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COSMIC-RAY TRANSPORT AND GAMMA-RAY EMISSION IN SUPERNOVA SHELLS

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ABSTRACT

We examine the mixing and transport of cosmic rays accelerated by a pulsar inside an expanding supernova remnant and the resulting high-energy γ-ray emission from nuclear interactions of these accelerated particles in the shell. Rayleigh-Taylor instability at the interface between a pulsar wind cavity and the inner supernova envelope is assumed to be the mixing mechanism. We apply our analysis to the model of Gaisser, Harding, & Stanev, where protons are accelerated at the reverse shock in the pulsar wind. We estimate the instability time scale from the dynamics of the pulsar wind cavity, and model the injection, diffusion, and interaction of protons in the shell. The resulting γ-ray flux is lower than previous estimates due to proton adiabatic losses in the expanding pulsar wind. We find that the protons mix and diffuse only into the innermost regions of the envelope before interacting. Energy-dependent diffusion causes the higher energy γ-ray light curves to decay faster than those at lower energy.

Subject headings: cosmic rays: general — gamma rays: general — nebulae: supernovae remnants — particle acceleration

1. INTRODUCTION

Although young supernova remnants were suggested some time ago as possible sources of high-energy γ-rays through the nuclear interaction of accelerated particles (Berezinsky & Prilutsky 1978; Sato 1977; Shapiro & Silberberg 1979), there has been renewed interest in these sources following the explosion of SN 1987A in the Large Magellanic Cloud. A number of experiments have attempted to detect a γ-ray or neutrino signal from SN 1987A over a wide range of energies (see Harding 1989 for a review). So far, there has been no steady signal reported, with derived upper limits on the proton luminosity as low as 10^{39} ergs s^{-1}. There has been one report by the JANZOS group (Bond et al. 1988) of a 2 day transient signal above 3 TeV in 1988 January, coincident with a soft X-ray flare (Makino et al. 1988a, b). Theoretical models for γ-ray emission from young supernovae have also recently been developed and refined (Gaisser, Harding, & Stanev 1987, 1989, hereafter GHS; Berezinsky & Ginzburg 1987; Yamada et al. 1987). These models assume that acceleration of protons results from the presence of a pulsar deep inside the remnant. Gamma rays and neutrinos will result from the decay of neutral and charged pions, produced when the protons interact with nuclei in the envelope.

Thus far, the models have made highly simplified assumptions about the transport and distribution of protons in the envelope in order to calculate γ-ray fluxes and light curves. For example, Yamada et al. (1987) assume that the accelerated protons are not confined at all and freely propagate through the envelope. The resulting light curve peaks and decays within a year after the explosion. On the other hand, GHS have assumed that protons are confined by magnetic fields in the envelope, in which case the light curves can peak as late as 6 years after the explosion. They also showed that the light curves were very sensitive to the assumed distribution of protons in the envelope and to the degree of mixing of protons with gas.

In this paper, we study in greater detail the transport of protons in an expanding supernova envelope, based on the model of GHS. In that model, particles are accelerated at the reverse shock which forms in the relativistic wind from the pulsar as a result of the confinement of the wind by the envelope. If the protons are confined by the high magnetic field in the low-density cavity surrounding the pulsar, then production of γ-rays through nuclear interactions requires that the cosmic rays mix with the gas in the envelope by diffusion or bulk motion. Rayleigh-Taylor instability at the interface between the inner envelope and the pulsar wind cavity has been suggested as a possible mixing mechanism by Berezinsky & Ginzburg (1987) and by GHS. We explore this idea in more detail to develop a description of the dynamics of the pulsar wind cavity and the proton injection, mixing and diffusion in the expanding envelope. In § 2, we discuss the interaction of the pulsar wind with the envelope, using the model developed by Reynolds & Chevalier (1984), and the resulting Rayleigh-Taylor growth length and time scale. From these results, we derive in § 3 the injection spectrum of accelerated protons at the wind-envelope interface. In § 4, we discuss the diffusion and interaction of protons in the shell and calculate the proton distribution and γ-ray production by means of a Monte Carlo code. We present γ-ray light curves in this model for the case of SN 1987A. Although the focus of our results are on SN 1987A, the ideas developed here are intended to have a general application.

2. DYNAMICS OF THE PULSAR WIND CAVITY

In the model developed by Reynolds & Chevalier (1984, hereafter RC) to describe the interaction of a pulsar with a

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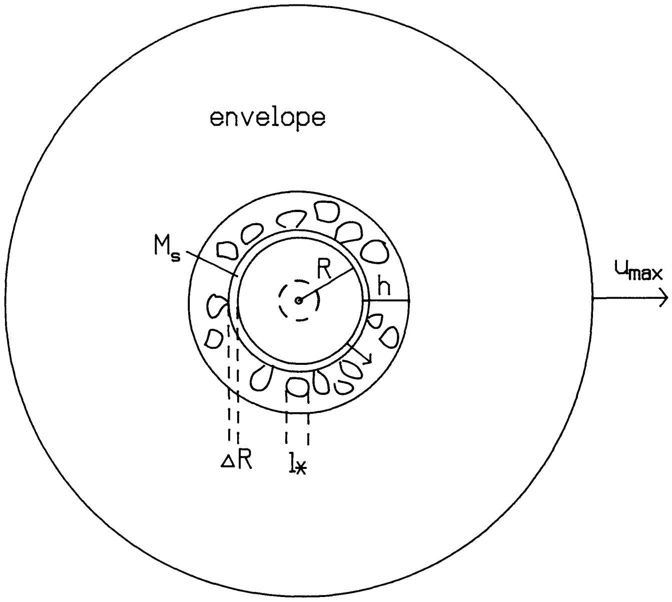

