

THE DISTRIBUTION OF CLUSTERS OF GALAXIES WITHIN $300 \text{ Mpc } h^{-1}$ AND THE CROSSOVER TO AN ISOTROPIC AND HOMOGENEOUS UNIVERSE

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ABSTRACT

Confirming an earlier, although preliminary result, we present for the first time firm evidence for the spatial convergence of a cosmological dipole (i.e., peculiar acceleration). The objects used in the analysis are clusters of galaxies from the Abell and ACO catalogs. The cluster dipole increases with distance up to $\sim 180h^{-1} \text{ Mpc}$, then bends over and stays flat well within the depth of the sample, while the monopole continues to grow linearly with distance up to the maximum sample depth, $300h^{-1} \text{ Mpc}$, where it reaches a level which is $\gtrsim 5$ times the dipole level. The still-growing monopole *proves* that the convergence of the dipole is real and is not due to depth incompleteness of the sample. The behavior of two such quantities is the one expected in an asymptotically homogeneous and isotropic universe and gives further three-dimensional (all-sky) support to a FRW model, besides the two-dimensional one from cosmic backgrounds.

After suitable corrections for the differences in the two catalogs of clusters of galaxies (Abell and ACO) limited at $|b| > 20^\circ$, the asymptotic dipole direction appears to be stable pointing in a direction $\approx 25^\circ$ off the CMB dipole apex. By *a posteriori* pushing the limit at $|b| > 10^\circ$ to include the effects of the Perseus cluster, we find that, after correction for the missing Galactic strip, the dipole direction is within only 6° – 10° of the CMB apex.

An analysis of the behavior of the dipole as a function of distance suggests that a nonnegligible fraction of the peculiar acceleration responsible for the LG motion comes from greater depths than concluded from previous studies of *IRAS* and optical catalogs of galaxies, with possible important consequences on the viable cosmological scenarios. From these data we estimate that $\lesssim$ one-third of the acceleration comes from $d \geq 60h^{-1} \text{ Mpc}$. The same data can also be used to estimate the density parameter Ω_0 . After rescaling the cluster dipole to monopole ratio by the value of the cluster-to-galaxy bias derived from the correlation functions ($b_{\text{cg}} \approx 3$), we obtain $\Omega_0 = 0.2$ – 0.4 for no or mild bias of galaxies ($b_{\text{gp}} \approx 1$ – 1.5). An $\Omega_0 = 1$ universe could still be accommodated by requiring a very large bias of galaxies with respect to matter, $b_{\text{gp}} \approx 2.7$ – 3 , which is, however, not supported by several large-scale observations.

Subject headings: cosmology — galaxies: clustering

1. INTRODUCTION

The study of galaxy peculiar motions with respect to the Hubble flow pioneered by the work of Rubin, Ford, & Rubin (1973) has received new interest from the observations of Dressler et al. (1987), which have prompted a flurry of observational and theoretical work. One of the critical issues is the amplitude and size of the mass fluctuations which give rise to the general peculiar flows and to the peculiar motion of the Local Group (LG), which is determined with respect to the (“Machian”) reference frame of the cosmic microwave background (CMB).

While most authors in the past years found evidence for a relatively “local” (i.e. within $40h^{-1} \text{ Mpc}$) origin of the accelerations which cause the LG peculiar motion (Strauss & Davis 1988, hereafter SD88; Lynden-Bell, Lahav, & Burstein 1989), already two years ago we first pointed out (Scaramella et al. 1991a, 1989, hereafter Paper I and SBCVZ89, respectively) that the large-scale distribution of clusters of galaxies suggests that a nonnegligible fraction of such acceleration may be due to much larger scales. Two recent papers (Rowan-Robinson et al.

1990 and Plionis & Valdarnini 1991, hereafter RR90 and PV91, respectively) also support both our earlier suggestion of a nonlocal convergence of the dipole and the importance of clusters as mass tracers. In this *Letter*, using a well-defined, complete, deeper ($300h^{-1} \text{ Mpc}$), volume-limited sample of clusters of galaxies (see Scaramella et al. 1991b, hereafter Paper II), we confirm the nonlocal convergence of the dipole and derive new estimates for the fraction of the local peculiar velocities which is due to large-scale mass inhomogeneities and for the value of Ω_0 . We also discuss the implications that the firm evidence we present for the dipole convergence has on the issue of isotropy of our observable universe. In our view, this aspect, which has been often taken for granted, has not always received an emphasis adequate to its importance.

