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TURBULENTLY GENERATED MAGNETIC FIELDS IN CLUSTERS OF GALAXIES

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ABSTRACT

The typical scale and velocity of the dominant, turbulent eddies excited by the motion of galaxies in clusters are determined from basic considerations which are valid if a steady state is attained. Hydrodynamic turbulence in the intracluster plasma presumably generates magnetic fields; our estimates of the typical scale and mean strength of these fields in cluster cores are about 10 kpc and at most a few 10^{-7} G. We discuss implications of fields with these properties on the detectability of diffuse hard X-ray emission from clusters, and on transport properties in the intracluster space.

Subject headings: galaxies: clustering — galaxies: intergalactic medium — magnetic fields — turbulence

1. INTRODUCTION

Magnetic fields are a well-established, general property of the interstellar (IS) medium in galaxies (see, e.g., the recent review by Sofue, Fujimoto, & Wielibinski 1986, and the book by Zel'dovich, Ruzmaikin, & Sokoloff 1983) and seem to be common also in the intracluster (IC) gas of clusters of galaxies (Rephaeli 1988). It is thought that galactic fields have their origin in extremely small seed fields, 10^{-18}–10^{-20} G, generated in the post-recombination era (Harrison 1970), which had been amplified by the dynamo action to the typical value of a few μG (e.g., Rees 1987).. One can easily identify the possible sources which contributed (nonnegligibly) to IC fields. Cluster galaxies, both normal and radio, certainly account at least in part for IC fields, through the dispersal of their magnetized IS media. Another possible source of magnetic energy is the conversion of kinematic turbulence, excited by the motions of the cluster galaxies, into magnetohydrodynamic modes (Jaffe 1980; Ruzmaikin, Sokoloff, & Shukurov 1989, hereafter RSS). Yet another contribution can come from compactification of cosmological fields in clusters, but this is perhaps only a minor source (Rephaeli 1988). The relative significance of these possible contributions can be assessed only through a quantitative determination of the strength and morphology of IC fields.

Interest in IC fields is obvious: first, their origin may give us some clues on the evolution of clusters and their gaseous cores. Although magnetic fields do not play appreciable dynamical role in clusters, their properties can be quite relevant to the state of the IC plasma. Transport properties of thermal, as well as relativistic, particles are significantly affected by the fields. And, of course, the fields are an essential ingredient in the phenomena of diffuse IC radio emission.

Direct observational evidence for IC magnetic fields comes from measurements of diffuse radio emission and from Faraday rotation. Detection of diffuse emission does not by itself yield the value of the *mean* IC field, B. This is the reason for the wide range of field values which have been obtained in analyses of radio data alone. For example, in the Coma cluster estimates of B span the range 0.03–3 μG (for references, see Rephaeli 1988). Radio emission is most likely synchrotron radiation from relativistic electrons traversing IC fields. Faraday rotation of the plane of polarization of radiation from sources seen through clusters yields information on the mean line-of-sight component of the field, weighted by the gas density. Rotation measures seem to have been determined in the directions to two clusters with deduced mean field values of $\simeq 0.2$, 2 μG (Vallee, Broten, & MacLeod 1987; Dreher, Carilli, & Perley 1987), and upper limits deduced in other clusters (Lawler & Dennison 1982; Hennessy, Owen, & Eilek 1989).

From the presence of relativistic electrons in the IC space we can predict that there will also be diffuse hard X-ray (HXR) emission from clusters: Compton scattering of these electrons by the cosmic microwave background (CMB) radiation boosts the energy of a typical CMB photon to X-ray energies. The spectra of clusters are dominated by thermal emission below 30 keV, so the power-law Compton component should be looked for at higher energies. From combined radio and HXR measurements we can determine the value of B. If HXR flux is not detected, then a lower limit on the value of B is obtained. Radio and *HEAO 1* HXR measurements have been recently analysed for six Abell clusters for which there is strong evidence of diffuse IC radio emission. The analyses yielded lower limits, in the range 0.04–0.15 μG, for the mean magnetic fields in the central regions of these clusters (Rephaeli, Gruber, & Rothschild 1987; Rephaeli & Gruber 1988).

Results of all current measurements and their analyses (which are not always free of simplifying assumptions) can be summarized by stating that deduced values of IC fields seem to fall in the range 0.03–3 μG. Rephaeli (1988) has estimated that mean field values in the central regions of gas-rich clusters are expected to be around 0.1 μG, if the field originates in the IS media of cluster galaxies. This estimate is based on the reasonably good assumption that the fields are frozen in these stripped media. No reliable estimate can be made if the fields originate in the remnants of cluster radio galaxies. On the other hand, relatively definite predictions have been claimed in the context of scenarios in which the fields are turbulently generated. Both Jaffe (1980) and RSS have assumed efficient conversion of the energy in turbulence to magnetic energy, with resulting mean field values of a few μG. It seems that there is roughly a difference of an order of magnitude between the predicted field values in these two canonical models of IS and in situ, turbulent origins.

