THE ASTROPHYSICAL JOURNAL, 378:307–314, 1991 September 1

CONSTRAINTS ON THE PHYSICAL PROPERTIES OF OPTICAL BULLETS IN SS 433

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Received 1990 August 21; accepted 1991 March 5

ABSTRACT

Possible mechanisms for continuously heating the Hα emitting bullets of SS 433 out to distances of 5×10^{14} cm and for turning off this emission at 10^{15} cm are discussed. Various observational constraints are used to establish bounds on permissible solutions in terms of the two key bullet parameters, mass M_B and angular radius θ seen from the central source. In particular, it is required that there should be sufficient heating only in the observed range of emission distance, that the lines neither be too broad nor exhibit deceleration, and that the bullets be dense enough to suppress forbidden lines and to emit enough Hα by recombination.

For convenience, the analysis is first carried out for the mathematically simplest case of uniform spherical bullets. For radiative heating of such bullets by starlight it is found that solutions exist only for very massive (10^{26} g) bullets with $\theta \sim 0.03$ radians ($n_e \sim 10^{10}$ cm^{-3}), which are highly implausible on the grounds of the large implied mean kinetic luminosity $\sim 10^{41}$ ergs s^{-1}. For heating of such bullets by collisions with the wind from the companion, a solution regime exists, for some wind densities, with more reasonable bullet masses $\sim 10^{24}$ g, densities $n_e \sim 10^{11.5}$ cm^{-3}, and narrow angular radii $\theta \sim 10^{-3}$ radians. Solutions exist only for wind mass-loss rates $\dot{M}_w \sim 10^{-6}$ to 10^{-3} $M_\odot$ yr^{-1}, the lower end of which range is fully compatible with SS 433 wind data and with typical wind parameters for an initially massive W-R star in a strongly mass-exchanging binary.

It is concluded that such collisional interaction is therefore the most likely mechanism for heating the optical bullets of SS 433. The narrow range of acceptable parameters not only allows good determination of wind and bullet properties but also may help explain the uniqueness and temporal variability of the SS 433 phenomenon at optical wavelengths.

The effects on these constraints of the bullets being fragmented rather than uniform, as suggested by theory, and being elongated rather than spherical, as required by data at other wavelengths, are then discussed. It is found that the main conclusions regarding acceptable θ and M_B are essentially unchanged but that bullets with the elongation suggested by the X-ray duty cycle only permit collisional heating solutions if they are substantially fragmented.

Subject headings: radiative transfer — stars: individual (SS 433) — stars: mass loss

1. INTRODUCTION

In the decade since the discovery of the jets of SS 433, this object has been the focus of extensive observational coverage at all wavelengths (see, e.g., reviews by Margon 1984; Vermeulen 1989; Brown, Collins, & Cassinelli 1991) and of intensive theoretical effort. These observations have served to confirm the likely correctness of the basic kinematic model of Abell & Margon (1979), which is essentially geometric in character, and to elucidate the temporal and spatial morphology of the jet material. On the other hand, there still remain many observational puzzles (e.g., Brinkmann, Kawai, & Matsuoka 1989), and theoretical work, while supportive of the qualitative picture of jet acceleration and collimation in a compact accretor, has as yet to come to terms with the fundamental physical problems posed by the system, most notably the power and rapidity required by jet acceleration and the accuracy of the jet speed governor (cf. Icke 1989, for example). Furthermore, even given a parametric description of the process of jet production, comparatively little has been done concerning the physics of how the jets emit the radiation by which they are observed, or the physical conditions in the jets required for such emission to be possible. Indeed most current thinking in this regard can already be found in the earliest postdiscovery papers, particularly that of Davidson & McCray (1980), when plausible system parameters were first estimated on the basis of minimal data. In the present paper we therefore revisit the problem of optical jet conditions in some detail, considering what constraints can be placed on the parameters describing them by combining various upper and lower bounds imposed by observations, and using up-to-date data. By this approach we will show that only a surprisingly narrow range of bullet parameter values are permissible for the system to appear as it does and that the mechanisms of energization of the optical jets are likewise restricted. These results may help explain the rarity and temporal variability of the SS 433 phenomenon at optical wavelengths. Before examining specific mechanisms for energizing the optical jet emission we must first state what we believe to be well established about the system and the assumptions on which our modeling will be based.

Certainly the best-established values relating to SS 433 are those defined in the kinematic model of Abell & Margon (1979), viz., the jet precessional cone angle, the system inclination, and the remarkably constant jet speed of 0.26*c* (cf. Collins

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& Newsom 1986). Second, the stationary spectrum of the system defines fairly well the bolometric luminosity (assumed isotropic) as $L_{bol} = 4 \times 10^{39}$ ergs s^{-1} (somewhat larger than estimated by Davidson & McCray 1980) and strongly indicates the presence of a dense stellar wind, first suggested by Margon et al. (1979) and quantified by Wagner (1983) as having a terminal speed of $v_w = 500$ km s^{-1} and a mass-loss rate of $\dot{M}_w = 3 \times 10^{-5}\ M_\odot$ yr^{-1} (if the near-equatorially observed wind values held isotropically), both values within a factor of 2. As regards the moving spectrum, a key property of this is that the Hα lines appear to be emitted by discrete transient "bullets," each of which maintains a constant velocity (to less than 1%) while radiating for only 1–2 days (Grandi 1981) at a rate $L_{H\alpha} \sim 10^{35}$ ergs s^{-1} per jet (cf. Davidson & McCray 1980; Wagner 1983). Since the time between such bullets is of the same order as the duration of the emission of each, the mean and peak values of $L_{H\alpha}$ are similar. The lifetime of these bullet emissions leads to the conclusion that the bullets cease to radiate after traveling a distance $D \sim 10^{15}$ cm, with a mean radiating distance $D \sim 5 \times 10^{14}$ cm (cf. Davidson & McCray 1980). Using dynamical model analysis of orbital phasing of photometric eclipse data and of light travel time from different jet regions, Collins & Newsom (1986) arrived at independent estimates of the mean emission distance of 5 (± 1.4) and 4 (± 3) $\times 10^{14}$ cm, respectively, so that this is quite well defined.

Two further well-defined constraints arise from the absence of forbidden lines in the spectrum, indicating a density $n > 10^7$ cm^{-3} (Davidson & McCray 1980; Margon 1984; J. Mathis 1987, personal communication), and the widths of the moving lines, equivalent to less than 10^3 km s^{-1} (cf. Margon 1984). The latter show that the jets cannot be expanding with a cone half-angle in excess of 0.03 radians (cf. Davidson & McCray 1980) and that the bullets cannot undergo a deceleration above $\sim 1\%$ during their optical emission lifetimes. In fact the trajectories of radio bullets (Hjellming & Johnston 1981, 1985; Vermeulen 1989) and the jet interaction with W50 show that there is little bullet deceleration even over much greater distances, but, as we will see below, jet interactions which would cause deceleration are likely to be confined to distances of the order of the optical emission distance.