2. SAMPLE AND METHOD OF ANALYSIS

We use as data base for our analysis the clusters listed in the Abell (1958) and ACO (Abell, Corwin, & Olowin 1989) catalogs. Details on the properties of these catalogs are discussed in Paper II. Here we will mainly consider a volume-limited

sample with radial distance $d \leq 300h^{-1}$ Mpc, because beyond this distance, the estimates for the Abell catalog become highly unreliable, $|b| \geq 20^\circ$, and $R \geq 0$ (~ 860 clusters).

For this sample we compute the “monopole,” $\mathcal{M}$, and the “dipole,” $\mathcal{Q}$, which is proportional to the peculiar acceleration on the LG, as $\mathcal{M} \equiv \sum_i w_i / r_i^2$ and $\mathcal{Q} \equiv \sum_i w_i \hat{r}_i / r_i^2$ (both are in units of $s^2 km^{-2}$). We tried various possible weights, each with or without the additional use of the inverse of the angular selection function derived in Paper II, $w_i = \phi^{-1}(\hat{r}_i)$, which takes into account the large-scale gradients present in the raw data. These weights are $w_i = 1$ (number); $w_i = (N_{g,i}/100)$ (cluster richness scaled to a Coma-like cluster—Coma has $N_g = 106$); $w_i = (N_{g,i}/100)^{3/2}$ (mass estimates through richness and scaled as before to Coma; the power law comes from a fit to the n^* values given by Lucy 1983 in his Table 2). The results obtained with these different weighting schemes are all qualitatively similar to each other. We then correct for differences and incompleteness of the two catalogs by subtracting from $\mathcal{Q}_{raw}$, computed directly from the data, the correction $\mathcal{Q}_{rand}$ obtained from the average of 100 random samples which have the same large-scale biases as the data have.

3. RESULTS

1. A very good convergence of $|\mathcal{Q}|$ is obtained well within the sample depth ($\approx 80\%$ complete in redshift within the convergence length, $d_{conv} \lesssim 180h^{-1}$ Mpc), while $\mathcal{M}$ keeps growing linearly with distance (see Figs. 1a and 1b). That is, the value of $|\mathcal{Q}|$ is basically unchanged by adding twice as many clusters as those (~ 300) responsible for the signal. As seen in Figure 1 of Scaramella, Vettolani, & Zamorani (1991c), the general behavior is essentially the same even discarding the $R = 0$ class clusters, i.e., by halving the sample.

2. The correction applied by subtracting the random samples (see § 2) changes little the value of $|\mathcal{Q}(d_{conv})|$ (see Fig. 1b) but nicely fixes the dipole direction: $\hat{\mathcal{Q}}$ stays almost constant in the distance range $d_{conv} \lesssim d \leq 300h^{-1}$ Mpc. It must be noticed that this direction (Fig. 1c, solid circles), which happens to be within $\approx 20^\circ$ – 30° from the direction of the CMB dipole (without extrapolation on the Galactic strip), is already determined by the clusters in the inner region ($d \leq 60h^{-1}$ Mpc) and

is in agreement with results from shallower samples. The contribution from the region $60 \leq d \leq 180h^{-1}$ Mpc is along the same direction ($\Delta\theta \approx 20^\circ$) and contributes for one-fourth to one-third of the final value (ϕ^{-1} without and with mass weight, respectively).

3. By the directional agreement shown in the previous point, we have been led to try to extend the sample to even lower Galactic latitude ($|b| > 10^\circ$), although in this region the sample is severely incomplete. This extension corresponds to adding ~ 40 clusters, among which the nearby Perseus cluster, which has an effect which is not negligible. The main change in the results is that now there is an even better directional agreement with the CMB apex (10° – 20°) (Fig. 1c, squares). If we now apply the extrapolation over the missing strip through spherical harmonics to the dipole order, we get $\hat{\mathcal{Q}}$ within 6° – 10° from the CMB direction.

4. We get an estimate for sampling errors through the bootstrap method (Efron 1979). From 100 cases we get a dispersion of $\sim 30\%$ for the dipole over monopole ratio, and a dispersion of $\sim 15^\circ$ for the final value of the direction of the dipole. These relatively large values reflect the obvious sensitivity of the final figures to the position and presence of the relatively few nearest clusters. Also, the probability of having from the region $60 \leq d \leq 180h^{-1}$ Mpc a shot noise dipole of the same characteristics as the observed one (same magnitude and direction within less than 30° from the signal from the inner region) is $P \sim 2 \times 10^{-3}$ (Scaramella, Vettolani, & Zamorani 1991d).

