# MULTICHANNEL ASTROMETRIC PHOTOMETER-BASED PARALLAXES OF EVOLVED STARS: $\chi$ CYGNI, 51 ANDROMEDAE, AND OP ANDROMEDAE 

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#### Abstract

Using data from Allegheny Observatory's Thaw/MAP astrometric system, absolute parallaxes are presented for $\chi$ Cygni ( $8.8 \pm 1.9$ mas), 51 Andromedae ( $15.5 \pm 1.3$ ), the newly discovered RS CVn variable OP Andromedae ( $6.1 \pm 1.5 \mathrm{mas}$ ), and selected stars in these fields. Using existing angular diameter measures and the new parallaxes, linear diameters for $\chi$ Cygni and 51 And are given. $\chi$ Cygni has a perturbation in declination having a period of 0.75 yr and an amplitude of $\pm 5 \mathrm{mas}$. Owing to the extreme variability of $\chi$ Cygni, no further characterization of the system is possible at this time. Other results of special interest include the detection of a perturbation in the motion of $\mathrm{BD}+47^{\circ} 466$. This system was found to have a parallax of $14.4 \pm 1.2$ mas. Subject headings: astrometry - stars: diameters - stars: late-type


## 1. INTRODUCTION

The Multichannel Astrometric Photometer (MAP)/Thaw long-focus astrometric system has been in regular use at Allegheny Observatory since 1986 (see Gatewood et al. 1991). The system reaches a precision of several thousandths of an arcsecond (mas) per observation (Gatewood et al. 1988) and is, therefore, capable of carrying out parallax studies at considerably greater distances than its photographic predecessors. The results presented here, 51 Andromedae ( $\pi=15.5$ mas $\pm 8 \%$ ), $\chi$ Cygni ( $\pi=8.8$ mas $\pm 21 \%$ ), OP Andromedae ( $\pi=6.1$ mas $+24 \%$ ) and BD $+47^{\circ} 466 \quad(\pi=14.4$ mas $\pm 8 \%$ mas $)$ underscore that fact.

The reductions of the raw MAP data to astrometric positions were carried out by the procedures currently standard at Allegheny (see Gatewood 1987). The transformation of these to star constants was by use of the central overlap technique (Gatewood \& Eichhorn 1973). One particularly attractive feature of the latter technique is that, because each reference star is subject to the same analysis as is the target object, star constants (including parallax) are produced for both the target and reference stars. This has an important advantage in addition to the obvious one of finding nearby stars among the reference objects (e.g., BD $+47^{\circ} 466$ in the 51 And field). It provides all the astrometric information necessary for the reduction (and future improvement) of the relative parallaxes to absolute (see next section).

The basic data for the correction to absolute parallax (in addition to some available in the literature) came from two sources. Photometry in UBVRI, the DDO system, and in uvby and $\mathrm{H} \beta$ systems were obtained from Castelaz \& Persinger (1989) and Persinger \& Castelaz (1990) whose data were obtained using the photometric system of the No. 2, 36 inch ( 0.9 m ) telescope at KPNO. Bruce Stephenson of the Warner \& Swasey Observatory of Case Western Reserve University, using $10^{\circ}$ objective prism plates obtained with Warner \& Swasey's Burrell Schmidt telescope ( $108 \AA \mathrm{~mm}^{-1}$ at $\mathrm{H} \gamma$ ), provided MK spectral classifications.

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### 1.1. The Method of Adjustment to Absolute Parallax

The method by which the MAP relative parallaxes have been adjusted to absolute has been evolving. That trend continues with this paper. Rather than simply discussing the new features, in the interest of clarity, we will review the whole process.
As pointed out by Gatewood (1989), because no constraint is placed upon the weighted mean parallax in the star constant solution, the system of equations converge upon relative rather than absolute parallaxes such that the sum of the weighted mean parallaxes of all the reference stars is zero. The difference between the mean absolute parallax (i.e., the true parallax) of the reference stars and their weighted mean relative parallax has been traditionally called the "correction to absolute," for by adding this constant to the relative parallax of each reference star the star's absolute parallax is obtained.

Clearly, an estimate of the correction to absolute may be obtained using any reference star whose absolute parallax is already known (here referred to as a "parallax standard"). That estimate is given by

$$
\begin{equation*}
\Delta_{\pi}=\pi_{a}-\pi_{r} \tag{1}
\end{equation*}
$$

where $\pi_{a}$ is the absolute parallax and $\pi_{r}$ is the relative parallax. If $N$ parallax standards are available, an even better estimate of the correction to absolute would be the mean of the $N$ individual estimates. Thus, if $\sigma$ is the standard error associated with an individual correction, the standard error of the mean of $N$ estimates is $\sigma /(N-1)^{1 / 2}$. This is basically the adjustment process used through 1989.

With this paper and Gatewood et al. (1990), the individual corrections to absolute are weighted and the weighted mean of the $N$ corrections is taken as the correction to absolute. The weight assigned to the $i$ th parallax standard is

$$
\begin{equation*}
W_{i}=\left(\frac{1}{\sigma_{i}}\right)^{2} \tag{2}
\end{equation*}
$$

where $\sigma_{i}$ is the estimated error in the correction to absolute obtained by propagating the errors in $\pi_{a}$ and $\pi_{r}$ through equation (1):

$$
\begin{equation*}
\sigma^{2}=\sigma_{a}^{2}+\sigma_{r}^{2} \tag{3}
\end{equation*}
$$

Here $\sigma_{a}$ and $\sigma_{r}$ are, respectively, the errors in the absolute and in the relative parallaxes. The error in the relative parallax (obtained from the star constant solution) varies somewhat from star to star, but that associated with the absolute parallax can vary substantially as will now be shown.

Spectroscopic and photometric data (specifically MK classifications and $U B V R I$ photometry) have been used to derive absolute parallaxes for the parallax standards. From these data the absolute visual magnitude ( $M_{v}$ ) and the visual extinction $A_{v}$ are determined. A "spectroscopic" parallax is then calculated using

$$
\begin{equation*}
\pi_{a}=10^{0.2\left(M_{v}-V-5+A_{v}\right)} \tag{4}
\end{equation*}
$$

Propagating the errors in $M_{v}, V$, and $A_{v}$ through equation (4), it is seen that the error in the absolute parallax is

$$
\begin{equation*}
\sigma_{\pi}=0.46 \pi_{a}\left(\sigma_{M_{v}}^{2}+\sigma_{V}^{2}+\sigma_{A_{v}}^{2}\right)^{1 / 2}, \tag{5}
\end{equation*}
$$

where $\sigma$ denotes the error in the subscripted quantity. Castelaz \& Persinger (1989) have shown that the errors in both $V$ and $A_{v}(0.03 \mathrm{mag})$ are significantly smaller than that associated with $M_{v}$. The error in $M_{v}$ generally ranges from 1.5 mag down to 0.1 mag depending upon spectral type and luminosity class of the parallax standard. Hence, to necessary accuracy, equation (5) becomes

$$
\begin{equation*}
\sigma_{\pi}=0.46 \pi_{a} \sigma_{M_{v}} \tag{6}
\end{equation*}
$$

While Castelaz \& Persinger were discussing absolute parallaxes derived from their $U B V R I$ and DDO photometry, the same conclusion applies to absolute parallaxes derived from $U B V$ photometry and MK classification through the visual inspection of spectrograms.