In order to model the energization of the bullets, we will have to adopt some geometry for them. At optical wavelengths, the SS 433 jets certainly exhibit bullet-like structures lasting about a day (Grandi 1981), though it is not known whether these are localized (approximately spherical) structures which glow for a day or elongated structures one day "long." Since the X-ray data (e.g., Stewart et al. 1989; Brinkmann et al. 1990) show the inner jets to be almost continuous, the jets must undergo some degree of longitudinal fragmentation in density and/or temperature as they propagate. For *mathematical simplicity and compact diagrammatic presentation* of results, to start with (§§ 2 and 3), we will model the case of homogeneous spherical *optical* bullets, but in § 4 will discuss the modifications of our analysis to allow for their likely elongated form and for the fragmentation commonly believed to be an essential feature of them. The essential problem addressed in this paper, then, is that of how energy is continuously supplied to the bullets, out to distances of 10^{15} cm, to enable them to radiate. (We will only be considering the global energy balance, averaged over the whole bullet structure.) The need for such energy supply was recognized early on (e.g., Davidson & McCray 1980) and is most simply stated as follows. Starting from Hα temperatures, the emission of just one Hα photon (~ 2 eV) by each hydrogen atom would completely cool the gas. Thus, in order to supply $L_{H\alpha} \sim 10^{35}$ ergs s^{-1}, without energy input to maintain the bullet temperature, would require the jets to carry 3×10^{46} atoms s^{-1} at $v_j = 0.26c$ (34 MeV per atom), corresponding to a kinetic luminosity $L_{kin} \sim 2 \times 10^{42}$ ergs s^{-1}. This is comparable to the power of a galactic nucleus and is highly implausible in terms of source energetics, lifetime (>3000 yr; Hjellming & Johnston 1981, 1985), and the dynamics of formation of W50.

The only evident *external* candidate for the necessary resupply of bullet thermal energy is absorption of radiation of the parent stars, which we discuss in § 2. The fact that L_{kin} is so large, however, also points to conversion of bulk kinetic energy as a strong candidate. This could arise either through loss of bulk bullet kinetic energy by interaction with a surrounding medium such as the stellar wind, as suggested by Davidson & McCray (1980) and discussed in § 3, or by dissipation of internal differential (turbulent) mass motions in the bullets themselves. An upper limit is set on the latter by the line widths, which show that they cannot exceed $\Delta v \sim 10^3$ km s^{-1}. In the bullet frame the turbulent energy supply therefore cannot exceed $(\Delta v/v_j)^2 L_{kin} \sim 10^{-4} L_{kin}$. For the value $L_{kin} \sim 10^{39}$ ergs s^{-1} suggested to distort W50, this would be just enough to supply Hα alone on the assumption that all of the line width is due to turbulent motion. We do not consider this marginal possibility further here.

In considering bullet heating, it is also important to recognize the dominant competing cooling processes, and we will take these to be the optical output of the bullets. While expansion cooling may be comparable to radiative in the hot inner (X-ray) jet (Watson et al. 1986; Begelman, Blandford, & Rees 1984), it is easy to show that it is negligible in the optical region. Even if expansion cooling is taken to operate at the maximum kinetic expansion speed of $10^{-2} v_j$, then the cooling rate (bullet thermal energy/bullet expansion time) would be less than $L_{H\alpha}$ for Hα emission distances. Furthermore, this kinematic expansion speed is far in excess of the sound speed at Hα temperatures, which is the one physically relevant to expansion cooling (i.e., for conversion of thermal to bulk motion).

2. RADIATIVE HEATING OF BULLETS

The degree to which the stellar radiation flux heats a bullet depends solely on the fraction of L_{bol} intercepted by it. For this estimation we will assume L_{bol} to be radiated essentially isotropically. We note, however, that if the bullets are ejected from a funnel, in a thick disk or giant star (Collins, Brown, & Cassinelli 1990), above which the radiation flux is much higher than $L_{bol}/(4\pi r^2)$, then we will be underestimating the effectiveness of radiative heating. Such a "searchlight beam" mechanism is intriguing but raises a number of broad questions detailed discussion of which is beyond the scope of this paper. In particular, Davidson & McCray (1980) point out that this would further increase the total energy demands of the system. On the other hand, if radiation in the funnel is to be the dominant mechanism for initial acceleration of the bullets (cf. Icke 1989), and especially if bullets are as massive as our analysis below suggests in the radiative case, then the short acceleration time inferred by Watson et al. (1986) certainly demands a very large radiation flux in the funnel. One observable consequence of this situation would be that funnel precession would cut off radiative heating of the bullet after a fraction of order $\theta/2\pi$ of the precessional period, viz., ~ 1 day (as first suggested by Calvani & Nobili 1981).

We will also assume for geometric simplicity that all of the heating radiation originates in the source of the jets, whether this be a giant star or a massive accretion disk (cf. Collins et al. 1990). Photometry (Kemp et al. 1986) and polarimetry (Carlaw 1988) in fact suggest that a significant fraction of $L_{\rm bol}$ originates in the companion to the jet source. If this is so, then in principle there should be a synodic (precessional-orbital) phase-dependent difference between the radiative heating of the blue and red jets. At the distance of the optical emission ($\gg$ component separation) this effect will be small, and the two sources of radiation will be seen virtually as one point source, but in the inner X-ray jet it could be important.

In the isotropic case, then, the total radiative heating rate for one bullet will be

$$P_{\rm rad} = \epsilon \theta^2 L_{\rm bol}/(4\gamma^4) , \tag{1}$$

where γ is the bullet Lorentz factor at $0.26c$, θ is the bullet conical half-angle in radians, and ϵ is the fraction of the incident radiative power absorbed by the bullet. Assuming that successive bullets do not shadow one another, then ϵ is related to the optical depth τ of a single bullet by

$$\epsilon = 1 - e^{-\tau} , \tag{2}$$

of which we will consider only the limiting optically thick and thin cases.