We will now discuss in turn the two main points of this work, that is the convergence of the dipole, which is important by itself, even with no reference to the final direction and/or the specific depth, and the effects of large-scale inhomogeneities on the LG, for which these issues are instead crucial.

4. THE CROSSOVER TO ISOTROPY AND HOMOGENEITY

Much effort has been devoted in the recent past to study the inhomogeneities and anisotropies in the distribution of galaxies surrounding us, especially in connection with the problem of large-scale peculiar flows (see Gunn 1988 for a review). The basic goal has been to determine the depth, d_{conv} ,

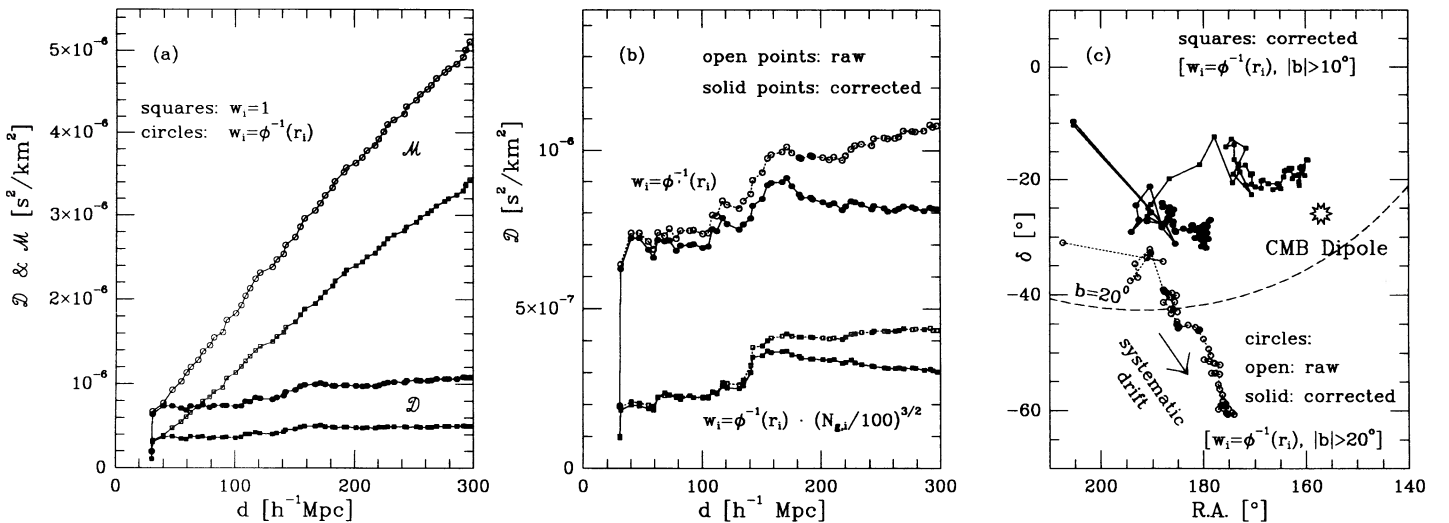


FIG. 1.—In (a) we show monopole and dipole behavior with depth. In (b) we show the effect of the correction for random samples and of the mass weighting on the $\mathcal{Q}$ value. In (c), the systematic drift present in the raw data is shown to be properly taken into account by the subtraction of random samples. We also show the effect of extending the sample to low latitudes ($|b| \geq 10^\circ$).

that contains all the matter fluctuations which are responsible for the peculiar motion of the LG with respect to the CMB rest frame. The quantity d_{conv} can be obtained in principle by considering the behavior of $\mathcal{Q}(d)$ of the matter distribution: $\mathcal{Q}(d > d_{\text{conv}}) \equiv \mathcal{Q}(d_{\text{conv}})$. It must be noticed that $d_{\text{conv}}(\mathbf{x}_{\text{obs}})$ is a function of the position $\mathbf{x}_{\text{obs}}$ of the cosmic observer and is much larger than the size of typical ($\delta\rho/\rho \sim 1$) matter fluctuations.

The value of d_{conv} (together with its probability distribution over the ensemble of cosmic (observers) has great theoretical relevance in testing different spectra of cosmic perturbation (e.g., Juszkiewicz, Vittorio, & Wise 1990 and Lahav, Kaiser, & Hoffman 1990). Here we want also to stress that *the proof of the very same existence of d_{conv} for a set of cosmic objects strongly suggests the very important consequence that it has been operationally reached the crossover point to an isotropic and homogeneous universe* (the homogeneity is also supported by the linear behavior of $\mathcal{M}$ with distance; for a general discussion of the problem see Stoeger, Ellis, & Hellaby 1987 and Ellis 1988; strictly speaking, the present result is still compatible with a cellular structure of appropriate scale lengths).

