THE ASTROPHYSICAL JOURNAL, 380:476–483, 1991 October 20

FRAGMENTATION OF ELONGATED CYLINDRICAL CLOUDS. II. POLYTROPIC CLOUDS

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Received 1991 March 18; accepted 1991 May 6

ABSTRACT

We present a numerical study of the fragmentation of elongated, polytropic, axially symmetric clouds. Results have been obtained for various ratios of length to diameter L/D, polytropic constant γ, and initial Jeans number J_0 (ratio of the absolute value of gravitational to thermal energies). We introduce initial density perturbations on the axis of otherwise uniformly dense cylinders and determine the maximum number of fragments $N_{f\max}$ that can form and grow for combinations of L/D, γ, and J_0. At fixed values of L/D and γ, $N_{f\max}$ increases with J_0, then saturates at a finite value $N_{f\max} = N_{\max} \approx 2L/D$ at large J_0 for $\gamma \geq 1$. At fixed values of L/D and J_0, $N_{f\max}$ decreases when γ increases. Since the saturation value $N_{\max}$ is independent of γ, reaching saturation at high γ requires a larger initial Jeans number. The number of fragments formed can exceed $2L/D$ for $\gamma < 1$. We show that J_N, the initial Jeans number required for forming N fragments, scales approximately as γ^2 for $0.9 \leq \gamma < 4/3$ and $N < N_{\max}$. This relation does not hold for $\gamma > 4/3$. Implications of these results on the minimum stellar mass are discussed.

Subject headings: hydrodynamics — nebulae: general — numerical methods — stars: formation

1. INTRODUCTION

The process of cloud fragmentation and collapse leading to star formation is still poorly understood. Most multidimensional analyses of this problem considered spherical clouds. However, it is becoming clear that other geometries ought to be considered. This paper is one in a series that considers cylinders as the initial cloud geometry. The interest in this geometry is more than academic since recent observations have shown that molecular cloud cores are mostly elongated (Myers et al. 1991). In some cases these cores are associated with extended filaments initially described by Schneider & Elmegreen (1979). The effects of magnetic fields on these clouds are not clear since field lines parallel (e.g., L1755; Goodman et al. 1990), perpendicular (e.g., L204; McCutcheon et al. 1986), and at arbitrary angles to some elongated clouds are observed. It thus appears that magnetic fields, at least in some cases, do not dominate the dynamical evolution of clouds (Myers & Goodman 1991). *IRAS* data also show evidence for elongated structures in the interstellar medium which are not gravitationally bound (Low et al. 1984; Magnani, Blitz, & Mundy 1984, 1985).

The collapse of an interstellar cloud to stellar dimensions involves changes of many orders of magnitude in density, size, and temperature. Accurate modeling of the complete temperature-density relation requires a sophisticated treatment of the radiative transfer, yet for certain ranges of density simplified methods can be used. The temperature-density relation has been calculated for a collapsing interstellar cloud consisting of pure hydrogen (Bodenheimer 1978; Tohline 1982). The total range of density is divided in intervals, and in each one the relation is approximated by $\partial \log T/\partial \log \rho \approx c$, where T is the temperature, ρ is the density, and c is a constant which takes a different value in each interval. Thus, the temperature-density relation can be transformed into a polytropic equation of state $P = K\rho^\gamma$, where $\gamma = c + 1$ (assuming an ideal gas law). During collapse, c increases from negative to positive values. Consequently, the cloud goes through three different physical regimes: a cooling, an isothermal, and a heating regime, corresponding to $\gamma < 1$, $\gamma = 1$, and $\gamma > 1$, respectively. During the cooling phase ($\gamma < 1$[4]), collisional excitations followed by radiative deexcitations increase with density, lowering the thermal energy. This compressional cooling continues until the density approaches 10^{-19} g cm^{-3} (Larson 1973; Gerola & Glassgold 1978). The temperature then approaches 10 K, and a balance between the collisional excitations and radiative deexcitations maintains the gas in an isothermal regime ($\gamma = 1$). The third regime occurs when radiative excitations followed by collisional deexcitations become important so that the cloud is no longer optically thin (this is true for a pure hydrogen composition; in the interstellar medium dust is the main contributor to opacity at these densities). Increases in density then raise the cloud's opacity, trapping the radiation and raising the cloud's temperature. The actual rate of heating depends on the cloud composition, which varies during collapse as the molecular hydrogen is dissociated. It is nearly constant within each interval. The value of γ is 7/5 for molecular hydrogen, 1.1 for partly dissociated molecular hydrogen, and 5/3 for atomic hydrogen.

The fragmentation of polytropic clouds has been the subject of several studies (Cook & Harlow 1978; Boss 1980, 1981). Rotating spherical clouds was the only case considered, and although fragmentation was attained with $\gamma = 7/5$, very high initial Jeans numbers J_0 (the ratio of the absolute value of the gravitational to thermal energies) were required. This result is corroborated by Tohline's (1981) analytical investigation of MacLaurin spheroids.

