THE ASTROPHYSICAL JOURNAL, 378:157–162, 1991 September 1

REINVESTIGATION OF THE BINARY FREQUENCY IN THE OPEN CLUSTER IC 4665

NIDIA MORRELL[1] AND HELMUT A. ABT
Kitt Peak National Observatory, National Optical Astronomy Observatories,[2] Box 26732, Tucson, AZ 85726
Received 1990 December 27; accepted 1991 March 5

ABSTRACT

We measured the radial velocities of 15 bright B3–A2 stars in the cluster IC 4665, using a CCD. Only four of the 15 stars are spectroscopic binaries, giving a binary frequency of 27%. Four of the six sharp-lined stars ($V \sin i \leq 50$ km s^{-1}) are binaries, and none of the more rapidly rotating stars are binaries with $K \geq 10$ km s^{-1}. Statistically, nearly all of the stars with low projected rotational velocities are slow rotators, and most are binaries; two of the four binaries may rotate synchronously. In two of the four binaries, the mass ratios are near 1.0, as one would expect for binary formation in three-body interactions after many crossing times. The measured cluster velocity dispersion is only 1.6 km s^{-1}, but most of that is still probably due to measuring errors.

Subject headings: clusters: open — stars: binaries

1. INTRODUCTION

We are continuing to explore the frequencies of spectroscopic binaries and the mean stellar rotational velocities in open clusters to understand whether there are real differences in binary frequencies (and in mean rotational velocities) between various clusters or whether the observed differences are due to small-number statistics. If the former is true, how does the formation of binaries depend upon cluster parameters and age? And again if the former is true, is there a correlation between these two quantities, as one would expect from tidal breaking or a partition of angular momentum between orbital and rotational motions?

There have been disagreements (e.g., Abt & Sanders 1973; Crampton, Hill, & Fisher 1976) about the validity of some measurements and the reality of an inverse correlation between binary frequency and rotation, so obviously better measurements are needed. But it seems relevant that the two clusters, Pleiades and α Persei, that are known to have extremely large mean rotational velocities (Kraft 1967) are probably deficient in binaries among their B stars relative to other clusters. The Pleiades (Abt et al. 1965) definitely lacks short-period binaries (zero out of 15 B stars) and the data for the α Persei cluster are incomplete, but Heard & Petrie (1967) found only two possible velocity variables (at a 1% confidence level) out of 65 B- and A-type stars.

We have been observing with a CCD the α Persei stars to improve and complete the Petrie & Heard (1969) study, and we wanted to observe a cluster of a similar age but with normal rotational velocities as a control. IC 4665 was selected for that purpose. In retrospect, it may have been optimistic to select only one cluster because of small-number statistics, so we will not draw any far-reaching conclusions in this paper, but merely present the results for this cluster.

The question of the binary frequency in IC 4665 has had an erratic history. On the basis of a few low-dispersion spectra per star, R. J. Trumpler (unpublished) and Abt & Snowden (1964) suspected that many of the cluster members are spectroscopic binaries. The estimates were 85% (Trumpler) and 63% (Abt and Snowden). Then Abt, Bolton, & Levy (1972) concluded that probably all of the brighter stars are spectroscopic binaries. Crampton et al. (1976) could not confirm that. They discovered that the Abt et al. observations during two observing runs deviated systematically. The difficulty was that Abt et al. were using a new spectrograph still in the process of alignment and they were unable to observe (bright) standard stars to monitor the changes. Crampton et al. concluded that six or seven of the 13 brightest stars are probable binaries, giving a frequency of about 50%. Only one single-lined binary (Kopff 49) has a large velocity amplitude, and both studies (Abt et al.; Crampton et al.) agree on its period of 10.53 days.