It is quite important to know if the mean IC field values should be O(0.1) or O(1) μG. Besides implications on the evolution of the gaseous cores of clusters and the strength of diffuse IC radio emission, knowledge of B has a direct bearing on the detectability of IC HXR emission. If $B = O(0.1)$ μG, then we can safely predict that such emission should be detected by the next generation X-ray telescopes, whereas if $B = O(1)$ μG, the predicted emission is too weak to be detected. This stems from the steep dependence of the predicted HXR emission on B (Rephaeli 1988).

Motivated by the need to clarify the matter, we have obtained a realistic estimate of the predicted strength of IC fields generated through the turbulence excited by galaxies in clusters. Our estimates indicate that the process is less efficient than previously claimed, capable of producing fields whose mean values probably do not exceed ~ 0.2 μG. We discuss the details of our estimates and some of their implications.

2. STEADY STATE TURBULENCE AND EQUIPARTITION MAGNETIC FIELDS

The general scenario, as considered in particular by RSS, is as follows: Motion of galaxies through IC gas excites hydrodynamic turbulence, which is expected since molecular viscosity is significantly reduced in the presence of a magnetic field, even much weaker than the field ejected from galaxies. Turbulent eddies transfer energy to magnetohydrodynamic modes, and a steady state is attained corresponding to an equipartition between turbulent and magnetic field energy densities. The scenario (whose substantiation necessitates detailed calculations, and use of some unknown quantities) may not be fully realizable, but for our purposes here we can logically presume its general correctness. If the conversion of turbulent energy to magnetic field energy is not as efficient as claimed by RSS, then our main conclusion will only be strengthened.

The transsonic motion of a galaxy through IC gas creates a turbulent wake. The free decay of turbulence with time results in a decrease of the amplitude of the turbulent velocity as well as an increase in the scale of the largest length scale associated with the turbulent velocity field. According to RSS, a simplified treatment can yield the geometry of the wake, typical scales, and velocities of the turbulent eddies. Jaffe (1980) considered such a wake to spread until the turbulent velocity becomes smaller than the Alfvén velocity. Thereafter, the turbulence spreads as an Alfvén wave until merging with similar waves from wakes of other galaxies. RSS, in a more detailed analysis, considered the turbulent wakes to spread until merging. They performed averages of the turbulent velocity amplitude, and of the turbulence length scale, over the volume of the wake, and considered the resulting averages to represent steady state values at *any* spatial location. Their analysis leads to a scale of ~ 20 kpc and a velocity of ~ 400 km s^{-1}. Presumably, magnetic field generation occurs on scales smaller than the turbulence length scale.

We find the treatment of the steady state turbulence by means of volume averaging over isolated, noninteracting, *decaying* wakes to be questionable. If steady state is attained at all, it will certainly be a result of the cumulative turbulent stirring by all galaxies in the cluster core. We will first determine if galactic motions can indeed establish a steady state turbulence throughout the entire core volume on a time scale shorter than the cluster age. As we find this to be possible, we then proceed to obtain limits on the values of the mean magnetic field that could be generated by this turbulence.

2.1. *Steady State Turbulence in Cores of Clusters*

First, note that hydrodynamic turbulence can be created in the IC space, as both the usual and magnetic Reynolds numbers are sufficiently high for typical values of the relevant parameters. We wish to find the characteristics of the steady state turbulence generated in the IC gas by the motion of the galaxies in the core. As will be seen below, the dominant length scale of the velocity turbulence is much smaller than the cluster core radius. This allows us to treat the turbulence as homogeneous and isotropic, with only a weak dependence of the core values of the turbulent mean quantities on the distance r from the cluster center. (In § 3 we will comment on the predicted behavior outside the core.)

The turbulent velocity field is defined by the spectral function of the square turbulent velocity, $F(k)$, were $k = \pi l^{-1}$ is the wavenumber associated with a length scale l. The ensemble average (which, in case of ergodic and time-independent turbulence, equals also the time average) of the square of the fluctuating velocity on a scale $l = \pi k^{-1}$, $v^2(k)$, is given by (e.g., Batchelor 1973; Hinze 1975)

$$v^2(k) = \int_k^\infty F(k')dk' . \tag{1}$$

The mean-squared velocity, $v^2 = v^2(k_0)$, is obtained by setting the lower limit in equation (1) equal to k_0, the smallest wave number (largest scale) present in the turbulence. The value of k_0 is determined by the characteristics of the source—the motion of galaxies in the present case. Kinetic energy of the galaxies is fed predominantly into the large scales, $l \sim l_0 = \pi/k_0$, and is transferred in a cascade process—due to nonlinear eddy interaction—to smaller scales. The cascade terminates at a characteristic dissipation scale, small enough for microscopic viscosity (molecular viscosity or, as in our case here, magnetic viscosity) to dissipate the hydrodynamic turbulent energy. In the presence of sufficiently strong magnetic fields the hydrodynamic turbulence will transform to Alfvén waves, starting at a scale (higher than the dissipation scale) at which the turbulent velocity is equal to the Alfvén velocity.