Previous studies of the collapse of elongated isothermal cylindrical clouds with initially uniform density has shown the development of two density maxima on the symmetry axis on either side of the equatorial plane (Bastien 1983; Rouleau & Bastien 1990, hereafter RB). The dynamical collapse time is shorter for the two subcondensations than for the entire cloud

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[4] It should be noted that a value of $\gamma < 1$ is not strictly the thermodynamical $\gamma \equiv C_P/C_V$. It should be considered to be an effective γ that relates the density and pressure in the polytropic equation of state.

if the Jeans number exceeds a critical value $J_{2-\mathrm{frag}}$ (RB). Thus a binary system will form if the subcondensations can avoid colliding at the cloud's center. The collision can be avoided if one considers more realistic conditions, i.e., a nonzero angular momentum (Bonnell et al. 1991). RB found an empirical relation relating the critical Jeans numbers for collapse (J_c, defined as the minimum value of J_0 such that the Jeans number increases monotonously as collapse proceeds) of polytropic and isothermal clouds,

$$J_c(L/D, \gamma) \simeq \gamma^2 J_c(L/D, \gamma = 1) . \quad (1)$$

The critical Jeans number J_c is undefined for $\gamma > 4/3$ because the thermal energy increases faster than the absolute value of the gravitational energy (see § 4.2). Bonnell & Bastien (1991) have shown that when rotation is included, the spontaneous fragmentation of elongated cylinders forms two toroidal subcondensations. Each subcondensation is expected to be unstable to fragmentation via non-axisymmetric density perturbations (Norman & Wilson 1978). The γ^2 scaling relation for J_c was shown to be valid for rotating clouds as well.

The fragmentation of elongated cylinders into more than two subcondensations occurs in the presence of initial axisymmetric density perturbations (Bastien et al. 1991, hereafter Paper I). Contrary to expectations, the maximum number of fragments $N_{f\max}$ able to form in a collapsing cylinder (determined by increasing the initial number of perturbations N_i) initially increases with J_0 and eventually saturates at a value $N_{f\max} = N_{\max} \approx 2L/D$ at large J_0. This result was somewhat unexpected on the basis of previous theoretical studies (Stodolkiewicz 1963; Larson 1985). The main goal of this paper is to extend the study of elongated cylindrical clouds (Bastien 1983; RB; Bonnell & Bastien 1991; Paper I) to the polytropic case. More specifically, we want to determine the effects of the polytropic constant on the maximum number of fragments that can form during the collapse of a cylindrical cloud as a function of J_0 and L/D. It is thus a direct extension of Paper I, which considered only the isothermal case. In view of the rather unexpected results of Paper I, we felt that an exhaustive study of parameter space was warranted.

In § 2, we describe the numerical simulations and briefly discuss the code used. In § 3, we present the number of fragments obtained for 2 values of L/D, and discuss the results in § 4. We also compare these results to those obtained previously for the isothermal case (Paper I) and for the polytropic case with a two-dimensional code (RB). Conclusions are presented in § 5.

2. CALCULATIONS

The numerical simulations were carried out with the smooth particle hydrodynamics (SPH) code used in Paper I. For a detailed description of the method, we refer the reader to the review paper by Benz (1990). The particularities of our code, such as the choice of numerical parameters, are given in Paper I. For the calculations reported here, as in RB, a polytropic equation of state

$$P = K\rho^{\gamma} , \quad (2)$$

was used with the constant K set to $(R_g/\mu)T_0\rho_0^{1-\gamma}$, where R_g, μ, T_0, ρ_0, and γ are the gas constant, the mean molecular weight, the initial temperature and density of the cloud, and the polytropic constant, respectively. With this value of K, the equation of state of a perfect gas is satisfied, and with $\gamma = 1$ we recover the isothermal equation of state used in Paper I. The specific internal energy of the cloud is

$$u = K\rho^{\gamma-1} , \quad (3)$$

and the sound speed is

$$c_s = \left(\frac{\partial p}{\partial \rho}\right)^{1/2} \propto \rho^{(\gamma-1)/2} . \quad (4)$$

The collapse of elongated uniform cylindrical clouds does not lead directly to their spontaneous fragmentation into more than two fragments (Bastien 1983). In order to initiate multiple fragmentation, density perturbations must be introduced. The form of the perturbations used is the same as in Paper I,

$$\rho = \rho_0\left[1 + A\left(\sin^2 \frac{\pi N_i z}{L}\right)\left(1 - \frac{2r}{D}\right)^2\right] ,$$
$$0 \le r \le D/2 , \quad 0 \le z \le L , \quad (5)$$

where z and r are the axial and radial coordinates, L and D are the length and diameter of the cylinder, ρ_0 is the background density, A is the amplitude of the perturbations, and N_i is the initial number of perturbations. At the start of a run, all velocities are set to zero and the internal energy is adjusted, while keeping the total mass constant, in order to produce a given value of J_0. As in Paper I, we set $A = 1$.

We use the results of RB, who also considered various polytropic constants, in order to check those presented in this paper. We estimate J_c (with 2000 particles) for the cases $L/D = 1$, $\gamma = 1.1$ and $\gamma = 1.2$, and compare the results with those obtained by RB using a two-dimensional finite difference code with an adaptive grid (the YAQUI code). RB calculated the values of J_c in two modes: a high-resolution mode and a low-resolution one. This latter mode, in order to save computer time, used only $\sim\frac{1}{4}$ of the number of cells considered in the more precise mode. We find $1.28 < J_c < 1.30$ for $\gamma = 1.2$, in good agreement with the result of YAQUI ($J_c = 1.29$). For $\gamma = 1.1$, we find $1.05 < J_c < 1.11$, in rough agreement with the value $J_c = 1.13$ obtained with YAQUI in the low-resolution mode. The agreement would be better with a high-resolution YAQUI calculation as this tends to lower J_c by a few percent (RB performed high-resolution calculations for only a few cases). Since these critical Jeans numbers are of primary importance in determining the outcome of the cloud's evolution, obtaining consistent values from the two algorithms is a major requirement. A large number of numerical tests show that $N_{\max}$ is real and not the result of some numerical artifact: increasing the number of particles from $\simeq 1000$ to $\simeq 5000$ does not change the value of $N_{\max}$ (Paper I). A comparison of two finite difference codes and a SPH code in a study of fission of rotating polytropes revealed that SPH can adequately treat the same problems with 8 times fewer particles than the number of cells required for the finite difference codes. Since these codes assume a symmetry across the equatorial plane and through the rotation axis while SPH does not, the factor of 8 is an underestimate (Durisen et al. 1986).

