The Astrophysical Journal, 378:93–105, 1991 September 1

A STATISTICAL STUDY OF THE CORRELATION OF GALACTIC SUPERNOVA REMNANTS AND SPIRAL ARMS

Zongwei Li,[1] J. Craig Wheeler,[2] Frank N. Bash, and W. H. Jefferys
Department of Astronomy, RLM 15.308, The University of Texas, Austin, TX 78712-1083
Received 1989 November 27; accepted 1991 February 5

ABSTRACT

The distribution of observed giant H II regions and supernova remnants (SNRs) in Galactic longitude is analyzed with Monte Carlo models to determine the possible correlation with spiral arms or disk populations. The novel feature of the current analysis is that it depends mainly on the angular distribution of the H II regions and SNRs and hence is independent of distance. The SNRs seem to require a long disk scale length ($\gtrsim 5$ kpc). The possible correlations of the SNR distribution with the old stellar disk and the average radial distribution of the H I, H_2, and total gas density in the Galaxy is discussed. Our models incorporate selection effects in a parameterized way and suggest that ~ 1000 SNRs exist in the Galaxy, with ~ 850 remaining to be discovered. There is no substantial evidence that SNRs are especially concentrated in spiral arms as they would be if they were selectively born in spiral arms or if the high gas density in the arms were the dominant selection effect for the production of long-lived remnants. On the basis of the current evidence, the hypothesis that the shell-type remnants that dominate the SNRs sample arise from old disk SN Ia events cannot be strongly rejected. The H II region distribution is not especially well fitted with any of our models but suggests a radial scale length in the solar neighborhood of ~ 3.5 kpc, substantially less than the SNR distribution.

Subject headings: galaxies: structure — nebulae: H II Regions — nebulae: supernova remnants

1. INTRODUCTION

Based on statistical studies of external galaxies, our Galaxy is expected to produce Type I and Type II supernovae in nearly equal number (Tammann 1982), although recent evidence suggests that Type II may dominate (Evans, van den Bergh, & McClure 1989). There are as of this writing 155 identified Galactic supernova remnants. There is no certain, direct way to determine which of the Galactic supernova remnants are associated with Type I events and which with Type II. Light curves of Tycho's supernova in 1572 and Kepler's supernova in 1604 and the great apparent brightness of SN 1006 suggest that these events may have been of Type Ia (Clark & Stephenson 1977), but light curves alone may not differentiate them from Type II–linear (Doggett & Branch 1985) or Type Ib (see Strom 1988; Harkness et al. 1987; Porter & Filippenko 1987 and references therein). X-ray spectra of Tycho and SN 1006 and UV absorption studies of SN 1006 are consistent with carbon deflagration models of Type Ia (Hamilton et al. 1986; Hamilton & Fesen 1988). The interpretation of the fragmentary evidence for SN 1054 is debated (Chevalier 1977; Wheeler 1978) with the preponderance of feeling being that it was of Type II, perhaps of the linear variety (Swartz, Wheeler, & Harkness 1990). There is no direct evidence for any of the other 151 remnants as to the type of supernova that engendered it.

There are speculations to the effect that shell type remnants (the majority, $\sim \frac{3}{4}$ by number) may come from Type Ia events that are thought to leave no compact remnant and that the filled-center, Crab-like plerion remnants come from Type II (Weiler 1983). This notion is based on the idea that Type II supernovae come from massive stars that are theoretically expected to form neutron stars, and hence pulsars, such as observed in the Crab Nebula, and that putative Type Ia remnants (SN 1006, 1572, 1604) display no compact object. This potentially clean division is confused by the discovery of objects with a classical outer shell, but an inner nebula, presumably excited by relativistic particles from a pulsar (Helfand & Becker 1984, 1987). The extreme contrary view is that the formation of long-lived remnants is controlled not by the type of star that explodes, but by their gaseous environment and that virtually none arise from Type Ia events that preferentially explode at large scale heights and low gas density (van den Bergh 1988). Even in such a case it is relevant to ask whether or not the SNRs are tightly confined to spiral arms or whether or not they follow a particular gaseous component, e.g., molecular or atomic hydrogen.

There are other important questions that are indirectly related to the issue of the type of supernovae that make different SNRs. The rate of production of pulsars in the Galaxy is comparable to the estimated rate of occurrence of supernovae based on historical or extragalactic events (Lyne 1982; Tammann 1982; Narayan & Ostriker 1990). Equating pulsar production with Type II supernovae becomes difficult given evidence that most SNRs show no indication of neutron stars, despite the fact that pulsars with an estimated lifetime of $\sim 10^6$ yr should outlive SNR with lifetimes of $\sim 10^5$ yr. This is not simply a question of beaming of pulsar emission, because most SNRs also lack evidence for thermal surface radiation or X-ray synchrotron nebulae which should be isotropic (Nomoto & Tsuruta 1981; Helfand 1984). This poses two problems even if the rate of production of pulsars and SNRs are equivalent. One is why SNRs do not reveal more evidence of neutron stars. Perhaps $\frac{3}{4}$ of the SNRs leave no neutron star, or neutron stars that cool rapidly and also have little magnetic field so they do not generate pulsar radiation or synchrotron nebulae. The

[1] Postal address: Department of Astronomy, Beijing Normal University, Beijing, People's Republic of China.

[2] AURA Visiting Professor, National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc. (AURA) under cooperative agreement with the National Science Foundation.

complementary question is how to account for the rate of production of pulsars if the rate of production of pulsars observed to be associated with SNRs is too low. Perhaps many pulsars form with no associated optical outburst and no expanding remnant. These tentative explanations may be too facile, however, because they provide no ready explanation of how SNRs can selectively give rise to undetectable neutron stars while pulsars selectively manage to be born with no effect on the ISM.

Ilovaisky & Lequeux (1972) attempted to estimate distances to SNRs and to apply corrections for selection effects related to angular diameter, surface density, and flux to derive the SNR distribution. They argued for a rather flat distribution at <8 kpc and a sharp cutoff in the SNR distribution beyond 10 kpc. They concluded that the H I distribution extends to significantly larger radii than the SNR distribution with the SNR distribution more closely resembling the H II than the H I distribution. Ilovaisky & Lequeux estimated a local SNR density of ~ 0.4 kpc^{-2} and a total number of SNRs in the Galaxy of ~ 200. Clark & Caswell (1976) give very few SNRs near the Sun and none within 2 kpc in their sample "complete" to the source confusion limit corresponding to SNR diameter less than 36 pc. It is curious that while the sample of SNRs has only grown $\sim 50\%$ in the subsequent ~ 20 yr, the estimate of the local density and hence the total number of SNRs has gone up by a factor of several. Although there remains a sudden paucity of SNRs ~ 3 kpc beyond the Sun, current catalogs contain a large number near the Sun in the anticenter direction. This discourages the notion that there is simply a rather rapid, smooth decline in SNR density through the location of the Sun (see § 5). Whereas Ilovaisky & Lequeux took the rapid decline beyond the Sun literally, the current models suggest that the statistics of small number and selection effects which prevent the detection of distant SNRs may play a significant role and that the SNR distribution could be much flatter than suspected by Ilovaisky & Lequeux.

Kodaira (1974) reanalyzed the data and made a different type of empirical correction for selection effects, arguing that the major effect was source confusion and that it was independent of longitude. He derived a radial distribution that was much more peaked at small Galactic radii than Ilovaisky & Lequeux (1972). Beyond the Sun, Kodaira's distribution fell off rapidly, if somewhat less steeply than that of Ilovaisky & Lequeux. These differences were crucial to the interpretation. Ilovaisky & Lequeux, citing the correlation with H II and a z-distribution in the solar neighborhood similar to O and B stars and smaller than that of Cepheids, argued that the SNRs must come only from extreme Population I stars. Kodaira, on the other hand, argued that his distribution was much closer to the observed radial distribution of optical extragalactic supernovae in spiral galaxies and deduced that all supernovae, including Type Ia, must contribute to the SNR distribution. Tammann (1974) presented data from face-on Sc galaxies showing the radial distribution of the total optical supernova rate (6 Type I, 12 Type II, 15 unclassified). Tammann argued that this distribution closely followed that of Schmidt (1965) for the total mass density of the Galaxy. Converting Tammann's data to surface density of supernovae suggests that unlike the total mass density the surface density of SNRs may rise steadily toward the galactic center. The question of whether there is a peak in the surface density of optical supernovae in spiral galaxies, its location, and the rate of decline of surface density at larger radii is still subject to small number statistics in this sample, and so the question of the correlation of the radial distribution of optical supernovae and Galactic SNRs is still open. It would be of interest to re-examine the question of the distribution of optical supernovae with current data.