We wish to determine whether the motion of galaxies can indeed establish a steady state turbulence throughout the entire core volume. To do so one needs the turbulence spectrum, $F(k)$, and more specifically the values of v_0 and l_0. The time scale for buildup of a steady state turbulence from the motion of galaxies is $\sim l_0/v_0$, which must be shorter than the cluster age for such a state to be established. The value of v_0 implies a conservative upper limit on the presumably generated magnetic field $B_0 = (4\pi\rho v_0^2)^{1/2}$.

In the following we use simple dimensional arguments to obtain expressions for v_0 and l_0 in terms of the basic parameters characterizing conditions in the cores of rich clusters of galaxies. A more formal treatment that yields the spectral function $F(k)$ in the context of a specific model of turbulence is presented in the Appendix. As is shown there, the resulting l_0 and v_0 have the same functional dependence on the parameters as those derived by the dimensional arguments below. We consider this to be a clear indication of the generality of the results. The formal treatment also enables a more quantitative assessment of the time scale for buildup of a steady state.

Two conditions must be satisfied if steady state turbulence is attained: First, the characteristic time scale associated with the breakup of the largest eddies has to be equal to the time scale associated with the generation of turbulence. Second, the rate of input energy (per unit volume) into the turbulence must be equal to the rate at which galaxies lose energy by turbulent drag. These two conditions yield the values of l_0 and v_0.

Consider the first, temporal condition. We identify the time scale of galactic stirring that drives the turbulence with the mean, weighted time scale for a galaxy to pass through a volume of size $\sim l_0^3$, namely $\sim(n_G \pi l_0^2 v_G)^{-1}(r)$. Here n_G is the number density of galaxies, and v_G is their typical velocity, both functions of r. The lifetime of the dominant largest eddy is $\sim l_0/v_0$. Thus we have

$$n_G \pi l_0^2 v_G = \lambda_1 v_0/l_0 , \tag{2}$$

where λ_1 is a number of order unity whose exact value is determined by the spectrum of the turbulence.

The rate of energy density injected into the turbulence is $\sim \rho v_0^3/l_0$, where ρ is the IC gas mass density. Galaxies not only excite the turbulence, but are also dragged by it, losing energy at a rate $\sim n_G \pi a v_G^2 \rho v_t(a)$, where a is the effective radius of a galaxy for interaction with the IC gas, and $v_t(a)$ is the turbulent viscosity on the spatial length scale a, $v_t(a) \sim v(a)a$. Equating the two rates, we have

$$\rho n_G \pi a^2 v_G^2 v(a) = \lambda_2 \rho v_0^3/l_0 , \tag{3}$$

where λ_2 is a coefficient of order unity which depends on the exact form of the spectral function. Given a velocity spectrum we can express $v(a)$ in terms of v_0. For simplicity, we assume that the spectrum for $l < l_0$ can be approximated by the Kolmogorov spectrum $v(l) \propto l^{1/3}$. This assumption is valid for $l \lesssim 0.3 l_0$ (see the Appendix). Since, as we shall see below, $a \sim 0.1 \times l_0$, the approximation is reasonable. Using this relation in equation (3) we get

$$n_G \pi a^2 v_G^2 v_0 (a/l_0)^{1/3} = \lambda_2 v_0^3/l_0 . \tag{4}$$

Combining equations (2) and (4) yields

$$v_0/v_G = \lambda_1/\lambda_2 (a/l_0)^{7/3} , \tag{5}$$

which when used in equation (2) implies

$$l_0 = a \lambda_1^{3/8} \lambda_2^{-3/16} (n_G \pi a^3)^{-3/16} , \tag{6}$$

and

$$v_0 = v_G \lambda_1^{1/8} \lambda_2^{-9/16} (n_G \pi a^3)^{7/16} . \tag{7}$$

Equations (6) and (7) are identical up to factors of order unity, to equations (A12) and (A13), respectively, which were derived from $F(k)$ in the context of the specific model described in the Appendix.

It is of interest to note that the scaling assumptions of RSS lead to relations similar to equations (6) and (7), albeit with different powers and coefficients. However, here the (steady state) values of v_0 and l_0 are the turbulent velocity and length scale at *any* point in the core. Our approach to obtain relations between l_0 and v_0 is very different from that of RSS, who arbitrarily averaged over a volume of a *single decaying* wake in order to find relations between these two quantities, which they assumed to hold in a steady state throughout the core. This averaging procedure by itself leads to an overestimate of v_0. See § 3 for a more detailed comparison of our approach with that of RSS.