FIG. 1.—Expanding supernova shell containing pulsar wind cavity of radius R, which sweeps mass M_s from envelope into shell of width ΔR. Protons are accelerated at the pulsar wind shock (*dashed circle*). Bubbles of pulsar wind and cosmic rays of size l_* mix with material in the envelope in a shell of width h. (Not to scale).

supernova remnant, the pulsar wind blows a cavity around the pulsar by sweeping up material in the inner envelope. During phase I of the RC model, the pulsar wind is overtaking the material in the envelope and the swept-up mass shell increases (as we show in Fig. 1). If the density is assumed to be constant in the dense, slow-moving inner core of the envelope, the equations of momentum and mass of the shell, M_s, may be written,

$$M_s \ddot{R} = 4\pi R^2\{P - \rho_1[\dot{R} - v(R)]^2\} \tag{1}$$

$$\dot{M}_s = 4\pi R^2 \rho_1[\dot{R} - v(R)] , \tag{2}$$

with $\rho_1 = 3M_c/4\pi v_1^3 t^3$, where M_c and v_1 are the total mass and outer velocity of the slow-moving core material. With $v(R) = R/t$, the solution for M_s in phase I is $M_s = (4\pi/3)\rho_1 R^3$. The pressure inside the cavity, $P \simeq 3L_d t/4\pi R^3$, is just the accumulated energy density from the pulsar spin-down luminosity, L_d. RC find that the solution for the cavity radius during this phase is

$$R = \left(\frac{125}{99}\frac{v_1^3 L_d}{M_c}\right)^{1/5} t^{6/5} . \tag{3}$$

Phase I continues until all the slower moving material has been swept up or the pulsar energy output begins to decline. If the former occurs first, then phase II begins, during which the pulsar wind accelerates an expanding shell of constant mass. According to RC, Phase I ends at time

$$t_I \approx \frac{M_c v_1^2}{L_d} = 6.7 \times 10^3 \text{ yr} \left(\frac{M_c}{M_\odot}\right) v_8^2 L_{38}^{-1} , \tag{4}$$

where $v_8 = v_1/10^8$ cm s^{-1} and $L_{38} = L_d/10^{38}$ ergs s^{-1}.

Since the interactions of accelerated particles in the shell will only be important in the early evolution of the supernova remnant, when the density is high, we consider the phase I solution to be the most appropriate for the present calculation. Harding, Mastichiadis, & Protheroe (1990) investigated proton diffusion and γ-ray production for a supernova remnant in phase II of RC, which might apply to high-luminosity pulsars and a small core mass. In the case of SN 1987A, model 10HMM of Pinto & Woosley (1988) has a density profile that is nearly constant in the inner region of the envelope and falls off significantly in the outer regions. If we take this inner region to be the slow-moving core in the RC model, then $\rho_1 = 2.7 \times 10^{-14}$ g cm^{-3} $t_{\rm yr}^{-3}$, $v_1 = 1.25 \times 10^8$ cm s^{-1}, and $M_c = 3M_\odot$. This gives $t_I = 3 \times 10^4$ yr L_{38}^{-1} for the end of phase I, well beyond the epoch of nuclear interactions.

We assume, following GHS, that protons are accelerated at the pulsar wind shock which lies deep inside the pulsar wind cavity and are convected outward by the MHD flow of the shocked pulsar wind. After they have reached the outer edge of the cavity, the accelerated protons and the magnetic field that confines them may mix with gas in the envelope through Rayleigh-Taylor (RT) instabilities. The interface between the pulsar wind and inner envelope is unstable, because the pressure from the pulsar wind causes an effective outward acceleration and the envelope is much denser than the wind. Bubbles containing cosmic rays and tangled magnetic field will grow and penetrate the inner part of the envelope. Transport of these bubbles into the envelope together with leakage of neutral gas into the bubbles provide the mixing of cosmic rays and target material necessary for γ-ray production.