Because the B stars, at least, in IC 4665 have low rotational velocities (Abt & Chaffee 1967), they should provide a good control for a study of the α Persei cluster, and we should be able to obtain accurate radial velocities for them. For the B4–B8 V stars with $-1.5 \leq M_V \leq 0.0$, IC 4665 has $\langle V \sin i \rangle = 101 \pm 34$ (s.e. of the mean) km s^{-1}, while α Persei has 229 ± 36 km s^{-1} for such stars. Perhaps more striking are the results that 62% of these IC 4665 stars have $V \sin i \leq 50$ km s^{-1}, while only 9% of the α Persei stars have such low values.

IC 4665 has stars numbered by Kopff (1947) and a proper-motion study for membership by Vasilevskis (1955). It is relatively far ($b = 17°$) from the Galactic plane and is at a distance of 330 pc (Johnson 1957). Spectral types have been given by Abt & Levato (1975); the brightest star is a B3 IV. The cluster age is about 50×10^6 yr.

2. OBSERVATIONS

We observed 15 of the brightest cluster members. Not included were Kopf 23A and 23B (=ADS 10741), a 1″6 binary that could not always be observed as separate components. The observed stars have $V = 6.9$–9.3 mag. The stars are listed in Table 1 in order of decreasing brightness. The columns give the Kopff number, Henry Draper number, projected rotational velocities ($V \sin i$), spectral type, number of observations (n), the mean radial velocities (gamma velocities are given for the binaries), standard errors per observation (for the primaries in the SB2s), and conclusions about variability.

[1] Member of the Consejo National de Investigaciones Cientificas y Técnicas (CONICET) of the Argentine Republic; CONICET Fellow at KPNO, on leave from the Facultad de Ciencias Astrónomicas y Geofisicas de la Universidad Nacional de La Plata, Argentina.

[2] Operated by the Association of Universities for Research in Astronomy, Inc., under contract with the National Science Foundation.

TABLE 1

OBSERVING LIST

Kopff No.	HD	V sin i (km s^{-1})	Spectral Type	n	<ρ> (km s^{-1})	σ	Conclusions
62	161573	50	B3 IV	21	-13.8	±2.4	Constant velocity
73	161677	210	B5 IV	14	-11.0	±4.0	Constant velocity
64	161603	220	B5 IV	15	-13.3	2.6	Constant velocity
105	162028	30,10	B6 V	30	-12.7	59.3	SB2, P = $6^{d}.2013$
58	161572	200	B6 V	12	-13.8	3.0	Constant velocity
49	161480	25	B6 Vp(CII)	18	-11.8	31.3	SB1, P = $10^{d}.5302$
72	161660	35	B7 V	22	-11.	8.1	SB1, P = $10^{d}.79$
82	161733	40	B6 Vp(CII)	12	-11.9	2.7	Constant velocity
76AB	161698	80	B8.5 Vp(Hg,Mn)	12	-11.0	3.2	Constant velocity
32	161261	350	B8 V+shell	11	-12.9	1.2	Constant velocity
22	161165	240	B8.5 V	13	-14.4	3.8	Constant velocity
81	161734	225	B8 V	12	-10.7	3.9	Constant velocity
43	161426	185	A1 V	11	-14.6	2.7	Constant velocity
50	161481	≤50	A1 V+A2 V	17	-11.3	44.8	SB2, P = $11^{d}.4150$
102	161940	90	A2 V	10	-16.2	±1.7	Constant velocity
Mean		15			-12.7		