Because $\sigma_{\pi}$ is proportional to $\pi$, the contribution of $\sigma_{\pi}$ to $\sigma$ and, hence, to the weight in equation (2) is usually negligible. Thus, since $\sigma_{r}$ varies little from star to star, weighting would appear to make little sense. However, for less distant parallax standards, $\sigma_{\pi}$ can dominate $\sigma$ especially for red evolved stars where $\sigma_{M_{v}}$ can reach 1.5 mag and more (Castelaz \& Persinger 1989). In this regime, improper weighting can significantly alter the mean correction to absolute (e.g., the difference is about 2 mas for the $\chi$ Cygni field).

A further improvement employed here is the use of a consistency check on the absolute parallaxes, one first used by Persinger \& Castelaz (1990). This check has proven especially useful in the $\chi$ Cygni field for detecting gross errors in individual parallax standard distance estimates. The reader is referred to Persinger \& Castelaz for details, but basically the technique involves preparing a plot of parallax standard distance versus $A_{v}$. To the extent that absorbing material is uniformly distributed along the line of sight to the star field, it is expected that all points will fall near a line passing through $(0,0)$ and having a slope equal to the mean extinction per unit distance in the line of sight. A gross outlier likely indicates an incorrect absolute parallax (see the discussion of the $\chi$ Cygni field for examples).

It should be noted that this is ultimately a convergent technique. As our knowledge of $M_{v}$ as a function of MK type improves as parallax studies of target stars having many different MK types accumulate, it will eventually become possible to reanalyze the original observations using the resulting improved estimates of $M_{v}$ to obtain even better estimates of the distance to each parallax standard in each individual region. This, in turn, results in better corrections to absolute in each region and hence to a better estimate of each target star paral-
lax. The end result will be an even better knowledge of the relationship between $M_{v}$ and MK type. Repetitions of the cycle soon converge upon an internally consistent set of target star absolute parallaxes.

## 2. $\chi$ CYGNI

$\chi$ Cygni $\left(=\mathrm{BD}+32^{\circ} 3593=\right.$ HD 187796), discovered to be variable by Gottfried Kirch in 1686, was the third variable star discovered, following Algol and Mira Ceti. The spectrum of this Mira variable has been variously classified as K0 III, Se, S7.1e, S10.1e (Jaschek, Horacio, \& Sierra 1964) and S5 (Buscombe 1977). The luminosities of S-type Mira variables have proved hard to obtain with great accuracy. Their low number density in the Galaxy, and resulting great distances, has hampered both direct trigonometric measures of their individual parallaxes and statistical estimates through reflected solar motion (see, e.g., Stephenson 1978 and Yorka \& Wing 1979). Obtaining accurate parallaxes for Mira variables by traditional long-focus photographic astrometry has also been hampered by the extreme variability of these objects.
The nonlinear photometric response of the photographic plate coupled with guiding errors combine to yield observed positions that are a function of stellar magnitude. $\chi$ Cygni is especially annoying in this respect, for its light curve has an unusually large amplitude, its mean range in $m_{v}$ being 5.3-13.3 Merrill (1947) with reported extremes of +2.3 and 14.3 (Merrill 1953). The MAP, however, is well suited for astrometric studies of Mira variables, for its linear photometric response eliminates the need for magnitude terms in the field model. This fact is demonstrated in the residual versus magnitude plot of Figure 3 (to be discussed at greater length in a later section). Here, over a range of over 6 mag (5.5-12), $\chi$ Cygni's right ascension residuals show no sign of correlation with observed magnitude.
$\chi$ Cygni was placed upon the MAP parallax program prior to the 1986 season at the suggestion of Diane Turnshek, then on staff at Allegheny.

### 2.1. The Effect of Atmospheric Dispersion

Atmospheric dispersion poses yet another problem for astrometrists studying Mira variables. These stars show pronounced changes in both their absorption and emission spectra which correlate with phase in their light cycles (see, e.g., Merrill 1947, 1955). The interaction between changes in the star's intrinsic color and the wavelength dependency of atmospheric refraction will cause the observed position of the Mira to move with respect to the reference frame along a vertical circle passing through the star's true position. The magnitude of the observed shift is zero at the zenith but will grow with increasing zenith distance. Of course, the effect is minimal if very narrow bandpasses are used. But even with wider bandpasses, the effect could be removed with a field model featuring appropriate color terms. Unfortunately, the mean colors in the system bandpass of both the Mira and the reference stars must first be known, and that information is rarely, if ever, available.

In the case of MAP, the severity of this problem is greatly reduced by the MAP's relatively narrow bandpass ( $6425 \pm 250$ $\AA$, nominal). Nevertheless, care must be taken to ensure that atmospheric color effects remain acceptably small, for the MAP bandpass includes the strongly variable $\mathrm{H} \alpha$ emission line (see, e.g., Derviz \& Savanov 1978), and the TiO bands (the dominant absorption feature in the bandpass) change in
strength with the cyclic changes in the Mira's surface temperature.

A rough idea of the magnitude of the variable color-induced shift can be obtained as follows. Following Green (1985), the magnitude of the differential color effect in mas $\AA^{-1}$ is approximately

$$
\frac{d R}{d \lambda}=\left\{\frac{1.21 \times 10^{7} P}{\left[\lambda^{3}(273+T)\right]}\right\} \tan z
$$

where $P$ is the atmospheric pressure in millimeters of $\mathrm{Hg}, T$ is the temperature in ${ }^{\circ} \mathrm{C}, z$ is the zenith distance, and $\lambda$ is the wavelength in $\AA$. Because $\chi$ Cygni (decl. $+32^{\circ} 55^{\prime}$ ) passes within $8^{\circ}$ of the zenith as seen from Allegheny (latitude $+40^{\circ} 28^{\prime} 58^{\prime \prime}$ ), differential color refraction is greatly reduced. For average $\chi$ Cygni observing conditions at Allegheny ( $P=730, T=20$ ), at the center of the MAP bandpass ( $6425 \AA$ ) with $\chi$ Cygni on the meridian, $d R / d \lambda=0.015 \mathrm{mas} \AA^{-1}$. $\mathrm{H} \alpha$, at $6562 \AA$, is only $137 \AA$ from the center of the bandpass, and at maximum light (when $\mathrm{H} \alpha$ is strongest) the continuum appears to contribute at least as much light as does the $\mathrm{H} \alpha$ line (see, e.g., representative spectrograms in Merrill 1940) further reducing the ability of $\mathrm{H} \alpha$ to effect the mean color. However, even assuming that the mean color shifts by the full distance of $\mathrm{H} \alpha$ from the center of the bandpass ( $137 \AA$ ) the differential color effect is only about 2 mas.

Small as the estimated effect of atmospheric dispersion is, it is still significant compared to the expected parallax effect (order 10 mas ). Fortunately, the strongest parallax signal is in right ascension, while the differential color effect (near the meridian) is predominantly in declination. Further, because the component in right ascension changes sign as the star crosses the meridian, the observed $x$-parallax should be systematically unaffected by differential color refraction. Further improvements could be realized by avoiding observations made when the emission spectrum is strong (near maximum light). These predictions are amenable to testing with the data at hand and appear to be correct.