From the dispersion relation for waves at the interface between two fluids with a magnetic field in the plane of the interface (Chandrasekhar 1961), GHS find that the maximum growth rate occurs for bubbles of size l_*, given by

$$l_*^{-1} = k_* \approx \frac{2\pi g \rho_s}{B^2} , \tag{5}$$

where ρ_s is the matter density in the swept-up mass shell, B is the magnetic field in the pulsar wind cavity and g is the effective acceleration. The maximum RT growth time scale is

$$t_* \approx \frac{B}{g\sqrt{\pi\rho_s}} . \tag{6}$$

We can estimate g from the acceleration at the inner edge of the envelope in phase I of RC,

$$g \approx \ddot{R} = \frac{4\pi R^2 P}{M_s} \simeq \frac{L_d t}{R M_s} . \tag{7}$$

To estimate the density, ρ_s, we assume that the mass M_s is distributed over a shell of thickness ΔR,

$$\rho_s \approx \frac{M_s}{4\pi R^2 \Delta R} . \tag{8}$$

The density in the swept-up mass shell is therefore $\rho_s/\rho_1 = R/3\Delta R$. Following Chevalier (1977), we estimate the shell thickness ΔR by balancing the shell pressure, $P_s = (kT/\mu m_p)\rho_s$, with the pressure in the pulsar wind cavity, P, to give

$$\Delta R \simeq \frac{kT}{\mu m_p}\frac{M_s}{L_d t} R = 1.5 \times 10^{12} \text{ cm } t_{\rm yr}^{4/5} L_{38}^{-1/5} \times \left(\frac{T/\mu}{10^4 \text{ K}}\right)\left(\frac{M_c}{M_\odot}\right)^{1/5} v_8^{-3/5} , \tag{9}$$

where $\mu \approx 6$ is the mean atomic weight based on the composition of the model 10HMM inner envelope of Pinto & Woosley

(1988) and T is the temperature of the inner envelope. The density in the shell will not actually be constant but will decrease beyond $r = R$ with a scale height determined by the acceleration of the shell and its temperature. The scale height in this case is the same as the above expression for ΔR. We will assume that the density, $\rho = \rho_s$ is constant within ΔR.

Assuming equipartition between magnetic and particle energy density in the pulsar wind (i.e., $P = 2 \times B^2/8\pi$), as in GHS, to obtain an expression for the magnetic field in the cavity in terms of the pulsar luminosity,

$$B = \left(\frac{3L_d t}{R^3}\right)^{1/2} . \tag{10}$$

Equation (5) gives

$$l_* \approx 6\Delta R = 10^{13}\ \text{cm}\ t_{\text{yr}}^{4/5} L_{38}^{-1/5} \times \left(\frac{T/\mu}{10^4\ \text{K}}\right)\left(\frac{M_c}{M_\odot}\right)^{1/5} v_8^{-3/5} . \tag{11}$$

The maximum growth time scale is then

$$t_* \approx 0.2\ \text{yr}\ t_{\text{yr}}^{4/5} L_{38}^{-1/5}\left(\frac{T/\mu}{10^4\ \text{K}}\right)^{1/2}\left(\frac{M_c}{M_\odot}\right)^{1/5} v_8^{-3/5} . \tag{12}$$

Thus, we expect RT perturbations to become nonlinear in less than 1 yr.

Once the RT instability has reached the nonlinear phase, the bubbles will move out into the envelope at a speed

$$v_b \approx l_*/t_* = 1.7 \times 10^6\ \text{cm s}^{-1}\left(\frac{T/\mu}{10^4\ \text{K}}\right)^{1/2} \tag{13}$$

relative to the envelope material. The bubble formation rate can be estimated as the area of the cavity wall divided by the bubble area and growth time scale, $\dot{N}_b \approx 4\pi R^2/l_*^2 t_*$. The density of bubbles will therefore be $n_b \approx \dot{N}/4\pi R^2 v_b = l_*^{-3}$, so that they form in close-packed layers. Cosmic rays accelerated at the pulsar wind shock and trapped in the pulsar wind cavity will be convected out into the envelope at speed v_b in the bubbles to a distance

$$h = v_b t \approx 5 \times 10^{13}\ \text{cm}\ t_{\text{yr}}\left(\frac{T/\mu}{10^4\ \text{K}}\right)^{1/2} . \tag{14}$$

Gamma-ray production requires some degree of mixing of cosmic rays with material in the envelope. Since we expect the cosmic rays to be confined by the tangled magnetic fields inside the bubbles, mixing can occur only through the leakage of neutral material into the bubbles. We can estimate the time scale for leakage by writing the rate of mass flow into the bubbles as

$$\frac{dM}{dt} = 4\pi l_*^2 v_s(\rho_s - \rho_b) , \tag{15}$$

where v_s is the sound velocity and ρ_b is the density inside the bubbles. With $d\rho_b/dt = (3dM/dt)/(4\pi l_*^3)$, the above equation gives the leakage time scale

$$\tau \approx \frac{(\rho_s - \rho_b)}{\dot{\rho}_b} = \frac{l_*}{3v_s} \sim 8 \times 10^5\ \text{s}\ t_{\text{yr}}^{4/5} . \tag{16}$$

Thus the bubbles will rapidly fill with neutral matter from the envelope and mixing of cosmic rays and gas will occur for a distance h outside the pulsar wind-envelope interface. Since the outer edge of the envelope is $R_{\max} = 4.3 \times 10^{16}$ cm t_{yr} in the Pinto & Woosley (1988) model, $h/R_{\max} \ll 1$, and γ-ray production will take place in the innermost region of the envelope. Protons confined to the slowest moving, densest part of the envelope will have a higher nuclear interaction rate than protons distributed uniformly throughout the envelope (GHS), contributing to a higher γ-ray productivity. This must be balanced, however, with the larger attenuation of γ-rays coming from deep inside the envelope.

