LITHIUM IN HALO STARS: CONSTRAINING THE EFFECTS OF HELIUM DIFFUSION ON GLOBULAR CLUSTER AGES AND COSMOLOGY

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ABSTRACT

It has been proposed that helium diffusion can reduce age estimates for globular clusters through its effect on the main-sequence lifetime as well as on the morphology of isochrones near the turnoff. Using stellar evolutionary models with diffusion, both ours and from other work, we show that observations of lithium in extreme halo stars provide crucial constraints on the magnitude of the effects of helium diffusion. The flatness (and/or possible slight increasing slope) of the observed Li- T_{eff} relation (the Spite Li plateau) severely constrains diffusion Li isochrones, which tend to curve downward toward higher T_{eff} . Previously published age reductions of order 20%-30% seem to be ruled out by the Li flatness constraint from the current observations; however, age reductions of order 10% might be possible. Additional lithium observations are urgently required, particularly toward the hot edge of the plateau as well as beyond the cool edge, to define more securely the morphology of the Li- T_{eff} trend, and thus to gauge more precisely the possible effects of helium diffusion.

Subject headings: clusters: globular — diffusion — stars: abundances — stars: interiors — stars: Population II

1. INTRODUCTION

It has recently been suggested that helium diffusion in the outer layers of stars can lead to perhaps significant reductions in estimates for the ages of globular clusters. The magnitude of the effect depends sensitively on the depth of convection zones in turnoff halo stars; unfortunately, several factors still contribute to a sizable uncertainty in evaluating this depth. Our purpose in this paper is to propose observations involving lithium abundances in halo stars that could play a critical role in assessing and calibrating the effectiveness of helium diffusion.

Noerdlinger & Arigo (1980) and Stringfellow et al. (1983) proposed that helium diffusion could reduce age estimates for globular clusters by 25%, by reducing the main sequence lifetime of the stellar models. Deliyannis, Demarque, & Kawaler (1990; hereafter DDK) proposed that helium diffusion in the outer layers could reduce age estimates by as much as 25% by affecting the shape of isochrones at turnoff and reducing the size of their subgiant region. Proffitt, Michaud, & Richer (1990, hereafter PMR) confirmed the importance of diffusion in the outer layers, and to some degree the lifetime reduction. A detailed comparison of the different approaches will be given elsewhere; here, we restrict attention to the significance of the Li constraint.

2. SUMMARY OF THE EFFECTS OF DIFFUSION

Mechanisms often classified under microscopic diffusion (e.g., Vauclair 1983) include those for which an element diffuses downward relative to hydrogen (gravitational settling, thermal diffusion) or upward for species that are not fully ionized (radiative levitation). The latter is unimportant in halo stars (Michaud, Fontaine, & Beaudet 1984; DDK); the effects of the former become increasingly important with increasing mass fraction in a model. Diffusive motions are rendered inefficient by turbulent motions; hence, they are not important in con-

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vection zones. Nevertheless, the abundance in the convection zone (and thus also the surface) can be affected by diffusion out of or into the base of the convection zone. Conversely, it is also possible that rotationally induced mixing below the convection zone prevents diffusion from being efficient in the outer layers.

For now, we proceed by assuming conditions conducive to maximal diffusion, i.e., stability below the surface convection zone. Thus, diffusion is more effective at the base of a shallower convection zone; for a given model mass, the diffusion rates of He (or Li) out of the convection zone increase as the convection zone becomes shallower during main sequence (hereafter MS) evolution, and are negligible during the pre-MS. Furthermore, because more massive (higher T_{eff}) models have shallower convection zones at a given age, they experience more diffusion. This creates a downward curvature in the hotter portion of a lithium (or helium) isochrone in the Li- T_{eff} (or He- T_{eff}) plane (Fig. 1; DDK).