It should be noted that the calculations presented here were followed significantly further than in Paper I, allowing for a better determination of the final number of fragments. This tends to slightly increase the value of $N_{f\max}$ at all J_0 as the existence of fragments that are slow to develop is better ascertained. The added pressure forces due to the polytropic equation of state with $\gamma > 1$ produces less steep density profiles. The density falls off less rapidly from the fragments' maxima, increasing the density between the fragments. Thus fragments which in an isothermal calculation are well defined and separated tend to overlap.

3. RESULTS

We used our SPH code in order to run simulations with several values of L/D, J_0, and γ. Our results are summarized in Table 1 for $L/D = 2$ and Table 2 for $L/D = 5$. The first column in these tables gives the number of initial perturbations in the cylinder. The other columns show the number of fragments that form once collapse is well under way for a given J_0. These fragments would eventually become individual stars within the simplifying assumptions considered in this paper. The number of fragments formed increases with N_i until it reaches the maximum value $N_{f\max}$ which we indicate in boldface. The value of $N_{f\max}$ initially increases with J_0, then saturates at a finite value $N_{f\max} = N_{\max}$ which is found in the rightmost columns of the tables. Examples of the evolution of the cloud's density profiles are given in Paper I for the isothermal case. Increasing the polytropic constant decreases $N_{f\max}$ at small J_0,

TABLE 1

NUMBER AND NATURE OF THE FRAGMENTS FORMED WITH $L/D = 2$

N_i	$J_0 = 3$	$J_0 = 5$	$J_0 = 10$
	$\gamma = 1.0$		
4.........	**3**	...	...
5.........	2	4	...
6.........	2	3 + 2 c.f.	5
7.........	2	4	4 + 2 c.f.
8.........	...	**5**	5
9.........	...	4	**6**
10.........	...	...	5 + 2 c.f.
	$\gamma = 1.1$		
3.........	2	...	...
4.........	**3**	...	...
5.........	3	**4**	4
6.........	2	3 + 2 c.f.	**5**
7.........	...	2	4 + 2 c.f.
8.........	...	2	5
	$\gamma = 1.2$		
2.........	**2**	...	...
3.........	2	...	...
4.........	2	**3**	2
5.........	2	2 + 2 c.f.	4
6.........	...	...	**5**
7.........	...	...	4
	$\gamma = 1.3$		
2.........	**1**	...	...
3.........	...	2	...
4.........	...	**3**	2
5.........	...	2	4
6.........	...	2	**5**
7.........	...	...	2

NOTES.—The fragments in this table are classified as normal fragments or close fragments (c.f.). In the normal fragments we include always two binary fragments located at either end of the cylinder (see Paper I). Close fragments are on the point of being absorbed by binaries and their existence as independent fragments is doubtful. Boldface numbers indicate cases where saturation is reached.

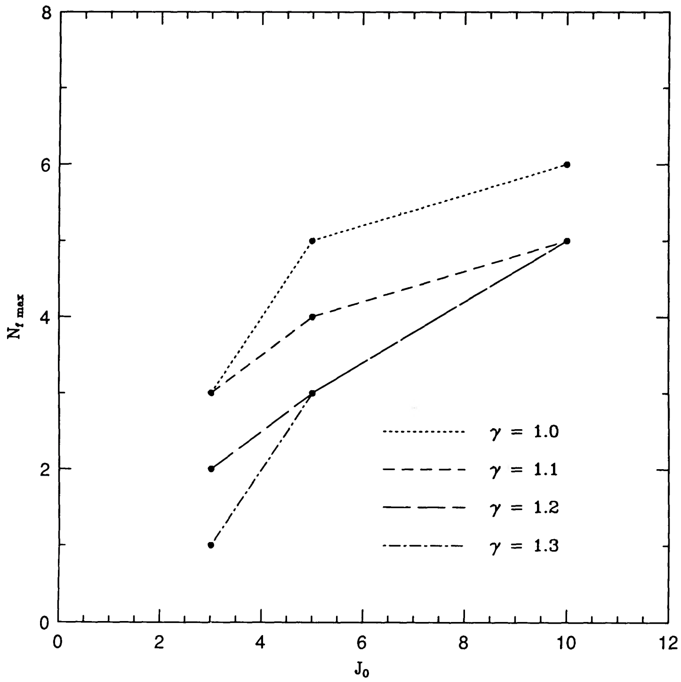

FIG. 1.—Maximum number of fragments formed as a function of the initial Jeans number for $L/D = 2$. The curves are parameterized by the polytropic constant.

but this has no effect on the saturation value $N_{\max}$ which depends solely on the geometry (i.e., L/D). Consequently, saturation is reached at larger J_0 for clouds with higher γ. Table 2 shows entries for $\gamma = 0.9$. In this case it was difficult to determine the maximum number of fragments for all but the smallest values of J_0. In fact, we found $N_{f\max} \geq 14$ for $J_0 = 4$.

The data shown in Tables 1 and 2 are plotted in Figures 1 and 2, respectively. The most striking feature in both figures is that at low J_0, $N_{f\max}$ increases with J_0, while at high J_0, $N_{f\max}$ saturates at a value $\approx 2L/D$. This saturation value is independent of γ for $\gamma \geq 1$. Only for $\gamma = 0.9$ was saturation not found at large J_0. In this case, $N_{f\max}$ was determined only for the two lowest values of J_0 and already exceeds the saturation value $N_{\max}$. This is explained below. Another important feature is the decrease of $N_{f\max}$ with increasing γ for a given value of J_0. The efficiency of fragmentation is somewhat reduced with higher values of γ.