An important question is then to what component, stellar or gaseous, does the currently deduced SNR distribution belong? Freeman (1970) found the exponential disk component of spiral galaxies to have a large scatter in radial scale lengths, from 1 to 5 kpc, despite the fact that most had a disk component with an identical central blue surface brightness. Freeman assigned the Galaxy a very small scale length, ~ 1.7 kpc, and a central surface brightness to the disk component that departed considerably from the unique value defined by the majority of spiral galaxies (by ~ 3.5 mag per square degree). Ignoring the latter, de Vaucouleurs and Pence (1978) assumed the Galactic disk to have the central surface luminosity unique to most spirals and found the disk radial scale length to be $h_d \sim 3.8$ kpc, corrected to $R_0 = 8.5$ kpc. This value effectively corresponds to $H_0 = 100$ km s^{-1} Mpc^{-1} and would be proportionately longer if H_0 were less. Bahcall & Soneira (1980), ignoring Freeman's Galactic scale length and the large general scatter, adopted the mean of Freeman's values for other Sbc galaxies of $h_d = 3.5$ kpc, again effectively normalized to $H_0 = 100$ km s^{-1} Mpc^{-1}. Thus depending on one's adopted value of H_0, a somewhat longer value of $h_d \sim 5$ kpc is not out of the question. More recent estimates have tended to favor such a value. Van der Kruit & Searle (1982) adopted a dimmer central disk surface brightness than de Vaucouleurs & Pence and derived $h_d \sim 5$ kpc. Van der Kruit (1986) examined old disk giants and derived a value $h_d = 5.5 \pm 1$ kpc. Habing (1988) investigated *IRAS* AGB stars and found a value of 4.5 kpc. Lewis & Freeman (1989) used the radial and tangential velocities of old disk K stars to derive $h_d = 4.37 \pm 0.32$ and $h_d = 3.36 \pm 0.62$ kpc, respectively, for $R_0 = 8.5$ kpc. The lower end of the range derived here for the SNR, $h_d \sim 5$ kpc, is thus not incommensurate with recent estimates of the scale length for the Galactic exponential stellar disk (see § 5). Inasmuch as even longer scale lengths for the SNR, $h_d \sim 10$ kpc, may be consistent with the observations, however, one cannot readily identify the distribution of SNR with the stellar disk.

The question of the correlation of the SNR distribution with the gaseous component of the disk is also of great interest. Gordon & Burton (1976) gave a surface density distribution for H I that is nearly flat from 4 to 11 kpc with a peak about a factor of 2 higher at 12–13 kpc and a steady decline beyond that with a scale length of ~ 3.4 kpc (Knapp, Tremaine, & Gunn 1978). Gordon & Burton argued that the CO, and hence presumably the H_2, distribution peaks between 4 and 6 kpc and declines rather steeply beyond that. Burton (1976) argued that the SNR distribution of Kodaira (1974) resembled that of the CO and other tracers of extreme Population I, and thus associated the SNR distribution with extreme Population I, despite Kodaira's conclusion to the contrary from his own analysis. Burton & Gordon give an estimate of the total surface density of hydrogen (H_2 + H I) which declines from $R \sim 4$ to 16 kpc with a scale length of ~ 3.5 kpc. If the SNR distribution extends significantly beyond the Sun, the question of the possible correlation with the H I rather than the H_2 component must be reexamined.

Ilovaisky & Lequeux (1972) noted that the SNR distribution has a significantly larger scale height beyond roughly the solar circle. They argued, however, that the growth in the scale height of H I (for which they did not allow) only enhanced the discrepancy between the radial distributions of the surface density of H I and that of the SNR. More recent results suggest that, like H I, the molecular component of the gas also has an increasing scale height with Galactic radius. The surface density of molecular gas and hence of the total gas component may thus decline less steeply at large radii than estimated by Gordon & Burton (1976). Mead, Kutner, & Evans (1990) argue that the average star formation rate per molecular cloud mass is only slightly lower in the outer Galaxy. It may thus still be true that SNR are closely associated with extreme Population I molecular material even though they are more prevalent beyond the solar radius than once thought.

All these issues motivate us to look for new ways to explore the question of the distribution of the Galactic SNRs. Failing a means to identify a specific remnant with a specific supernova type, we inquire as to whether there is a statistical means to determine the likelihood that supernova remnants are related to one type or the other, and in what proportion. The basis for our investigation is the observation that Type II supernova are tightly correlated with spiral arms in spiral galaxies (Maza & van den Bergh 1976; Huang 1987). The evidence, though not introvertible, is that Type Ia supernovae do not correlate with spiral arms, but are roughly of old disk population (Maza & van den Bergh 1976; Tammann 1982).

We seek, then, a method by which we can determine a correlation of SNR with spiral arms. We abandoned at the outset any attempt to use data on distances of SNR and the location of spiral arms. The distances of SNR are too uncertain to permit identification with a spiral arm, although such an exercise remains an eventual goal. Furthermore, there is considerable controversy over the location, the number, even the existence of spiral arms in our Galaxy (Bash 1981; Georgelin & Georgelin 1976). For this exercise we primarily use information on the position of the SNRs in Galactic coordinates. Spiral arms will in principle yield a characteristic angular distribution of column density or number counts, and such a distribution should be free, to first order, of uncertainties in distances.

As a test case we compared the angular distribution of SNRs and giant H II regions which are presumed to define the location of the spiral arms. To the eye there did seem to be a correlation. This raised the question as to whether there was any evidence for a population of SNRs which was not correlated with spiral arms, and this in turn demanded the sort of quantitative approach given below.

Preliminary results (Li, Wheeler, & Bash 1984) suggested a strong correlation of SNRs with spiral arms. We now understand that that conclusion was an artifact of a different, but interesting, fact that there are a relatively large number of SNRs in the Galactic anticenter region implying a radial scale length in excess of some estimates of that length appropriate to the stellar or gaseous disk (a brief report of this result is given in Li et al. 1988). As described in § 5.1, the earlier model solution forced the SNRs into the arms in order to produce an appropriate number of anticenter SNRs and fit the long disk scale length.

In § 2 we describe our data base for SNRs and giant H II regions. In § 3 we present techniques for the Monte Carlo simulation of the distribution of SNRs in the Galaxy, and in § 4 we discuss the Kolmogorov-Smirnov two-sample test to determine the likelihood of correlations. The applications of these techniques to the sample of SNRs is given in § 5, and our conclusions are summarized in § 6 along with a discussion of the potential of the application of these techniques to other astronomical data bases.

2. DATA BASE

Milne (1979) gave a catalog of 125 SNRs. Green (1984) gave a list of 145 SNRs. We added eight more candidates from catalogs of X-ray sources and from recent radio observations (Caswell & Barnes 1985; Green & Gull 1984; Reich et al. 1985; Green 1988). The total number of SNRs in this sample was 153. Green (1988) gives a revised list of 155 SNRs in which a number of older identifications are rejected and new objects are added. Table 1 gives a list of objects in Green (1988) which were not in the original list of 153, objects which were in the old list which are not in the new list, and objects specifically discussed and rejected by Green (1988) from his 1984 catalog. The principal difference is that the new list contains more objects from $l > 350°$ to $l < +40°$ and fewer from $l = 320°$ to $l = 350°$. In order to compare the distribution of giant H II regions and SNR we used the catalogs of Georgelin and Georgelin (1976), Smith et al. (1978), and Blitz (1982).

There are 147 SNRs in our sample for which some estimate of the distance is available. Most attempts to measure the distances of Galactic SNRs have been based on kinematics of H I absorption features or Hα filaments, but generally such techniques can provide only uncertain estimates (Green 1984). The empirical Σ-D relation (Caswell & Lerche 1979; Milne 1979) has often served to estimate the distances, even though it shows large scatter (but see Huang & Thaddeus 1985). In this work the distances are used only in an average sense to give a rough calibration of the influence of selection effects.

3. MONTE CARLO SIMULATION MODEL

We have developed a quantitative approach to investigate the questions of the correlation of SNRs with arm or disk populations. We use the observed angular distribution of SNRs and giant H II regions to form a cumulative distribution with respect to Galactic longitude. Two observed distributions can be compared and Kolmogorov-Smirnov (K-S) statistics

TABLE 1

DIFFERENCE IN CANDIDATE SUPERNOVA REMNANTS BETWEEN GREEN (1988) AND GREEN (1984) (POSITIONS IN GALACTIC LONGITUDE)

New in Green (1988)	Not in Green (1988)	Specifically Rejected in Green (1988)
0.0 + 0.0	2.4 + 1.4	2.4 + 1.4
0.9 + 0.1	22.0 + 0.0	
5.9 + 3.1	24.8 + 0.6	
6.4 + 4.0	35.6 − 0.0	
8.7 − 0.1	41.9 − 4.1	41.9 − 4.1
9.8 + 0.6	47.6 + 16.1	47.6 + 6.1
16.8 − 1.1	93.6 − 0.2	53.9 + 0.3
18.9 − 1.1	117.3 + 0.1	93.4 + 1.8
20.0 − 0.2	123.2 + 2.9	123.2 + 2.9
30.7 + 1.0	132.4 + 2.3	
31.5 − 0.6	193.a − 1.5	193.3 − 1.5
36.6 − 0.7	144.7 + 0.4	194.7 + 0.4
42.8 + 0.6	205.6 − 0.1	
45.7 − 0.4	287.8 − 0.5	287.8 − 0.5
54.1 + 0.3	322.3 − 1.2	322.3 − 1.2
57.2 + 0.8	338.1 + 0.4	
73.9 + 10.9	339.2 − 0.4	
179 + 2.6	342.1 − 0.1	343.0 − 6.0
330.2 + 1.0		
338.1 + 0.4		
357.7 + 0.3		

used to determine the probability that the two samples are not drawn from the same distribution. In addition, we have constructed Monte Carlo models in which sample objects are distributed in model spiral arms and a galactic disk in a prescribed fashion. The correlation of these models with observed distributions can be used in conjunction with K-S statistics in an affirmative fashion to establish a figure of merit for the goodness of fit.

We adopt a model Galaxy that consists of an exponential disk and a two-armed spiral pattern. The SNRs are placed in both the model arms and the disk and the scale length of the disk, the spiral pattern, and the relative fraction of SNRs in the disk and the arms is varied. Each model produces a predicted angular distribution along the Galactic plane as "observed" from the Sun. For example, when all the SNRs are placed in the spiral arms, a larger number of SNRs is predicted at Galactic longitudes that correspond to the arms lying tangent to the line of sight.