TABLE 2

RADIAL VELOCITIES OF IC 4665

Helio. JD 2440000+	ρ (km s^{-1})	Helio. JD 2440000+	ρ (km s^{-1})	Helio. JD 2440000+	ρ (km s^{-1})	Helio. JD 2440000+	(km s^{-1})
Kopff 22		8049.908	-10.9	7694.939	-47.6	Kopff 62	
7678.841	-14.2	8127.732	-14.9		+17.7	7678.668	-12.8
7679.798	-15.1	8128.734	-15.2	7695.908	-12.3	7678.682	-11.5
7680.818	- 9.1	8129.728	-11.1	7705.852	-58.8	7679.680	-11.2
7681.802	-12.3	8131.757	-15.3		+41.9	7680.682	-19.7
7694.851	-15.4	8179.585	-11.2	7765.727	+26.7	7681.662	-14.5
7695.845	-16.2				-64.9	7682.676	-15.3
7705.818	-12.5	Kopff 49		7766.723	+43.5	7694.677	-11.6
8048.896	- 9.3	7678.733	-17.5		-80.1	7695.662	-17.9
8127.800	-20.3	7679.739	+ 9.0	7994.996	+51.5	7696.660	-12.9
8129.796	-22.4	7680.753	+33.7		-75.7	7704.726	-11.8
8131.724	-14.5	7681.731	+38.9	7995.965	+50.0	7704.744	-13.8
8131.807	-11.8	7682.794	+24.8		-77.8	7765.758	-11.6
8177.598	-14.6	7694.748	-13.5	7996.980	+33.0	7766.752	-15.0
		7695.732	-40.6		-57.3	7992.856	-13.3
Kopff 32		7696.742	-60.6	8048.925	-29.3	7993.831	-15.3
7678.791	-14.7	7704.939	- 2.6		+20.3	7994.785	-11.2
7679.776	-12.8	7705.974	-29.4	8127.695	-59.7	7995.774	-14.8
7680.789	-11.8	7765.676	+36.8		+58.1	7996.774	-10.5
7681.774	-13.1	7766.673	+29.7	8128.696	-32.5	8048.667	-15.0
7694.821	-13.3	7993.875	-34.0		+27.1	8128.807	-16.1
7695.819	-11.1	7994.918	-13.6	8129.682	- 8.1	8130.635	-14.6
7705.772	-13.7	7995.877	+21.0				
8048.874	-14.7	7996.884	+34.0	Kopff 58		Kopff 64	
8049.848	-12.7	8048.786	+21.8	7678.722	-12.0	7678.693	- 8.6
8177.631	-11.2	8049.794	+35.1	7679.728	-10.4	7679.704	-17.1
8178.610	-12.3			7680.743	-13.5	7680.721	-14.7
		Kopff 50		7681.726	-13.7	7681.685	-10.7
Kopff 39		7678.989	-16.0	7682.982	-19.0	7682.708	-14.7
7679.945	-14.1	7679.904	-47.9	7694.735	-15.1	7694.704	-13.3
			+24.1	7695.720	-10.9	7695.687	-13.1
Kopff 43		7680.932	-76.1	7696.727	-11.7	7696.689	-10.8
7678.926	-16.6		+54.9	7704.922	-16.1	7704.798	- 9.9
7679.878	-17.2	7681.920	-75.4	8048.767	-15.9	7992.928	-17.7
7680.903	-13.5		+58.9	8049.766	- 9.7	7994.817	-15.9
7681.890	-16.4	7682.911	-62.1	8129.752	-17.5	7995.805	-14.3
7695.942	-18.8		+41.9				