3. INJECTION SPECTRUM IN THE SHELL

The spectrum of cosmic rays inside the pulsar wind cavity, and thus of cosmic rays entering the envelope from the cavity as described above, will differ from that produced by shock acceleration due to adiabatic deceleration in the cavity and convection out of the cavity. Cosmic rays trapped inside the cavity will suffer adiabatic energy losses,

$$\frac{dE}{dt} = -\frac{6E}{5t} , \tag{17}$$

and will be convected out of the cavity at the bubble speed v_b. The factor of 6/5 in the adiabatic energy loss rate is due to the acceleration of the shell by the pulsar wind ($R \propto t^{6/5}$). The total number per unit energy at energy E leaving the cavity and entering the envelope at time t will then be

$$Q^{\text{shell}}(E, t) = 4\pi R^2 n(E, t) v_b , \tag{18}$$

where n is the cosmic ray number density inside the cavity which is taken to be

$$n(E, t) = \frac{N(E, t)}{(4/3)\pi R^3} . \tag{19}$$

Thus,

$$Q^{\text{shell}}(E, t) = p(t)N(E, t) \tag{20}$$

where

$$p(t) = 3v_b/R . \tag{21}$$

$N(E, t)$ will be determined by the rate of production of cosmic rays, by adiabatic energy losses and by the rate of convection out of the cavity into the envelope. $N(E, t)$ must therefore obey the partial differential equation

$$\frac{\partial N}{\partial t} = Q - p(t)N + \frac{\partial}{\partial E}\left(N\,\frac{6E}{5t}\right) , \tag{22}$$

where $Q(E, t)$ is the rate of production by the accelerator of cosmic rays of energy E per unit energy. We will assume that the accelerator switched on at time t_{start} and produced a spectrum of cosmic rays

$$Q(E, t) = q_0 E^{-2} , \tag{23}$$

typical of shock acceleration extending up to a maximum energy $E_{\max}$. For these assumptions it is possible to solve equation (22) analytically. We will do this adopting a heuristic approach.

We will initially restrict our attention to protons with energies in the range E_0 to $(E_0 + \Delta E_0)$ at a particular time t_0. To be in this energy range at time t_0 protons produced at an earlier time t would have had to be produced with energies in the range E to $(E + \Delta E)$ where adiabatic energy losses imply

$$E = \left(\frac{t_0}{t}\right)^{6/5} E_0 , \tag{24}$$

$$\Delta E = \left(\frac{t_0}{t}\right)^{6/5} \Delta E_0 . \tag{25}$$

Let us define the total number of cosmic rays inside the cavity in this range of energies which varies with time as

$$\mathcal{N}(t) = N(E, t)\Delta E = N\left[\left(\frac{t_0}{t}\right)^{6/5} E_0, t\right]\left(\frac{t_0}{t}\right)^{6/5} \Delta E_0 . \tag{26}$$

Then, we can write a first order linear differential equation for $\mathcal{N}$:

$$\frac{d\mathcal{N}}{dt} = Q(E, t)\Delta E - p(t)\mathcal{N} . \tag{27}$$

We can write this in standard form,

$$\frac{d\mathcal{N}}{dt} + p(t)\mathcal{N} = q(t) , \tag{28}$$

where, for the assumed convection rate of cosmic rays out of the cavity and the assumed production spectrum of cosmic rays we obtain

$$p(t) = 10^{-8} t^{-6/5} L_{38}^{-1/5}\left(\frac{T/\mu}{10^4\ \mathrm{K}}\right)^{1/2} \times \left(\frac{M_c}{M_\odot}\right)^{1/5} v_8^{-3/5}\ \mathrm{s}^{-1} = at^{-6/5} , \tag{29}$$

$$q(t) = q_0 E_0^{-2}\left(\frac{t}{t_0}\right)^{6/5} \Delta E_0 = bt^{6/5} . \tag{30}$$

The solution is

$$\mathcal{N}(t) = e^{5\ at^{-1/5}}\left(\int bt^{6/5} e^{-5\ at^{-1/5}} dt + c\right) . \tag{31}$$

Making a change of variable to

$$x = 5\ at^{-1/5} , \tag{32}$$

one obtains

$$\mathcal{N}(t) = e^x\left[5b(5a)^{11} \int x^{-12} e^{-x} dx + c\right] . \tag{33}$$

We can determine the constant c from the initial condition,

$$\mathcal{N}(t_{\mathrm{min}}) = 0 , \tag{34}$$

where

$$t_{\mathrm{min}} = \max\left[\left(\frac{E}{E_{\mathrm{max}}}\right)^{5/6} t, t_{\mathrm{start}}\right] \tag{35}$$

takes into account the maximum energy of protons, E_{max}, that the accelerator is capable of producing. From equations (23)–(25) and (30), we have

$$N(E, t) = \frac{\mathcal{N}(t)Q(E, t)}{bt^{6/5}} . \tag{36}$$

Therefore, assuming $t_{\mathrm{start}} = 0$, we have

$$N(E, t) = Q(E)t^{6/5} 5(5a)^{11} e^x \int_x^{x_{\mathrm{max}}} x'^{-12} e^{-x'} dx' , \tag{37}$$