The minimum Jeans number required for forming N fragments is defined as J_N (notice that the values of J_N we determine might be slightly higher than the actual values because of the small number of values of J_0 considered in our calculations). The Jeans numbers J_1 and J_2 correspond to the critical Jeans numbers J_c and $J_{2\text{-frag}}$ calculated in RB, respectively (they are actually overestimates). Notice, however, that the fragments in RB evolved spontaneously while here we introduce initial perturbations in order to obtain multiple fragmentation. This is necessary because spontaneous fragmentation never forms more than two fragments.

4. DISCUSSION

In this section, we discuss six points relevant to our results: (1) the choice of the input parameters in our simulations; (2) the effects of the polytropic constant γ on the outcome of

TABLE 2

NUMBER AND NATURE OF THE FRAGMENTS FORMED WITH $L/D = 5$

N_i	$J_0 = 2$	$J_0 = 3$	$J_0 = 4$	$J_0 = 5$	$J_0 = 13.3$
			$\gamma = 0.9$		
3......	3	...	...	...	...
4......	2	...	...	...	...
5......	4	...	...	...	...
6......	**5**	5	...	...	...
7......	4 + 2 c.f.	6	...	...	...
8......	2	7	...	...	...
9......	2	8	8	8	...
10......	...	9	9	...	...
11......	...	8	10	10	...
12......	...	2	11	11	...
13......	...	9 + 2 c.f.	10 + 2	12	...
14......	...	10 + 2 c.f.	2	13	...
15......	...	**11**	...	12 + 2 c.f.	...
16......	...	10 + 2 c.f.	14	13	...
17......	...	2	...	...	...
18......	...	2	...	...	...
			$\gamma = 1.0$		
4......	3	...	...	...	...
5......	**4**	...	...	...	...
6......	2	5	...	...	...
7......	...	6	6	6	...
8......	...	**7**	7	7	8
9......	...	6	8	8	9
10......	...	2	**9**	9	**10**
11......	...	...	8 + 2 c.f.	**10**	2
12......	...	...	...	9 + 2 c.f.	2
			$\gamma = 1.1$		
3......	**2**	3	...	...	...
4......	2	2	...	...	...
5......	...	4	5	...	...
6......	...	**5**	5	...	...
7......	...	4	6	...	7
8......	...	2	7	7	8
9......	...	...	**8**	...	9
10......	...	...	7	9	**10**

N_i	$J_0 = 2$	$J_0 = 3$	$J_0 = 4$	$J_0 = 5$	$J_0 = 13.3$
			$\gamma = 1.1$		
11......	...	...	...	**10**	2
12......	...	...	...	9	...
			$\gamma = 1.2$		
2......	**2**	...	...	...	...
3......	2	**3**	...	...	...
4......	2	2	...	...	...
5......	2	...	...	...	5
6......	...	2	5	...	...
7......	...	...	**6**	...	...
8......	...	...	5 + 2 c.f.	...	8
9......	...	...	2	**8**	**9**
10......	...	...	...	7 + 2 c.f.	2
			$\gamma = 1.3$		
3......	**1**	**2**	...	...	...
4......	1	2	...	...	...
5......	1	2	4	...	...
6......	...	...	**5**	5	...
7......	...	...	2	6	7
8......	...	...	2	**7**	8
9......	...	...	...	6	**9**
10......	...	...	...	2	2
			$\gamma = 1.4$		
N	$J_0 = 3$	$J_0 = 4$	$J_0 = 5$	$J_0 = 6$	$J_0 = 13.3$
2......	**1**	**2**	...	...	...
3......	...	2	...	...	...
4......	...	2	2 + 2 c.f.	**4**	...
5......	...	2	**2**	3 + 2 c.f.	...
6......	...	...	2	2	...
7......	...	...	2	...	7
8......	...	...	...	...	**8**
9......	...	...	...	...	2
10......	...	...	...	...	2

NOTES.—The fragments in this table are classified as normal fragments or close fragments (c.f.). In the normal fragments we include always 2 binary fragments located at either end of the cylinder (see Paper I). Close fragments are on the point of being absorbed by binaries and their existence as independent fragments is doubtful. Boldface numbers indicate cases where saturation is reached.

fragmentation over the range of J_0 and L/D considered here; (3) the empirical γ^2 relation between the number of fragments formed and J_0 for small values of J_0; (4) the effect of the cloud's geometry on the minimum stellar mass; (5) magnetic fields as a physical justification for considering an effective $\gamma < 1$; (6) a comparison of our results with observations.