TABLE 2—*Continued*

Helio. JD 2440000+	ρ (km s^{-1})	Helio. JD 2440000+	ρ (km s^{-1})	Helio. JD 2440000+	ρ (km s^{-1})	Helio. JD 2440000+	ρ (km s^{-1})
7996.811	-13.0	8049.742	- 9.8	7694.793	- 8.9	7765.662	+68.0
8048.717	-12.3	8129.704	-15.9	7695.758	-10.8		-95.4
8179.659	-12.7	8177.680	-16.0	7696.773	-10.9	7766.655	+30.0
		8178.670	-16.1	7705.724	-14.7		-49.9
Kopff 72				7765.770	-14.0	7952.961	+15.0
7678.741	- 7.0	Kopff 76		7766.762	-11.1		-39.9
7679.745	- 1.9	7678.770	- 9.7	8048.833	- 7.2	7992.988	-82.5
7680.762	+ 5.8	7679.762	-11.8				+56.7
7681.738	+ 0.8	7680.776	- 9.4	Kopff 102		7993.957	-14.5
7682.812	- 2.9	7681.758	- 9.7	7679.924	-16.8	7994.872	+65.3
7694.756	-10.5	7682.942	-15.6	7680.950	-16.6		-94.0
7695.747	-17.9	7694.805	-12.5	7681.937	-15.5	7995.849	+43.7
7696.760	-29.0	7695.772	-10.0	7682.960	-19.6		-77.4
7704.961	- 3.2	7696.786	- 8.0	7694.962	-17.0	7996.856	-10.9
7765.695	- 3.1	7705.742	-16.6	7695.970	-15.0	8048.744	-87.5
7766.689	+ 5.5	7765.787	-13.9	7705.926	-15.3		+62.4
7993.999	- 2.4	7766.782	- 6.3	8127.664	-17.7	8048.950	-82.8
7994.970	+ 1.3	8048.850	- 8.1	8128.668	-14.1		+56.4
7995.927	- 3.0			8129.646	-14.4	8049.696	- 7.9
7996.934	- 8.7	Kopff 81				8049.959	+17.4
8048.801	+ 2.9	7678.878	- 9.3	Kopff 105			-38.2
8049.817	- 0.8	7679.841	- 8.7	7678.709	+62.9	8050.969	+69.8
8131.784	- 6.3	7680.849	- 8.1		-103.9		-88.2
8177.656	- 5.7	7681.837	- 7.0	7679.717	+ 3.7	8127.636	-35.9
8178.584	+ 3.1	7694.914	- 9.8		-73.3		+17.0
8224.552	-21.6	7695.880	-11.6	7680.732	-15.7	8127.827	-43.9
8225.560	- 2.3	7705.894	- 3.5	7681.697	-91.6		+24.3
		8127.770	-11.4		+42.5	8128.648	-74.7
Kopff 73		8128.774	-12.6	7682.757	-89.3		+61.0
7678.674	- 5.1	8129.777	-16.1		+65.7	8128.826	-88.0
7679.690	-12.9	8178.643	-17.9	7694.723	-96.7		+63.3
7680.704	-13.8	8179.624	-12.5		+68.6	8129.621	-77.1
7681.672	- 4.6			7695.706	-69.8		+56.8
7682.688	- 9.4	Kopff 82			+36.1	8129.812	-63.4
7694.689	-10.1	7678.754	-14.4	7696.710	+43.3		+45.0
7695.672	-13.5	7679.751	-16.0		-76.4	8131.696	+69.0
7696.673	- 5.7	7680.769	-13.2	7705.708	-15.8		-83.6
7704.776	- 9.2	7681.746	-11.9	7765.646	+69.0		
8048.688	-13.0	7682.879	- 9.2		-95.4		

The stars were observed with the Kitt Peak 0.9 m coudé feed telescope and coudé spectrograph. A grating-camera (No. 5) combination gives 15 Å mm^{-1} reciprocal dispersion. A Texas Instruments 800 × 800 pixel CCD chip allows observations of Hγ, λ4471 He I, and λ4481 Mg II at one time. We used the former line in broad-lined spectra and the latter two lines for sharp-lined spectra. We generally exposed to a S/N of 100; the resolution was 0.22 Å pixel^{-1}. Cross-correlations were made against velocity standards (HR 6031, 6035, 6092, 6603, 6787, 7287, 7426, and 7512) and an Fe-Ar hollow-cathode discharge tube was used to monitor changes. Initially, we used a slit aperture but later shifted to a fiber-optics scrambler, which reduced the guiding errors.

The velocities are listed in Table 2, which gives for each Kopff number the heliocentric Julian Date and radial velocity. Because only one to three lines per spectrum were used and because they were analyzed as a group, no internal standard errors are derived.

3. DISCUSSION AND CONCLUSIONS

Without values of the internal standard errors, we cannot apply the E/I (external to internal mean errors) test for variability (see Crampton et al. 1976). Instead we plotted in Figure 1 the standard errors as a function of rotational line broadening, $V \sin i$. It shows four stars (Kopff 105, 49, 72, and 50) with standard errors of 8–59 km s^{-1} and 11 stars with errors of 1.2–4.0 km s^{-1}. The former are spectroscopic binaries, and we derived orbital elements for them. The latter are probably constant-velocity stars; they have a mean standard error of 2.8 km s^{-1}. But without the low point at 350 km s^{-1} (Kopff 32, a shell spectrum with sharp hydrogen cores that yield accurate velocities), the remaining show a (more realistic) linear variation from 2.1 km s^{-1} at $V \sin i = 0$ to 4.1 km s^{-1} at $V \sin i = 350$ km s^{-1}.