In the optically thick case $\epsilon = 1$, and, taking Wagner's (1986) value $L_{\rm bol} = 4 \times 10^{39}$ ergs s^{-1}, we obtain

$$\log P_{\rm rad} = 2 \log \theta_{-2} + 34.9 , \tag{3}$$

where $\log \equiv \log_{10}$ and $\theta_{-2} = 10^{-2}\,\theta$ is the bullet conical half-angle in units of 10^{-2} radians, with similar notation for other variables. The absolute minimum radiative loss which must be offset by this heating is the observed output $L_{{\rm H}\alpha} = 10^{35}$ ergs s^{-1} of Hα itself, which will represent a fraction $\delta = L_{{\rm H}\alpha}/L_{\rm rad}$ of the total radiative cooling rate $L_{\rm rad}$. Requiring $P_{\rm rad} > L_{\rm rad}$, we obtain the first constraint on a radiatively heated bullet model, viz. (with $\delta = 1$),

$$\log \theta_{-2} > 0.03 \quad \text{(optically thick)} . \tag{4}$$

(The actual value of δ can range from 1, when the bullet is optically thin only to Hα to $\sim 1/20$ when Lyα becomes optically thin; cf. Davidson & McCray 1980; Osterbrock 1974, pp. 60–70.)

In the optically thin limit $\tau \ll 1$, $\epsilon = \tau$, which, through the center of a largely ionized spherical bullet, with absorption of any scattered radiation, is

$$\tau = 2 n_e \sigma_{\rm T} D\theta = \frac{3 M_B \sigma_{\rm T}}{2\pi D^2 \theta^2 m_p} = 1.9 \times 10^{-2} \frac{M_{B25}}{D_{15}^2 \theta_{-2}^2} , \tag{5}$$

where $D = 10^{15} D_{15}$ cm is the bullet distance from the source, $M_B = 10^{25}\ M_{B25}$ g is the total bullet mass (assumed to be hydrogen), m_p is the proton mass, and $\sigma_{\rm T}$ is the Thomson cross section. We use $\sigma_{\rm T}$ because the radiative heating is governed by the frequency averaged optical depth. Under the optical bullet conditions of low density, irradiated by a hot star, the Thomson depth is the relevant one as in hot stellar winds.

Here the density n_e (relevant to the forbidden-line constraint below) can be expressed as

$$n_e = \frac{M_B/m_p}{4\pi D^3 \theta^3/3}$$
$$= 1.4 \times 10^9 \ {\rm cm}^{-3} \times \frac{M_{B25}}{(\theta_{-2} D_{15})^3} . \tag{6}$$

In this limit we thus obtain

$$P_{\rm rad} = 3\sigma_{\rm T} L_{\rm bol} M_B/(8\pi\gamma^4 D^2 m_p) , \tag{7}$$

which, as expected, depends only on M_B and on D but not on θ.

Thus we find the condition for radiative heating $P_{\rm rad}$ to exceed $L_{{\rm H}\alpha}$ is (again with $\delta = 1$)

$$\log M_{B25} > 1.79 + 2 \log D_{15} \quad \text{(optically thin)} . \tag{8}$$

So long as the bullet remains optically thick, the condition that radiative heating suffice depends only on θ, all regions in the plane ($\log M_{B25}$, $\log \theta_{-2}$) above the θ_{-2} value given by equality in equation (4) satisfying this constraint. As D increases and τ falls to ~ 1, the radiative heating rate becomes independent of θ but is proportional to M_B/D^2, falling off quickly with distance. At any given D the condition $P_{\rm rad} > L_{{\rm H}\alpha}$ is then represented by a vertical left exclusion boundary line in the ($\log M_{B25}$, $\log \theta_{-2}$)-plane, which moves to the right with increasing D according to $M_B \sim D^2$.

In Figure 1 we show the combined optically thick/thin condition (eqs. [4] and [8] joined smoothly through an intermediate point at $\tau = 1$) that $P_{\rm rad} > L_{{\rm H}\alpha}$ at the mean emitting distance D of 5×10^{14} cm in order to supply the optical emission there. This appears in the form of an L-shaped boundary, marked "insufficient radiative heating," above and to the *right* of which the system parameters must lie. Second, in Figure 1 we show a similar boundary for $D = 10^{15}$ cm. We know that Hα bullets fade by this distance, and, attributing this fading to the onset of optical thinness, we then require that the system parameters lie to the *left* of the vertical boundary in Figure 1 marked "radiative heating lasts too long."

$L_{\rm bol}$ delivers momentum p, as well as energy, to the bullet at a rate of order $\epsilon L_{\rm bol} \theta^2/4c$, and we require that this not change the bullet speed by $\Delta v/v_j > \sim 10^{-2}$ during its Hα radiating lifetime $\Delta t_{\rm rad}$, which we know to be not less than the interval Δt_B between bullets. Then

$$\frac{\Delta v}{v_j} = \frac{\Delta p}{p} = \frac{\epsilon L_{\rm bol} \theta^2 \Delta t_{\rm rad}}{4 M_B v_j c} < \frac{\epsilon L_{\rm bol} \theta^2 \Delta t_B}{4 M_B v_j c}$$
$$= \frac{\epsilon v_j \theta^2 L_{\rm bol}}{8 c L_{\rm kin}} < 3 \times 10^{-6} \theta_{-2}^2 \frac{L_{\rm bol}}{L_{\rm kin}} , \tag{9}$$

which is very small (unless $L_{\rm bol}$ is highly beamed in the bullet direction). We therefore do not include this very weak constraint in our diagram of $\log M_{B25}$ versus $\log \theta_{-2}$ for the radiative heating case.

A further, and strong, radiative constraint which applies equally to the collisional heating case treated in § 3, is that the bullet must be dense enough for the recombination rate to provide the observed Hα output at the mean radiating distance. For relevant parameters the recombination coefficient at Hα temperatures given by Osterbrock (1974) will result in a maximum Hα luminosity

$$L_{{\rm H}\alpha} = 1.2 \times 10^{-13} \delta\, h\nu_\alpha \frac{M_B^2}{m_p^2 D^3 \theta^3} , \tag{10}$$