4.1. *Choice of Input Parameters*

Prasad, Heere, & Tarafdar (1991) argue that most clouds are not in equilibrium but are simply contracting, bouncing back and contracting again. In our formalism, the effective J_0 of such clouds is too small to be able to create one fragment (i.e., $< J_c$). However, the existence of internal dissipative processes and the presence of a time-dependent local environment would result in a slow variation of J_0 with time. The cloud will not collapse to stellar densities until J_0 is raised above J_c, and this delay reduces the star formation rate. This result is interesting because the star formation rate in the solar neighborhood would be too large if clouds were collapsing on a time scale comparable to the free-fall time appropriate to their density. The problem is that if stars are indeed formed in clouds that have a slowly changing effective J_0, then the cloud would collapse as soon as $J_0 > J_c$, resulting in the formation of a single star. The existence of binary and multiple stars argues against this scenario, as it requires initial Jeans numbers J_0 at least larger than $J_{2\text{-frag}}$. Cloud collisions, at least in some environments (i.e., starburst regions), could suddenly raise the effective J_0 from a value lower than J_c to a value higher than $J_{2\text{-frag}}$, leading to the formation of a binary or multiple system. External effects caused by, e.g., a nearby OB association or a supernova explosion could also raise J_0 above $J_{2\text{-frag}}$. The aggregation of matter in a magnetically dominated cloud (Shu, Adams, & Lizano 1987) can lead either to (1) a supercritical regime where the gravitational energy overwhelms the magnetic field or (2) a subcritical regime where the magnetic field decays by ambipolar diffusion. In the supercritical regime the collapsing cloud will have a high J_0 whereas in the subcritical regime the cloud could eventually either become supercritical or fragment quasi-statically. Once collapse is initiated (i.e., when $J_0 > J_c$), the Jeans number increases throughout the isothermal phase until $\gamma > 4/3$. This implies that clouds that trap a significant portion of the released gravitational energy ($\gamma > 1$) have a large Jeans number. Additionally, once fragmentation occurs, each fragment will itself have a large Jeans number.

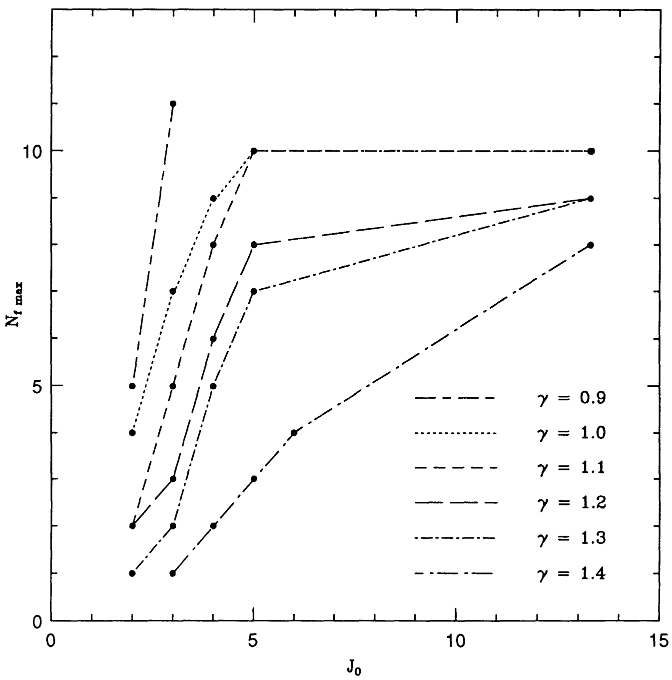

FIG. 2.—Same as Fig. 1 for $L/D = 5$. Not shown is the lower estimate of $N_{f\,\max}$ (≥ 14) derived for $J_0 = 4$ and $\gamma = 0.9$ (see Table 2).

Hence, large Jeans numbers are relevant in studying hierarchical fragmentation. Therefore, we consider mainly Jeans number $J_0 \geq J_{2-\rm frag}$ in this paper.

We point out that observational estimates of the initial Jeans number of a given cloud are difficult. Cloud masses determined by assuming virial equilibrium must be rejected because it supposes a priori an initial Jeans number. Usually, one measures the column density and derives a mass by assuming that the size of the cloud along the line of sight is an average of its dimensions in two perpendicular directions in the plane of the sky. Uncertainties of at least a factor of two on the mass are expected. In addition to the mass, the density and temperature distributions are required to accurately determine the Jeans number. The problems encountered when estimating density and temperature distributions in star forming regions have been discussed by Walmsley (1988). These problems affect directly the determination of the Jeans number.

Clouds with large J_0 are very unstable to collapse. Such clouds will collapse on a time scale $t_{\rm ff} \propto J_0^{-1/2}$. Therefore, the probability of observing clouds with large J_0 is small. Conversely, clouds with $J_0 \lesssim J_c$ exist for much longer time scales and are therefore much more likely to be detected. Hence, even though clouds with large J_0 cannot be nearly as common as those observed with $J_0 \simeq J_c$ (if clouds with large J_0 were common, the star formation rate would be much greater than observed), the lack of detection of such unstable and short-lived objects is not a convincing proof of their inexistence.

Myers et al. (1991) argue that most molecular clouds are elongated. They found typical elongations to be in the range 2–3. Allowing for projection effects, actual elongations $L/D \leq 5$ are fairly common. Clouds with larger elongation exist, but these are not so common. Possible values for the polytropic constant γ and their significance are discussed in §§ 1 and 4.5. In this paper, we explore the physically interesting range, from 7/5 to less than 1.

4.2. *Effects of the Polytropic Constant* γ

4.2.1. $\gamma \geq 1$

The fragmentation of elongated cylindrical clouds is not drastically impeded by a polytropic equation of state. Even for relatively high values of γ (1.3 and 1.4), fragmentation occurs relatively easily. Although the number of fragments is reduced for low J_0, the saturation value for high J_0 is unaffected. This is understandable for $\gamma > 1$ since the thermal energy and hence the pressure forces increase throughout the collapse. Fragmentation is impeded when the pressure forces become comparable to gravity. An arbitrarily large J_0 implies a very low thermal energy relative to gravity and thus an effective reduction of pressure forces. By the time these pressure forces are sufficiently high to compete with gravity, fragmentation has already occurred. Hence, fragmentation of clouds with high J_0 is unaffected by the value of γ. To test this hypothesis, one calculation was performed with $J_0 = 20$, $L/D = 5$, $N_i = 10$, and $\gamma = 1.4$. Ten fragments formed in this calculation, implying convergence of $N_{f\,\max}$ at high J_0 independently of the value of γ.