What is the effective radius of a galaxy for interaction with the IC gas? Galaxies within the cores of rich clusters are mostly ellipticals. These have relatively small amounts of IS gas, probably due to appreciable tidal interactions (particularly during the early stages of cluster evolution) and ram pressure stripping by the IC gas. The detection of X-ray emission from normal ellipticals (for a review, see Fabbiano 1989), even if in clusters, does not necessarily imply the presence of a large amount of interstellar hot gas in these galaxies. We do not yet know exactly what fraction of the emission is from binary X-ray sources; neither do we know if the gas is in a stable, hydrostatic equilibrium, or, rather, if it is on its way out in the form of a galactic wind. Note also that the relatively few spirals seen in the cores of rich clusters may only be projected there. Even if in the core, spirals probably lost most of their gas before settling there (see Lea & De Young 1976, and the review by Dressler 1984). Thus, the characteristic radius at which the IS gas (thermal or magnetic) pressure is comparable to the external is expected to be small. Although a quantitative estimate cannot be readily given, we can obtain an upper limit on the value of a by demanding that the slow-down time of galaxies due to hydrodynamical drag, $\tau_D = M/(2\rho a^2 v_G)$, be longer than the age of the universe. This requirement leads to the bound $a \leq 4.5$ kpc for $M = 10^{11}\ M_\odot$, $v_G = 10^8$ cm s^{-1}, and $\rho = 5 \times 10^{-27}$ g cm^{-3}.

If the pressure of interstellar matter is sufficiently small, it is more appropriate to take for the effective size of a galaxy (mass M) as "seen" by the IC gas the gravitational accretion radius

$$a = 2GMv_G^{-2} \simeq 0.86 M_{11} v_8^{-2} \text{ kpc} , \tag{8}$$

with M_{11} and v_8 denoting the galaxy mass and velocity in units of $10^{11}\ M_\odot$ and 10^8 cm s^{-1}, respectively. The fact that the accretion radius is much smaller (by an order of magnitude) than a typical geometrical radius of a (normal) cluster galaxy is significant to our discussion here. In spite of the uncertainty in estimating a realistic value for a, it is reasonable to expect it to be in the rough range spanned by the latter two values. (Other considerations pertinent to the value of a are given in § 3.)

Substituting this value of a in equations (6) and (7) we calculate the following values for l_0 and v_0:

$$l_0 = 10.0 \lambda_1^{3/8} \lambda_2^{-3/16} v_8^{-7/8} M_{11}^{7/16} n_3^{-3/16} \text{ kpc} , \tag{9}$$

$$v_0 = 3.2 \lambda_1^{1/8} \lambda_2^{-9/16} v_8^{-13/8} M_{11}^{21/16} n_3^{7/16} \text{ km s}^{-1} . \tag{10}$$

We have scaled the galaxy number density to 10^3 Mpc^{-3}, a typical value in the cores of rich Abell clusters (e.g., Coma—Rood et al. 1972; Kent & Gunn 1982). (Note the weak dependence of l_0 and v_0 on λ_1 and λ_2.) The corresponding relations in the Appendix (eqs. [A14] and [A15]) lead to values which are not too different from the above.

The time scale for energy input into the turbulence is $\sim l_0/v_0 \sim 3.1 \times 10^9$ yr. Using the results of the Appendix we obtain even a longer time scale of $\sim 7.5 \times 10^9$ yr. Thus, it is not obvious that a steady state turbulence (generated by the motion of galaxies) should have been attained at all by the present epoch. But since a may be higher than 1 kpc (the above time scales are lower by a factor of ~ 4 if $a = 4.5$ kpc), and in accord with our aim to obtain realistic upper limits on turbulently generated IC fields, we will nonetheless *assume* that steady state had been established, realizing that our results will only be strengthened otherwise.

2.2. *Equipartition Magnetic Fields*

The appearance of chaotic magnetic fields in a conducting, turbulent fluid (Batchelor 1950) is not surprising, although a quantitative description of the ensemble-averaged properties of the fields is rather involved. For this, one needs to solve for the induced fields, follow the various amplification and decay channels, and perform statistical averages. Such analyses have been carried out (see the review by Zel'dovich et al. 1988), including RSS′ most recent treatment of turbulently generated IC fields. As we have stated, it is not our aim in this paper to challenge previous results on the magnetohydrodynamics level, but rather to show that previous estimates of the resulting fields are unrealistically high, essentially due to the assumed high velocity values.