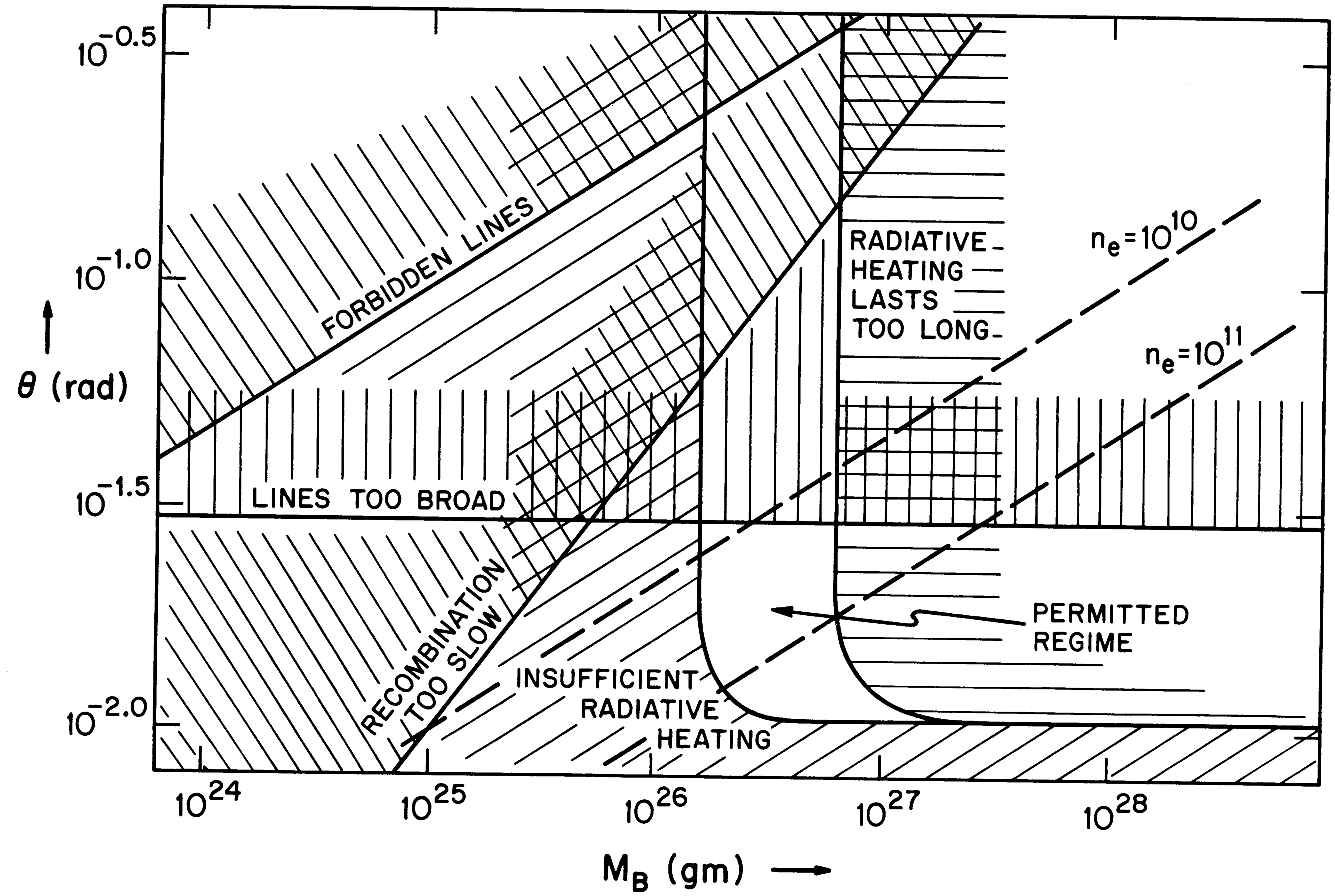

FIG. 1.—Constraint diagram for the radiatively heated spherical bullet model in the (M_B, θ)-plane, for the value of $L_{bol} = 4 \times 10^{39}$ ergs s^{-1} given by Wagner (1986).

where $h\nu_\alpha$ is the Hα photon energy. For the same parameters as above, the condition that $L_{H\alpha} > 10^{35}$ ergs s^{-1} then yields the constraint (for $\delta = 1$)

$$\log M_{B25} - 1.5 \log \theta_{-2} - 1.5 \log D_{15} > 0.45 , \quad (11)$$

shown in Figure 1 as "recombination too slow."

Two further constraints, again applicable to either heating mechanism and already cited in § 1, are that $\theta < 0.03$ rad because of the observed line width, or

$$\log \theta_{-2} < 0.48 \quad (12)$$

(shown in Fig. 1 as "lines too broad"), and that $n_e > 10^7$ cm^{-3} to suppress forbidden lines, or, using equation (6),

$$\log M_{B25} - 3 \log \theta_{-2} - 3 \log D_{15} > -2.1 , \quad (13)$$

shown in Figure 1 (for $D_{15} = 0.5$) and labeled "forbidden lines." Also superposed on Figure 1 are dashed lines of constant bullet density (eq. [6] again at $D = 5 \times 10^{14}$ cm) labeled with their n_e values.

Inspection of Figure 1 shows that the forbidden-line constraint is much weaker than once thought. However, the other conditions do constrain permissible values of M_B and θ. Specifically, though there is a substantial domain where L_{bol} can heat the bullets at $D = 5 \times 10^{14}$ cm and let them fade at $D = 10^{15}$ cm, the line-width restriction on θ permits solutions only for bullets of $n_e \sim 10^{10}$–10^{11} cm^{-3}, so that they are marginally optically thick, corresponding to very massive bullets of $M_B \sim 10^{26}$–10^{27} g. Such bullets would have a kinetic energy of $\sim 10^{46}$ ergs each, which, with a firing interval less than 1 day, corresponds to a mean $L_{kin} > 10^{41}$ ergs s^{-1}, in excess of plausible system power requirements or effects on W50. Furthermore, in the presence of both wind collisional heating and (isotropic) radiative heating, collisional heating would be totally predominant for the wind parameters given by Wagner (1983). *It seems therefore that radiative bullet heating is not a likely mechanism,* unless the central radiation source is anisotropic and beamed along the jet direction, as might occur above a jet acceleration funnel in a star or above a thick disk.

3. COLLISIONAL HEATING OF BULLETS IN THE STELLAR WIND

Davidson & McCray (1980) first suggested collisional interaction with a stellar wind as a means of energizing Hα from SS 433 bullets. Jet/wind interaction was also discussed by Begelman et al. (1980), but in terms of bullet heating by X-rays generated in a shock interface between the bullet and wind fluids. We will not concern ourselves here with such secondary processes as may be involved in energy redistribution through the bullet, but only with the gross rates of primary energy and momentum input to it from the wind. The existence of a substantial wind, first noted by Margon et al. (1979), was confirmed and quantified by Wagner (1983), who found a terminal speed $v_w \sim 500$ km s^{-1} and a wind strength along the line of sight corresponding to a mass-loss rate of $\dot{M}_w \sim 3 \times 10^{-5}$ $M_\odot$ yr^{-1} if the wind were emitted isotropically with this line-of-sight column density. It is very likely that the wind in the polar direction relevant to jet interaction will differ considerably from equatorial wind conditions because winds in rotating and binary systems in general tend to be equatorially flattened (e.g., Poe, Friend, & Cassinelli 1989), and, in SS 433, because of the sweeping action of the jets. We will therefore parameterize the polar wind in terms of a speed v_w (cm s^{-1}) $= 10^8 v_{w8}$ and a wind density corresponding to a mass-loss rate $\dot{M}_w$ ($M_\odot$ yr^{-1}) $= 10^{-6}\dot{M}_{w-6}$ if the wind were isotropic.