The fragmentation of elongated clouds with higher values of γ occurs for relatively low values of J_0. Indeed, two fragments are formed for $\gamma = 1.4$ and $J_0 = 4$. In contrast with the relatively easy fragmentation of elongated clouds, the fragmentation of polytropic rotating spherical clouds (Boss 1980, 1981) requires very large values of J_0 (~ 60) and a significant amount of rotational energy.

4.2.2. $\gamma < 1$

A polytropic equation of state with $\gamma < 1$ implies that both the thermal energy and the sound speed decrease during collapse (see eqs. [3] and [4]). As the second speed decreases, subregions loose contact with each other more easily and are thus able to fragment. As collapse proceeds, the sound speed decreases further allowing even more fragmentation. Thus it is only for clouds with low J_0 that the subregions do not loose contact and hence the number of fragments remains finite. The geometrical effects that induce a saturation value at high J_0 are less important due to the decreasing sound speed of the gas. In fact, the maximum number of fragments seems to follow closely the maximum expected if one calculates the number of Jeans masses contained in the initial cloud (see Paper I), at least for the values computed here.

4.3. *Empirical* γ^2 *Relation*

RB showed that J_c, and possibly $J_{2-\rm frag}$, scale as γ^2 for a given L/D (eq. [1]). Figure 3 shows that this relation can be extended successfully to higher fragment numbers for $L/D = 5$. RB and Bonnell & Bastien (1991) have shown that equation (1) is valid for all values of L/D. We thus expect that our results for $L/D = 5$ will also be valid for other values of L/D. Figure 3*a* shows saturation as a function of J_0 for the case $\gamma = 1.0$. The other panels show similar curves for other values of γ. In order to check if equation (1), which was derived by RB, applies to our results, we plotted the number of fragments calculated using

$$J_N(L/D, \gamma) \simeq \gamma^2 J_N(L/D, \gamma = 1) , \qquad (6)$$

which is a generalization of equation (1) to an arbitrary number of fragments. A comparison of the two curves in each panel of Figure 3 demonstrates that the above relation is valid for low J_0. At higher values of J_0, the saturation value of $N_{f\,\max}$ (i.e., $N_{\max}$) is independent of γ. To further demonstrate the validity of the γ^2 relation in the low J_0 regime, we plotted on

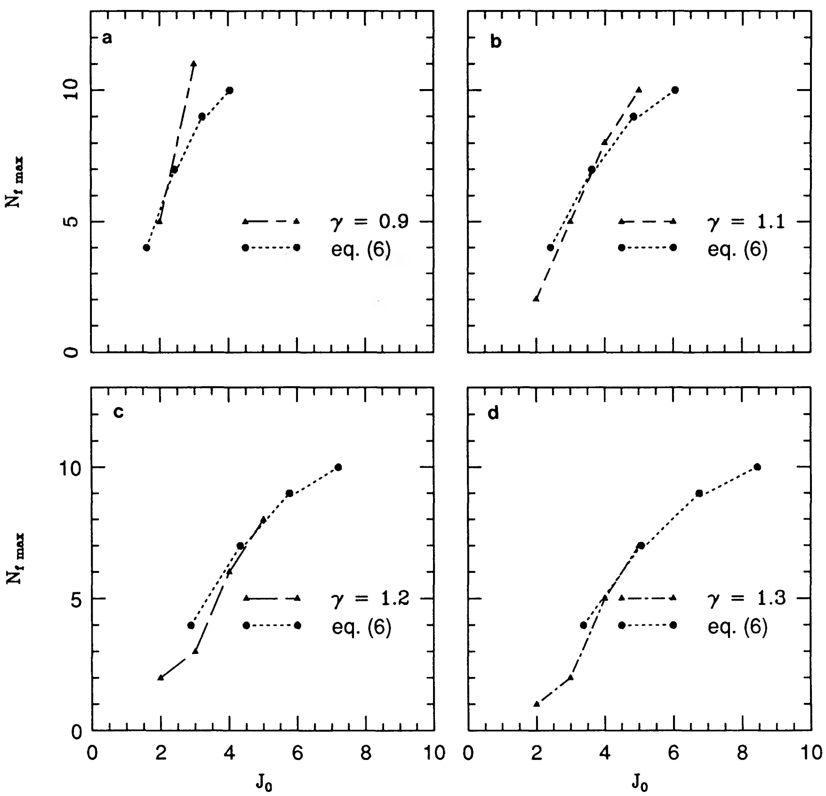

FIG. 3.—Panels (*a*)–(*d*) show $N_{f\max}$ as a function of J_0 for $\gamma = 0.9$, 1.1, 1.2, and 1.3, respectively, for $L/D = 5$. Also shown are the values predicted by eq. (6).

Figure 4 the number of fragments formed against the value of J_0/γ^2 for $L/D = 5$ and for several values of γ. The values shown are for the cases where $N_{f\max}$ still depends primarily on J_0 (i.e., low J_0). The curves are almost superposed for the values considered between $\gamma = 0.9$ and 1.3, with only slight deviations at both ends. The difference in these curves at high J_0/γ^2 is due to the saturation of $N_{f\max}$ which is independent of γ. For small values of J_0/γ^2, deviations are easily explained by the discretization of $N_{f\max}$ (i.e., $N_{f\max}$ can only be an integer). The case $\gamma = 1.4$, the only value above 4/3 we considered, clearly does not agree with equation (6).