For if a steady state turbulence is sustained by the motion of galaxies in the core at a level corresponding to an energy density $\rho v_0^2/2$, then it is clear that the strength of the mean field cannot exceed $B_0 = (4\pi\rho v_0^2)^{1/2}$. Taking a value of 3.2 km s^{-1} for v_0, and a gas number density of 3×10^{-3} cm^{-3} (a typical value in the cores of rich clusters), we compute $B_0 = 0.1$ μG. The field will most likely be generated on scales smaller than l_0, so that its value on the scale l_0 will be lower. Specifically, in the dynamo model of RSS, $v_0 = 400$ km s^{-1} yields a magnetic field of 2 μG, which corresponds to an Alfvén velocity of 80 km s^{-1}. Thus, the deduced field seems to be associated with an Alfvén velocity which is only $v_0/5$. Adopting this reduced efficiency, our value of $v_0 = 3.2$ km s^{-1} would imply $B = 0.02$ μG. Accounting for the uncertainty in the value of a by taking our deduced upper limit of 4.5 kpc, we obtain $v_0 = 28$ km s^{-1} (11 km s^{-1}, if we use the results of the Appendix), leading to the more conservative uper bounds of $B = 0.2$ μG (or 0.1 μG). This optimally high mean IC field will be expected to be correlated on scales $\sim l_0$, which from our estimates in equations (9) and (A14) is around 10 kpc (20 kpc, if $a = 4.5$ kpc).

It is important to realize that B_0 is essentially a volume average over cells of size l_0 in the core region of radius r_c. An average along a line of sight through the core is obtained by multiplying B_0 by the square root of the mean number of cells of size l_0 along the line, namely $(r_c/l_0)^{1/2} \sim 4$, where we have taken $r_c = 300$ kpc (also very typical of rich clusters). Thus, we expect a mean field value of 5×10^{-2} μG along a line of sight.

In our estimates, we have taken typical values for galactic mass and velocity. It is easy to use instead an average value for the mass, by integrating over the mass function of cluster galaxies. We have done so using the mass function suggested by Miller (1983); our final results are only slightly different than the above values. The value of 10^3 km s^{-1} for the mean velocity of galaxies in rich clusters is very appropriate. In our numerical estimates we have taken $\lambda_1 = \lambda_2$, but due to the weak dependence it is very unlikely that doing so resulted in a considerable underestimate of v_0. The treatment in the Appendix suggests that the above value of v_0 is indeed not an overestimate.

3. DISCUSSION

Intuitive as well as more formal (in the Appendix) arguments led us to the first conclusion, that the attainment of a steady state hydrodynamic turbulence—excited by the motion of the cluster galaxies—is only of marginal likelihood. Our second and perhaps more important conclusion is that even if such a state is reached, the turbulently generated magnetic fields *throughout the entire core volume* have values of at most 0.2 μG, significantly lower than the few μG fields previously estimated by Jaffe (1980) and RSS. It should be emphasized that we cannot exclude higher field values in regions of few kiloparsecs around the galaxies themselves. Nor can we exclude the possibility of more intense fields generated by some other mechanism (e.g., via strong hydrodynamical turbulence, perhaps as a result of infall of a subcluster onto the cluster core—Loewenstein & Fabian 1990).

Our approach to the characterization of the basic properties of steady state hydrodynamic turbulence in the cores of clusters is quite different from those of Jaffe (1980) and RSS. In both of these previous treatments, the estimated fields are significantly higher, by a factor of 10. Jaffe's higher value for the mean field in the Coma cluster is based on assuming that the turbulence is generated by 50 galaxies whose individual mass is 10^{12} $M_\odot$. Even if galaxies have massive halos, it is very unlikely that the halos can be retained in the cores of rich clusters. From an intuitive point of view, the packing of this number of massive galaxies, whose typical radius must be O(100) kpc (or higher, if the halo mass density falls off more steeply than r^{-2}), in a 250 kpc core is impossible, if the halos are to remain intact. Indeed, tidal interactions between galaxies (and mean-field interactions during the violent relaxation phase of cluster formation) have disrupted the extended galactic halos. This has been seen in many N-body simulations of cluster formation (e.g., Richstone & Malmuth 1983; Merritt 1984; and the review by Dressler 1984). Moreover, in the central regions of rich clusters the gravitational drag on a 10^{12} $M_\odot$ galaxy is strong enough to cause infall to the center in less than a Hubble time (Rephaeli & Salpeter 1980). Note also that Jaffe's estimate does not hold if the massive galaxies are spread out throughout (even the central region of) the cluster: the gas density is much lower outside the core, and, therefore, the efficiency of conversion of turbulent to magnetic energy is much reduced. We conclude that the deduced field (in Jaffe's estimate) is much lower if the more typical value of 10^{11} $M_\odot$ is taken.