All four binaries have $V \sin i \leq 50$ km s^{-1}. If similar binaries existed among the broad-lined stars, they would easily have been discovered with our accuracy. Thus we have the first result, namely that four of the six stars with $V \sin i \leq 50$ km s^{-1} are binaries, while none of those with $V \sin i > 50$ km s^{-1} are binaries with large velocity amplitudes (above a limiting $K \geq 10$ km s^{-1}). If we had 14 stars with $V = 200$ km s^{-1} and axes distributed randomly, only 0.4 star would have a projected $V \sin i \leq 50$ km s^{-1} and 1.9 stars would have $V \sin i \leq 100$ km s^{-1}. Instead we observe six and eight stars, respectively, with those projected rotational velocities. Therefore most of the stars with $V \sin i \leq 100$ km s^{-1} actually have $V \leq 100$ km s^{-1} and are genuine slow rotators. Therefore, the bulk of the

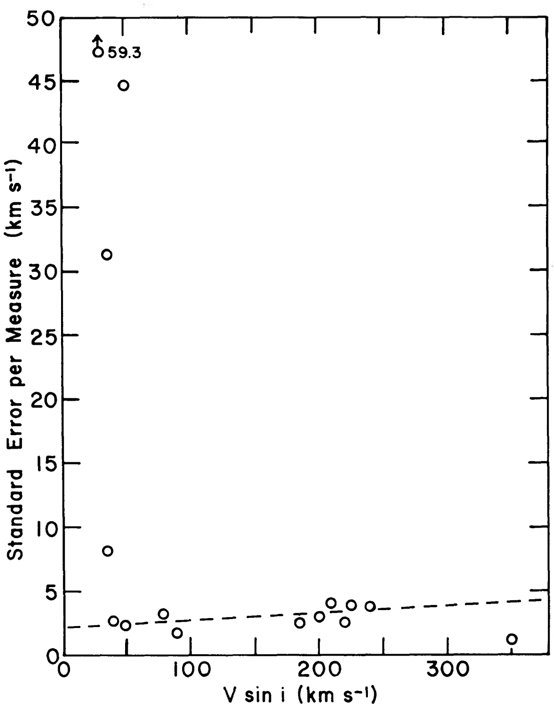

FIG. 1.—For each of the 15 stars measured in IC 4665, we plot the velocity dispersion against the projected rotational velocity. The four known binaries have $\sigma = 8$–59 km s^{-1}; the apparently constant-velocity stars have $\sigma = 1.2$–4.0 km s^{-1}. The dashed line indicates the mean measuring error, $\sigma = 2.1 + 0.0057$ ($V \sin i$), ignoring the shell star at $V \sin i = 350$ km s^{-1}.

slow rotators are in spectroscopic binaries, and none of the rapid rotators are in short-period binaries.

The mean cluster velocity is -12.7 km s^{-1} with a dispersion of 1.6 km s^{-1} per star. Because the true velocity dispersion is probably less than 1 km s^{-1}, the bulk of the dispersion is still instrumental.

The orbital elements are listed in Table 3 and the velocity curves are shown in Figures 2–5. The elements were derived from period searches and multiple least-squares solutions for the elements. Most of the rows in Table 3 are self-explanatory. The last row gives the mean observed minus computed ($O-C$) velocities. For the SB1's, they agree well with the velocity dispersions for the constant-velocity stars, namely in the range 1.2–4.1 km s^{-1}; for the SB2's, they are about twice as high, which is understandable for double lines that are partly blended at times.