Consider the effect of atmospheric dispersion upon the observed right ascension of a target star whose effective wavelength (in the system's bandpass) is shorter than the effective wavelength of the reference stars used to define the coordinate frame (all assumed to be of the same color for simplicity). Because atmospheric dispersion displaces the target's observed position toward the zenith, its effect in right ascension will be to displace the target toward the meridian. Thus when observing to the east of the meridian the $O-C$ residuals in right ascension will be negative, and when observing to the west, they will be positive. Further, because the effect is proportional to the tangent of the zenith distance and because observations are made near the meridian, the magnitude of the residuals will increase nearly linearly with hour angle. Clearly, a target whose effective wavelength is longer than that of the reference stars will behave similarly except that the signs of the residuals on either side of the meridian are reversed. In both cases, the magnitude of the displacement at a given hour angle will be proportional to the wavelength difference between effective color of the target and that of the reference frame.

Figure 1 shows the expected right ascension displacements as a function of hour angle for $\chi$ Cygni assuming first its mean wavelength is $150 \AA$ shorter than the mean of the reference stars (dashed line) and then that it is $150 \AA$ longer (solid line).

Now consider the situation in which the target's effective


Fig. 1.-Expected displacements in right ascension as a function of hour angle for $\chi$ Cygni due to atmospheric dispersion under the assumption that (solid line) its effective wavelength is longer than that of the mean of the reference stars by $150 \AA$ and then under the assumption (dashed line) that it is $150 \AA$ shorter.
wavelength varies with time between the two extremes. If the errors of measurement are small with respect to the atmospheric displacements, the observed residuals would be expected to lie within the area bound by the dashed and solid lines of Figure 1. The slope of the area boundaries would yield an estimate of the range in target color. If the errors of measurement are not small with respect to the displacements (the case here), the bow tie figure may not be apparent. In this case, the variance associated with a position measurement $V_{m}$ is given by the variance of the residuals within the bin centered on hour angle $=0$. The variance of the residuals within selected hour angle bins (e.g., within the boxed area centered on $-50^{\mathrm{m}}$ in Fig. 1), $V_{t}$ includes $V_{m}$ and the variance due to the full range of color variation, $V_{c}$. The value of $V_{c}$ is then given by $V_{t}-V_{m}$. In either event, the mean color difference between the target and frame can be estimated from the slope ( $m$ ) of the best-fit line:

$$
V_{x}=m H,
$$

where $V_{x}$ is the right ascension residual and $H$ is the hour angle.

### 2.2. The Data Set

In all there exist 76 MAP observations obtained on 63 different nights spanning the epochs 1986 June 9 to 1990 June 30. Preliminary to the final reductions, this data set was used to obtain the star constants position, proper motion, and parallax.

Figure 2 shows the right ascension residuals plotted against mean hour angle of the east/west scans. The solid line is a linear fit to the data and has a slope of only $-0.4 \mathrm{mas} \mathrm{hr}^{-1}$ indicating that the mean color of $\chi$ Cygni is very nearly equal to the mean color of the reference frame. The standard deviation of the residuals contained in the $\pm 10$ minute hour angle bins centered on $-40^{\mathrm{m}},-20^{\mathrm{m}}, 0,+20^{\mathrm{m}}$, and $+40^{\mathrm{m}}$ are, respectively, $7.5,5.5,5.8,6.7$, and 7.4 mas. If the standard deviation of the hour angle $=0$ bin is representative of the actual measuring error and the increased dispersion in the hour angle $= \pm 40^{\mathrm{m}}$ bins is real, then a variable color-induced random displacement (due to atmospheric dispersion) in the observed positions of $\pm 5.1$ mas at hour angle $= \pm 40^{\mathrm{m}}$ is


Fig. 2.-For the full $\chi$ Cygni data set, the residuals in right ascension to a star constant solution for position, proper motion, and parallax plotted against the mean hour angle of the right ascension observation. The line is a least-squares fit to all the data and has a slope of $\mathbf{- 0 . 4}$ mas $\mathrm{hr}^{-1}$. See text for discussion.
implied. This, however, is probably a slight overestimate, for it does not take into account the effect of degraded seeing with increasing zenith distance nor does it allow for the resulting mismatch of seeing disk size with ruling line spacing (see Gatewood 1987), both of which tend to increase the random error.

In Figure 3 the resulting residuals are plotted against observed stellar brightness (obtained from the same MAP data). For magnitudes brighter than 5.2 in right ascension, there appears to develop a trend toward negative residuals of order 7 mas (there might be a similar trend for magnitudes fainter than 11.5 ; however, this is much less certain owing to the rapidly increasing random error in the measured positions with the onset of photon starvation). The trend at brighter magnitudes appears too strong to result from atmospheric dispersion alone and may indicate that instrumental color effects are at work as well (though it is worth noting that all but one of the bright and one of the faint observations were taken at hour angles in excess of $20^{\mathrm{m}}$ indicating that the trend at brighter


Fig. 3.-For the full $\chi$ Cygni data set, the residuals in right ascension to a star constant solution for position, proper motion, and parallax plotted against the target's observed magnitude (relative to AO 758, taken to be of magnitude 7.6 in the Thaw/MAP bandpass). Note the tendency toward negative residuals when the magnitude of $\chi$ Cygni exceeds 5.2.
magnitudes may have been exaggerated somewhat by atmospheric dispersion).

In order to control atmospheric dispersion and any possible instrumental color effects, a subset of the full data set was finally selected for this study which included only those observations for which the mean hour angle of the two R.A. scans was $20^{m}$ or less (atmospheric dispersion random error component: $\pm 2.5$ mas) and for which the magnitude of $\chi$ Cygni lay between 5.2 and 11.5 (removing those observations suspected of showing instrumental color effects). That left 43 MAP observations obtained on 40 separate nights covering the interval 1986 June 9 to 1990 June 30. Much of the attrition came from observations that were intended to be one of a pair that straddled the meridian, but for one reason or another, the companion observation was not available (clouded out, failed in prereduction due to wind shake, poor seeing, electronic problems, observer error, etc.) though some observations simply exceeded the $20^{\mathrm{m}}$ limit.
The $20^{m}$ hour angle restriction, while somewhat arbitrary, basically was chosen to hold the differential color effect to a minimum in right ascension while maintaining a large enough set of observations to obtain a meaningful parallax.

### 2.3. Star Constant Solution

The results for the $\chi$ Cygni region appear in Table 1, stars 757-765. $\chi$ Cygni is star 763. The standard errors on the zeroepoch position and the proper motions apply to the approximate midepoch of the observations, J1988.0. For reasons discussed earlier and because the right ascension residuals appear free of certain systematic trends found in the declination residuals (see next section), the parallax quoted for $\chi$ Cygni in Table 1 is that obtained from the R.A. scans alone. The quoted parallaxes are absolute, having been obtained by adding to the relative parallaxes an adjustment factor of 4.2 mas (see next section), the weighted mean correction based upon the spectroscopic and relative parallaxes of the reference stars excluding stars 759,760 , and 765 (see below).
Residuals to an intermediate-stage solution for position and proper motion for star $760\left(=\mathrm{BD}+32^{\circ} 3589\right)$ show signs of a perturbation in right ascension (see Fig. 4). Accordingly, it is not used as a reference star in this study. The star constants (except for the parallax) appearing in Table 1, being obtained from a model allowing only for position and proper motion, represent the mean motion of the object. The parallax is obtained from a separate solution fitting a star constant model of the form

$$
\begin{align*}
& \xi(t)=\xi_{0}+\mu_{\xi} t+\pi P_{\xi}+A t^{2} \\
& \eta(t)=\eta_{0}+\mu_{\eta} t+\pi P_{\eta}+B t^{2} \tag{7}
\end{align*}
$$

Because this object lies near the extreme southern border of the MAP field, its calculated positions are rather sensitive to slight changes in the reference frame caused by the interaction between small random errors in the adopted reference star constants and observations in which one or more reference stars were not observed. Accordingly, the solution for this object's star constants used only observations for which all reference stars were observed. The quadratic terms were found to be $A=0.00278 \pm 0.00073 \mathrm{yr}^{-2}$ and $B=-0.00101$ $\pm 0.00069 \mathrm{yr}^{-1}$.