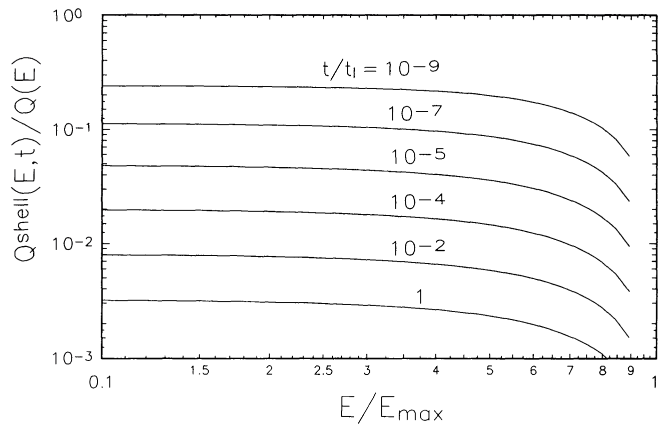

FIG. 2.—Ratio of rate of convection of cosmic rays into supernova envelope to rate of production by accelerator. Numbers attached to curves give the supernova age in units of the phase I time scale t_I.

where

$$x_{\mathrm{max}} = \left(\frac{E}{E_{\mathrm{max}}}\right)^{-1/6} x . \tag{38}$$

In terms of the time, t_I, at which phase I ends (eq. [4]), the dimensionless variable x is

$$x(t) = 0.24\left(\frac{t}{t_I}\right)^{-1/5} v_8^{-1}\left(\frac{T/\mu}{10^4\ \mathrm{K}}\right)^{1/2} . \tag{39}$$

In Figure 2 we plot the ratio of the rate at which cosmic rays of energy E are convected into the shell to the rate at which they are produced by the accelerator

$$\frac{Q^{\mathrm{shell}}(E, t)}{Q(E)} = x^{12} e^x \int_x^{x_{\mathrm{max}}} x'^{-12} e^{-x'} dx' , \tag{40}$$

assuming the accelerator turns on soon after neutron star formation, i.e., $t_{\mathrm{start}} \simeq 0$, and $L_{38} = 1$, $v_8 = 1.25$. In this case $t_I = 3 \times 10^4$ yr. We also take $\mu = 6$ from the inner shell composition in model 10HMM and $T = 5000$ K for $t \leq 700$ days, $T = 2000$ K for $t > 700$ days, as is indicated by observed infrared line ratios (Moseley et al. 1989). At 1–10 yr after the explosion, the spectrum of cosmic rays convected into the envelope is down by more than a factor of ~ 10 compared with that at production. Thus, our predictions will be significantly lower than those of GHS because of this lower rate of cosmic rays entering the envelope from the cavity.

4. TRANSPORT OF COSMIC RAYS IN THE SHELL

As mentioned, the cosmic rays accelerated at the pulsar wind shock will eventually be convected into the supernova envelope, being carried by bubbles of tangled magnetic field. These bubbles will fill rapidly with material from the envelope and will start coalescing. This will result, in effect, in the formation of a region of thickness h (eq. [14]) just outside the pulsar wind cavity containing cosmic rays, tangled magnetic fields and matter. One then expects high-energy interactions to occur there, leading to the production of energetic photons (either directly from $\pi^0 \to 2\gamma$, or indirectly from $\pi^\pm \to \mu^\pm \to e^\pm$ and consequent electron radiation). However, not all cosmic rays entering the envelope will interact promptly. Instead they will start diffusing and suffer adiabatic deceleration, and some will escape from the envelope without interacting. Therefore, in order to calculate the γ-ray flux we need to calculate first the interaction rate of the injected cosmic rays.

The thickness of the diffusion region is much less than the cavity radius, $h \ll R$, and so we need consider the diffusion in only one (the radial) direction. Thus, in order to calculate the cosmic-ray interaction rate we have written a Monte Carlo code that simulates the cosmic-ray diffusion by a one-dimensional random walk (Chandrasekhar 1943; Owens & Jokipii 1977a, b). At a particular SN age t_i we inject protons from the energy distribution given by relation (40). We take a time step Δt ($\ll t_i$) and we let the protons undergo a random walk in position with step

$$\Delta r_{\rm diff} = \pm (2K_p \Delta t)^{1/2} , \tag{41}$$

where K_p is the diffusion coefficient. The diffusion occurs only inside the shell of thickness h (eq. [14]). We assume that in this shell the magnetic field is still given by the equipartition value (eqn. [10]). Since the magnetic field is tangled in this region, we assume that protons will diffuse with the minimum diffusion coefficient

$$K_p = r_g c/3 , \tag{42}$$

where r_g is the proton gyroradius. Apart from the random walk, protons will be convected outward along with the SN ejecta. Thus we take