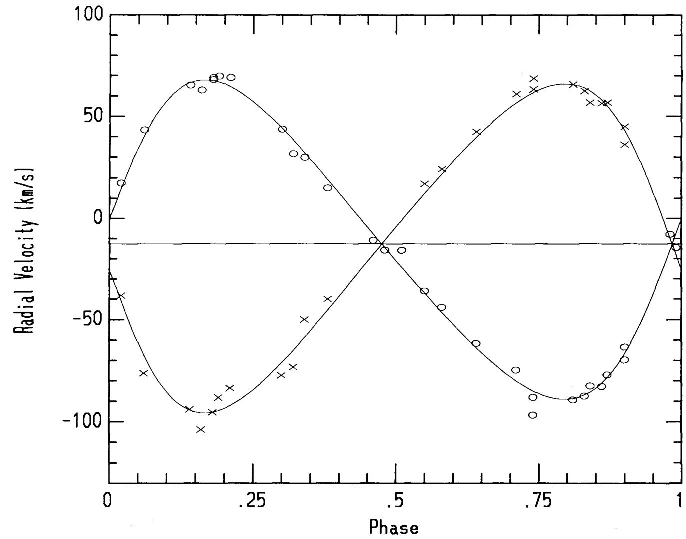

FIG. 2.—For Kopff 105, the 30 radial velocities for the primary (*circles*) and secondary (*crosses*) are compared with the computed velocity curves for a period of 6.2013 days.

Comments on the individual binaries are as follows:

Kopff 49.—Our period is the same as that derived by Abt et al. and Crampton et al., and the elements are similar.

Kopff 50 and 105.—Crampton et al. were unable to resolve the line doubling (at their 30–40 Å mm^{-1} dispersion). Abt et al. detected the doubling, but the new elements supersede theirs. Neither system is expected to be eclipsing from the low values of $\mathcal{M} \sin^3 i$, and neither is a known variable, or is listed in the catalog of suspected light variables (Kukarkin et al. 1982).

Kopff 72.—Our period and elements agree with those of Crampton et al., but only our measures were used in the solution.

Comments on stars considered by us to be constant in velocity are as follows:

Kopff 22 and 62.—We do not confirm Crampton et al.'s

TABLE 3

BINARY ORBITAL ELEMENTS

Element	Kopff 49	Kopff 50	Kopff 72	Kopff 105
P (days)	10.5302	11.4150	10.792	6.2013
	±0.007	±0.0001	±0.005	±0.0006
T_0 (JD 2,440,000+)	7428.83	7995.82	7686.0	8049.81
	±0.03	±0.03	±0.6	±0.04
K (km s^{-1})	49.2 ± 0.7	62 ± 1	16 ± 2	78.5 ± 0.9
		71 ± 1		80.9 ± 0.9
γ (km s^{-1})......................	−11.8	−11.3	−11	−12.7
	±0.5	±0.6	±1	±0.5
e	0.00	0.04	0.23	0.207
	±0.01	±0.01	±0.08	±0.009
ω	...	10	16	278
		±21	±2	±2
$a_1 \sin i$ (10^6 km)	7.12	...	2.31	...
$f(\mathcal{M})$ ($\mathcal{M}_\odot$)	0.130	...	0.0042	...
$\mathcal{M}_1 \sin i$ ($\mathcal{M}_\odot$)	...	1.48	...	1.24
$\mathcal{M}_2 \sin i$ ($\mathcal{M}_\odot$)	...	1.30	...	1.20
$O-C$ (km s^{-1})	2.8	4.4	3.2	5.0