### 2.4. Adjustment to Absolute Parallax

Many of the photometric data upon which the following adjustment rests were obtained from Castelaz \& Persinger

TABLE 1
Star Constants

| AO <br> Number | d | $\underset{(\mathrm{mag})}{V}$ | $E(B-V)$ | $\begin{gathered} \pi \\ \text { (mas) } \end{gathered}$ | RA (2000) |  | $\begin{gathered} \text { Proper Motion (RA) } \\ \left(\mathrm{s} \mathrm{yr}^{-1}\right) \end{gathered}$ | Decl. (2000) | Proper Motion (decl.) $\left(\operatorname{arcsec} \mathrm{yr}^{-1}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 757 | 2 | 9.20 | 1.64 | 4.0 | $19^{\text {h }} 48^{\mathrm{m}} 50.96529$ |  | . 000212 | $32^{\circ} 40^{\prime} 20 \prime 0514$ | . 00222 |
|  |  |  |  | 1.3 |  | . 00005 | . 000031 | . 0010 | . 00040 |
| 758 | 2 | 7.63 | 1.40 | 4.6 | 1949 | 3.36459 | . 000569 | 324757.2344 | . 01081 |
|  |  |  |  | 1.1 |  | . 00003 | . 000021 | . 0007 | . 00028 |
| 759 | 2 | 9.74 | 1.22 | 5.5 | 1949 | 40.92480 | -. 000820 | 325325.6644 | -. 00081 |
|  |  |  |  | 1.4 |  | . 00006 | . 000039 | . 0010 | . 00044 |
| 760 | 2 | 8.14 | -0.01 | 1.6 | 1950 | 4.07929 | . 000324 | 323824.6272 | -. 01096 |
|  |  |  |  | 2.2 |  | . 00008 | . 000054 | . 0017 | . 00070 |
| 761 | 2 | 9.20 | 0.23 | 5.9 | 1950 | 8.22357 | . 000280 | 33811.6220 | . 02923 |
|  |  |  |  | 1.1 |  | . 00003 | . 000023 | . 0007 | . 00029 |
| 762 | 2 | 10.26 | 1.58 | 0.4 | 1950 | 8.97963 | -. 000158 | 325824.2532 | . 01215 |
|  |  |  |  | 1.4 |  | . 00005 | . 000033 | . 0010 | . 00042 |
| 763 | 2 | Var | Var | 8.8 | 1950 | 33.86404 | $-.002070$ | 325450.7490 | -. 03380 |
|  |  |  |  | 1.9 |  | . 00007 | . 000048 | . 0032 | . 00066 |
| $764 \ldots \ldots \ldots$. | 2 | 8.80 | 0.37 | 3.3 | 1951 | 29.07809 | . 004027 | $\begin{array}{lll}33 & 6 & .5180\end{array}$ | . 03204 |
|  |  |  |  | 1.3 |  | . 00006 | . 000043 | . 0010 | . 00051 |
| 765 | 2 | 10.39 | 1.19 | 5.8 | 1951 | 30.08572 | . 000099 | 325238.0045 | -. 00877 |
|  |  |  |  | 1.2 |  | . 00005 | . 000035 | . 0009 | . 00041 |
| $816 \ldots \ldots \ldots$. | 2 | 5.92 | 1.20 | 6.1 | 136 | $27.17169$ | -. 001904 | $484322.4586$ | $-.01321$ |
|  |  |  |  | 1.5 |  | $.00002$ | $.000025$ | $.0003$ | $.00024$ |
| $817 \ldots \ldots \ldots$. | 2 | 8.78 | 0.17 | 0.5 | 137 | 9.49234 | . 000408 | 484242.9950 | $-.00682$ |
|  |  |  |  | 0.9 |  | . 00005 | . 000051 | $.0006$ | $.00051$ |
| 818 | 2 | 6.76 | $\ldots$ | 14.4 | 134 | 40.07632 | . 016217 | $48 \quad 929.6935$ | . 00397 |
|  |  |  |  | 1.2 |  | . 00009 | . 000067 | $.0010$ | . 00065 |
| $819 \ldots \ldots \ldots$. | 2 | 3.57 | 1.28 | 15.5 | 137 | $59.50911$ | . 005858 | 483741.1305 | -. 11705 |
|  |  |  |  | 1.3 |  | $.00007$ | . 000073 | $.0009$ | $.00071$ |
| $820 \ldots \ldots \ldots$. | 2 | 10.00 | 0.60 | 6.3 | 138 | $13.93660$ | . 002614 | 482311.0964 | $.03209$ |
|  |  |  |  | 0.9 |  | $00005$ | . 000052 | $.0006$ | . 00052 |
| $821 \ldots \ldots \ldots$. | 2 | 10.02 | 0.14 | 0.1 | 138 | 37.81906 | . 000410 | 482817.9309 | $-.01373$ |
|  |  |  |  | 1.1 |  | . 00005 | . 000063 | $.0007$ | $.00060$ |
| $822 \ldots \ldots \ldots$. | 2 | 9.04 | 0.05 | 2.4 | 138 | 42.09070 | $-.001864$ | $485417.7489$ | $-.00726$ |
|  |  |  |  | 0.9 |  | . 00005 | . 000050 | $.0006$ | $.00050$ |
| $823 \ldots \ldots \ldots$. | 2 | 10.45 | 1.21 | -2.4 | 138 | 53.39951 | -. 001012 | $4841 \quad 6.9576$ | $-.00490$ |
|  |  |  |  | 1.3 |  | . 00007 | $.000073$ | $.0009$ | $.00072$ |
| $824 \ldots \ldots \ldots \ldots$ | 2 | 9.32 | -0.03 | 0.4 | 139 | 51.59708 | $-.001767$ | $484316.9655$ | -. 01358 |
|  |  |  |  | 0.9 |  | . 00004 | . 000047 | $.0006$ | . 00046 |
| 825 | 2 | 10.12 | 0.57 | 6.6 | 140 | 1.20715 | -. 001526 | $484815.8829$ | $-.03805$ |
|  |  |  |  | 1.1 |  | . 00006 | . 000062 | $.0008$ | $.00060$ |

Notes.-All standard errors, for example those of the positions, are strictly internal and do not allow for the zero-point errors of the reference system. The positional epoch for these star constants is J1988.0 (the approximate central epoch of the observations) while the orientation epoch is J2000. Column "d" denotes the device used to gather the astrometric data. A blank indicates photographic plates measured on either the USNO SAMM or on a MANN measuring machine, " 1 " indicates plates measured on the AO Theiss machine and 2 indicates that the data were obtained with the MAP. Star 763 is $\chi$ Cygni, 816 is OP Andromedae, and 819 is 51 Andromedae. Quoted parallaxes are absolute. For stars 757-765, the correction to absolute parallax was 4.2 mas and the precession for +50 yr , at the target object, is 1 m 924 in R.A. and 7.69 in declination. For stars $816-825$, the correction to absolute parallax was 4.3 mas and the precession for +50 yr at AO 819 is 3.076 in R.A. and 15.25 in decl. The star constants for AO 818 result from a separate adjustment allowing for acceleration (see text).