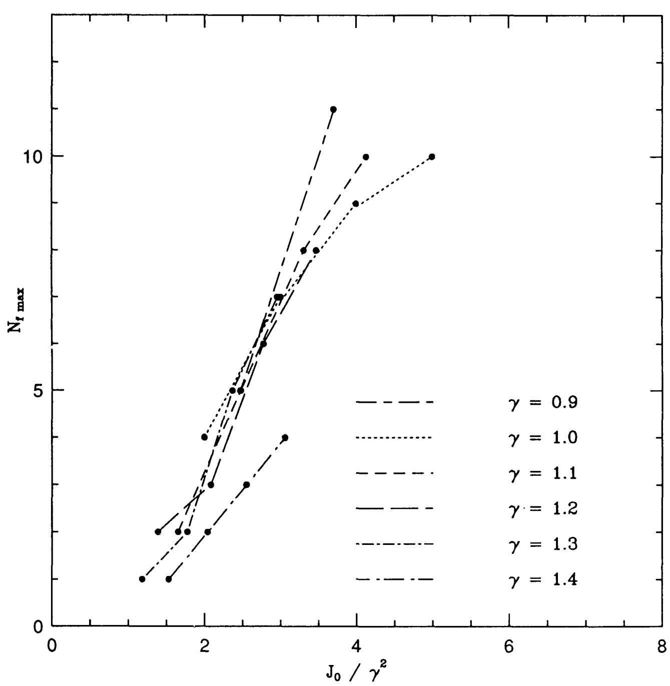

FIG. 4.—Maximum number of fragments formed as a function of J_0/γ^2 for $L/D = 5$.

The different behavior of the case $\gamma = 1.4$ can be understood using the following argument. The gravitational energy of a $\gamma > 1.2$ polytropic sphere is given by (Chandrasekhar 1939, p. 101)

$$E_g = -\frac{3}{5-n}\frac{GM^2}{R}, \tag{7}$$

where G is the gravitational constant, M is the cloud's mass, R is the cloud's radius, and $n = 1/(\gamma - 1)$ is the polytropic index. Therefore, the dependence of the gravitational energy on the density is

$$E_g \propto \rho^{1/3}, \tag{8}$$

$$\frac{\partial E_g}{\partial \rho} \propto \rho^{-2/3}. \tag{9}$$

The dependence of the specific internal energy (see eq. [3]) on the density is

$$\frac{\partial u}{\partial \rho} \propto \rho^{\gamma-2}, \tag{10}$$

which for $\gamma = 4/3$ gives the same dependence as the gravitational energy. Thus, at $\gamma = 4/3$, the thermal and the absolute value of the potential energies increase at the same rate, and the Jeans number (and also the Jeans mass) remains constant throughout the evolution. On the other hand, the Jeans number increases during collapse when $\gamma < 4/3$. RB also noted that the γ^2 empirical relation is inadequate when $\gamma \geq 4/3$. Equation (6) holds in the small J_0 region and breaks down for higher values of J_0. For a discussion of the physical ideas behind this relation the reader is referred to Bonnell & Bastien (1991).

4.4. *Minimum Stellar Mass*

The Jeans mass,

$$M_J = \left[\frac{3}{4\pi}\left(\frac{3R_g}{2G}\right)^3\right]^{1/2}\left(\frac{T}{\mu}\right)^{3/2}\rho^{-1/2}, \tag{11}$$

is the minimum mass necessary for a spherical cloud or cloud region to undergo collapse (e.g., Tohline 1982). For a polytrope, the temperature T varies as

$$T \propto \rho^{\gamma-1}, \tag{12}$$

thus the Jeans mass varies as

$$M_J \propto \rho^{(3\gamma-4)/2}. \tag{13}$$

Therefore,

$$\frac{\partial M_J}{\partial \rho} \propto \frac{3\gamma-4}{2}\rho^{(3\gamma-6)/2}, \tag{14}$$

implying that as the density increases during collapse, the Jeans mass either increases, remains constant, or decreases depending on whether $\gamma > 4/3$, $\gamma = 4/3$, or $\gamma < 4/3$, respectively.

Low & Lynden-Bell (1976) and Rees (1976) estimated the minimum mass that could eventually fragment. They based their calculations on opacity effects. The Jeans mass decreases when the fragment is optically thin. As contraction proceeds, the opacity increases implying that more and more of the released gravitational energy remains in the cloud in the form of thermal energy. This results in a concomittant increase in the value of γ. Eventually γ exceeds 4/3 and thus the Jeans mass increases. Hence the Jeans mass goes through a minimum ≈ 3–$6 \times 10^{-3}\ M_\odot$. We wish to compare this estimate with the predictions of our calculations. We showed that increasing the value of γ while keeping J_0 constant leads to a decrease in $N_{f\max}$. The results illustrated in Figures 1 and 2 can be expressed as

$$\frac{\partial N_{f\max}}{\partial \gamma} \leq 0, \tag{15}$$

$$\frac{\partial N_{f\max}}{\partial J_0} \geq 0. \tag{16}$$

Increasing the value of γ affects the dependence of the thermal and gravitational energies on the density. For $\gamma > 4/3$, the thermal energy increases faster than the absolute value of the gravitational energy, thus the Jeans number decreases. Thus, as contraction proceeds, the ability to fragment further is strongly impeded. In essence, our calculations show that fragmentation depends on the number of initial perturbations and/or the geometry and not so much on the Jeans mass (see Fig. 5 of Paper I). As γ increases, the discrepancy shown in Figure 5 of Paper I becomes more acute, as fragments more massive than the Jeans mass form. It thus follows that the estimates given by Rees (1976) and Low & Lynden-Bell (1976) are underestimates of the real minimum mass. Contrapuntally, for $\gamma < 1$, the number of fragments formed is unaffected by the geometrically induced saturation at high J_0. Indeed, the final number of fragments is relatively well estimated, at least for the values of $N_{f\max}$ determined in this paper, by the number of Jeans masses contained in the cloud (see Paper I) when $\gamma < 1$. This is due to the fact that the cloud can "forget" its initial geometry since the sound speed decreases during collapse.