Helium is abundant and diffuses relatively rapidly; therefore, its diffusion can affect stellar structure (Vauclair, Vauclair, & Pamjatnikh 1974; Michaud et al. 1984) and evolution (Noerdlinger & Arigo 1980; DDK). In halo star models with turnoffs near the hot end of the Spite Li plateau (defined in § 4), convection zones become extremely thin with evolution: the convection zone recedes from a ZAMS depth of order 1% (by mass fraction) to a depth in the range 10^{-3} - 10^{-6} (or less) at turnoff, depending on the stellar parameters. Diffusive models develop an increasingly hydrogen-rich envelope, yet their core luminosity, and thus the energy that must be transported through the envelope, is (roughly) similar to that of standard models. Thus, the outermost layers of diffusive models are larger and the surface temperature is smaller relative to standard models. During post-turnoff evolution most of the diffused helium will be dredged up into the convection zone. resulting in only a minor change in the position of the giant branch. Helium diffusion in the outer layers thus lowers age estimates by reducing the size and possibly the shape of the subgiant region in an isochrone; a lower-aged diffusion isochrone is required to fit a given cluster.

3. LITHIUM AS A TRACER

Although the surface helium abundance cannot be measured directly, the effects of helium diffusion can be traced by observations of surface lithium, which diffuses in nearly the same way. The properties of the Spite lithium plateau (§ 4) severely restrict the amount of diffusion-induced curvature that can be tolerated in a lithium isochrone (DDK).

Lithium is in fact an ideal tracer of diffusion in extreme halo stars, not only because it is relatively easy to observe, but also because their initial abundance seems to have remained (uniformly) close to that produced in the big bang (§ 6.2 in Deliyannis 1990; Deliyannis, Pinsonneault, & Demarque 1991b) even while the abundance of other elements such as iron increased by orders of magnitude (Spite et al. 1987; Rebolo, Beckman, & Molaro 1987; § 7 in Deliyannis 1990); the halo stars thus form a sample of old stars with a uniform initial Li abundance. (By contrast, for example, the tracer beryllium, which also diffuses roughly similarly to helium, has evolved during the evolution of the halo [Ryan et al. 1990; Deliyannis & Pinsonneault 1990].) A sample with nonuniform initial Li abundances would be far less useful in constraining helium diffusion theory; it is thus crucial to take great care in isolating a uniform sample. One danger is that if Galactic Li enrichment occurs, it need not do so uniformly. Evidence that such enrichment has indeed occurred is manifest when considering depletion from both standard halo stellar models (DDK) as well as from rotational models (§ 9 in Deliyannis 1990; Pinsonneault, Deliyannis, & Demarque 1991). Furthermore, several viable Li enrichment mechanisms have been proposed (see § 7 in Deliyannis for a recent review), and the pattern seen in progressively more metal-rich stars can be interpreted, at least in part, as Galactic Li enrichment (Rebolo, Molaro, & Beckman 1988; DDK). If one restricts attention only to extreme halo stars which are not likely to have experienced Galactic Li enrichment, it is possible to have a sample of stars with very similar (or even the same) initial Li abundance that can be used effectively to constrain helium diffusion theory.

4. THE LITHIUM OBSERVATIONS

Lithium observations in halo dwarfs were pioneered by Spite & Spite (1982; hereafter SS) who discovered the striking properties described below. Many subsequent studies have repeatedly confirmed these properties and have expanded the original SS sample.² Employing both kinematical and chemical selection criteria,³ DDK defined group A stars to be those extreme objects that are least likely to have formed out of material enriched in lithium by Galactic processes.

The group A stars exhibit the following properties (Fig. 1a):

1. A remarkably uniform lithium plateau (the Spite plateau) from ~ 6400 to 5500 K, possibly with a slight increasing slope toward hotter $T_{\rm eff}$.

2. Progressively less lithium for stars progressively cooler than 5500 K.

The plateau severely restricts the amount of downward curvature that a Li isochrone can have toward hotter T_{eff} : the current observations tolerate no more than a 0.10 dex (or conservatively, 0.15 dex) difference in the Li abundance between

² Spite, Maillard, & Spite (1984); Spite & Spite (1986); Boesgaard (1985); Hobbs & Duncan (1987); Rebolo, Molaro, & Beckman (1988); these and SS have been compiled in DDK. Some very extreme group A stars from Spite et al. (1987), Rebolo et al. (1987), and Hobbs & Pilachowski (1988) have been compiled in Deliyannis & Demarque (1991).

³ [Fe/H] ≤ -1.3 and $V_{LSR} \leq 160$ km s⁻¹ unanimously for all sources quoted in DDK.

the hot edge ($\sim 6400-6300$ K) and the middle (~ 5800 K) of the plateau. The depleted detections in the cool stars are also important for constraining theory: plausible stellar models must be able to reproduce both the plateau and the depleted cool star detections. Future observations can also test dependence of the Li depletion on metallicity as predicted by theory (Deliyannis & Demarque 1991).