$$\Delta r_{\rm conv} = v(r)\Delta t , \tag{43}$$

where v is the velocity of the homologous expansion of the SN envelope. We calculate the grammage Λ traversed by the proton in Δt

$$\Lambda = c \Delta t \rho(r, t_i) , \tag{44}$$

where $\rho(r, t_i)$ is the matter density of the envelope at radius r and time t_i, and we allow the possibility of the cosmic ray interacting with probability $P_{\rm int} = (1 - e^{\Lambda/\Lambda_{\rm int}})$, $\Lambda_{\rm int}$ being the mean interaction length of protons in the envelope material. We chose Δt such that $P_{\rm int} = 0.1$. We have assumed that the density in the region between R and $R + \Delta R$ (i.e., inside the swept-up mass region) is given by relation (8), while from $R + \Delta R$ up to $R + h$ it is given by the 10HMM model of Pinto & Woosley (1988). Furthermore, for the proton-nucleus interaction cross sections used to calculate $\Lambda_{\rm int}$, we follow GHS and use the results by Denisov et al. (1972).

If the proton interacts within Δt, then the energy of the cosmic ray and the time of the interaction is tabulated, and a new energy is given to the cosmic ray sampled uniformly between 0 and the energy before interaction (mean inelasticity = 0.5). If the proton does not interact, then its energy is appropriately reduced to account for adiabatic losses. We use Δt to increment the SN age, i.e., $t = t_i + \Delta t$, and time dependent quantities such as the matter density ρ, the magnetic field B and the shell thickness h, are evolved. We repeat the whole procedure until the proton either escapes or its energy becomes less than some minimum energy. In order to reproduce the net outward flow of cosmic rays implied by equation (20), particles diffusing inward into the cavity are reflected back out by a perfectly reflecting boundary placed at the cavity radius. Protons are assumed to escape freely from the region on reaching the outer boundary $r_{\rm out}$, (where $r_{\rm out} = R + h$).

The interaction rate for the injected cosmic ray spectrum at time t_i is a two-dimensional distribution in energy and time, i.e., the cosmic-ray interaction rate is of the form $R(\gamma, t; t_i)$ where obviously $t > t_i$. Since the interaction rate at a particular SN epoch t_0 depends on contributions from cosmic-ray injections at earlier epochs, the interaction rate at epoch t_0 will be given by

$$\mathscr{R}(\gamma, t_0) = \int_{t_{\rm start}}^{t_0} dt_i \, R(\gamma, t_0; t_i) , \tag{45}$$

where $t_{\rm start}$ is the earliest epoch of cosmic-ray injection. In practice we have replaced the above integration with a sum. We binned energy in logarithmic intervals of width 0.25 and time in logarithmic intervals of width 0.15. The need to use logarithmic intervals for the time is justified by the fact that the cosmic-ray interaction rate depends on the density of the ejecta which in turn has a steep dependence on time ($\propto t^{-3}$). The cosmic-ray interaction rate $\mathscr{R}(\gamma, t_0)$ is then used to calculate the γ-ray and neutrino spectrum at each SN epoch.

The production spectra of γ-rays, neutrinos, and electrons were obtained by numerical integration over the energy spectrum of interacting protons (see, e.g., Stecker 1971 for the method of calculating γ-ray spectra and, e.g., Dermer 1986 for electron spectra). We have used an energy-dependent nucleon-nucleon inelastic cross section and fits to the inclusive cross section for pion production originally based on ISR data and lower energy data (Gaisser et al. 1978; Protheroe 1982) which have been updated to agree with more recent data at higher energies. The production spectrum of γ-rays was obtained from the π^0 spectrum taking account of the simple two-body decay process $\pi^0 \rightarrow 2\gamma$ (see Stecker 1971 for kinematics of two- and three-body decays). The production spectra of electrons and neutrinos were obtained by treating the charged pion and muon decays (two- and three-body, respectively). Because of spin effects the muon decay is not isotropic in the muon rest frame (see, e.g., Dermer 1986). The effect on the electron and neutrino energy spectra is small in our case, and we have taken this into account in an approximate way.

Absorption of the γ-rays by pair production with matter in the envelope was included, as in GHS, using the density and composition distribution of model 10HMM of Pinto & Woosley (1988). Photon absorption of γ-rays by radiation is important only at early epochs. Following Protheroe (1987), we find that IR radiation provides enough opacity for the total absorption of 1 TeV γ-rays emitted from the inner part of the shell only for $t < 200$ days, after which they can escape essentially unabsorbed by photon-photon collisions.