The two component wind models of Poe et al. (1989), for several W-R stars in binaries, have values of $\dot{M}_{w-6}/v_{w8}$ in the range 0.3–6, with higher values corresponding to spherically symmetric winds, and lower to strong equatorial flattening. The wind values reported by Wagner (1983) for SS 433 are very similar to those of the W-R star CV Ser, which has a mass of 13 $M_\odot$, spherical equivalent $\dot{M}_w = 3 \times 10^{-5}$ $M_\odot$ yr^{-1}, and a polar flow corresponding to $\dot{M}_{w-6}/v_{w8} = 0.55$. According to the evolutionary models of Maeder & Meynet (1987), such a star could have had an original mass of 60 $M_\odot$ and so provided ample mass transfer to power SS 433.

For simplicity we will assume a hydrogen wind, but we point out how results would be modified by a substantial helium content, very likely to be present for the components of SS 433. We will further suppose the wind to originate in the companion to the jet source. If this were not the case, then a bullet of $\theta > 10^{-2}$ moving through a wind of $\dot{M}_w = 10^{-6}$ $M_\odot$ yr^{-1} would run into 10^{23} g of dense inner wind material before becoming collisionally thin at a distance of $\sim 10^{15}$ cm (for the typical M_B we find below). This would cause significant bullet deceleration between the X-ray and Hα emission regions, contrary to observations. Analogous to the radiatively heated case, we note that the geometry we are assuming should result in precession-orbit synodic phase-dependent differences between the heating of the red and blue jets, particularly in the inner jet regions.

At optical bullet distance D, large compared with the stellar separation, the electron density of the (ionized) hydrogen wind is

$$n_w = \frac{\dot{M}_w}{4\pi m_p v_w D^2} = 3 \times 10^4 \text{ cm}^{-3} \frac{\dot{M}_{w-6}}{v_{w8} D_{15}^2} . \tag{14}$$

Because $v_j \gg v_w$ and $\theta \ll 1$, the energy and momentum delivered by wind particles to the bullet is largely through its front end (Davidson & McCray 1980; Brown et al. 1988) of area $\pi D^2\theta^2$; as in the radiative case we will neglect screening of bullets by adjacent ones. The wind proton flux seen by the bullet is $n_w v_j$, and the momentum and energy of each wind proton are $m_p v_j$ and $0.5 m_p v_j^2$, respectively. The rate of deposition in the bullet depends on the bullet's collisional thickness as measured by the ratio N/N_0, where N is the electron column density through the bullet and N_0 is the column density required to stop a wind proton. When $N > N_0$, wind particles will stop completely in the bullet, which will therefore absorb all the energy and momentum incident on its area, while if $N < N_0$, only a fraction $\epsilon_{\rm coll}$ will be absorbed, given by (Brown 1972; Emslie 1978)

$$\epsilon_{\rm coll} = 1 - (1 - N/N_0)^{1/2} \sim N/2N_0 \quad \text{for } N \ll N_0 . \tag{15}$$

Through the center of a bullet we have

$$N = 2D\theta n_e = \frac{3M_B}{2\pi m_p D^2\theta^2} = 2.8 \times 10^{22} \text{ cm}^{-2} \frac{M_{B25}}{D_{15}^2 \theta_{-2}^2} . \tag{16}$$

For a single proton of speed v_j (34 MeV) the value of N_0 in an ionized hydrogen target is 6×10^{22} cm^{-2} (e.g., Emslie 1978), contrary to the value of 10^{24} cm^{-2} used by Davidson & McCray (1980). However, for an overall neutral wind the equal energy loss rate of the electron accompanying each proton reduces N_0 to 3×10^{22} cm^{-2} for the pair. (In the case of a helium wind the energy loss rate of a nucleus is $Z^2 = 4$ times higher than for a proton with a further contribution of $Z = 2$ times the proton value added by the two accompanying electrons. On the other hand, the energy carried is a factor of 4 higher than for a proton, so that the resulting N_0 value for neutralized helium is 4/3 times the hydrogen value. Following this argument shows that a helium wind gives results differing from hydrogen only by factors of order unity, although the wind electron density [14] will be lower if $\dot{M}_w$ comprises He rather than H.) Another factor affecting N_0 is that in a *neutral* hydrogen bullet it is increased by a factor ~ 2.6 (Brown 1973; Emslie 1978), but as in § 2 we will take the bullet to be mainly ionized and adopt

$$N_0 = 3 \times 10^{22} \text{ cm}^{-2} . \tag{17}$$

Writing the collisional heating rate as

$$P_{\rm coll} = \epsilon_{\rm coll} \pi D^2 \theta^2 n_w v_j \times 0.5 m_p v_j^2 , \tag{18}$$

using equation (14) for n_w, and setting

$$N/N_0 = 0.94 M_{B25}/(D_{15}^2 \theta_{-2}^2) , \tag{19}$$

we find that in the *collisionally thick regime*

$$P_{\rm coll} = 3.8 \times 10^{36} \text{ ergs s}^{-1} \frac{\dot{M}_{w-6}\theta_{-2}^2}{v_{w8}} , \quad N > N_0 , \tag{20}$$

which for a given wind depends only on θ because the increasing bullet area offsets the decline in n_w with D. In the *collisionally thin regime*,

$$P_{\rm coll} = 1.8 \times 10^{36} \text{ ergs s}^{-1} \frac{\dot{M}_{w-6} M_{B25}}{v_{w8} D_{15}^2} , \quad N < N_0 , \tag{21}$$

which depends only on M_B and D, independent of θ.

As a bullet moves outward, it will thus experience a constant heating rate $P_{\rm coll}$ until it becomes collisionally thin. This rate will suffice to power $L_{\rm H\alpha} = 10^{35}$ ergs s^{-1} (with $\delta = 1$) only if

$$\log \theta_{-2} > -0.95 - 0.5 \log (\dot{M}_{w-6}/v_{w8}) . \tag{22}$$

When the bullet becomes collisionally thin, this is modified to

$$\log M_{B25} > -1.25 - \log (\dot{M}_{w-6}/v_{w8}) + 2 \log D_{15} \tag{23}$$

at distance D.