It is impressive that each observational study independently confirmed these properties, even though each study employed different instrumentation, analysis, and atmospheric models. While a statistical analysis of the pleateau stars in the Li- $T_{\rm eff}$ plane suggests that they are consistent with no intrinsic dispersion (DDK), an analysis in the equivalent width-color plane suggests that there might exist a small intrinsic dispersion (Deliyannis & Pinsonneault 1991). This conclusion is important in the present context because diffusion, by itself, cannot cause dispersion in the Li abundances from star to star of a given mass ($T_{\rm eff}$). Note that when progressively less extreme (than group A) stars are considered, progressively more dispersion is observed (DDK); this may be due to variable Galactic Li enrichment, or to more complex stellar interior depletion processes, or both.

Observers independently agree that the uncertainty in $[Li]^4$ for most plateau stars is ≤ 0.1 dex and slightly more for cool stars. In the quantitative analysis below, we adopt the weights discussed in Deliyannis & Demarque (1991; see also § A in DDK).

5. DISCUSSION

5.1. Application of the Li Flatness Constraint: Diffusion Overestimated

To illustrate the nature of the lithium constraint and the necessity for new observations, we employ diffusion models computed as described in § Vb of DDK, as well as the models of PMR.

First, it is worth emphasizing the high quality of the fit that standard isochrones can yield. Figure 1a shows a 16.5 Gyr, Z = 0.0001 ($\alpha = 1.5$) standard Li isochrone, which reproduces in detail the morphology of the observations including the depletion in the cool stars, the slope at the cool end of the plateau, and the possibly lesser slope toward higher $T_{\rm eff}$. A weighted least squares fit to the detections yields an initial abundance of 2.17 and a reduced chi-squared (χ^2/ν) of 1.03, with a probability $P(\chi^2/\nu) = 0.45$ of obtaining a higher value of χ^2/ν . This is very close to the ideal values of $\chi^2/\nu = 1$ and P = 0.5⁵ Thus, assuming that observers have estimated their uncertainties properly, this standard isochrone can be interpreted as providing a near-ideal fitting function to the data. Although this in itself does not ensure the accuracy of the standard models, it strongly suggests that to be compelling, alternate stellar evolutionary scenarios (e.g., diffusion) must provide at least as good a fit to the data.

⁴ Lithium abundances are given by number relative to hydrogen on a logarithmic scale where hydrogen is 12, and are represented by notation "[Li]"; i.e., $[Li] \equiv 12 + \log (N_{Li}/N_{H})$. We also employ the usual notation for relating iron abundances to solar values: $[Fe/H] = \log (Fe/H)_{*} - \log (Fe/H)_{\odot}$.

⁵ Fitting the standard isochrone of PMR yields an initial abundance of 2.17, $\chi^2/\nu = 1.49$ and P = 0.025, just barely an acceptable fit at the 1.96 σ level. The quality of this fit may simply be due to lack of resolution in the cooler part of their isochrone rather than to deficiencies in their models. Unfortunately, the PMR isochrone does not extend to sufficiently cool T_{eff} to cover the important depeleted detections for $T_{eff} < 5400$ K; if its extension behaves like our isochrone, then including these stars would make the fit even worse.

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FIG. 1.—(a) Observations of lithium by number relative to hydrogen on a logarithmic scale (*left*) where hydrogen is 12, in extreme halo dwarf stars. The error bar shows the effect on [Li] of a representative relative uncertainty in T_{eff} . Also shown is a standard Li isochrone of mean metallicity, illustrating depletion factor D = Li(t)/Li(initial) due to nuclear destruction of Li (*right scale*). The agreement with observation is excellent: the isochrone reproduces the morphology of the detections in detail. (b) In sharp contrast to (a), diffusive Li isochrones conflict with observation; in particular, they exhibit more downward curvature toward higher T_{eff} than is tolerated by the observations.