The γ-ray light curves from SN 1987A for proton luminosities $L_p = 10^{38}$ and 10^{40} ergs s^{-1} are plotted in Figures 3 and 4. The light curves were calculated including contributions from both π^0 decay and radiation by electrons assuming the accelerator outputs luminosity L_p in protons distributed over 8 decades up to 10^{17} eV. Electrons with energies above ~ 0.1 TeV rapidly lose their energy by synchrotron radiation in the local magnetic field (eq. [10]) and there is no significant contribution of synchrotron photons to the γ-ray flux at either 1 GeV or 1 TeV. For electrons with energies below ~ 0.1 TeV bremsstrahlung energy losses dominate and these electrons contribute approximately the same flux to the 1 GeV γ-ray light curve as π^0 decay. For comparison, we also show in Figures 3 and 4 the γ-ray light curves that would result if the cosmic rays convected into the envelope interacted immediately and lost all their energy to pion production (*dashed curves*). The γ-ray flux does not increase linearly with L_p due to the dependence of the

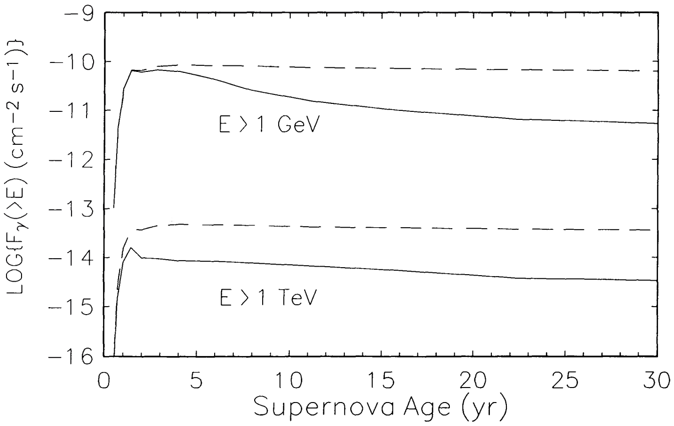

FIG. 3.—GeV and TeV γ-ray light curves for SN 1987A, assuming a proton luminosity of 10^{38} ergs s^{-1}, showing results of a full treatment (*solid curve*) and assuming all cosmic rays entering envelope interact immediately (*dashed curve*).

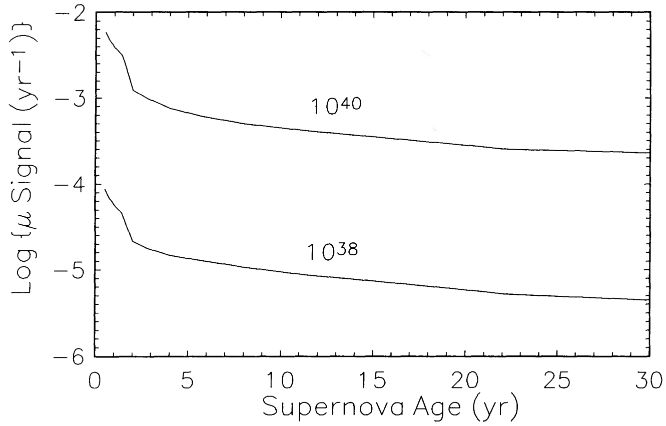

FIG. 5.—The rate of upward-going muons with energies above 2 GeV induced by neutrinos from SN 1987A plotted against supernova age for a detector of area 1000 m^2. Curves are labeled with proton luminosity.

proton injection rate into the shell on the pulsar luminosity through equations (29) and (39). As illustrated in Figure 2, the injection rate generally scales with t/t_I, the ratio of supernova age to the Phase I time scale (the time during which the pulsar wind accelerates the shell). As the pulsar luminosity increases, t_I decreases, and the proton injection rate relative to the accelerator output at a particular supernova age decreases, but more slowly than the accelerator output increases (we have assumed $L_p \approx L_d$).

The signal due to muons above 2 GeV expected in a neutrino telescope is given by

$$S_\nu(t) = \int_0^\infty P(E_\mu > 2 \text{ GeV}, E_{\nu_\mu}) F(E_{\nu_\mu}, t) dE_{\nu_\mu} , \quad (46)$$

where $P(E_\mu > 2$ GeV, $E_{\nu_\mu})$ is the probability that a neutrino of energy E_{ν_μ} produces an upward going muon of energy >2 GeV which passes through the detector (see, e.g., Gaisser & Stanev 1985) and $F(E_{\nu_\mu}, t)$ is the differential flux of neutrinos at time t. The signal calculated in this way for SN 1987A is plotted in Figure 5 and is clearly not detectable with the current neutrino telescopes. Even if $L_p = 10^{40}$ ergs s^{-1} DUMAND, with a proposed collecting area of $\sim 10^5$ m^2 (Learned 1986), would have detected only ~ 0.7 muons yr^{-1} in the first year.