In Figure 2 we show (for $\dot{M}_{w-6} = v_{w8} = 1$) criteria (22) and (23) joined smoothly for the mean Hα emitting distance $D_{15} = 0.5$, the system having to lie above and to the *right* of the L-shaped boundary marked "insufficient collisional heating." We now invoke the onset of collisional thinness as the mechanism by which $L_{\rm H\alpha}$ is turned off at $D > 10^{15}$ cm (because n_w declines with distance) and require that inequality (23) should be reversed at $D_{15} = 1$. This means that the system must lie to the *left* of the line in Figure 2 marked "collisional heating lasts too long." (In their treatment Davidson & McCray 1980 made the ad hoc suggestion that turnoff of $L_{\rm H\alpha}$ was due to a "density bounded wind" curtailed at 10^{15} cm by an unspecified physical mechanism. One process which might bring a physical basis to this suggestion is that the outer wind regions will be swept away by the jets each time they pass by. Since this will occur at

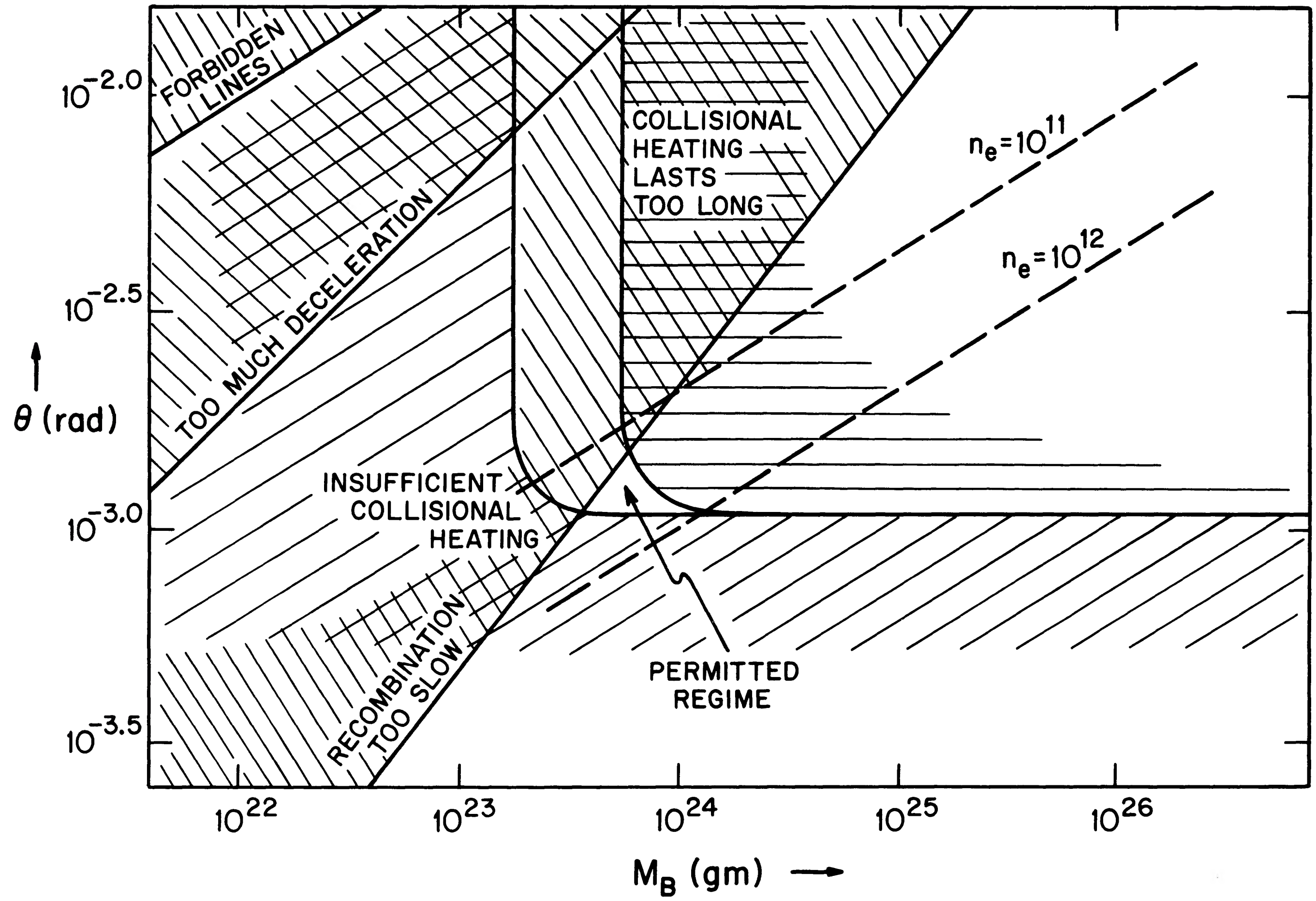

FIG. 2.—Constraint diagram for the collisionally (wind) heated spherical bullet model in the (M_B, θ)-plane, for wind parameter value $\dot{M}_{w-6}/v_{w8} = 1$. Note the shift in origin relative to Fig. 1, though the scales are the same. The "lines too broad" constraint is in this case off the top of the θ scale shown; cf. Fig. 1.

intervals of $T \sim 162$ days, we should expect the wind encountered by a jet to be depleted farther out than a distance $\sim v_w T \sim 1.6 \times 10^{15} v_{w8}$ cm, just where bullets do in fact fade. This effect will enhance that of collisional thinness considered here.)

The condition that the wind interaction should not decelerate the bullet observably is most simply written in terms of momentum conservation:

$$\Delta v_j/v_j = \Delta M_B/M_B , \tag{24}$$

where ΔM_B is the wind mass swept up by the bullet. Since momentum delivery declines rapidly once $N < N_0$, we can approximate

$$\Delta M_B = m_p \int_0^{D_0} n_w(r)\pi r^2 \theta^2 dr = \dot{M}_w \theta^2 D_0/(4v_w) , \tag{25}$$

where D_0 is such that $N = N_0$ in equation (16), so that

$$\Delta M_B/M_B = 1.6 \times 10^{-3} \dot{M}_{w-6} \theta_{-2}/(v_{w8} M_{B25}^{1/2}) . \tag{26}$$

Requiring $\Delta v_j/v_j < 10^{-2}$ then yields the constraint

$$2 \log \theta_{-2} - \log M_{B25} < 1.6 - 2 \log (\dot{M}_{w-6}/v_{w8}) , \tag{27}$$

which is shown in Figure 2 as the upper left-hand boundary, "too much deceleration." Also shown on Figure 2 are the general constraints discussed in § 2 (line width, forbidden lines, and recombination time) and dashed lines of constant bullet n_e.