3.1. *Distribution of SNRs in the Galactic Disk*

Following Bahcall and Soneira (1980), we adopted an exponential stellar distribution for the Galactic disk of the form

$$\rho \propto \exp\left[-(r - R_0)/h_d\right] \quad (1)$$

where R_0 is the distance of the Sun from the Galactic center and h_d is the radial scale length of the stellar disk. The following values are adopted: $R_0 = 8.5$ or 10 kpc, $r_{\rm min} = 3.55$ kpc, $r_{\rm max} = 16.45$ kpc (12 kpc in some models), where $r_{\rm min}$ and $r_{\rm max}$ are the minimum and maximum radii in our model stellar disks and h_d is in the range 3–9 kpc.

With no selection effects we evaluated the normalization constant $A(h_d)$

$$A(h_d) = 1\Big/\left[\exp\left(\frac{R_0 - r_{\rm min}}{h_d}\right) - \exp\left(-\frac{r_{\rm max} - R_0}{h_d}\right)\right]h_d\,, \quad (2)$$

such that

$$\rho(h_d, r) = A(h_d)\exp\left[-(r - R_0)/h_d\right]\,. \quad (3)$$

To establish the distribution of SNRs in the Galactic disk we then write,

$$\int_{r_{\rm min}}^{r} \rho(x, h_d)dx = F_1(r)\,, \quad (4)$$

where $0 \leq F_1(r) \leq 1$ is a dimensionless measure of the integral of the density chosen randomly in the Monte Carlo simulation. We solve equation (4) for the radial distance from the Galactic center to give

$$r = R_0 - h_d \ln\left[\exp\left(\frac{R_0 - r_{\rm min}}{h_d}\right) - F_1(r)\frac{A(h_d)}{h_d}\right], \quad (5)$$

and assign an angle with respect to the line of centers to the Sun as

$$\theta = 2\pi \cdot F_2(\theta)\,, \quad (6)$$

where F_2 is again a dimensionless function of θ to be randomly chosen. With r and θ from the Monte Carlo simulation, one obtains the longitude of the SNR, l, and the distance, d, of the model SNR from the Sun.

To introduce the influence of selection effects we adopted a function $S(d)$,

$$S(d) \propto \frac{S_2^2 + S_0^2}{S_2^2 + d^2}\,, \quad (7)$$

where S_0 and S_2 are constant parameters, and d is the distance from the Sun. This function is intended to describe the decreasing probability of detection of a SNR which falls off like $1/d^2$ due to the luminosity falling below a detection threshold. Another simple selection effect model is to take a similar function in which the terms vary linearly with d, corresponding to a selection effect on the angular size which is crudely proportional to $1/d$. With the addition of the selection effect term, the integral in equation (4) ceases to be analytic and the distribution of SNRs must be established by the select and reject method. We make no attempt to incorporate different selection effects for shell-type and Crab-type remnants, although this is probably the case (Green 1988; Helfand et al. 1989).

3.2. *The Distribution of SNRs along the Spiral Arms*

We calculated the distribution along spiral arms with no selection effects using a two spiral arm model,

$$r = r_0 e^{a(\theta - \theta_0)}\,, \quad (8)$$

(Bash 1981). Here r_0 is 5.62 kpc, θ_0 is chosen to be zero, and $a = \tan i$, where i is the pitch angle. The values of $r_{\rm min}$ and $r_{\rm max}$ are the same as for the Galactic disk models.

We first assumed that the SNR were uniformly distributed per unit length λ along the spiral arms so that:

$$\lambda = \lambda_{\rm min} + (\lambda_{\rm max} - \lambda_{\rm min})F_1(r)\,, \quad (9)$$

$$\lambda_{\rm max} = \frac{(1 + a^2)^{1/2}}{a} r_{\rm max}\,, \quad (10)$$

and

$$\lambda_{\rm min} = \frac{(1 + a^2)^{1/2}}{a} r_{\rm min}\,, \quad (11)$$

so that

$$r = \frac{a}{(1 + a^2)^{1/2}}\lambda\,, \quad (12)$$

and

$$\theta = \theta_0 + \frac{1}{a}\log\left(\frac{r}{a}\right)\,. \quad (13)$$

From r and θ, the values of l and d for each model SNR are again obtained. In other models we also required a decrease in probability density along the arms according to the same exponential law used for the disk population (eq. [3]) with radial scale length h_a. This necessitated the use of the select and reject method once again. We also spread the points in a Gaussian distribution perpendicular to the arms so that they were not infinitesimally thin. A selection effect function, equation (7), was also added in the same fashion as for the disk models.

In order to reduce numerical fluctuations, a typical Monte Carlo model is run to produce 1000 sample points. Table 2 gives the parameters and the range of values used in the Monte Carlo calculations.

4. KOLMOGOROV-SMIRNOV TWO-SAMPLE TEST OF SUPERNOVA REMNANTS, GIANT H II REGIONS, AND MONTE CARLO MODELS

The Kolmogorov-Smirnov two-sample test (K-S test; Press et al. 1986) is a test of whether two independent samples have been drawn from the same population. The two-tailed test is

TABLE 2

PARAMETERS FOR MONTE CARLO CALCULATION OF SUPERNOVA DISTRIBUTION

Parameter	Definition	Value
P	"Probability" that two samples are drawn from the same distribution	0–1
f	Fraction of SNRs in spiral arms	0–1
i	Opening angle of logarithmic spiral arms	$-6°$ to $-10°$
h_d	Radial scale length of disk surface density distribution	3.0–9 kpc
h_a	Radial scale length for logarithmic spiral arms	3.0–9 kpc
S_0	Size of selection-free region around Sun	1–3 kpc
S_2	Scale length of selection effect ($\propto d^{-2}$)	1–5 kpc
w	Gaussian width of arms	0.5 kpc
r_0	Initial radius of spiral	5.62 kpc
θ_0	Initial angle of spiral	0
r_{min}	Minimum radius of spiral arms and disk	3.55 kpc
r_{max}	Maximum radius of spiral arms and disk	16.45 kpc
R_0	Distance of Sun from Galactic center	8.5, 10 kpc

sensitive to any kind of difference in distributions from which the two samples were drawn.

The two-sample test is applied to the cumulative distributions of two sets of sample values. If the two samples have, in fact, been drawn from the same population, then the cumulative distributions of both samples may be expected to be fairly close to each other, inasmuch as they both should show only random deviations from the population distribution. If the cumulative distributions of the two samples are "too far apart" at any point, this suggests that the samples come from different populations. To apply the K-S test, we make a cumulative frequency distribution for each sample of observations (H II regions, SNRs) and for the Monte Carlo models. For example, we use the distribution of H II regions in Galactic longitude as observed and as generated in a Monte Carlo model. We then subtract one cumulative function from the other. The K-S test focuses on the maximum size of these deviations.

In the classical use of the K-S test to compare two observed distributions, one can determine that probability that the distributions are not drawn from the same sample when the distributions deviate substantially. The figure of merit cannot be interpreted as a probability that two samples are or are not drawn from the same sample in the opposite limit when the samples are similar. That is, the K-S test can be used to establish the probability that two distributions *are not* drawn from the same distribution if the sample cumulative distributions differ considerably, but it cannot be used to assign a probability that they *are* drawn from the same distribution if the cumulative distributions are similar. One can measure the magnitude of departure, but not goodness of fit.

The situation is different in principle for the comparison of the theoretical Monte Carlo models with the observational data. In this case, one has the capability of perturbing a parameter and reestablishing the values of the variables that maximize the figure of merit. In this way, the standard error of a parameter can be estimated directly from the derivatives of the likelihood function at its minimum. In our case we have used the Kolmogorov-Smirnov statistic to determine the best fit. The details of this procedure are given in the Appendix.

5. RESULTS

5.1. *Simple Estimate of the Radial Scale Length of the SNR Distribution*

Li et al. (1984) presented statistical evidence that they interpreted as favoring a distribution of SNRs almost entirely in the spiral arms. We now recognize that this conclusion was an artifact of the models in which the radial scale length of the disk distribution, h, was constrained to be 3.5 kpc, and the model SNRs were distributed at constant density per unit length along the arms. These assumptions forced the models to respond to the large number of observed SNRs in the outer Galaxy by putting nearly all the model SNRs in the arms to reproduce the outer Galaxy SNR distribution. We have now relaxed the constraint that $h = 3.5$ kpc for the SNR disk distribution.

Inspection of Figure 1 shows that the observed sample of SNRs for which there are distance estimates contains a relatively large number which fall toward the Galactic anticenter. Recall that the SNR distances are quite uncertain and selection effects are present that get worse at large distances. Nevertheless, it is very instructive to focus attention on the nearby remnants for which the selection effects should not be too severe. Two circles of radius 2.5 kpc centered 2.5 kpc toward the Galactic center and 2.5 kpc toward the anticenter display 28 and 17 SNR, respectively, giving surface densities of 1.4 ± 0.3 and 0.9 ± 0.2 kpc^{-2}, respectively. This distribution

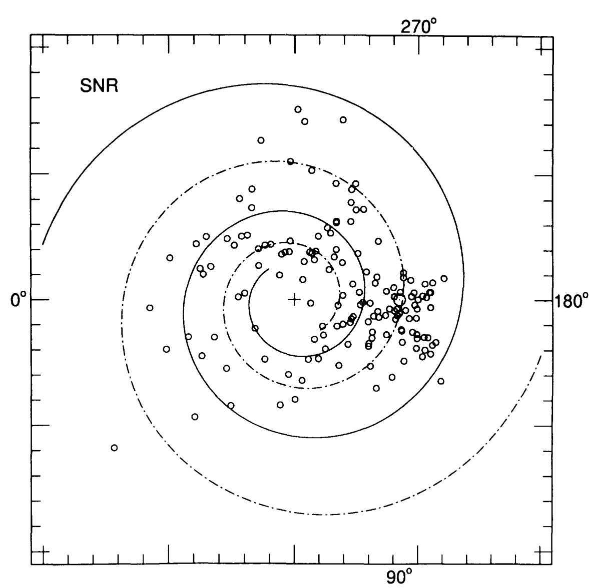

FIG. 1.—Radial plot of ~155 supernova remnants with estimated distances superposed on a two-arm logarithmic spiral pattern with opening angle $i = -8°$. The location of the Sun is marked with a larger circle at an assumed distance from the Galactic center of 8.5 kpc.