Fig. 4.-(a) Residuals in right ascension for AO 760 resulting from an adjustment for star constants: position, proper motion, and parallax. Each data point represents one Thaw/MAP observation. (b) Residuals in declination for AO 760.

TABLE 2
Auxiliary Data for the Stars of the $\chi$ Cygni Field

| Number | BD | $\begin{gathered} \pi_{a} \\ \text { (mas) } \end{gathered}$ | $\stackrel{\sigma_{a}}{(\mathrm{mas})}$ | $\underset{\text { Class }}{\text { MK }}$ | $V$ | $A_{v}$ | $M_{v}{ }^{\text {a }}$ | $\sigma_{M v}{ }^{\text {a }}$ | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 757 | $+32^{\circ} 3574$ | 1.6 | 1.6 | M1 III | 9.22 | 0.77 | -0.3 | 1.9 | 1, 2, 3 |
| $758 . . . . . . . . . .$. | + $32^{\circ} 3578$ | 3.4 | 3.0 | K3 III | 7.51 | 0.15 | 0.0 | 1.5 | 1, 4 |
| $759 . . . . . . . . . .$. | $+32^{\circ} 3586$ | 1.3 | 0.9 | K2 III | 9.74 | 0.26 | +0.3 | 1.5 | 1, 5 |
| $760 . . . . . . . . . .$. | + $32^{\circ} 3589$ | 0.7 | $\cdots$ | B1 Vnn | 8.14 | 0.84 | -3.5 | $\ldots$ | 6 |
| $761 . . . . . . . . . .$. | $+32^{\circ} 3590$ | 4.9 | 1.0 | A7 V | 9.21 | 0.15 | +2.5 | 0.3 | 2, 7, 8 |
| $762 . . . . . . . . . .$. |  | 1.1 | 0.7 | K4 III | 10.24 | 0.65 | -0.1 | 1.5 | 1,2 |
| 763 ............ | + $32^{\circ} 3593$ | ... | ... | S | Var | ... | $\ldots$ | ... | $\chi$ Cyg |
| $764 . . . . . . . . .$. | $+32^{\circ} 3600$ | 5.5 | 1.3 | F0 V-IV | 8.81 | 0.21 | +2.3 | 0.4 | 2, 7 |
| $765 . . . . . . . . . .$. | ... | 1.0 | 0.7 | K2 III | 10.32 | 0.11 | +0.2 | 1.5 | 1, 2, 5 |

${ }^{\text {a }}$ Unless otherwise noted, listed values of $M_{v}$ and some values of $\sigma_{M_{v}}$ were derived from data in Allen 1973.
Notes.-(1) MK type and $A_{v}$ by Castelaz \& Persinger 1990. (2) Photometry from Castelaz \& Persinger 1989. (3) Castelaz \& Persinger 1990 also gives K4 I but regards this as inconsistent with the observed value of $A_{v}$. (4) $U B V$ photometry by Hopp 1979. Kukarkin et al. 1981 list as suspected irregular variable, but five seasons of MAP photometry show no evidence for variations in excess of 0.4 mag . (5) Absolute parallax not consistent with observed $A_{v}$. (6) Astrometry shows indication of duplicity. MK types: B0.5 V, Roman 1978 and B2 Vnn Stephenson 1988a. (7) MK type by Stephenson 1988a. (8) Buscombe 1977 gives A5 III, but Stephenson 1988 rules out this luminosity class.
(1989). In the interim they have made significant improvements in their algorithms (see Persinger \& Castelaz 1990). Using the new algorithms, they have reanalyzed the data for the $\chi$ Cygni field and made the results available for use here. These results supercede those in the original paper.

Auxiliary information for each star is given in Table 2 (see table notes for sources). For parallax standards, the absolute parallaxes, their associated errors, and the values of the quantities from which they were derived are included as well.

The calculations leading to the weighted mean correction to absolute parallax were performed as described earlier, and the results are summarized in Table 3.

### 2.5. Interstellar Extinction

In Figure 5 are plotted observed visual extinction versus distance for all the stars in the $\chi$ Cygni field, save the target itself. The crosses represent the parallax standards, and the corresponding distances are from the absolute parallaxes of Table 1. The solid line is a least-squares fit to these five data points. No attempt was made to constrain the line to pass through the origin (as it should). The fact that the $A_{v}$ intercept is only 0.03 mag is reassuring. A least-squares fit forcing the line to pass through the origin yields a slope of $0.87 \pm 0.15$ $m a g \mathrm{kpc}^{-1}$ with a standard error of unit weight in $A_{v}$ of 0.17 mag. This is a value considerably lower than the mean value for the Galactic plane of $1.9 \mathrm{mag} \mathrm{kpc}^{-1}$ given in Allen (1973). It does, however, agree with the value obtained by Fitzgerald (1968) in his study of extinction along the Galactic plane. His

TABLE 3
Summary of the Calculations Leading to the Weighted Mean Correction to Absolute in the $\chi$ Cygni Field

| Number | $\pi_{r}$ <br> (mas) | $\sigma_{r}$ <br> (mas) | $\pi_{a}$ <br> (mas) | $\sigma_{a}$ <br> (mas) | $\Delta_{\pi}$ <br> (mas) | $\sigma_{\Delta}$ <br> (mas) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $757 \ldots \ldots \ldots \ldots$ | -0.2 | 1.1 | 1.8 | 1.6 | 2.0 | 1.9 |
| $758 \ldots \ldots \ldots \ldots$ | 0.4 | 0.8 | 3.4 | 3.0 | 3.0 | 3.1 |
| $761 \ldots \ldots \ldots \ldots$ | 1.7 | 0.8 | 4.9 | 0.7 | 3.2 | 1.1 |
| $762 \ldots \ldots \ldots \ldots$ | -3.8 | 1.2 | 1.1 | 0.7 | 4.9 | 1.4 |
| $764 \ldots \ldots \ldots \ldots$ | -0.9 | 1.0 | 5.5 | 1.0 | 6.4 | 1.4 |
| $\left\langle\Delta_{\pi}\right\rangle \ldots \ldots \ldots$ | $4.2 \pm 0.7$ mas |  |  |  |  |  |
| $\sigma_{1} \ldots \ldots$ | 1.1 mas |  |  |  |  |  |

Figure 5 shows that out to 500 pc and in the direction of $\chi$ Cygni ( $l^{\prime \prime}=68^{\circ}, b^{\prime \prime}+3.2$ ), $E(B-V)$ ranges from 0.1 to 0.2 . Assuming a ratio of total to selective absorption of 3.1 (Massa \& Savage 1988), in magnitudes per kpc, $A_{v}$ becomes $0.6-1.2$, in good agreement with the value found here.