RSS cite the unusually high Faraday rotation measures in the Cygnus cluster reported by Dreher et al. 1987, as evidence for 2 μG field. (It should be noted though that this value is quite uncertain, as it is not clear whether the rotation occurs in the IC space proper; see Rephaeli 1988). Their estimate of 400 km s^{-1} for v_0 indeed allows the dynamo generation of the assumed 2 μG field. However, we believe that this value of v_0 is an overestimate, resulting in part from an inappropriate averaging of l_0 and v_0 over the volume of a wake, and assuming that the resulting values apply throughout the core volume.

Specifically, RSS demand that at any given time the generated wakes fill out completely the core volume. They postulate that all the kinetic energy in wakes (when they merge) is available for maintaining steady state turbulence throughout the core volume. Since the presumed steady state is a result of the cumulative turbulent stirring by all galaxies in the core, it is difficult a priori to assess the validity of this postulate. A simple check is to use their preferred value of 10 kpc for a in our formula for v_0. We find that $v_0 = 80$ km s^{-1} (for 32 km s^{-1}, if eq. [A15] is used), which should be compared with RSS′ value of 400 km s^{-1}. This shows that even if the wakes do fill completely the core, this method leads to an overestimation by a factor $\sim$25–100 of the turbulent energy in steady state. Note also that from RSS′, expression for the length of a single decaying wake, it follows explicitly that if a is lower than their assumed value of 10 kpc, then individual wakes are long and sufficiently narrow that they fill up only part of the core volume. In comparison, our steady state treatment applies for any value of a. It is a self-consistent treatment in the sense that the energy input into the turbulence is self-regulated by the turbulence itself, through the turbulent viscosity. Therefore, for any value of a we can characterize the steady state turbulence, regardless whether single decaying wakes merge within the core to fill it completely or not. (As has been done above, our approach also allows explicit estimation of the time scale for reaching a steady state.)

The significant difference is also due to the much lower value we have taken for the effective radius of a galaxy for interaction with the IC gas. Unlike RSS, who assumed that this radius is simply the geometrical radius of the galaxy, we have given (above) reasons to believe that it is more realistic to identify it with the accretion radius (Note that the particular expression for a in eq. [8] [e.g., Ruderman & Spiegel 1971] includes the factor 2 which is ignored sometimes.) Nevertheless, it is not impossible that cluster galaxies have larger hydrodynamical cross sections. The upper limit of 4.5 kpc we set on the value of a, together with the 0.2 overall efficiency factor we have discussed above, leads to an upper limit of 0.2 μG on the value of B. It should be emphasized that all estimates for the coherence scale yield values in the range 10–20 kpc.

In all of our estimates we have referred to conditions in the cores of rich clusters. How far out of the core can we expect effective amplification of magnetic fields by hydrodynamic turbulence? The time scale of field amplification is $\tau \sim l_0/v_0$, or about 3.1×10^9 yr for $v_0 = 3.2$ km s^{-1} and $l_0 = 10$ kpc. From equations (9) and (10) it follows that $\tau \propto n_G^{5/8}$, and if we adopt (an analytical fit to) the King profile (Rood et al. 1972), $n_G \propto (r/r_c)^{-3}$, then $\tau \sim 3.1 \times 10^9 (r/r_c)^{15/8}$ yr. Thus, τ is comparable to or higher than the age of the universe already at $r \sim 2r_c$. We conclude that hydrodynamic turbulence is not expected to reach steady state outside the cores of clusters, and, therefore, that magnetic field amplification will be limited essentially to the core region. (Note also that field propagation out of the core is practically negligible, due to the very low value of v_0.)

We have already mentioned the issue of detectability of Compton-produced IC HXR emission as an example for the immediate relevance of the exact strength of the mean IC field. Specifically, if $B \lesssim 0.2$ μG, we predict that HXR fluxes from rich, not too distant clusters (like Coma and A2319 in the analysis by Rephaeli et al. 1987) will be detected at the 10^{-6} (cm^2 s keV)$^{-1}$ level at energies in the range 40–50 keV. If, however, $B \sim 2$ μG, then (diffuse IC) HXR emission will be at least 100 times lower, and therefore will not be detected even by the (next generation) detectors aboard the *GRO* and *XTE* satellites. In this regard it should also be noted that HXR (balloon) measurements of the Coma cluster have also been made by the Frascati group, which repeatedly claimed (Bazzano et al. 1984, 1990) detection of Coma at a flux level higher by factors 3–8 than the A4 2 σ upper limit of Rephaeli et al. (1987), We are not convinced that the emission originates in the IC space of Coma. However, if it does, then this can be construed as an observational evidence that the mean IC field in Coma is much smaller than the values advocated by Jaffe (1980) and RSS.