We present a diffusive Li isochrone (16.5 Gyr, $\alpha = 1.7$) in Figure 1b. The most striking difference between this and the standard isochrone is the high degree of downward curvature toward hotter $T_{\rm eff}$. The data, on the other ahnd, show no evidence of curvature, and in fact exhibit a slight increasing slope with hotter $T_{\rm eff}$. The fit of the diffusive Li isochrone to the data (see below) is poor also in another respect: the portion at the cool end of the plateau is forced to be above most of the stars to compensate for the hot portion curving below the stars. More quantitatively, a weighted least-squared fit yields an initial abundance of 2.39, and a χ^2/ν of 2.37 with a $P = 1.2 \times 10^{-6}$ probability of obtaining a higher value.⁶ Assuming that the observers have estimated their uncertainties properly, these values become a measure of the quality of the fit. We find that the diffusive lithium isochrone is not an appropriate fitting function, at the 4.5 σ confidence level (also at the 4.5 σ level for PMR). (Note that isochrones with lower α yield still worse fits.)

These results depend critically on how well observers have estimated their uncertainties. For example, in the unlikely event that the real uncertainties are all twice as large as what has been assumed here, then the diffusive Li isochrones become acceptable. However, several reasons suggest that the relative uncertainties are not difficult to estimate in the halo star context. The Li line is weak and falls on the linear part of the curve of growth. In metal-poor stars it is unblended, and it is not difficult to establish the location of the continuum; therefore the equivalent widths should be accurate. In fact, equivalent widths of the same star given by different observers

⁶ Encouragingly, although the models differ in the details of their computation, the diffusion isochrone of PMR (15 Gyr) yields very similar results: initial abundance = 2.40, χ^2/ν = 2.28 and $P = 6.5 \times 10^{-6}$; again, the isochrone does not cover two of the cool star detections, and the fits would probably be worse if it did.

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generally agree well with each other (Deliyannis & Pinsonneault 1991). There is also the transformation to the Li- $T_{\rm eff}$ plane to consider, with additional possible uncertainties that might be introduced, both random and systematic, e.g., due to the use of model atmospheres. However, possible systematic errors between different observational studies seem to behave very much like what the observers claim (e.g., use of a higher $T_{\rm eff}$ does indeed result in higher Li, approximately by the amount that is claimed).

We must conclude, then, that diffusive models discussed here overestimate the effects of diffusion. If the amount of diffusion were to be smaller and preferentially so for the edge of the Spite plateau, then perhaps the Li flatness constraint can be met. Several factors could be responsible for the overestimate. First, there are intrinsic uncertainties in the calculations for the rates of diffusion itself; second, we may be neglecting mechanisms that inhibit diffusion (e.g., rotationally induced turbulence); and third there are uncertainties in the "background" stellar structure on which diffusion acts. The first seems the least important at present (e.g., by comparing diffusion rates between this work and PMR at a given convection zone depth). The third depends on many factors, including the choice for opacities and composition, treatment of convection and choice of mixing length, uncertainties in the equation of state, and details in the treatment of the atmospheric boundary condition, all of which can affect the depth of the convection zone.

5.2. Opacities and Mixing Length

Indeed, the diffusion overestimate might be caused in large part by an underestimate of the depth of the convection zone. For example, at the hot edge of the plateau a higher opacity results in a steeper temperature gradient, a deeper convection zone, and thus less diffusion. Opacities recently computed at Livermore (Iglesias, Rogers, & Wilson 1990) are higher than the Los Alamos opacities (Huebner et al. 1977; Huebner 1978; used by PMR), which in turn are higher than Cox-Stewart opacities (Cox & Stewart 1969, 1970; used here). The Livermore enhancement peaks at 2.7×10^5 K. On an isochrone, the temperature at the base of the convection zone of progressively higher mass ($T_{\rm eff}$) models comes progressively closer to this temperature; such behavior in the opacities may thus be just what is needed to flatten out the Li isochrones.

Another possibility is hinted at from our diffusive solarcalibrated models: The model radius is sensitive to the value of the mixing length (α); thus the solar radius constrains the choice for α in solar models. Diffusive solar models, in which the envelope Y decreases slightly with time, require a larger α to match the solar radius. (A larger convective efficiency compensates for the otherwise larger outer layers.) A larger α might then also be appropriate for halo star models, which assume a smaller Y to begin with (and which decreases with time). For a given mass, larger α implies a deeper convection zone and higher T_{eff} [e.g., Table 7, § IIIa(ii) and § IIIb(ii) in DDK]. Compounding this, a higher α model at a given T_{eff} has a lower mass, and thus an even deeper convection zone, further reducing the amount of diffusion. Diffusion might also require an increasing α with time, and thus a still deeper convection zone. There may be other reasons as well, possibly related to surface rotation and magnetic fields, for supposing that α might be an increasing function with time (Tayler 1986).