As mentioned earlier, stars 759 and 765 were not used as parallax standards because their observed $A_{v}$ was not consistent with their MK classification. Their classification is largely based upon DDO photometry. Persinger (1990) has pointed out that their dereddened C4547, C4245, and C4142 colors place them at points in the C4547-C4245 and C4142-C4245 planes where stars of class K2 III and K2 IV are to be found (see Figs. 2 and 3 of McClure \& Forrester 1981), making their luminosity classification rather uncertain. For the above


Fig. 5.-Observed visual extinction vs. distance for the stars in the $\chi$ Cygni field except $\chi$ Cygni itself. The regression line is fitted to the stars marked with crosses. No attempt was made to constrain the line to pass through the origin. So constraining the line results in a mean line-of-sight extinction of $0.87 \pm 0.15$ $\mathrm{mag} \mathrm{kpc}{ }^{-1}$. The boxes represent Castelaz \& Persinger's (1990) initial values for AO 759 and 765. That they appear to have far too little extinction for their estimated distances indicates an error in their assigned luminosity classes (see text). The asterisks indicate the locations using the Thaw/MAP observed distances for stars AO 759, 765 (follow dashed lines) and 760. AO 760 appears to be binary.


Fig. $6 a$


Fig. $6 b$

Fig. 6.-(a) Residuals to the star constant adjustment in right ascension for $\chi$ Cygni. The motion modeled only position, proper motion, and parallax. (b) Residuals to the star constant adjustment in declination for $\chi$ Cygni. Note the large runs in residuals within some years (especially 1988).
reasons, neither star is used as a parallax standard. Their locations in the $A_{v}$ versus distance plot of Figure 2 are marked with squares.

Since the correction to absolute was derived without the aid of stars 759,760 , and 765 , their parallaxes as determined with the MAP (see Table 1) are independent of uncertainties in their MK type. If all has gone well, their distances should now be more in line with their observed $A_{v}$. In Figure 5, the asterisks mark the locations indicated by the trigonometric parallaxes. Stars 759 and 765 may be identified by tracking back along the dashed lines connecting them to their locations using their MK types. Clearly, their distances determined by MAP are in much better agreement with their observed $A_{v}$. As for AO 760, Stephenson and Roman both classify it as an early B dwarf. Stephenson (1988a) classifies it B2 Vnn and Roman (1978) classifies it B0.5 V. Stephenson also notes that its absorption lines are obviously broadened by stellar rotation with no sign of emission lines. Blanco et al. (1970) list $V=8.14$ and $(B-V)=0.01$. The intrinsic $B-V$ (type B 1 V assumed) is -0.28 (Allen 1973) which implies that $A_{v}=0.84$. Using this
value and the distance corresponding to the observed trigonometric parallax to place this object on the distance- $A_{v}$, plane, it is clear that its location is consistent (albeit with considerable uncertainty) with the indicated extinction as well.

### 2.6. Discussion: $\chi$ Cygni

The residuals from the star constant solutions in right ascension and in declination appear as a function of time in Figure 6. Those in right ascension appear random, showing no obvious trends, but those in declination have a tendency to show largeamplitude runs within individual observing seasons. A search for periodicities in the residuals yielded negative results except for a short range of periods from approximately 0.7 yr to 0.8 yr . In Figure 7 are shown phase plots for the residuals in both coordinates using a midrange period of 0.75 yr . The right ascension residuals still appear to be reasonably random, but those in declination are distinctly sinusoidal.
That color changes are the source of the 0.75 yr periodicity in the declination residuals cannot be totally ruled out at this time. But, given that the photometric period is $1.114 \mathrm{yr}(=407$


Fig. $7 a$


FIG. $7 b$

Fig. 7.-(a) A phase plot of the $\chi$ Cygni right ascension residuals of Fig. $6 a$; assumed period: 0.75 yr . No systematic trends are evident. (b) A phase plot of the $\chi$ Cygni declination residuals of Fig. $6 b$; assumed period: 0.75 yr. The systematic trends (apparently sinusoidal) are clearly seen.
days), it is not clear how this is translated into a 0.75 yr periodicity.

Semipermanent variations in surface brightness such as long-lived star spots seem even less likely. With an approximate radius of 283 solar radii (see below) and a mass believed to be on the order of one solar mass (Allen 1973), a rotation period of 0.75 yr would imply an equatorial velocity well in excess of the circular satellite velocity.

The most likely explanation appears to be that $\chi$ Cygni is a member of a binary star system. If this is true, it must be kept in mind that the perturbation seen in Figure 7 represents only the motion of the system's photocenter. Further, because $\chi$ Cygni is so strongly variable, there is a good chance that the observed photocenter exhibits significant motion along the radius vector between $\chi$ Cygni and its companion. Even if the actual barycentric motion were being observed (i.e., that companion's luminosity is negligible at all observed brightnesses of $\chi$ Cygni), the standard errors on the parallax, orbital period $\pm 0.34 \mathrm{yr}$ ( $=\frac{1}{2}$, the FWHM of the residuals' power spectrum), and photocentric semimajor axis $5.3 \pm 1.5$ mas (from a special solution for the geometric orbital elements) combine to yield errors on the individual masses (for assumed values of the mass ratio) that are somewhat larger than the computed masses themselves. Thus, further characterization of the system is not possible at present.

In the long term, however, the variability of the primary will prove an asset rather than a liability. When more observations carefully restricted to the meridian have been gathered, it will become possible to group the data into sets within which the range in observed magnitude of $\chi$ Cygni (and presumably effective color) is small. Separate star constant solutions (including orbital terms) for each set should yield identical (and correct) values for all star constants except the photocentric semimajor axis (whose value is a function of the ratio of the brightnesses of the two stars). A comparison of the observed photocentric semimajor axis size with the observed apparent magnitude of $\chi$ Cygni will then yield the apparent magnitude of the companion.

The distance modulus, allowing for a visual extinction of $0.10 \pm 0.03 \mathrm{mag}$ (that appropriate for a star at the observed distance of $\chi$ Cygni with the mean extinction of $0.87 \pm 0.15$ mag $\mathrm{kpc}^{-1}$ obtained earlier), is $5.38 \pm 0.46$. In their 1979 paper, Yorka \& Wing adopted $m_{v}=3.4$ at maximum (one of the brighter maxima). For this value, the above parallax implies that $M_{v}=-1.98 \pm 0.47$ in reasonable agreement with the conclusion of Yorka and Wings (1979) that for S-type Miras in general $M_{v}$ (at maximum) lies between -1.5 and -2.0. It should be kept in mind, however, that their result is not terribly well established owing, as they point out, to the overall paucity of data. Furthermore, Wing (1988) makes the point that, because $\chi$ Cygni's visual magnitude at maximum light varies so greatly from cycle to cycle, $M_{v}$ is not a welldefined quantity for this star.

Christou \& Worden (1980), observing in essentially the MAP bandpass, have reported an angular diameter of $23.1 \pm 4.1$ mas (epoch: 1976 June 13-17). This can now be converted into a linear diameter. With $\pi=8.8 \pm 1.9$ mas, the radius of $\chi$ Cygni becomes $(1.97 \pm 0.55) \times 10^{8} \mathrm{~km}(=283 \pm 79$ solar radii), a value believed typical for Mira variables of type M4-M7 having periods in the range 220-500 days (Allen 1973). The standard error on the linear diameter is now about equally divided between that due to the parallax and that due to the angular diameter. Thus, only a modest improvement will be realized with future improvements in the parallax.

