surface brightness profiles, compared to only 44% of the total population (of galaxies with measured redshifts) in these clusters which exhibit truncated profiles. This result is interpreted as evidence that the bound galaxies are on eccentric orbits focused on the D/cD. However, it is difficult to evaluate the weight one should assign to this result. Of the 42 galaxies (excluding the two D/cD galaxies) with available surface photometry listed in BS II Table 6, only 28 have measured velocities listed in their Tables 1, 2, and 4. Of these 28, only 10 have $|v_{\rm rel}| < 300$ km s^{-1}. Furthermore, the majority of galaxies with available surface photometry in this sample come from A2107, the cluster which the Indicator test suggests is the least likely to have an associated bound population. Nevertheless, further investigations of this nature are extremely important, as they provide complementary information to the kinematic evidence.

Pooling of the available redshifts for a large number of individual clusters provides a possible tool for establishing the existence of bound populations in at least some clusters. However, we have pointed out that several previous analyses of pooled samples of relative velocities have adopted a faulty null hypothesis. Simulations of random draws from clusters with a wide range of dispersions confirm that the expected distribution of such a mixture is not Gaussian, but one that is substantially peaked in the center (due to the contribution from low-dispersion clusters), and heavier tailed than Gaussian (due to the contribution from the high-dispersion clusters). We demonstrate that the expected distribution is well described by a one-parameter functional form, the Logistic. We have argued that a Logistic model (with scale parameter $b = 438$ km s^{-1}) provides an excellent description of the expected velocity distribution when galaxies from clusters with dispersions typical of the entire population of rich clusters (quantified by ZHG) are pooled, and thus may serve as a useful null distribution for future comparison.

Double Gaussian fits (with three free parameters, not counting the means) can also provide excellent fits to pooled data sets, which is not surprising. However, the excellence of these fits should not be taken as prima facie evidence for the existence of bound populations, due to the inadequate null hypothesis. The χ^2 obtained by CH for their Double Gaussian fit is 2.6 (2 degrees of freedom), with parameters $\alpha = 0.60$, $\sigma_1 = 250$ km s^{-1}, and $\sigma_2 = 1400$ km s^{-1}. The χ^2 obtained from a comparison of the normalized Logistic model from our simulations to the CH data is 6.6 (6 degrees of freedom), also an excellent description. BS II report a rather low reduced χ^2_ν (1.72) for a Double Gaussian fit to their pooled cluster data (they do not report the number of degrees of freedom), with parameters $\alpha = 0.20$, $\sigma_1 = 120$ km s^{-1}, $\sigma_2 = 1200$ km s^{-1}. The χ^2 obtained from a comparison of the normalized Logistic model from our simulations to the BS II data is 11.8 (8 degrees of freedom), also an excellent description. A powerful exploratory data analysis tool, the Double Root Residual (DRR) plot, has been used to show that the pooled CH and BS II samples are consistent with Logistic models. Thus, in contrast to the conclusions of CH and BS II, we find that no convincing kinematic evidence exists for the observation of bound populations in a large sample of clusters of galaxies.

BS II argue that differences in (radial) selection effects between their data and that of CH are sufficient to explain the differences in mixture model parameters between the two studies. While we agree that selection effects differ, the *primary* difference between the CH and BS II data lies in the mix of cluster dispersions from which they sample. In both studies, the best-fit mixture models include a second-component velocity dispersion which is large compared to the majority of rich clusters. In fact, only a small number of high-velocity galaxies (outliers?) are necessary to arbitrarily inflate the second-component dispersion in mixture models such as these. The first-component dispersion, as well as the inferred mixture proportions, are strongly correlated with the estimate of the second-component dispersion. Future researchers of this problem need to carefully consider the mixture of input cluster dispersions, test for the presence of bound populations in clusters of similar dispersions, or compare to the expected distribution of relative velocities from a pooled sample using a model like the Logistic.

We have considered the existence of statistically significant velocity offsets of D/cD galaxies in 14 clusters where they have been previously claimed. We show that canonical techniques often underestimate the confidence intervals which should be associated with an estimate of the cluster central location in velocity space. Improved estimators of confidence intervals on location, based on a bootstrap resampling technique, are generally longer and (often) asymmetric compared to the canonical intervals. As a result, we find supporting evidence for statistically significant offsets in only four of these clusters, three of which have 40 or more available redshifts per cluster. Other clusters (with typically 10–30 available redshifts) are either not statistically significant, or are only marginally so. Because we have restricted our analysis to include only those clusters with previously suspected D/cD velocity offsets, our result serves primarily to warn that this phenomenon may not be as common as might have been otherwise assumed. An analysis of a large sample of D/cD clusters with on order 50 available redshifts per cluster is required before a clean statistical answer to the question "what fraction of D/cD galaxies exhibit significant velocity offsets?" can be obtained.

Among those clusters in the present sample which do exhibit significant D/cD velocity offsets, we suspect (following Beers 1986 and ZHG) that unrecognized substructure is the cause for the offsets. The significant offsets are typically no more than 50% of the inferred dispersion of their parent clusters. The observed relative velocities of gravitationally bound subclusters in several clusters of galaxies with complex structures (e.g., A98, Beers, Geller, & Huchra 1982; A2440, Beers et al. 1991) is on order 500–1000 km s^{-1}. If such structures went unrecognized, we would have inferred global dispersions for A98 and A2440 on order 1000 km s^{-1}. The "observed" velocity offsets of the D galaxies in each of these clusters would be on order 0.5–1.0.

A FORTRAN program, ROSTAT (V1.2), which implements the robust and resistant location and scale estimators, normality tests, and bootstrap techniques employed in this paper, is available to any interested party. Persons desiring a current copy of ROSTAT should contact T. C. B. by mail or electronic mail (BITNET: BEERS@MSUPA; INTERNET: BEERS@MSUPA.PA.MSU.EDU). We would like to thank J. Schombert for discussions concerning the Bothun & Schombert (1990) analysis and an anonymous referee for suggestions which improved the final text.

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Note added in proof.—After this paper was submitted, we received a preprint of the paper "A Search for Bound Satellite Populations Around Central Dominant Galaxies in Clusters," by Merrifield & Kent (Merrifield, M. R., & Kent, S. M., AJ, 101, 783 [1991]). These authors reach a conclusion which is consistent with the present study, that is, that the best estimate for the fraction of cluster members projected within 20 kpc of a central dominant galaxy that constitute a bound population is on order 5%, with an upper limit on order 20%.