5.3. Future Directions

Observationally, we have seen that Li observations at the hot edge of the plateau are particularly important in constraining the effects of helium diffusion; yet, they are currently few in number. We thus propose that additional observations are required there, as well as below 5500 K, to define more securely the morphology of the halo Li abundances. Fortunately, there now exist several surveys identifying numerous previously unknown halo objects (e.g., Beers, Preston, & Schechtman 1985; Carney et al. 1990; Ryan 1989; Schuster & Nissen 1989). The Li flatness constraint applies most directly to the more metal-poor clusters, such as M92, because only the correspondingly metal-poor stars are hot enough (solid circles in Fig. 1) to constrain the downward curvature of the diffusion isochrones. Understanding of diffusion in these clusters may then perhaps be extended to more metal-rich systems. Regardless, acceptably fitting globular cluster diffusion isochrones (in the HR diagram) must also pass the restrictions imposed on their lithium isochrone counterparts.

Theoretically, more work is required to study in detail the dependence of diffusion on various stellar parameters, the implications for globular cluster dating from isochrone fitting the ΔV method, and the $\Delta(B-V)$ method, and the interaction of diffusive motions with rotationally induced motions. It is also possible that only a small amount of diffusion suffices to yield a change in age estimates. Preliminary work at Yale using the Bahcall & Loeb (1990) approximation for helium diffusion suggests that there might exist parameter combinations that simultaneously give good fits to globular cluster CM diagrams, pass the Li flatness constraint, and still yield a substantial age reduction (Chaboyer et al. 1991a, b). In the case of M92, one of the oldest clusters, the age reduction is about 2 Gyr (from 16 to 14 Gyr).

5.4. Primordial Li Abundance and Cosmology

Finally, we consider implications for the primordial Li abundance. Since the isochrones discussed here overestimate the amount of diffusion, they also overestimate the initial Li abundance. We believe that a conservative upper limit to the initial Li abundance, due to diffusive effects alone, is 2.35 (DDK stated 2.36)⁷; the work of Chaboyer et al. (1991a, b) hints even this value is unduly conservatively high. The implications for

⁷ In this regard, certain aspects of the work of PMR warrant a few more remarks. PMR did not derive an initial Li abundance; they simply placed the initial abundance of their isochrone at 2.5, which is misleadingly high for several reasons. Their plot is not a fit, and shows their isochrone passing over the vast majority of observations. Recall that our weighted least-squares fit of their isochrone yields a lower initial value (and also looks more balanced). Furthermore, although the stars plotted by PMR were "taken from the compilation of" DDK, they include more than just the DDK group A dwarfs. PMR have included subgiants, which may experience post-turnoff dilution and thus should not be plotted together with dwarfs. In addition, less extreme stars have been included, some of which have slightly higher abundances. If the higher abundances are a result of Galactic Li evolution, then they should be ignored when fitting Li isochrones because such fits will yield a misleadingly high initial Li abundance. (One such star in particular, at $T_{eff} = 5660$, [Li] = 2.3 in Fig. 3 of PMR, is only one of two observations below 5900 K that lie above the PMR isochrone.) If, on the other hand, the higher abundances are due to differences in stellar depletion from similar initial abundances, then diffusion theory cannot by itself account for the observed differences. It is possible that other mechanisms, which can reproduce these differences, may yield far more depletion than does diffusion; for example, the models with rotationally induced mixing of Pinsonneault, Deliyannis, & Demarque (1991) produce dispersion in Li naturally.

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big bang nucleosynthesis then hardly differ from those derived using standard models (Deliyannis et al. 1991a, b). Far more important might be the possibility that rotationally induced mixing could reduce substantially the initial abundance in halo stars in a uniform manner by up to nearly an order of magnitude (Pinsonneault, Deliyannis, & Demarque 1991); if correct,

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the implications for big bang cosmology may be profound (Deliyannis et al. 1991a, b).

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