## 3. 51 ANDROMEDAE

51 Andromedae ( $=\mathrm{BD}+47^{\circ} 467$ ), a relatively bright giant ( $V=3.57$ ) universally reported to be of MK type K3 III, has been the subject of a number of parallax studies. Jenkins (1952) lists two parallaxes, $7 \pm 18$ mas (s.e.) from McCormick and $29 \pm 12$ mas (s.e.) from Allegheny giving a weighted mean parallax of $22 \pm 10$ mas. This star was placed upon the MAP parallax program in 1986 at the request of John Bachall as part of his ongoing study of the space motions of $K$ giants.
In addition to 51 Andromedae, the field contains several other interesting stars. AO $816\left(=\right.$ BD $\left.+47^{\circ} 460\right)$ is the RS CVn variable OP Andromedae. Apparently no previous parallax exists for this star. AO 817 ( $=\mathrm{BD}+47^{\circ} 465$ ) is the visual binary ADS 1263. The system was discovered by Aitken in 1904 at which time his micrometer measures showed the separation and position angle to be 0.4 and $46^{\circ}$, respectively (Aitken 1932). Baize (1972), in addition to visual magnitudes for the components of 9.0 and 9.2, gives separations and position angles for the epoch 1971.11 of $0^{\prime \prime} .52$ and $32^{\circ}$. Thus these stars form a relatively fixed pair. Like OP And, this star appears to have no previous parallax determination.

The field was observed on 47 nights over the seasons 19861989 for a total of 60 MAP observations.

### 3.1. The Star Constant Solution

The star constants for the 51 Andromedae region appear in Table 1, stars 816-825. 51 Andromedae is AO 819. The standard errors for the zero-epoch position and for the proper motion refer to the approximate midepoch of the observations, J1988.0. The quoted parallaxes are absolute, having been obtained by adding to the relative parallaxes an adjustment factor of 4.3 mas (see next section).

The reference stars used in calculating the star constants were AO 817 and 820-825. A preliminary adjustment for position and proper motion disclosed a perturbation in the motion of AO $818\left(=\mathrm{BD}+47^{\circ} 466\right)$ visible in both right ascension and in declination (see Fig. 8). Thus, in subsequent adjustments this object was treated as a target and deleted from the list of reference stars. The star constants appearing in Table 1 were obtained by fitting a model of the form of equation (7) to the observed positions. The resulting quadratic terms were $A=$ $-0.00089 \pm 0.00073 \mathrm{yr}^{-2}$ and $B=-0$ ". $00452 \pm 0 \prime 000071$ $\mathrm{yr}^{-1}$.

### 3.2. Adjustment to Absolute Parallax

Much of the photometric data upon which the following adjustment rests was obtained from Persinger \& Castelaz (1990). Auxiliary information for each star is given in Table 4 (see table notes for sources). For parallax standards, the absolute parallaxes, their associated errors, and the values of the quantities from which they were derived are included as well.

The calculations leading to the weighted mean correction to absolute parallax were performed as described earlier, and the results are summarized in Table 5.

### 3.3. Interstellar Extinction

In Figure 9 are plotted observed visual extinction versus distance for all the parallax standards in the 51 And field. The distances were obtained from the individual absolute parallaxes in Table 5. The regression line is fitted only to the data points marked with crosses. The regression line has a slope of $0.72 \pm 0.11 \mathrm{mag} \mathrm{kpc}{ }^{-1}$. Fitzgerald (1968) gives (to 500 Pc ), $0.1<E(B-V)<0.2$ or, with a ratio of total to selective extinc-


Fig. $8 a$


Fig. $8 b$

Fig. 8.-(a) Residuals in right ascension to an adjustment for star constants: position, proper motion, and parallax for AO 818. Each point represents one Thaw/MAP observation. (b) Residuals in declination for AO 818.
tion of 3.1 (Massa \& Savage 1988) and normalizing to 1 kpc , a mean visual extinction of $0.62-1.2 \mathrm{mag} \mathrm{kpc}^{-1}$, a result in agreement with that obtained here.

For AO 824, both the distance implied by the Stephenson MK classification (the box in Fig. 9) and that for the Persinger \& Castelaz classification (the triangle) are plotted. That the regression line is a poor fit to either point is to be expected for, at a Galactic latitude of -13.5 , even the minimum of the two distance estimates place the star over three extinction scale heights below the Galactic plane.

### 3.4. Discussion: 51 Andromedae

51 Andromedae is AO 819. The Thaw/MAP parallax, $15.8 \pm 1.3$ mas, is in agreement with the weighted mean of the two previous determinations ( $22 \pm 10 \mathrm{mas}$ ) and represents nearly an order of magnitude improvement in accuracy. Adopting the mean visual extinction of the last section ( $0.72 \pm 0.11 \mathrm{mag} \mathrm{kpc}^{-1}$ ) and the above parallax, the distance

TABLE 5
Summary of the Calculations Leading to the Weighted Mean Correction to Absolute in the 51 Andromedae Field

| Number | $\stackrel{\pi_{r}}{\text { (mas) }}$ | $\stackrel{\sigma_{r}}{\text { (mas) }}$ | $\begin{gathered} \pi_{a} \\ \text { (mas) } \end{gathered}$ | $\underset{\text { (mas) }}{\sigma_{a}}$ | $\begin{gathered} \Delta_{\pi} \\ \text { (mas) } \end{gathered}$ | $\underset{\text { (mas) }}{\sigma_{\Delta}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 820 | 2.1 | 0.8 | 6.2 | 1.4 | 4.1 | 1.6 |
| 821 | -4.2 | 0.9 | 1.8 | 0.4 | 6.1 | 1.0 |
| 822 | -1.8 | 0.7 | 1.4 | 0.3 | 3.2 | 0.8 |
| 824 | -4.0 | 0.7 | 0.4 | 0.1 | 4.5 | 0.7 |
| 825 | 2.3 | 1.0 | 5.3 | 1.2 | 3.0 | 1.6 |
| $\left\langle\Delta_{\pi}\right\rangle$ | $4.3 \pm 0.5 \mathrm{mas}$1.2 mas |  |  |  |  |  |
| $\sigma_{1} \ldots \ldots$. |  |  |  |  |  |  |

modulus of 51 And becomes $4.06 \pm 0.18$ mag. Adopting the mean magnitude of the many listed in SIMBAD, $V=3.57$ the absolute magnitude of 51 And is $M_{v}=-0.49 \pm 0.18$, a value essentially in agreement with (but about 1 mag brighter than)

TABLE 4
Auxiliary Information for the Stars of the 51 Andromedae Field

| Number | BD | $\begin{gathered} \pi_{a} \\ \text { (mas) } \end{gathered}$ | $\stackrel{\sigma_{a}}{(\mathrm{mas})}$ | MK <br> Class | $V$ | $A_{v}{ }^{\text {a }}$ | $M_{v}{ }^{\text {a }}$ | $\sigma_{M v}$ | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 816 | $+47^{\circ} 460$ | $\ldots$ | $\ldots$ | K1 III-IVe | 5.92 | $\ldots$ | $\ldots$ | $\ldots$ | 1, 2, 3 |
| 817 | $+47^{\circ} 465$ | $\ldots$ | $\ldots$ | A1 V | 8.78 | $\ldots$ | $\ldots$ | $\ldots$ | 3, 4, 5 |
| 818 | $+47^{\circ} 466$ | ... | ... | F4.5 V | 6.76 | $\ldots$ | $\ldots$ | $\ldots$ | 6,7 |
| 819 | $+47^{\circ} 467$ | $\ldots$ | $\cdots$ | K3 III | 3.57 |  | $\cdots$ |  | 7 |
| 820 | $+47^{\circ} 469$ | 6.2 | 1.4 | F7 V | 10.00 | 0.25 | 3.7 | 0.5 | 4, 8 |
| 821 | $+47^{\circ} 471$ | 1.8 | 0.4 | A1 V | 10.02 | 0.34 | 1.0 | 0.5 | 3, 4, 9 |
| 822 | $+47^{\circ} 493$ | 1.4 | 0.3 | B8.5 IV | 9.04 | 0.50 | -0.7 | 0.4 | 3, 4, 10, 11 |
| 823 | .. | $\ldots$ | $\ldots$ | K5 | 9.04 | ... | ... | ... | 4, 8 |
| 824 | $+47^{\circ} 476$ | 0.4 | 0.1 | B2.5 IV-III | 9.32 | 0.65 | -3.6 | 0.6 | 3, 4, 11, 12 |
| 825 | $+48^{\circ} 499$ | 5.3 | 1.2 | F6 V | 10.12 | 0.25 | 3.5 | 0.5 | 3, 4, 8 |