Two different approaches have mainly been used to attack this problem. The first approach, essentially two-dimensional, is the study of the “dipole” of the surface distribution of the objects in the available samples (i.e., flux-limited IRAS galaxy samples [Yahil, Walker, & Rowan-Robinson 1986; Meiksin & Davis 1986] or a merge of optical catalogs, [Lahav, Rowan-Robinson, & Lynden-Bell 1988]). This approach *assumes that the data in the sample have a distribution in depth which is substantially deeper than d_{conv}* . On the basis of agreement between the direction of $\mathcal{D}_{\text{surf}}$ and that of the CMB, the above hypothesis has been inferred to be true *a posteriori*. However, the agreement in direction is subject to various uncertainties due to the different possible methods of correcting for obscuration zones and by itself does not necessarily prove the achievement of convergence of $\mathcal{Q}(d)$, whose apparent flattening instead can be due just to sparse sampling and/or incompleteness of the samples at large distances. An effect like this is seen in Figure 6 of Lynden-Bell et al. (1989), which clearly shows that the dipole starts to flatten only at a distance where also the monopole is already flattening because of incompleteness of the sample (cf. also Fig. 10 of PV91).

The second approach is to use three-dimensional samples, and it has been applied to IRAS galaxies (SD88; Yahil 1988; RR90), to X-ray-selected clusters (Lahav et al. 1989), to a handful of AGN (Miyaji & Boldt 1990), and to optically selected clusters in our earlier (Paper I) and present considerations (see also PV91). On the basis of the analysis of a sample of relatively bright IRAS galaxies (hereafter IRAS I), SD88 claimed convergence of the IRAS dipole essentially within $d = 40h^{-1}$ Mpc and excluded a significant effect from the Great Attractor (GA). However, this sample suffers from severe depth incompleteness. RR90, using a significantly deeper but sparse sample of IRAS galaxies (hereafter IRAS II), show that the IRAS II dipole, after a first flattening in the distance range $40 < d < 70h^{-1}$ Mpc starts to grow again up to $\sim 100h^{-1}$ Mpc, and then remains flat up to $\sim 150h^{-1}$ Mpc. In comparing this result with ours, two points are worth noticing: first, the RR90 sample is significantly incomplete beyond $100h^{-1}$ Mpc (see their Fig. 2) (for instance, for $100 \leq d \leq 300h^{-1}$ Mpc the number of galaxies in their sparse sample is slightly smaller than the number of clusters). Second, the quasi plateau in the behavior of $|\mathcal{Q}(d)|$ of clusters for $50 < d < 100h^{-1}$ Mpc (Paper I and Fig. 1b) resembles that of IRAS I and IRAS II in the same

range of distance. We also note that the number of clusters that we used here is more than twice the number used by PV91 in their analysis of a cluster magnitude-limited sample, which is substantially incomplete beyond $d \sim 200h^{-1}$ Mpc. The basic problem in all these analyses is the connection between the unobservable matter distribution and that of the cosmic tracers used, i.e., the different amount of biases. This is of course the weakest point in trying to map medium and large scales with clusters of galaxies. These, in fact, can trace only part of the galaxy distribution (the densest spots), and very sparsely. Indeed, rich clusters do not trace our nearby surroundings, e.g., Virgo. Clusters are also more correlated among themselves than galaxies and therefore than matter. However, the nonzero galaxy-cluster correlation function (Seldner & Peebles 1977) suggests that each rich cluster has a large cloud of galaxies related to it and hence traces a mass perturbation much larger than the typical cluster virialized core.

All the above considerations, however, just strengthen the principal point we make here. Indeed, *the fact that the distribution of clusters of galaxies is proved to become homogeneous and isotropic on scales which, although large, are significantly smaller than the depth of the sample, implies that the same happens to the matter distribution, because the former constitutes a biased subset of the latter*.

In other words, we can conservatively conclude that $d_{\text{conv}}^{(\text{matter})} \leq d_{\text{conv}}^{(\text{clusters})}$, where we proved $d_{\text{conv}}^{(\text{clusters})}$ to be finite. This statement can be falsified only if (1) *clusters are less correlated than matter* (i.e., a strong antibias such to have matter more clumped than clusters) and/or (2) *the majority of the clusters listed in the catalogs and that have $d < 300h^{-1}$ Mpc are spurious and do not correspond to physical concentrations*. Both cases are in contrast with experimental evidence.