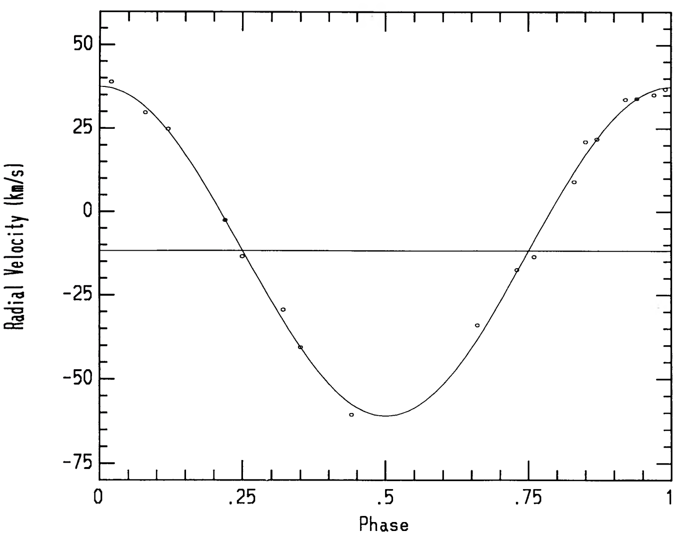

FIG. 3.—For Kopff 49, the 18 measured velocities are compared with the velocity curve for a circular orbit and period of 10.5302 days.

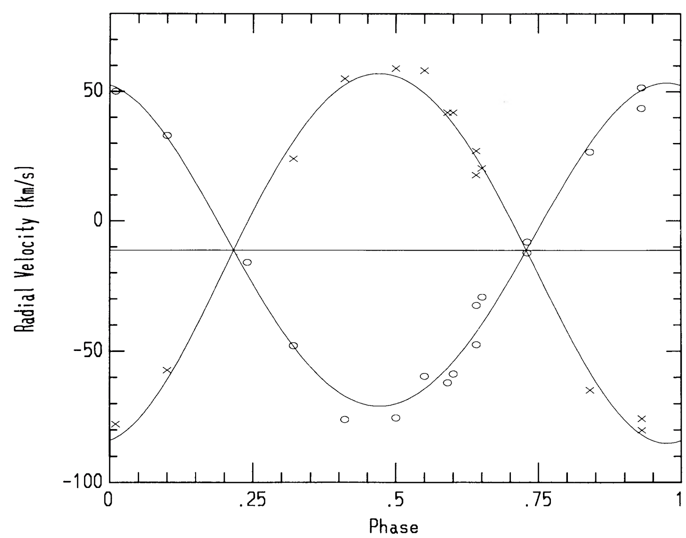

FIG. 5.—For Kopff 50, the 17 radial velocities of the primary (*circles*) and secondary (*crosses*) are compared with a computed curves with a period of 11.4150 days.

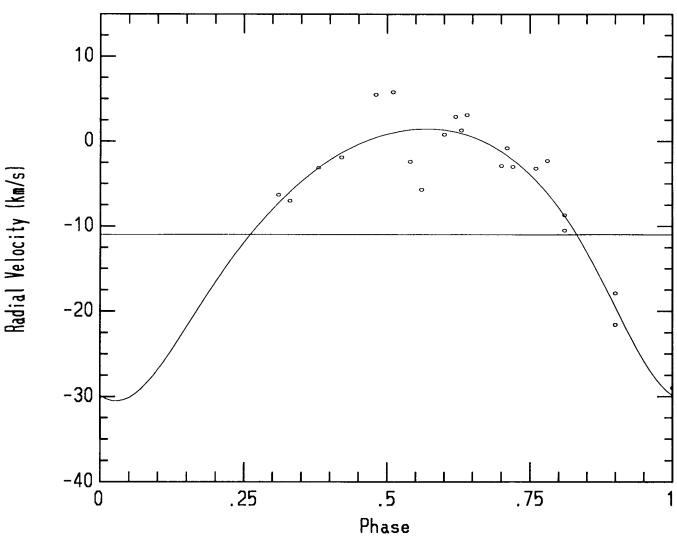

FIG. 4.—For the small-amplitude binary Kopff 72, the 22 velocities are compared with a velocity curve of period 10.792 days.

variability or orbital elements. Our external errors, E, of 3.8 and 2.4 km s^{-1} are smaller than theirs of 9.5 and 4.6 km s^{-1}, respectively. Thus Kopff 62 = HD 161573 may still be a velocity standard, as was suggested by Petrie (1953).