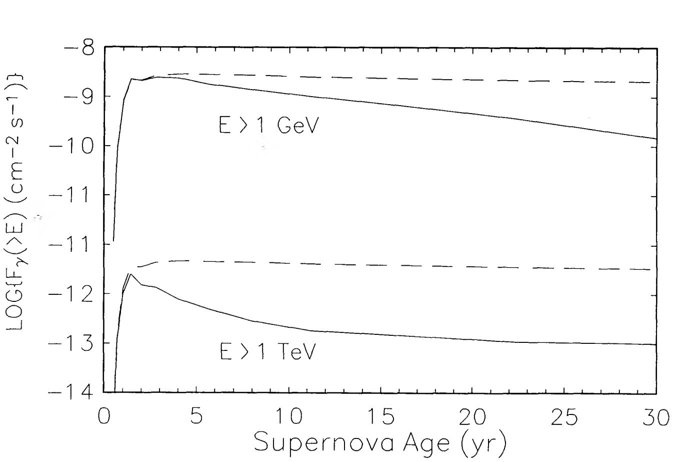

FIG. 4.—Same as Fig. 3 for a proton luminosity of 10^{40} ergs s^{-1}

5. CONCLUSIONS

We have investigated the mixing and transport of cosmic rays accelerated inside an expanding supernova remnant in greater detail than previous studies. In particular, we have developed a model of diffusion and interaction of accelerated protons in the shell, based on the picture that the accelerated protons are injected into the shell by the formation of cosmic-ray bubbles through the Rayleigh-Taylor instability. We find that the rate of proton injection into the shell is significantly lower than the rate of production by the accelerator. This is due to adiabatic losses of the accelerated protons as they diffuse out of the pulsar wind cavity to the inner edge of the shell and to the rate of convection of protons into the shell via the instability. Neither of these effects were considered in previous calculations. As a result, the peak γ-ray flux predicted by our model is at least an order of magnitude lower than that predicted by GHS at 1 TeV.

We find that mixing of cosmic rays does not extend far into the shell so that the protons diffuse and interact only in the inner regions of the expanding envelope. Furthermore, we find that the γ-ray light curves are energy dependent, primarily because protons of higher energy diffuse further out into the shell before interacting. The high-energy protons therefore interact in regions of lower density and higher expansion velocity, so that the interaction rate drops below the expansion rate sooner. The high-energy light curves thus decay faster. In addition electron bremsstrahlung contributes much more to the flux at 1 GeV than at higher energies.

The sensitivity of the current ground-based Cherenkov telescopes to a steady signal above 1 TeV is $\sim 10^{-11}$ photons cm^{-2} s^{-1}. This is well above our predicted flux in Figures 3 and 4 from SN 1987A, even assuming a proton luminosity of 10^{40} ergs s^{-1}. The predicted peak flux above 1 GeV is somewhat closer to the sensitivity limit of the EGRET telescope on GRO, which is 5×10^{-8} photons cm^{-2} s^{-1} above 100 MeV, but would still be undetectable. The bolometric light curve of SN 1987A has showed a slow-down in decline, possibly a leveling off, starting around day 800 (Boucher, Danziger, & Lucy 1991). Even if this effect is completely due to energy input by a pulsar, its luminosity could not be much above 10^{38} ergs s^{-1} (Nomoto et al. 1991). The prospects for detection of SN 1987A do not seem as optimistic as in earlier models. However, depar-

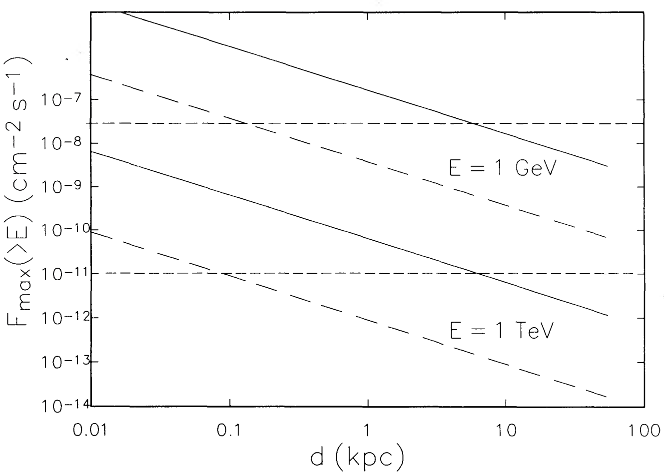

FIG. 6.—Peak γ-ray flux above 1 GeV and 1 TeV from a supernova at distance d for proton luminosity 10^{38} ergs s^{-1} (*dashed lines*) and 10^{40} ergs s^{-1} (*solid lines*). The horizontal dashed lines indicate the ground-based telescope flux limit above 1 TeV and the EGRET sensitivity above 1 GeV.

tures from spherical symmetry, which seem to be suggested by X-ray and γ-ray observations of radioactive decay lines (Dotani et al. 1987; Matz et al. 1988; Gehrels, Leventhal, & MacCallum 1988), could increase the rate of injection of protons into the envelope and increase the high-energy γ-ray flux over what we have predicted. Also, a proton acceleration spectrum that is steeper than the assumed index of 2 could increase the predicted flux around 100 MeV (see GHS), since a greater fraction of the energy would appear in the pion-decay peak.

It may be possible to detect high-energy γ-ray emission from proton acceleration in supernovae occurring in our own Galaxy. Figure 6 shows the predicted peak flux above 1 TeV and 1 GeV as a function of distance from the supernova compared to the flux sensitivity of ground-based Cherenkov telescopes and of EGRET. Supernova remnants with proton luminosity around 10^{40} ergs s^{-1} could be detected within 10 kpc, but those with proton luminosity of 10^{38} ergs s^{-1} could only be detected within ~ 100 pc.

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