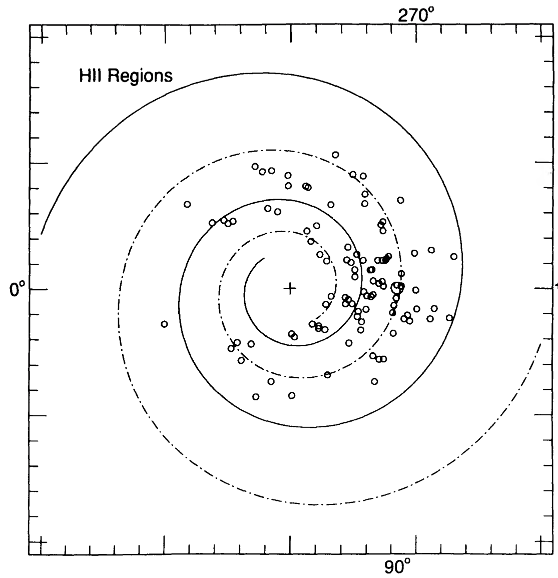

FIG. 2.—Same as Fig. 1, but for 100 giant H II regions

cannot be fitted satisfactorily with a scale length of 3.5 kpc which would demand a drop in surface density by a factor of 4.2 over the 5 kpc spanning the centers of the sampling circles. The observed decline implies a scale length of roughly 10 ± 3 kpc. By contrast, the same exercise with the giant H II regions of Figure 2 gives 25 and five objects toward the Galactic center and anticenter respectively, for surface densities of 1.3 ± 0.3 kpc^{-2} and 0.25 ± 0.1 kpc^{-2}, quite consistent with a scale length of 3.5 kpc. Taken at face value, this observation suggests that the distribution of SNRs has a significantly larger effective radial scale length than the disk stars (~ 5 kpc; see § 1), an interesting conclusion in itself. As we shall see in the next section, the SNR observations are best fitted by a model with disk component scale length $h_d \sim 5$–9 kpc.

The problem with earlier models was that a disk model with a scale length $h_d = 3.5$ kpc (and no selection effects) places far too few SNRs at large distances from the Galactic center so the K-S test rejected all models with pure disk distributions. On the other hand, a model with uniform population per unit length along the arms (and again no selection effects) did place some points in the anticenter direction. Given this choice, the K-S test was forced to choose strongly, but artificially, models in which the SNRs were in the spiral arms.

Since the spiral arms are only a small perturbation on the total stellar density, the models will be more nearly self-consistent if the same scale length is chosen for both arm and disk populations, as has been done for the models reported below. There is a possibility that arm and disk populations have different radial scale lengths, and we have also explored this possibility.

In our judgment the large scale length for the SNRs in the solar neighborhood is probably real. As illustrated in this section, it is confirmed, independent of our model, by the number of SNRs just inside and just outside the position of the Sun. Nevertheless, this conclusion, and surely any quantitative value, is sensitive to selection effects as discussed in the next section.

5.2. *The Influence of Selection Effects*

Inspection of Figure 1 shows that there is a clump of SNRs within 3 or 4 kpc of the Sun of roughly constant surface density and then a more sparse distribution at larger distances. There is an abrupt cutoff beyond a line about 3 kpc beyond the Sun normal to the Galactic center direction, despite the high surface density of objects closer to the Sun in the anticenter direction, as argued above. Near the Sun there is a marked bias with fewer SNRs in direction $315° > l > 270°$ than $90° > l > 45°$, but for the distribution at larger distances the opposite is true. There is a zone of avoidance in a segment of the Galaxy on the opposite side of the Galactic center.

There are two sorts of selection effects that undoubtedly strongly influence this distribution; one is the failure to detect or recognize SNRs because of their low surface brightness or small angular diameter or related effects; the other is a failure to search all Galactic longitudes uniformly for SNRs (Green 1988). Clearly, the overall effect suggests that there are far more SNRs in the Galaxy than have been detected. If the surface density quoted above for the solar neighborhood, ~ 1 kpc^{-2}, is correct, then there should be of order 900 SNR in the 850 kpc^2 represented in Figure 1, assuming a uniform surface density distribution. This estimate will be refined below. One of the conclusions of this study will be that the present sample of ~ 150 SNRs is not yet adequate to reach firm conclusions by the techniques we have developed. Clearly an effort to discover the other ~ 1000 SNRs that appear to remain undetected would be very valuable.

We have not generally attempted to correct for biases in the search patterns, although this effect is surely present. There are only six known SNRs from Galactic longitude 210° to 290°, versus 23 from 70° to 150°. This is probably related to the fact that the former region is in the southern Galactic hemisphere and not searched as thoroughly to low surface brightness (Reich et al. 1988). We found that the results of the K-S test were somewhat sensitive to the Galactic longitude at which the cumulative sums were started if we included the region 210°–260° in the Monte Carlo models. This was not the case in the models calculated in which this region was explicitly excluded from the Monte Carlo models. Models excluding points from 210° to 260° tended to be less sensitive to the choice of the spiral arm fraction, f, than the ones that included points in this region (see next subsection). We also checked the effect of this region by applying the Kuiper (1960) test on the observed distribution in comparison with a Monte Carlo model in which no allowance was made for it. The Kuiper test found the model to be an acceptable fit to the data.

For the bulk of the models, we have accounted for selection effects in a very simple way. We have adopted a selection-free zone of radius S_0 around the Sun in an attempt to describe the fairly uniform concentration of SNRs observed there. Beyond S_0 we adopt the simple function given in equation (7) that corresponds crudely to a $1/r^2$ bias with scale length S_2. We have also included ad hoc the zone of avoidance beyond the Galactic center. Figure 3 shows the selection effect function, equation (7), we have adopted for several values of the parameter S_2. The lower curve shows an assumed radial distribution for SNRs with a scale length of 5 kpc. The Sun is located at $R_0 = 8.5$ kpc. Figure 4 shows the convolution of the selection effect function and the radial distribution.

The statistical models discussed below tend to be insensitive to the choices of the parameters S_0 and S_2. In order to reduce the number of free parameters in the Monte Carlo models, we have constrained S_0 and S_2 by choosing the values which, by eye, seem to give the best reproduction of the observed radial distribution given in Figure 1. All the distances used in con-

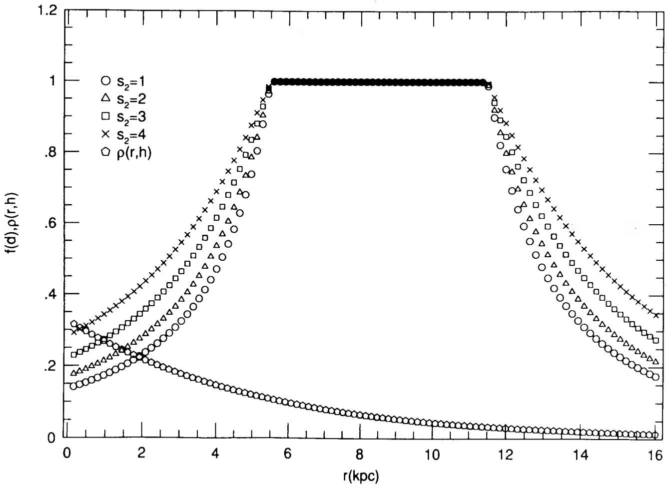

FIG. 3.—The assumed selection effect function as a function of Galactic radius. There is an assumed selection-free region of radius $S_0 = 3$ kpc and then a decline according to eq. (7) which approximates a $1/r^2$ selection effect. The scale length for the selection effect is given for $S_2 = 1.0$ (*circles*), 2.0 (*triangles*), 3.0 (*squares*), and 4.0 (*crosses*) kpc, respectively. Also shown (*pentagons*) is the radial surface density distribution corresponding to an exponential disk with scale length $h = 5$ kpc (see text).

structing Figure 1 are uncertain, so we attempt to reproduce only the gross features. In preliminary models we also investigated selection effects that scaled like $1/d$, but there was not a great deal of difference in the results, so they have not been explored in depth.

Although we confine any quantitative arguments to the comparison with the longitudinal distribution, the radial distribution can also be used to provide crude constraints on the physical parameters of the models. As an example, a Monte Carlo model with no selection effects, a distribution of 160 points in the disk (spiral arm fraction $f = 0$), and a radial scale length of $h_d = 3.5$ kpc causes the points to bunch toward the Galactic center and give a poor representation of the number of events observed toward the anticenter of the SNR distribution of Figure 1. There are also insufficient points near the Sun in comparison to the H II region distribution of Figure 2. A distribution of 160 points confined to (smeared) spiral arms ($f = 1$; opening angle $i = -8°$) with uniform population per unit length of arm (formally arm radial scale length $h_a = 1000$ kpc in the model) does place some points toward the anticenter, but is again a poor representation of the observed radial distribution. These examples show that it is important to include selection effects and a proper arm distribution in the Monte Carlo models.