Kopff 43.—We do not confirm (at $E = 2.7$ km s^{-1}) the Crampton et al. evidence (at $E = 7.1$ km s^{-1}) for variability.

Kopff 64.—Crampton et al. reported asymmetric lines and possible variability ($E = 6.9$ km s^{-1}). We find no evidence for variability ($E = 2.6$ km s^{-1}).

Note that for the two double-lined binaries (Kopff 105, 50), the mass ratios are nearly 1.0, while for the very young Orion Nebula cluster (age of 10^6 yr), all four binaries have mass ratios (Abt, Wang, & Cardona 1991) of roughly $\frac{1}{3}$. These results are consistent with binary formation by capture in three-body interactions: massive stars will initially pair with the more prevalent low-mass stars, giving small mass ratios, but after many generations or cluster crossing times, they will tend to pair with other massive stars (Aarseth & Hills 1972).

We are left with only four spectroscopic binaries out of 15 stars observed, giving the smallest estimate to date of the binary frequency (27%). This shows that improved data tend not to confirm all earlier photographic data and that we should be cautious about comparing these results with photographic ones unless one considers only binaries with relatively large velocity amplitudes ($K > 20$ km s^{-1}). The reasons for our smaller external errors (average of 2.8 km s^{-1}) for constant-velocity stars then by Crampton et al. (4.2 km s^{-1}) is due to our higher S/N (by a factor of 1.5–2), higher dispersion or resolution (by a factor of 2–3), and perhaps our automatic measuring technique.

Finally we can ask whether synchronization of rotational and orbital motions occurs in any of the four binaries. If we allow a ± 10 km s^{-1} error in the measured rotational velocities, then Kopff 49 (with $V_{orb} = 10.53$ days and $V_{rot} \leq 7.9 \pm 3.9$ days) and 105 (with $V_{orb} = 6.20$ days and $V_{rot} \leq 5.9 \pm 3.0$ days) may be synchronous. However, Kopff 72 (with $V_{orb} = 10.70$ days and $V_{rot} \leq 4.6 \pm 1.9$ days) and 50 (with $V_{orb} = 11.42$ days and $V_{rot} \leq 2.4 \pm 0.6$ days) are not quite synchronous. These results are consistent with Levato's (1976) empirical results for evolved main-sequence stars. What we cannot tell is whether the tendency for near-synchronization was caused during binary and star formation or was achieved by tidal effects during their main-sequence existence.

REFERENCES

Aarseth, S. L., & Hills, J. G. 1972, A&A, 21, 255
Abt, H. A., Barnes, R. C., Biggs, E. S., & Osmer, P. S. 1965, ApJ, 142, 1604
Abt, H. A., Bolton, C. T., & Levy, S. G. 1972, ApJ, 171, 259
Abt, H. A., & Chaffee, F. H. 1967, ApJ, 148, 459
Abt, H. A., & Levato, H. 1975, PASP, 87, 849
Abt, H. A., & Sanders, W. L. 1973, ApJ, 186, 177
Abt, H. A., & Snowden, M. S. 1964, ApJ, 139, 1139
Abt, H. A., Wang, R., & Cardona, O. 1991, ApJ, 367, 155
Crampton, D., Hill, G., & Fisher, W. A. 1976, ApJ, 204, 502
Heard, J. F., & Petrie, R. M. 1967, in IAU Symposium 30, Determination of Radial Velocities and Their Applications, ed. A. H. Batten & J. F. Heard (London: Academic), 179

Johnson, H. L. 1957, ApJ, 126, 121
Kopff, E. 1947, Astr. Nach., 274, 69
Kraft, R. P. 1967, ApJ, 148, 129
Kukarkin, B. V., et al. 1982, New Catalogue of Suspected Variable Stars (Moscow: Nauka)
Levato, H. 1976, ApJ, 203, 680
Petrie, R. M. 1953, DAO, 9, 297
Petrie, R. M., & Heard, J. F. 1969, DAO, 13, 329
Vasilevskis, S. 1955, AJ, 60, 384