Fields frozen in the ejected IS media of cluster galaxies are diluted in strength to an estimated IC value of about 0.1 μG (Rephaeli 1988). The ejection of the fields, either by ram pressure stripping or supernovae, does not preserve the large-scale morphology of IS fields. Therefore, the 10 kpc field component in spiral galaxies is not expected to survive in the IC space. Galactic fields have also a cellular component of a fluctuating field with a coherence length of about 100 pc. This scale expands by a factor ~ 5 (Rephaeli 1988) when reaching the IC space, and constitutes there the seed field amplified by the turbulence. The end result of this process is a spectrum of fields on all scales from ~ 1 kpc to $l_0 \sim 10$ kpc, such that on the original scale of ~ 1 kpc the field is still mainly IS in origin. The real morphology of IC fields may even be more complex, with features on all scales from a few 100 pc to a few tens of kpc. The degree of Faraday rotation depends sensitively on the morphology of the fields; a calculation of the rotation of the plane of polarization of radiation traversing a region with turbulent magnetic fields has been recently presented by Crusius-Watzel et al. (1990).

We end with an implication of the expected complex morphology of IC fields on heat conductivity in the hot IC plasma. If the correlation length of the field is smaller than the mean free path, δ, of an electron between binary collisions, then heat conduction is reduced below the rate calculated using the standard Spitzer (1962) formula. In the hot IC plasma, δ is O(10) kpc, a value which should be compared with a representative correlation scale most relevant to the motion of electrons. Although the spectrum of values of the field coherence scale may be wide, it is the small-scale end of the spectrum which is relevant here. The direction of the electron motion is significantly changed after traveling a distance which is typically comparable to $l_m \sim 1$ kpc. Therefore, δ is drastically reduced, even along field lines, since heat conduction is now a result of random walks with a characteristic step size $\sim l_m$. However, conduction will generally be determined also by motions across field lines and so will be effectively negligible. Turbulent heat diffusion, however, is equivalent to a random walk with a step size l_0 and a velocity v_0. While more important than the reduced heat conduction in the presence of magnetic fields, still it is smaller than calculated from the Spitzer formula by a factor $\sim v_0/v_t < 3 \times 10^{-2}$, where v_t is the thermal speed in the gas. (Turbulent heat diffusion will thus be effective over scales of ~ 20 [or ~ 80 kpc, if $a = 4.5$ kpc], in a time comparable to the age of the universe.) If based on these considerations alone, we would expect the IC gas not to be isothermal, as is indeed indicated by X-ray observations.

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APPENDIX

In the text we applied largely qualitative considerations to describe the steady state of hydrodynamic turbulence which is presumed to be reached in clusters of galaxies. Here in the context of a more formal treatment, based on a model for turbulence (Canuto, Goldman, & Chasnov 1987, 1988), we obtain the spectral function $F(k)$, and from it l_0, v_0, and the corresponding value of the magnetic field. The model has been tested successfully against experimental laboratory data, as well as in astrophysical applications.

The basic equation in the model is a balance of energy rates

$$\epsilon(k) = \int_{k_0}^{k} [\tau_s(k')^{-1} + \nu k'^2] F(k') dk' = [\nu_t(k) + \nu] \int_{k_0}^{k} F(k') k'^2 \, dk' , \tag{A1}$$

where $\epsilon(k)$ is the total energy per unit time and unit mass fed by the source in the wavenumber range k_0–k, ν is the effective microscopic viscosity, and $\tau_s(k)$ is the time scale characterizing the net energy input from the source. The right-hand side is the sum of the energy per unit time and unit mass dissipated in the above range by microscopic viscosity, plus the energy per unit time and unit mass cascaded to wavenumbers larger than k. The latter is due to nonlinear eddy interactions, modeled here by the turbulent viscosity $\nu_t(k)$ which is determined by interactions with eddies of wavenumbers larger than k:

$$\nu_t(k) = \int_k^\infty F(k')\tau_c(k')dk' , \tag{A2}$$

where $\tau_c(k)$ is the eddy correlation time scale. The latter time scale characterizes the nonlinear eddy interaction and is related to $\nu_t(k)$ by

$$\nu_t(k) = \gamma\tau_c(k)^{-1}k^{-2} , \tag{A3}$$

with $\gamma = [2/(3K_0)]^3 = 0.1(K_0/1.44)^{-3}$; K_0 is the universal Kolmogorov constant (see, e.g., Batchelor 1973; Hinze 1975). Equations (A1), (A2), and (A3) determine $F(k)$ for any given $\tau_s(k)$; for more details and a discussion of boundary conditions, see Canuto et al. (1987). The value of γ was determined by the requirement that in the inertial range of wavenumbers the normalization of the solution $F(k)$ will agree with that of the Kolmogorov spectral function

$$F_K(k) = 2K_0\epsilon^{2/3}k^{-5/3} , \tag{A4}$$

where ϵ is the total energy per unit time per unit mass fed into the turbulent velocity field by the source. Also note that the turbulent viscosity is, by definition, a decreasing function of k. From equation (A1) follows that its maximal value, attained at k_0, is $\nu_t(k_0) = \tau_s(k_0)^{-1}k_0^{-2}$.