We computed the radial distribution for three disk models ($f = 0$) with increasing scale length $S_2 = 1$, 3, 5 kpc for the selection effect parameter and a radius $S_0 = 3$ kpc for the selection free region. All had a radial scale length of $h_d = 7$ kpc. The model with $S_2 = 1$ kpc showed a fair concentration of objects near the Sun but was somewhat underpopulated larger distances. The model with $S_2 = 5$ kpc had somewhat too many objects at large distances and definitely left the solar neighborhood underpopulated. A compromise with $S_2 = 3$ kpc did a fair job near and far. Decreasing S_0 at constant S_2 forces objects to cluster closer to the Sun. None of these models yielded the distinct demarcation of objects within and beyond a distance of about 3 kpc beyond the Sun. The origin of this concentration, and hence a bias on our estimate of the scale length of the radial distribution, may be in more subtle selection effects we have not attempted to model. All these pure exponential disk models with modest scale lengths have a bias against the anticenter direction which will tend to cause the K-S test to reject them.

Other models explored the extreme opposite assumption by assuming all the objects are associated with spiral arms ($f = 1$). The influence of the model selection effects is to force a goodly number of points into the anticenter segment of the spiral arm which crudely satisfies the requirement that this angular range be populated and results in a somewhat higher measure of statistical angular correlation than the pure disk models.

As an illustration of the effect of selection effects on the apparent radial distribution, Figure 5 presents an intermediate model for which half the sample objects are in the disk and half in the spiral arms with a radial scale length of 5 kpc. A value

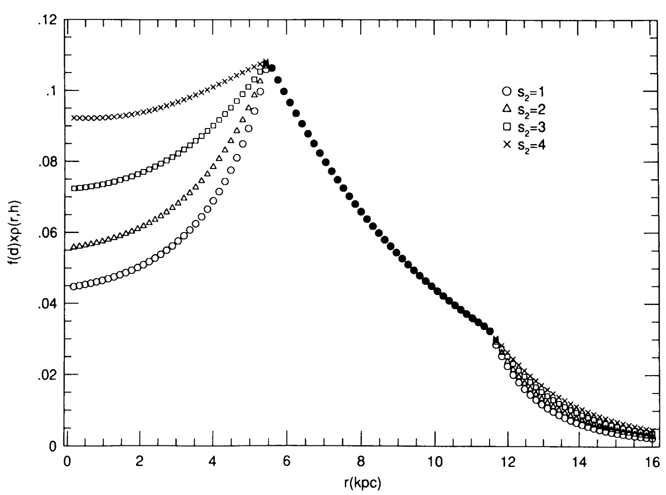

FIG. 4.—The convolution of the selection effect function of eq. (7) and Fig. 3 with the radial distribution of scale length $h = 5$ kpc.

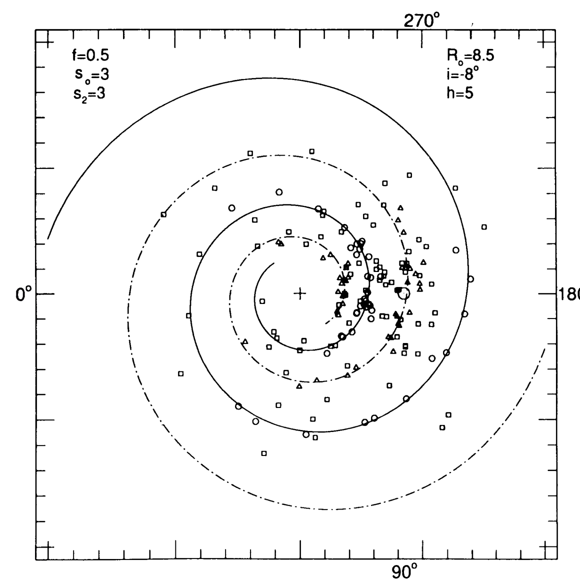

FIG. 5.—Model containing 150 points with half the points in the exponential disk (*squares*) and half in the spiral arms (*circles* and *triangles*) (i.e., $f = 0.5$) with radial scale length $h = 5$ kpc, opening angle $i = -8°$, the Sun at $R_0 = 8.5$ kpc from the Galactic center, selection-free radius $S_0 = 3$ kpc, and selection effect scale length $S_2 = 3$ kpc.

$S_2 = 1$ kpc tends to better reproduce the distribution near the Sun and $S_2 = 3$ kpc, as illustrated in Figure 5, that at larger distances. A value $S_2 = 5$ kpc tends to put too many points at large distances. The addition of the spiral arm objects tends to produce an anticenter demarcation as the observations suggest. Figure 5 gives surface densities of 1.8 ± 0.3 and 0.5 ± 0.2 kpc^{-2} for circles of radius 2.5 kpc centered a like distance toward the center and anticenter. These "observed" densities give an implied radial scale length of 3.7 ± 0.8 kpc, consistent within the uncertainty with the formally assumed value of $h = 5$ kpc. This result gives, however, too strong a radial gradient compared to the observations as also shown by the convolution of selection effect function and exponential disk of Figure 4. Either the radial scale length of the SNR distribution is even longer than 5 kpc or there is some bias against discovery of SNRs rather near the Sun in the direction of the Galactic center.

5.3. *Comparison of Observed* H II *and SNR Distributions*

For the cumulative distribution functions of the H II regions and the sample of 155 SNRs we derive (see eq. [A2] and the following discussion in the Appendix) a probability that they are drawn from the same distribution of $P = 0.55$. Since we are comparing two observed distributions we can only conclude that the SNRs and H II regions are not necessarily drawn from *different* distributions. We can also compare these distributions with representative Monte Carlo models. For a typical Monte Carlo model with 1000 data points, radial scale length for arms and disk of 5 kpc, selection-free radius $S_0 = 3$ kpc, and selection effect scale length $S_2 = 3$ kpc, we find $P \lesssim 0.6$ for the H II regions and $P \gtrsim 0.6$ for the SNRs for all values of the disk fraction, *f*. These results suggest that the H II regions and SNRs may be distributed differently. For the same parameters otherwise, but reducing the radial scale length to 3 kpc we find $P \lesssim 0.2$ for the H II regions, and $P \lesssim 0.4$ for the SNRs. This suggests that the SNRs, especially, may prefer longer scale length. These points will be investigated below.

In general, the basic trends with respect to model parameters for H II regions are similar to those for the SNRs, but the probabilities for any combination of parameters tends to be smaller and never greater than about 0.6. The reason for our inability to find any model that satisfactorily reproduces the H II regions, arm or disk, large scale length or small, is not clear.

5.4. *Comparison of Observed SNRs with Monte Carlo Models*

We compared the cumulative distribution functions for the old SNR sample of 153, the revised sample of 155 (which differs by about 30 objects; see Table 1), and the model with 1000 points and disk radial scale length, $h_d = 7$ kpc. This model agrees better with the revised sample at $l \sim 50°$–$100°$ because of the net reduction of objects at large l and the net addition of objects at small positive l. We find $P = 0.2$ for the older sample compared to $P = 0.9$ for the revised one. The main point of this illustration is that the results are somewhat sensitive to the sample. For the remainder of the paper we will deal only with the revised sample, bearing in mind that, at best, the analysis is only as good as the sample and that inasmuch as the technique has promise it will be more fully realized as the sample grows in size and certainty.

As mentioned in § 1, there is some controversy over the number, never mind the orientation of the arms. We have adopted a two-arm spiral and, of course, if that model is flawed in some fundamental way, so is our analysis of the fraction of SNRs in the arms. Study of other grand spiral galaxies like M81 suggest, however, that the model is quite representative in that case. We have computed models varying the opening angle, *i*, of the logarithmic spiral. To the degree to which the arms contribute strongly ($f \sim 1$) a value of $i \sim -8°$ is favored. The model presented by Bash (1981) favored $i \sim -6°$. The larger value here, if significant at all, could be explained by a natural tendency for the finite lifetime of the supernova progenitors to allow them to depart from the locus of the spiral arm shock in which they were born. Such an effect would be equivalent to a larger opening angle. We have also checked the effect of varying the initial phase angle of the arms. Increasing $\theta_{\min}$ is roughly equivalent to adopting a larger value of $|i|$ because a spiral arm will be at a larger distance from the Galactic center at a given location. Varying $\theta_{\min}$ from π to $1.5\,\pi$ produced a negligible effect on the results.

The results are generally insensitive to the choices of S_0 and S_2 in the range $S_0 \sim S_2 \sim 2$–3 kpc. Table 3 gives the probability P for a variety of models of the SNR distribution that explore the sensitivity to the disk and arm scale lengths, h_d and h_a, and the fraction of the SNR in the spiral arms, *f*. In this series of models the scale length in the arms and disk were taken to be identical and no model points were included in the interval 210°–260°. Table 3 shows, as did the eyeball analysis given at the beginning of the section, that models with a scale length of 3 kpc, comparable to that commonly assigned to the exponential stellar disk, tend to be rejected for most choices of parameters. The favored value of the scale length is ~5–9 kpc.