It can be seen from Figure 2 that, within the uncertainties of our treatment, only a rather small solution regime exists for bullet masses $M_B \simeq 10^{23.5}$ to $10^{24.5}$ g and angles $\theta \sim 10^{-2.8}$ to $10^{-3.0}$ radians, with densities $\sim 10^{11}$–10^{12} cm^{-3}, the latter figures corresponding to a mean $L_{\rm kin} \sim 10^{39}$ ergs s^{-1} just sufficient to inflate W50, and 100 times less than in models heated by isotropic stellar radiation.

It should also be noted that even this small solution regime only exists for a somewhat limited range of wind conditions. In particular, as the parameter $\dot{M}_{w-6}/v_{w8}$ is *increased*, the point of intersection of the vertical line in Figure 2 given by equation (23) with $D_{15} = 1$, and the horizontal line given by equation (22), moves down and leftward. This point eventually crosses the "recombination too slow" constraint line given by equation (11), for $\dot{M}_{w-6}/v_{w8} > 5$ (i.e., $\dot{M}_w \sim 2 \times 10^{-6} M_\odot$ yr^{-1} for the v_w reported by Wagner 1983). For denser winds the bullet mass, $M_B < 10^{23}$ g, required for (collisionally thin) heating to turnoff by 10^{15} cm, and the bullet angle $\theta > 10^{-3.3}$ required to provide enough (collisionally thick) heating at $10^{14.5}$ cm, combine to give a bullet too tenuous for sufficiently rapid recombination. At the other extreme, as $\dot{M}_{w-6}/v_{w8}$ is *decreased*, the horizontal line moves up until it crosses the "lines too broad" constraint (eq. [12])—this is off the top of the scale on Figure 2 (cf. Fig. 1)—when $\dot{M}_{w-6}/v_{w8} = 10^{-2.9}$ (i.e., $\dot{M}_w \sim 4 \times 10^{-10} M_\odot$ yr^{-1}) with $M_B = 10^{26}$ g and $\theta = 10^{-1.5}$. For less dense winds, the jet angle has to be so large to provide heating that the lines are too broad. Thus very low density winds are possible, but they demand large bullet masses, comparable to those needed for radiative heating, and posing the same problems. Wind densities near the upper end of the solution range require more credible bullet masses and are comparable to those reported in similar W-R binaries by Poe et al. (1989).

The fact that conditions just sufficient for bullet heating to be observed optically with an adequate recombination rate, but without much deceleration, are so sensitive to wind and bullet parameters could well be a factor in explaining the rarity of the SS 433 optical line phenomena. Other SS 433's may exist where jet and wind conditions are such as to result in too little excitation or too much deceleration for the moving lines to be observed optically. Likewise, fairly small temporal variations in the SS 433 wind and jets could result in rather large changes in the optical emission, as observed.

4. MODIFICATION OF RESULTS FOR BULLET FRAGMENTATION AND ASPHERICAL GEOMETRY

Thus far we have considered primarily constraints from optical data and have for simplicity analyzed the case of bullets which are uniform and spherical. However, it is commonly proposed that the bullets must in reality be highly fragmented into small "pellets" with densities much higher than the mean density in, and filling only a small fraction of the overall bullet volume. In addition, X-ray data on the inner jets and radio data on the outer jets indicate that the emission sources (bullets) are in fact elongated. We consider here the implications of these two geometric modifications for our constraints on the global bullet parameters derived above, and discuss briefly some facets of how the optical bullet conditions relate physically to those in these other emission regimes.

Aside from the large-scale formation of "bullet" structures in the optical region (see § 1 and below), small-scale jet fragmentation has been suggested on theoretical grounds, such as thermal instability (Bodo et al. 1985; Brinkmann et al. 1988). Historically this was also invoked to help overcome one of the problems with the Lyman line-locking model (Milgrom 1979), namely, that a radiation flux sufficient to provide the acceleration power would ionize the jet material unless its recombination rate were greatly enhanced by density clumping. If a spherical bullet of radius R, mass M_B, comprises $\mathcal{N}$ pellets of radius r spread uniformly through its volume, the effects of fragmentation can be described in terms of the volume filling factor $\xi = \mathcal{N}(r/R)^3$ and the area covering factor $\eta = \mathcal{N}(r/R)^2$, interrelated by $\xi/\eta = r/R$ and $\xi/\eta^{3/2} = 1/\mathcal{N}^{1/2}$. It is clear that $\xi \le \eta$ (with both equal to unity for a uniform bullet) and that $\xi \le 1$, whereas η can take any value from 0 to ∞. It is also clear that if $\eta > 1$ the total radiative or collisional heating of the bullet is the same as in §§ 2 and 3, while if $\eta < 1$ the heating will be reduced. On its own this factor would shift the "insufficient heating" lines in Figures 1 and 2 in such a way as to further restrict, or even remove, the "allowed regime." On the other hand, the decrease of ξ will enhance recombination and so relax the "recombination too slow" constraint in both cases. We will henceforth restrict ourselves to the most favorable heating model regime $\eta > 1$, $\xi \le 1$, but note that this assumption further restricts the likelihood of the observed SS 433 conditions being achieved to those jets with sufficiently many fragments $\mathcal{N} = \eta^3/\xi^2 > 1/\xi^2$, where ξ is as assessed below. The net effect of introducing such fragmentation on our analysis is to replace $n^2 V$ by $n^2 V/\xi$ (where n is now the volumetric mean density), hence the addition of a term 0.5 log ξ to the right-hand side of equation (11).

As regards bullet elongation, the radio images of the jets at $D_R > 10^{15}$ cm (Hjellming & Johnston 1981, 1985) show that jet "bullets" are still considerably elongated at these distances, with a lower bound on the length/radius ratio $S_R > 10$ set by the instrument beamwidth. X-ray data on the inner jet (Watson et al. 1986; Stewart et al. 1987, 1989) show that the inner jet ($D_x \sim 10^{12}$ cm) must be emitted almost continuously, i.e., the bullets there are highly elongated, because the X-ray line and continuum emission was seen essentially whenever data were taken (Stewart et al. 1989; Brinkmann et al. 1990). The data were not such as to be inconsistent with some modulation of the inner jet conditions, which might lead to "discrete" optical bullet formulation, on time scales of the order of a day (Grandi 1981). (It must be emphasized that the X-ray source length $\sim 10^{12}$ cm found by Watson et al. is much shorter than the total length $\sim v_j \times 1$ day of such a bullet, but this is presumed to be because of the rapid cooling of the X-ray–emitting material.) However, these data certainly preclude the emission of the inner jet material in the form of single spheres at intervals ~ 1 day, since such objects would only be visible in X-rays for a fraction $\sim 10^{12}$ cm/10^{15} cm $\sim$ 100 s/1 day $\sim 10^{-3}$ of the time. Such spheres are also excluded for other reasons. First, they would be about (1 day/100 s)/$\theta \sim 10^5/\theta_{-2}$ times denser than for quasi-steady jet ejection, at the same mean mass-loss rate, and not compatible with the optically "thin" X-ray spectrum observed. Second, the implied *instantaneous* source power during their 10^{12} cm/$v_j \sim 100$ s acceleration would have to be 1000 times greater than the already huge mean $L_{\rm kin}$ discussed earlier. If the X-ray bullets are $\sim 10^{15}$ cm long, then these *cannot* each become spherical optical bullets of radius $D\theta \sim 10^{13}\theta_{-2}$ cm without matter redistribution along the jet at speeds comparable to v_j, totally inconsistent with observed moving line widths.