In the case at hand, we take the rate characterizing the source, $\tau_s(k)^{-1}$, to be the rate by which a spherical region of radius $l = \pi/k$ is traversed by the cluster galaxies. This is the product of the inverse of the crossing time of the region $v_G/(2l)$ times the probability of a galaxy to be in the above region $4\pi/3n_G l^3$, thus

$$\tau_s(k) = \tau_0(k/k_0)^2 ; \qquad \tau_0 = \tau_s(k_0) = (2\pi/3v_G n_G l_0^2)^{-1} , \tag{A5}$$

where the largest scale $l_0 = \pi/k_0$ is yet to be determined. We are interested in the turbulent spectrum in the wavenumber range in which the microscopic viscosity is small compared to the turbulent viscosity. This range consists of the large scale eddies and the Kolmogorov inertial range. With this expression for $\tau_s(k)$ the solution for the spectral function, using equations (A1), (A2), and (A3), is (Canuto et al. 1987)

$$F(k) = 4\gamma^{-1}\tau_0^{-2}k^{-3}\frac{b^3}{3b - (k_0/k)^2} , \tag{A6}$$

where

$$b(k) = \gamma\tau_0/\tau_c(k) \tag{A7}$$

is the solution of

$$2b^3 = (k/k_0)^2 + b^2(k/k_0)^{-2} , \tag{A8}$$

with $b(k_0) = 1$.

The total energy per unit time and unit mass fed by the source, $\epsilon = \epsilon(\infty)$, is found to be (Canuto et al. 1987)

$$\epsilon = (2\gamma)^{-1}\tau_0^{-3}k_0^{-2} . \tag{A9}$$

It is easy to check that for large k values this solution tends indeed to $F_K(k)$ of (A4). For $k = 2.5k_0$ the deviation is 7.7%, and for 5 k_0 it is below 1%. In order for $F(k)$ to be fully determined we need to find l_0. To do so, note that in (the assumed) steady state, the rate of energy input per unit volume into the turbulent IC gas must equal the rate of energy per unit volume lost by the galaxies due to interaction with the turbulence (as manifested by the turbulent viscosity). Thus

$$\rho\epsilon = \rho\lambda n_G \pi a v_G^2 \nu_t(k_a) , \tag{A10}$$

where the left- and right-hand sides are the above energy rates, respectively. The IC gas density is ρ, λ is a dimensionless coefficient of order unity, a is the effective radius of a galaxy for interaction with the IC gas, and $\nu_t(k_a)$ is the turbulent viscosity at the wavenumber $k_a = \pi/a$. As will be seen below $(k_a/k_0) = l_0/a \sim 10$, thus (A3), (A7), and (A8) yield, to a precision better than 1%,

$$\nu_t(k_a) = \nu_t(k_0)2^{-1/3}(k_a/k_0)^{-4/3} = 2^{-1/3}\tau_0^{-1}k_0^{-2}(l_0/a)^{-4/3} , \tag{A11}$$

where we have substituted $\nu_t(k_0) = \tau_0^{-1}k_0^{-2}$ to get the second equality. Using this and ϵ given by equation (A9), we obtain from equation (A10), after some algebra,

$$l_0 = 0.8a(\gamma/0.1)^{3/16}\lambda^{3/16}(n_G\pi a^3)^{-3/16} . \tag{A12}$$

This completes the determination of the spectral function $F(k)$ in terms of the relevant cluster parameters. Using this $F(k)$ in equations (1) we obtain the rms value of the turbulent velocity on the scale l_0,

$$v_0 = v(k_0) = [0.7F(k_0)k_0]^{1/2} = 0.4v_G(\gamma/0.1)^{1/16}\lambda^{9/16}(n_G\pi a^3)^{7/16} . \tag{A13}$$

We see that equations (A12) and (A13) are identical, up to factors of order unity to equations (6) and (7), which were derived from essentially dimensional arguments, without carrying out the detailed solution for $F(k)$.

Substituting the value of a from equations (8) in equations (A12) and (A13) results in

$$l_0 = 8.2(\gamma/0.1)^{3/16}\lambda^{3/16}v_8^{-7/8}M_{11}^{7/16}n_3^{-3/16}\ \mathrm{kpc}\ , \tag{A14}$$

$$v_0 = 1.3(\gamma/0.1)^{1/16}\lambda^{9/16}v_8^{-13/8}M_{11}^{21/16}n_3^{7/16}\ \mathrm{km\ s^{-1}}\ , \tag{A15}$$

which agree up to factors of order unity with equations (9) and (10). The corresponding time scale for energy input into the turbulence is $\tau_0 \sim 1.2 l_0/v_0 \sim 7.5 \times 10^9$ yr.

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