Table 3 shows that for $R_0 = 8.5$ kpc, models with $h_d = h_a = 5$ kpc favor spiral arm fraction $f \lesssim 0.4$ while those with $h_d = h_a = 7$ kpc tend to favor $f \gtrsim 0.6$. This result is again driven by the observed distribution in the anticenter direction. The disk component of the models puts more points in the anticenter direction with increasing h_d. For the model arm geometry we have chosen, points beyond the solar circle tend to be concentrated along the outer arm between ~30° and 120° (see Fig. 5). The result is that for small scale length, $\lesssim 5$ kpc, none of the models work well independent of *f*. For $h \sim 5$ kpc, the disk component does an adequate job of reproducing the anticenter distribution, and the arm component tends to be rejected because it would load the quadrant 30°–120° in favor of the anticenter disk component. For longer scale lengths, however, the disk component tends to load the anticenter too heavily. In this circumstance, it becomes advantageous to increase the

TABLE 3

PROBABILITY OF AGREEMENT BETWEEN OBSERVED DISTRIBUTION OF 155 SNRs AND VARIOUS MONTE CARLO MODELS[a]

	$h_d = h_a$ (kpc)			
f	3	5	7	9
0.0.........	0.0	0.9	0.5	0.3
0.1.........	0.4	0.9	0.7	0.5
0.2.........	0.1	0.9	0.6	0.5
0.3.........	0.3	~1.0	0.7	0.2
0.4.........	0.2	~1.0	0.8	0.4
0.5.........	0.1	0.6	0.8	0.7
0.6.........	0.3	~1.0	0.9	0.5
0.7.........	0.2	0.7	~1.0	0.8
0.8.........	0.3	0.6	~1.0	~1.0
0.9.........	0.1	0.3	~1.0	~1.0
1.0.........	0.4	0.8	0.9	0.8

[a] $R_0 = 8.5$ kpc; $S_0 = S_2 = 3$; $i = -8°$; $n = 1000$.

arm fraction and thus to remove anticenter points and redistribute them between $\sim 30°$ and 120°.

The role of the spiral arm parameter is further illustrated by considering models in which points are allowed in the range 210°–260°, where there is a presumed search deficiency in the southern hemisphere. These models tend to favor $f \sim 0.5$. Inspection of the models shows that the disk component fills this angular range rather uniformly, as expected. The arm component puts points along the model arm passing near the solar location (for $R_0 = 8.5$ kpc) and hence selectively at $\sim 270°$ with relatively few at somewhat smaller angles even though they are not formally excluded by the model. The models thus tend to favor some arm component to adjust for the paucity from $\sim 210°$ to 260° but also some disk component to keep an appropriate proportion of events in the anticenter direction.

We conclude that the figure of merit, P, of the statistical models does give some constraints on the gross distribution of SNRs, and we suggest some credence be given to the tendency to favor large scale lengths. The arm parameter f, however, seems to serve primarily as simply another parameter to fine-tune the basic distribution for some details. It is not then at all clear that the spiral arms in the adopted model have any relation to observed spiral arm structure in the SNR distribution. On the other hand, if the SNRs were strongly and selectively concentrated in spiral arms, the present analysis might have detected some evidence of that, and this does not seem to be the case.

5.5. *Total Number of SNRs in the Galaxy*

Helfand et al. (1989) discussed the problem of selection effects on supernova remnants and the total number of SNRs in the Galaxy. They divided the Galaxy into 13 zones in distance and Galactic longitude, assuming a distance of the Sun from the Galactic center of 8.5 kpc and an outer radius of 12 kpc, and discussed the distribution of the observed SNRs. Table 4 gives the geometrical regions of their study and the number of known SNRs in the sample of 155 in appropriate regions. Our Monte Carlo models allow us to attempt to reproduce this regional distribution of SNRs. This exercise is a somewhat more detailed version of the use of figures like Figure 5 to constrain the selection effect parameters to reproduce the gross radial distribution.

For a given set of parameters, we can calculate a model with selection effects to see how many SNRs it predicts within our assumed selection-free zone of radius S_0 around the Sun. The model is constrained to have a total number of points (155) equal to the observed sample. With this constraint, the parameters of the model can in principle be adjusted to give the correct number within the selection-free region. The same model can then be run with no selection effects to see what the total sample size must be in order to reproduce the same number within S_0 as did the model with selection effects and a sample size of 155. This serves to normalize the calculation and to give a quantitative estimate of the total number of SNRs in the Galaxy. We can then independently check to see how the model with 155 points distributes those points in the geometrical regions defined by Helfand et al. Table 4 also shows the results of the distribution for a model with $R_0 = 8.5$ kpc, $i = -8°$, $S_0 = S_2 = 3$ kpc, $h_a = h_d = 5$ kpc, $\theta_{min} = 1.25\,\pi$, and $f = 0.5$ for the model with selection effects and two different models with no selection effects of total sample size 800 and 1000.

The number of observed SNRs 3 kpc from the Sun is estimated to be ~ 37. In Table 4 the model with 155 points and selection effects included gives just this number. It also gives reasonably good agreement with observations in the regions defined by Helfand et al. The model with 800 points and no selection effects comes the closest to reproducing the number in the normalizing region with $S_0 = 3$ kpc. It gives 43 versus 52 for the model with 1000 points. On the other hand, the model with 1000 points gives a better agreement in Helfand's region A. It gives 34 compared to the observed number of 32, whereas the model with 800 gives only 27. One might argue that because distances are so uncertain and Helfand's region A is defined principally by angle in Galactic longitude that region A represents a better region with which to normalize. This then allows us to estimate that there are ~ 1000 SNRs in the Galaxy with an "uncertainty" of $\lesssim 20\%$. This compares with our crude estimate of 900 given earlier based on a constant density of ~ 1 kpc^{-2} and a radius of $R_{max} = 16.45$ kpc. Helfand et al. estimate a total of 360 based on a constant surface density and a local density of 0.8 kpc^{-2} in region A and $R_{max} = 12$ kpc. They estimate ~ 590 after correcting for some radial gradient in the Galaxy. Table 4 also shows the distribution of these 590

TABLE 4

COMPARISON OF MODELS WITH $S_0 = S_2 = 3$ kpc, $i = -8°$, $h_d = h_a = 5$ kpc, AND $R_0 = 8.5$ kpc WITH THE DISTRIBUTION OF HELFAND ET AL. 1989

					MODEL			
ZONE	l	R (kpc)	AREA (kpc^2)	KNOWN	155	800	1000	Helfand
O	...	<3	...	37	37	43	52	...
A	$90° < l < 270°$	8.5–12	40.2	32	39	27	34	32
B	$45 < l < 90$	8.5–12	47.8	18	9	21	44	38
C	$45 < l < 90$	<8.5	29.3		11	41	46	43
D	$30 < l < 45$	<8.5	22.2	15	7	59	82	41
E	$30 < l < 45$	8.5–12	14.8		3	18	19	12
F	$30 < l < 330$	<8.5	124.0	60	42	420	532	245
G	$30 > l > 0$	8.5–12	9.9		3	25	35	8
H	$330 < l < 360$	8.5–12	9.9		2	21	15	8
I	$30 < l < 330$	8.5–12	40.2		5	46	50	32
J	$315 < l < 330$	<8.5	22.2	13	12	68	88	41
K	$315 < l < 330$	8.5–12	14.8		2	15	10	12
L	$270 < l < 315$	<8.5	29.3	17	12	42	51	43
M	$270 < l < 315$	8.5–12	47.8		10	35	30	38
Totals			452.4	155	155	800	1000	590

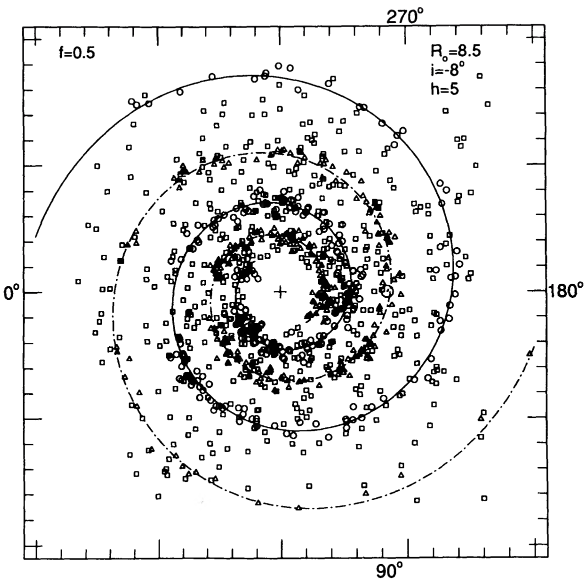

FIG. 6.—The distribution of the ~ 1000 SNR predicted to be present in the Galaxy ($R_0 = 8.5$ kpc, $i = -8°$, $h_a = h_d = 5$ kpc, $S_0 = S_2 = 3$ kpc, $f = 0.5$).

events as predicted by Helfand et al. We believe our estimate of 1000 to be more quantitative and hence more accurate. Figure 6 gives a representative plot of the radial distribution of the ~ 1000 SNRs we predict in the Galaxy.

6. CONCLUSIONS

Perhaps the most interesting conclusion of this study and the most distinct departure from earlier related studies is the notion that the radial scale length of the SNRs is rather long, $\sim$5–9 kpc. This range results from the detailed models presented here, but it is also consistent with a simple examination of the number of SNRs in the vicinity of the Sun. The lower end of the range derived here is consistent with current estimates of the radial scale length of the old stellar disk, ~ 5 kpc, so the SNRs could sample the stellar disk distribution. The upper end of this range is not commensurate with the stellar disk. If the SNRs or some portion of them, specifically the low surface brightness events, have such a long radial scale length, then the process that makes stars that lead to these SNRs may differ significantly from the process that makes the stars in the disk. There is also a suggestion here that the radial scale length of the SNR significantly exceeds that of the H II regions which is consistent with ~ 3.5 kpc.