${ }^{\text {a }}$ Unless otherwise noted, listed values of $M_{v}$ and some values of $\sigma_{M_{v}}$ were derived from data in Allen 1973.
Notes. (1) Photometry by Haggvist \& Oja 1970. Give $B-V=1.21$. (2) RS CnV variable OP And. (3) Also Stephenson 1988b. (4) Photometry from Persinger \& Castelaz 1990. (5) Binary ADS 1263. Baize 1972 gives visual magnitudes of 9.0 and 9.2. Stephenson 1988b gives A2 V. Persinger \& Castelaz 1990 give B9.5 V, $A_{v}=0.50$ and $\pi=0$ " 003 but did not take note of the duplicity. (6) MAP shows perturbation. Stephenson 1988b gives F6 V, and Persinger \& Castelaz 1990 give F3 V. (7) Photometry: mean of those listed in SIMBAD or in SIMBAD without reference. (8) MK type and $A_{v}$ (if given) by Persinger 1990. (9) Stephenson 1988b gives A0 V. Persinger \& Castelaz 1990 give A2 V. (10) Stephenson 1988b gives B9 III-IV. Persinger \& Castelaz 1990 give B8 V. (11) Intrinsic $B-V$ assumed that of a dwarf of same spectral type. (12) Stephenson 1988b gives B3 III, while Persinger \& Castelaz 1990 give B1.5 V.


Fig. 9.-Mean visual extinction along the line of sight vs. distance for selected stars in the 51 Andromedae field. The regression line (constrained to pass through the origin) was fit to the parallax standards marked with a cross. The resulting line-of-sight mean extinction is $0.72 \pm 0.11 \mathrm{mag} \mathrm{kpc}^{-1}$. The distance implied for AO 824 implied by Stephenson's MK classification (box) and that implied by the Persinger \& Castelaz classification (triangle) are indicated as well. This star was not used to determine the mean extinction because it lies over three extinction scale heights above the Galactic plane.
the mean for its assigned spectral type and luminosity class (Allen 1973).

Hutter et al. (1989) have measured the angular diameter of 51 And with the Mark III Michaelson interferometer at Mount Wilson which can now be converted to an accurate linear diameter. They used a passband of $200 \AA$ centered on $6740 \AA$ and baselines of 12.0 and 8.3 m . Angular diameters were computed for a uniform disk model $\theta_{u d}$ and for a limb-darkened model $\theta_{\mathrm{ld}}$ based upon the model atmospheres of Carbon \& Gingerich (1969). They give, for the epochs 1987 July-October, $\theta_{\text {ud }}=4.6$ $\pm 0.4$ mas and $\theta_{1 \mathrm{~d}}=4.9 \pm 0.4$ mas. Combined with the parallax obtained here, the corresponding linear diameters for 51


Fig. $10 a$

And are $D_{\mathrm{ud}}=(4.17 \pm 0.54) \times 10^{7} \mathrm{~km}$ and $D_{\mathrm{ld}}=(4.73 \pm 0.57)$ $\times 10^{7} \mathrm{~km}$ with the errors in the parallax and angular diameter measures contributing equally to the error in the computed linear diameter. Interpolating between radii given in Allen (1973) for stars of type K0 III and K5 III, it is seen that the expected diameter for a star of MK type K3 III is $3.0 \times 10^{7}$ km . Allowing for the slightly elevated absolute magnitude an expected linear diameter of $4.7 \times 10^{7} \mathrm{~km}$ is obtained, a value in good agreement with the values computed above.

## 4. DISCUSSION: OP ANDROMEDAE

For AO $816\left(=\mathrm{BD}+47^{\circ} 460=\mathrm{HD} 9746=\mathrm{OP}\right.$ And), Haggkvist \& Oja (1970) gave $V=5.92$ and $B-V=1.21$. Bidelman (1983) found strong Ca II emission in its spectrum and suggested it might be a RS CVn-type variable. Barksdale et al. (1984) found that AO 816 was indeed variable having an amplitude of 0.9 mag , with no strict periodicity and that it showed signs of a secular brightening of $0.00045 \mathrm{mag}_{\mathrm{day}}{ }^{-1}$. Later Boyd \& Genet (1985) gave a rough periodicity of 50 days.
RS CVn stars are binary systems having a F IV or G IV secondary and a K-G subgiant primary. Orbital periods generally are of the order of 50 days. Many are eclipsing systems, but star spots on the cool star can produce up to a 0.2 mag variation as well. The spot signal drifts in phase with respect to the eclipse signal. The pair is assumed to be in synchronous rotation except for the surface band where the spots are located which has its own rate (as in the Sun). About $20 \%$ are found at high Galactic latitudes, and some are X-ray sources as well.

In view of the known binary nature of the system, it is tempting to see if orbital motion can be seen in the residuals. A power spectrum of the residuals to the star constant solution (no orbit included) show a strong peak in both the $X$ and $Y$ residuals at a frequency of $7.69 \pm 1.5 \mathrm{yr}^{-1}$ (FWHM) yielding a period of $47.5 \pm 9.3$ days, about that expected for the system. A phase plot of the residuals in each coordinates against the above period appear in Figure 10. A signal of approximate


Fig. $10 b$

Fig. 10.-(a) Phase plots (assumed period, 47.5 days) of the residuals in right ascension to an adjustment for star constants: position, proper motion, and parallax for the RS CVn variable star OP Andromedae ( $=\mathrm{AO} 816$ ). (b) Companion declination residual phase plot to Fig. $8 a$. Though small, $1-2$ mas amplitude trends appear to be present. The period was arrived at through a power spectrum analysis which indicated identical periods in both coordinates of $47.5 \pm 9.3$ (FWHM) days, a period consistent with that found by other investigators.
amplitude 2 mas appears to be present in both coordinates, though such a small signal could be spurious.

Stephenson (1988b) classifies this star K1 III-IVe. The Thaw/MAP parallax ( $6.1 \pm 1.5$ mas) combined with the mean extinction found earlier ( $0.72 \pm 0.11 \mathrm{mag} \mathrm{kpc}^{-1}$ ) indicate a distance modulus of $6.19 \pm 0.61 \mathrm{mag}$. Using the combined apparent magnitude for the components given by Haggvist \& Oja (1970), $M_{v}=-0.27 \pm 0.61$, a value essentially in agreement with the primary's observed MK type of K1 III-IVe (Stephenson 1988b). The secondary, being 2-3 mag fainter than the primary, contributes little light to the system.

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