Since the evolution of bullet geometry with D is uncertain we will simply parameterize the elongation of the optical bullet by a "stretch factor" S equal to the optical bullet length/radius. (An upper bound to S is set by the fact that the differential radial velocity, with respect to the observer, between the ends of the radiating bullet due to source precession in period T should not violate the observed line broadening. However, this cannot exceed $\Delta\lambda/\Delta\lambda_{\rm max} = \Delta t/4T$, which is less than 0.02 for a bullet of duration $\Delta t = 1$ day.) Since the heating and deceleration processes we have discussed depend only on the bullet *column* density, the associated constraints will be unchanged by S, as will the line-width constraint. However, increase of S from 1 will reduce the bullet density by a factor S and so render more severe the recombination rate and forbidden-line constraints, i.e., will act in the opposite direction from increasing fragmentation (decreasing ξ). Combining the two effects, we find that equations (11) and (13) are then replaced by

$$\log M_{B25} - 1.5 \log \theta_{-2} - 1.5 \log D_{15} > 0.45 + 0.5 \log (S\xi) , \tag{28}$$

$$\log M_{B25} - 3 \log \dot{\theta}_{-2} - 3 \log D_{15} > -2.1 + \log (S\xi) . \tag{29}$$

As S is increased, for fixed ξ, these additional terms further restrict the permitted solution regime, which disappears entirely when either of the lines (28) and (29) crosses the bottom right-hand corner of the solution regimes in Figures 1 and 2.

This happens first for the recombination constraint, requiring the following conditions for solutions to exist with the above wind and radiation parameters and $D = 10^{14.5}$ cm:

$$\text{radiative heating:} \quad S\xi < 4 \times 10^4 , \tag{30}$$

$$\text{collisional heating:} \quad S\xi < 40 . \tag{31}$$

The largest possible value of S is when the bullets approach continuous jets so that their length l is $\sim D$, so that $S = l/R < D/R = 1/\theta$ and hence, by inequality (30), $\xi < 4 \times 10^4 \theta$ for the radiative case, and by inequality (31) $\xi < 40\theta$ for the collisional case. Figure 1 shows that no radiative solution requires $\theta < 10^{-2}$, so that even for near-continuous bullets the existence of a radiative solution only demands $\xi < 400$ which is always true. For the collisional case the corresponding results are $\theta < 10^{-3}$ and $\xi < 4 \times 10^{-2}$. Thus maximally elongated collisionally heated bullets are only possible if they are substantially fragmented.

Finally, it is important for future work to consider briefly the relationship between the physical conditions we find for the optical regime and those reported by Stewart et al. (1989) in the X-ray regime. First, we note that in a near-continuous jet of constant θ, the mean density should decline as D^{-2} or a drop of a factor of 10^{-5} between the X-ray and optical distances. For highly elongated bullets (whether radiatively or collisionally heated), we see from Figures 1 and 2 that the optical bullet mean density in our analysis is about 10^9 cm^{-3}, which would imply an X-ray source mean density of about 10^{14} cm^{-3}, only slightly higher than that found by Stewart et al. (1989) from the X-ray data. Second, the bullet masses we find are about 10^{24} g in the collisional case and about 10^{27} g in the radiative case, corresponding to $L_{\rm kin} = 3 \times 10^{38}$ ergs s^{-1} and 3×10^{41} ergs s^{-1}, respectively for bullet emission once per day. The former mass is a little lower and the latter much higher than the value ($10^{24.5}$ g) found from the emission measure results of Stewart et al. (1989), their value being an overestimate, since it assumed unit filling factor. Third, the jet opening angle θ would be expected to be of the order of v_t/v_j, where v_t is the proton thermal speed when the jet emerges from confinement. That is $\theta \sim 10^{-2}(T/10^8 \text{ K})^{1/2}$. The X-ray temperature varies very rapidly near the source, and its estimation is complicated by possible nonthermal emission, but it appears to be at least 10^8 K (Stewart et al. 1989). The corresponding $\theta = 10^{-2}$ is about what we obtained for *spherical* radiatively heated optical bullets but, even for $T = 10^9$ K, is well below that for radiatively heated elongated bullets (Fig. 1). On the other hand, it *is* consistent with the angles required in the collisionally heated case for very elongated optical bullets. (These comparisons are of course valid only if the jet angle does not change with distance due to the effect of the wind.)

Overall, therefore, we conclude that the various constraints we have applied from the optical data on SS 433 bullets are consistent with models in which the optical bullets are heated externally by collisions with the observed system wind, but only for a narrow range of bullet mass M_B and angle θ. Collisional heating satisfies the optical constraints only for M_B around 10^{24} g and θ in the range 10^{-2} to 10^{-3} radians, the higher value, together with fragmentation at filling factor $< 10^{-2}$, being required for consistency with the near-steady X-ray line data. These optical bullet parameters are also broadly consistent with the extrapolation of the X-ray line source parameters to large distances. Because of the small bullet angle, radiative heating by isotropic stellar radiation alone could work only for M_B values 100 times higher, for which the simultaneous heating by wind collisions would make the bullets glow for much too long, and which demand a much higher $L_{\rm kin}$. Anisotropic stellar radiation only becomes as effective as wind collisions if a power equal to $L_{\rm bol}$ is concentrated in about 1% of the sphere. The value of $L_{\rm kin}$ corresponding to a collisionally heated bullet once per day is about 3×10^{38} ergs s^{-1} only a little below the usual rough estimates based on the elongation of W50. One factor which would increase this is the inclusion of wind depletion beyond 10^{15} cm referred to in § 3, which would relax the "collisional heating lasts too long" constraint on M_B.

We gratefully acknowledge the financial support of this work by NASA grant NAGW-422 (J. P. C.), a University of Wisconsin Brittingham Visiting Professorship, the Nuffield Foundation, and the University of Glasgow (J. C. B.), and a UK SERC Research Grant (J. C. B.) and Visiting Fellowship (G. W. C. II).

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