This long radial scale length also suggests that the association of the SNR distribution with the gaseous components of the Galaxy must be reexamined. The SNR distribution might be commensurate with the observed distribution of H I alone. This is in contrast to the argument of Burton (1976) who concluded the H I had a radial scale length that significantly exceeded that of the sample of SNRs known at that time. SNRs could correlate with H I if long-lived SNRs are strictly a result of environment. It is possible that whereas too low a density allows the SNRs to expand and dissipate too quickly, too high a density accelerates the evolution or otherwise renders it unrecognizable (Wheeler, Mazurek, & Sivaramakrishnan 1980; Shull 1980). Since atomic gas surrounds molecular clouds, occasional association of SNRs with molecular matter may be only incidental.

On the other hand, if the molecular component extends to larger Galactic radii than previously thought and expands to larger scale height beyond the solar circle, then the SNR distribution might follow this distribution. This is basically Burton's (1976) premise extended with modern observations of both SNRs and CO. Huang & Thaddeus (1986) show that over half (seven of 11) of a sample of SNR are associated with large molecular cloud complexes. This sample was concentrated toward the anticenter ($70° < l < 210°$) and hence encourages the notion that even beyond the solar circle SNRs continue to follow the molecular gas. If the filling factor of molecular material is low with respect to neutral gas, this would suggest that the SNR distribution is a measure of the birth sites of the supernovae, and hence association with molecular matter is evidence of a young intrinsic underlying population.

Another aspect of the current study that is relatively model independent is the estimate of the total number of SNRs in the Galaxy. The current models suggest that the Galaxy contains a rather large number of SNRs, ~ 1000, and hence that ~ 850 remain to be discovered. Similar numbers follow from simple extrapolation of the SNR density near the Sun. It will be very difficult to detect the hundreds of undiscovered SNRs because of problems of source confusion, but we note that low-frequency surveys for which H II regions appear in absorption against the Galactic plane background may prove effective (Gorham 1990).

If all these SNRs live for a mean lifetime of $10^5\, \tau_{\rm SNR,5}$ yr, then they should be produced at a rate of $\sim 10^{-2}\, \tau_{\rm SNR,5}^{-1}$ yr^{-1} in the Galaxy as a whole and $\sim 1.3 \times 10^{-11}\, \tau_{\rm SNR,5}^{-1}$ pc^{-2} yr^{-1} locally in the solar neighborhood. These rates are consistent with estimates for the birth rates of supernovae and for pulsars. As noted in § 1, however, a simple equivalence of rates does not answer the important associated questions. If SNRs are produced at the same rate as pulsars but the majority of SNRs show no sign of neutron stars, then one must still account for why neutron stars do not frequently appear in SNRs and for how most pulsars are born in the absence of a SNR.

This study was initiated with the intention of seeking a correlation of SNRs with spiral arms, but no such correlation emerged. There are many reasons to expect the correlation of some component of SNRs with spiral arms. Massive stars should be born and die in spiral arms. Type II supernovae and the H II regions with which they are associated (Huang 1987) are observed to occur selectively in spiral arms of other galaxies. If Type II supernovae occur more frequently than Type Ia supernovae in the Galaxy (Evans et al. 1989), then the SNRs should be selectively in the arms. Some SNRs seem to be correlated with the molecular gas even beyond the Solar circle. Whether this is because the molecular matter is the stellar birthsite or merely a measure of a region of enhanced gas density, this component of SNRs should be concentrated in spiral arms. Even if the total SNR distribution simply follows the total gas density, the gas tends to be concentrated in spiral arms as well and so should the SNRs. This does not seem to be the case.

The simplest hypothesis is that many SNRs do exist in spiral arms, but our means to show that are inadequate. The insensitivity of the current results to the arm fraction parameter, f, may simply reveal the failure of an undoubtedly simplistic model, or the inadequate nature of the current sample. We cannot preclude, however, that the SNR distribution is predominantly a disk, rather than an arm distribution. This might mean that most SNRs come from stars sufficiently small in mass ($\lesssim 8\ M_{\odot}$; Biermann & Tinsley 1974) that they can drift out of the arms. Based on the current results, we specifically cannot rule out the hypothesis that the shell-type remnants are from disk population Type Ia supernovae. As remarked above, however, the SNR distribution might have a radial scale length that considerably exceeds that of the stellar disk, so the SNR distribution cannot be simply identified with the stellar disk. Furthermore, Type Ia supernovae may be a bulge rather than a disk population (Wheeler 1990, 1991). In such circumstances, Type Ia supernovae may not contribute substantially to the long-lived SNRs, even if the SNRs are not strongly concentrated in spiral arms. Van den Bergh (1988) argues that Type Ia supernovae do not contribute to long-lived SNRs, noting that there are few SNRs in the bulge at $R \lesssim 3$ kpc (see Fig. 1) which is plausibly gas-poor but rich in Type Ia supernovae. If this is the case, then we are again left with no explanation of why most of the known SNRs are shell-type remnants with no evidence of a central compact object.

There remains the serious question of the degree to which the current results are affected by selection effects. There is no question that the local density of SNRs and the number toward the anticenter have increased substantially since the early 1970s. It is also true that many of the latter, in particular, are low surface brightness objects that are more easily detectable toward the anticenter where the background confusion is less (Reich et al. 1985; Green 1988). This could give an artificial bias toward long radial scale length.

There are two factors to be considered when evaluating the significance of the low surface brightness SNRs. One concerns sheer numbers, the basis for the current analysis, and the other is the physical state of these SNRs. The first point is that the very existence of SNRs toward the anticenter is significant. Even if they are somehow a separate population of low surface brightness objects, they must either selectively populate the anticenter, effectively having a long radial scale length, or there must be a very large number of them to have a modest scale length and to still exist toward the anticenter in relatively large number.

The second point concerns why these SNRs have low surface brightness. They might be of low surface brightness because they have exploded in a low-density region. This in turn could be related to the fact that the scale height of the gas grows with increasing distance from the Galactic center. The surface density could be appreciable at large radii but the volume density low because of the large scale height. An alternative question is whether the low surface brightness SNRs might represent a physically different population of SNRs, particularly the Type Ia supernovae that selectively explode in lower density environments and rapidly dissipate. Important related questions are again the relative number of low surface brightness SNRs and their radial and vertical distributions. All these factors reinforce the notion that the distribution of the SNRs, especially toward the anticenter, is an important topic for further observational and theoretical consideration. The nature of the low surface brightness objects merits special attention.

This work was initiated when inspection of the data suggested a correlation of SNRs and H II regions. We have no ready explanation as to why we have been able to compute successful models for SNRs for some reasonable choices of parameters, but not for the H II regions. A possible suggestion is that the sample of supposedly giant H II regions is, in fact, badly contaminated with smaller, more randomly placed H II regions. Conti and Vacca (1990) suggest that spiral arm structure can be discerned in the distribution of Wolf-Rayet stars. We have not yet attempted to model the Wolf-Rayet distribution in any detail, but it remains of interest to do so. We note that the radial density gradient (e.g., at $l \sim 210°$ in Fig. 1 of Conti & Vacca) suggests a rather short scale length. Another set of objects which are amenable to study by the techniques we have developed here are the *IRAS* 25 μm and 60 μm sources, an interesting fraction of which may be SNRs (Gorham 1990). These will be the subject of future investigations.

Nakai & Sofue (1982) showed that a sample of 19 SNRs in M31 generally correlated with the distribution of H I gas and perhaps even more closely with H II regions, including those in the bulge which are not necessarily correlated with large H I column densities. It would be of interest to reexamine this galaxy with more recent data on SNRs and the distribution of CO.

The work reported here was originally funded by the NSF US-China Exchange program. It has been supported in part by NSF, the University of Texas University Research Institute, McDonald Observatory, and the National Natural Science Foundation of China. We are grateful for discussions of stars, supernova remnants, and molecular clouds and their distributions with John Caswell, John Dickle, Marc Kutner, Don Terndrup, and Sidney van den Bergh. J. C. W. gives special thanks to AURA and to Sidney Wolff and the staff at KPNO for hospitality while this paper was completed.

APPENDIX

KOLMOGOROV-SMIRNOV TWO-SAMPLE TEST OF OBSERVED AND MODEL DISTRIBUTIONS

Let $C_m(X)$ be the observed cumulative distribution function of one of the samples, $C_m(X) = K/m$, where K is the number of data points equal to or less than X. Let $C_n(X)$ be the observed cumulative distribution function of the other sample, that is, $C_n(X) = K/n$. One then constructs the parameter

$$D^{\max}_{m,n} = \text{maximum}\ |C_m(X) - C_n(X)| \tag{A1}$$

for a two-tailed test.

After evaluating the variable $D_{m,n}$, a figure of merit can be assigned. One defines

$$L(X) = (2\pi)^{1/2} X^{-1} \sum_{\nu=1}^{\infty} e^{-(2\nu-1)^2\pi^2/8X^2}\ , \tag{A2}$$

where $L(X)$ is the limiting cumulative distribution function of $n^{1/2}D_n$. If $D_{m,n}$ is the maximum of the difference $|C_m(X) - C_n(X)|$ between the empirical distribution of two samples of sizes m and n, respectively, then $L(X)$ is also the limiting cumulative distribution function of $(mn/(m+n))^{1/2}D_{m,n}$. Smirnov (1948) gives a table of $L(X)$ for various values of m, n as a function of $D_{m,n}$. $L(X)$ established in this way is large, near unity, when the distributions differ greatly. In this limit, $L(X)$ can be regarded as the probability that the two samples are not drawn from the same distribution, and $P \equiv 1 - L$ can be regarded as the probability that the two samples are drawn from the same distribution.