4.5. *Fragmentation in the Presence of Magnetic Fields*

The effect of a magnetic field can be simulated as a scalar pressure term if we assume that the field lines are tangled (see Prasad et al. 1991)

$$P = B^2/8\pi, \tag{17}$$

where B is the field strength and P the magnetic pressure. The field can be scaled to the local density according to

$$B \propto (\rho/\rho_0)^k, \tag{18}$$

where $1/3 < k < 2/3$. Hence, if the pressure term is dominated by the magnetic field then $2/3 < \gamma < 4/3$. There are some doubts, however, as to the possibility of finding such tangled field in the clouds we are interested in. They are likely not to be so tangled (see Myers & Goodman 1991), resulting in nondiagonal terms in the pressure tensor. Hydrodynamical calculations performed with magnetic fields, involving the collapse of rotating and nonrotating spherical clouds, can be found in Scott & Black (1980), Dorfi (1982), and Benz (1984). The study of the fragmentation of magnetized molecular clouds remains to be done.

4.6. *Comparison with Observations*

A comparison with observations was presented in Paper I for the isothermal case. The elongated shape of clouds observed recently by Myers et al. (1991) supports the relevance of our results.

Another point of comparison is to relate the outcome of the fragmentation process in the simulations to the observations of groups of stars. Most stars are found to be in binary or multiple systems. However, multiple systems have only a few components: those with more than four stars are rather exceptional. This seems to imply that for most clouds, the saturation limit $N_{\max}$ is not reached. Hence, the initial Jeans number would not be very large. Offsetting this effect is the limited long-term stability of multiple nonhierarchical systems. Therefore, if such systems are indeed formed, they would not survive for very long, so that they would be difficult to find.

One expects to find embedded infrared young stars in the cores of molecular clouds. With the recent advent of large infrared arrays, searches for these infrared sources should allow for a better comparison with our results. Small groups of such systems should be frequent according to our interpretation of the numerical simulations presented here.

5. CONCLUSIONS

In this paper, we investigated the fragmentation of elongated axisymmetric polytropic clouds. We determined the maximum number of fragments $N_{f\max}$ that form as a function of the geometry L/D, the initial Jeans number J_0, and the polytropic constant γ by varying the initial number of perturbations N_i. At fixed L/D and γ, $N_{f\max}$ increases with J_0, then saturates at a finite value N_{max} at large J_0. This result, which was obtained in Paper I for the isothermal case, remains valid for all the various values of γ we considered, except for $\gamma = 0.9$ where saturation is not obtained. In any case, if saturation does occur at larger J_0 for $\gamma = 0.9$, the saturation value will not incorporate the same dependence on the initial geometry L/D. The saturation value is $N_{max} \approx 2L/D$, independent of γ for $\gamma \geq 1$.

At fixed L/D and J_0, $N_{f\,max}$ decreases when γ increases. The effect of the pressure forces is greater at large γ, making fragmentation more difficult. For $\gamma > 4/3$, the Jeans mass increases with density. Fragmentation is still possible but $N_{f\,max}$ is strongly reduced compared to cases with lower values of γ. For $\gamma < 1$, the cloud cools as it collapses. Fragmentation becomes so efficient that the number of simulations required to determine $N_{f\,max}$ becomes prohibitive. Since the saturation value N_{max} is independent of γ for $\gamma \geq 1$, reaching saturation at high γ requires higher J_0 to compensate for the inhibiting effect of γ on fragmentation.

The minimum Jeans number J_N required for forming N fragments scales as γ^2 (see eq. [6]). This result was obtained by RB in a different context (one or two fragments, and no initial perturbations). The γ^2 relation is valid as long as $0.9 \leq \gamma < 4/3$ and $N_{f\,max} < N_{max}$ (i.e., J_0 small). For a given N, J_N is much larger for $\gamma > 4/3$ than predicted by equation (6). This was not unexpected since a transition between different physical regimes occurs at $\gamma = 4/3$. At large J_0 and $\gamma \geq 1$, the number of fragments saturates at $N_{max} \approx 2L/D$, independent of γ. Therefore, equation (6) is not valid in that regime.

The major constraint on the number of fragments at high J_0 for $\gamma \geq 1$ is the cloud's initial shape. Hence, the determination of the number of fragments by the number of Jeans masses contained in the cloud is an overestimate (see Fig. 5 of Paper I). Consequently, the determination of the minimum stellar mass based on opacity-related arguments ignoring geometrical effects is an underestimate. For $\gamma < 1$, the sound speed decreases with collapse such that when J_0 is large, subregions can loose contact with each other and "forget" the cloud's initial shape. Fragmentation then proceeds, constrained only by the number of Jeans masses (i.e., independent subregions) in the cloud. Geometrical constraints are therefore less important for the case $\gamma = 0.9$.

The fragments formed in these calculations would merge into a single object given a sufficient amount of time. In more realistic conditions the specific angular momentum would be sufficient for the stars to miss each other. Detailed calculations of fragmentation in rotating cylinders are presented in follow up papers. Bonnell et al. (1991, Paper III) studies the case where the cylinder rotates end over end and Martel et al. (1991, Paper IV), generalizes the problem to arbitrarily oriented angular momenta.

The authors are grateful to the Conseil de Recherche en Sciences Naturelles et en Génie (CRSNG) du Canada for financial assistance. J. P. A. thanks the Canadian Institute for Theoretical Astrophysics for his fellowship. W. B. acknowledges partial support from the Swiss National Science Foundation. This research would not have been possible without the new CNI (Calcul Numérique Intensif) facilities of the Université de Montréal.

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