When comparing an observed distribution with a Monte Carlo model, the Kolmogorov-Smirnov statistic can be used to determine a best fit and to assign a figure of merit that can be interpreted as a probability that the two distributions agree. A standard deviation for this fit can also be assigned. According to Goodman (1954), the statistic

$$S = 4N(D_{m,n}^{\max})^2 , \tag{A3}$$

where $N = mn/(m+n)$, $D_{m,n}^{\max}$ is the Kolmogorov-Smirnov statistic (eq. [14]), and m, n are the two sample sizes, has a sampling distribution which is approximated by a χ^2 distribution with 2 degrees of freedom as $N \to \infty$. In our case, the values of m and n are sufficiently large for this to be a valid approximation. It follows that we can associate with any value of $D_{m,n}^{\max}$ a value S which is approximately distributed as a χ^2 distribution. Since the probability density function of the χ^2 distribution with 2 degrees of freedom is given by

$$P(\chi) = \frac{1}{2} \exp\left(-\frac{\chi^2}{2}\right) d\chi^2 , \tag{A4}$$

which is, therefore, the likelihood function for this problem, it follows that $S = -2\ln(\text{likelihood}) + \text{const}$. According to Meyer (1975), if S is a function of the data, X_i, and a set of parameters to be fit, a_i,

$$S = S(X_i; a_1, a_2, a_3 \ldots a_n) , \tag{A5}$$

then the components of the covariance matrix for the adjustment of the parameter fit to the data are the inverse of the corresponding Hessian matrix of second partials of $S/2$, or,

$$\sigma_{i,j} = \left[\frac{\partial^2(S/2)}{\partial a_i \partial a_j}\right]^{-1} . \tag{A6}$$

Therefore, if we consider the distribution of S near its minimum (the value we choose by minimizing D), then for small increments δa_i to a parameter a_i, we expect S to vary as

$$S = S_0 + \delta a^2/\sigma^2 , \tag{A7}$$

where σ^2 is the variance of δa_i. By solving this equation for σ^2 given S, S_0 and δa_i^2, we can in principle estimate σ^2. For some parameters of the present problem the value of $a = 1 - f$ which minimizes S is physically unrealistic (corresponding to a value of the fraction in the spiral arms, f, greater than 1). There is, however, an alternative approach to estimating σ^2, which is to consider the quantity $Q = S^{1/2}$, using equation (A7). A simple calculation shows that

$$dQ/da_i = (1 + S_0\, \sigma^2/\delta a_i^2)^{-1/2}/\sigma , \tag{A8}$$

so that as δa_i increases, the slope of $Q(X)$ approaches $1/\sigma$. Thus we plotted Q as a function of spiral arm fraction f and found that (except near $f = 1$) it gave very nearly a straight line, whose slope then yielded the required value of σ.

In a test case we also used a statistical test developed by Kuiper (1960) which is specifically developed to test angular distributions in a way that is independent of the starting point of the cumulative sum. The results were consistent with the K-S test.

REFERENCES

Bahcall, J. N., & Soneira, R. M. 1980, ApJS, 44, 73
Bash, F. N. 1981, ApJ, 250, 551
Biermann, P., & Tinsley, B. M. 1974, PASP, 86, 791
Blitz, H. S. 1982, ApJS, 49, 183
Burton, W. B. 1976, ARA&A, 14, 275
Caswell, J. L. 1985, AJ, 90, 1076
Caswell, J. L., & Barnes, P. J. 1985, MNRAS, 216, 753
Caswell, J. L., & Lerche, I. 1979, MNRAS, 187, 201
Chevalier, R. A. 1977, ARA&A, 15, 175
Clark, D. H., & Stephenson, F. R. 1977, The Historical Supernovae (New York: Pergamon)
Clark, D. H., & Caswell, J. L. 1976, MNRAS, 174, 267
Conti, P. S., & Vacca, W. D. 1990, preprint
de Vaucouleurs, G., & Pence, W. D. 1978, AJ, 83, 1163
Doggett, B. J., & Branch, D. 1985, AJ, 90, 2303
Evans, R., van den Bergh, S., & McClure, R. D. 1989, ApJ, 345, 752
Freeman, K. C. 1970, ApJ, 160, 811
Georgelin, Y. M., & Georgelin, Y. P. 1976, A&A, 49, 57
Goodman, L. A. 1954, Psych. Bull., 51, 160
Gordon, M. A., & Burton, W. B. 1976, ApJ, 208, 346
Gorham, P. W. 1990, ApJ, 364, 187
Green, D. A. 1984, MNRAS, 209, 449
———. 1988, Ap&SS, 148, 3
Green, D. A., & Gull, S. F. 1984, Nature, 312, 527
Habing, H. 1988, A&A, 200, 40
Hamilton, A. J. S., & Fesen, R. A. 1988, ApJ, 327, 178
Hamilton, A. J. S., et al. 1986, ApJ, 300, 698
Harkness, R. P., et al. 1987, ApJ, 317, 355
Helfand, D. J. 1984, ApJ, 277, 710
Helfand, D. J., & Becker, R. H. 1984, ApJ, 283, 154
———. 1987, ApJ, 314, 203
Helfand, D. J., Velusamy, T., Becker, R. H., & Lockman, F. J. 1989, ApJ, 341, 151
Huang, Y. L. 1987, PASP, 99, 461
Huang, Y. L., & Thaddeus, P. 1985, ApJ, 295, L13
———. 1986, ApJ, 309, 804
Ilovaisky, S. A., & Lequeux, J. 1972, A&A, 18, 169
Kodaira, K. 1974, PASJ, 26, 255
Knapp, G. R., Tremaine, S. D., & Gunn, J. E. 1978, AJ, 83, 1585
Kuiper, N. H. 1960, Nederl. Akad. Wetensch. Proc. Ser. A, 63, 38
Lewis, J. R., & Freeman, K. C. 1989, AJ, 97, 139
Li, Z. W., Wheeler, J. C., & Bash, F. 1984, in Stellar Nucleosynthesis, ed. C. Chiosi & A. Renzini (Dordrecht: Reidel), 49
Li, Z. W., Wheeler, J. C., Bash, F. N., & Jefferys, W. H. 1988, in IAU Colloquium 101, Interaction of Supernova Remnants with the Interstellar Medium, ed. R. S. Roger & T. L. Landecker (Cambridge: Cambridge Univ. Press), 485
Lyne, A. G. 1982, in Supernovae: A Survey of Current Research, ed. M. J. Rees & R. J. Stoneham (Dordrecht: Reidel), 405
Maza, J., & van den Bergh, S. 1976, ApJ, 204, 519

Mead, K. N., Kutner, M. L., & Evans, N. J. 1990, ApJ, 354, 492
Meyer, S. L. 1975, Data Analysis for Scientists & Engineers (New York: John Wiley & Co.), 324–330
Milne, D. K. 1979, Australian J. Phys., 32, 83
Nakai, N., & Sofue, Y. 1982, PASJ, 34, 199
Narayan, R., & Ostriker, J. P. 1990, ApJ, 352, 222
Nomoto, K., & Tsuruta, S. 1981, ApJ, 250, L19
Nugent, J. J., Jensen, K. A., Nousek, J. A., Garmire, G. P., Mason, K. O., Walter, F. M., Bouyer, C. S., Stern, R. A., & Reigler, G. R. 1983, ApJS, 51, 1
Porter, A. C., & Filippenko, A. V. 1987, ApJ, 93, 1372
Press, W. H., Flannery, B. P., Teukolsky, S. A., & Vetterling, W. T. 1986, Numerical Recipes (Cambridge: Cambridge Univ. Press), 472
Reich, W., et al. 1985, A&A, 151, L10
Schmidt, M. 1965, in Stars and Stellar Systems, Vol. 5, Galactic Structure, ed. A. Blaauw & M. Schmidt (Chicago: Univ. of Chicago Press), 513
Shull, J. M. 1980, ApJ, 237, 769
Smirnov, N. 1948, Ann. Math. Statist., 19, 280
Smith, L. F., et al. 1978, A&A, 66, 65
Strom, R. G. 1988, MNRAS, 230, 331
Swartz, D. A., Wheeler, J. C., & Harkness, R. P. 1991, ApJ, 374, 266
Tammann, G. A. 1974, in Supernovae and Supernova Remnants, ed. C. B. Cosmovici (Dordrecht: Reidel), 155
———. 1982 in Supernovae: A Survey of Current Research, ed. M. J. Rees & R. J. Stoneham (Dordrecht: Reidel), 371
van den Bergh, S. 1988, JRASC, 82, 179
van der Kruit, P. C. 1986, A&A, 157, 230
van der Kruit, P. C., & Searle, L. 1982, A&A, 110, 61
Weiler, K. W. 1983, Observatory, 103, 85
Wheeler, J. C. 1978, ApJ, 225, 212
———. 1990, in Supernovae, ed. J. C. Wheeler, T. Piran, & S. Weinberg (Singapore: World Scientific), 1
———. 1991, in Frontiers of Stellar Evolution, ed. D. L. Lambert (San Francisco: ASP), in press
Wheeler, J. C., Mazurek, T. J., & Sivaramakrishnan, A. 1980, ApJ, 237, 781