5. EFFECTS OF LARGE-SCALE INHOMOGENEITIES ON THE LOCAL GROUP AND COSMOLOGICAL PARAMETERS

The determination of the fraction of the induced peculiar acceleration on the LG which is of nonlocal origin, i.e., comes from beyond $40h^{-1}$ Mpc, is still an open and debated issue. On one side there are the above mentioned results from IRAS I and optical dipoles, which claim this fraction to be quite negligible, thus implying a locally very homogeneous universe and excluding the presence of “nearby” large mass concentrations, such as the GA. Then there is the GA model itself (Lynden-Bell et al. 1988; Dressler & Faber 1990) which places the attention on a medium-distant ($d \cong 40\text{--}50h^{-1}$ Mpc) large ($R \sim 30\text{--}40h^{-1}$ Mpc) structure. We instead pointed out (SBCVZ89) the further presence of a distant structure, $d \cong 140h^{-1}$ Mpc, which is extremely rich in the number of clusters (Vettolani et al. 1990) and has the peculiarity of being aligned with both the direction of large-scale flows and the GA. This fact suggests that we are embedded in a very large filamentary structure, a fact that is likely to cause a bootstrap effect of the acceleration due to the two main concentrations in that direction. We had also tentatively estimated on the basis of the relative number of clusters the contribution of the further concentration to be slightly less than that from the foreground one, which is located consistently at the position of the GA (SBCVZ89), and at the same time pointed out how this estimate lacked both a self-consistent normalization (i.e. contributions from other areas of the sky) and a global one (i.e., the zero point, due to the absence of local structures in the cluster catalogs).

Here we confirm our previous result on the first of such

estimates: the fraction of the final value of $|\mathcal{Q}(d_{\text{conv}})|$ due to $d > 60h^{-1}$ Mpc is $\lesssim 30\%$ (see the discussion following Paper I and Fig. 1b). This suggests that $\approx 200\theta$ km s $^{-1}$ of the LG motion are possibly due to very large scales, where $0 < \theta \leq 1$ is the fraction of the peculiar acceleration which is due only to the structures traced by clusters. Similar conclusions about a deeper dipole convergence have been recently suggested also by RR90 and PV91. The latter authors independently performed an analysis similar to the one presented in this section, with conclusions in general agreement with those we derive below. The need for nonlocal contributions to the LG acceleration has also been suggested on various grounds by Plionis (1988), by Willick (1990), and by Shaya, Tully, & Pierce (1990).

To have more detailed estimates about the peculiar velocities possibly due to large scales, we can take two different approaches. The first is to consider the physical masses of the clusters and their extended halos of galaxies in terms of a common reference mass, and, once assuming a value of Ω_0 , assume the rest of the matter to be smoothly distributed. We proceed as in Paper I. The peculiar velocity v_p , induced by a peculiar acceleration, A_p , is $v_p = \frac{2}{3}H_0^{-1}\Omega_0^{-0.4}A_p$ (Peebles 1980). Using as reference mass that of a Coma-like cluster, $M^{(C)}$ (see § 2), we have that $A_p = H_0^2 GM^{(C)}\mathcal{Q}$. From this we obtain a relationship between Ω_0 and $M^{(C)} \equiv M_{15}^{(C)} \times 10^{15}h^{-1} M_\odot$ as a function of the fraction of peculiar velocity due to clusters and their halos.

Quantitatively, we have $M_{15}^{(C)} \approx 10^{-2} \theta v_p \Omega_0^{0.4} h^{-1} \text{ s km}^{-1}$ where we have used $\mathcal{Q} \approx 3.6 \times 10^{-7} \text{ s}^2 \text{ km}^{-2}$ (mass and ϕ^{-1} weight; see Fig. 1b). This translates in $M_{15}^{(C)} \approx 2, 4, 6$ when $\theta v_p = 200, 400, 600 \text{ km s}^{-1}$ and $\Omega_0 = 1$ (clusters would respectively trace 6%, 12%, 18% of the total matter), while for $\Omega_0 = 0.2$ one would have $M_{15}^{(C)} \approx 1, 2, 3$ (and 15%, 30%, 45%). If we consider the signal extrapolated in the Galactic strip, we have that the above values for $M_{15}^{(C)}$ are lowered by about one-third. Especially in a low-density universe these values compare well

with current estimates of the mass of Coma Cluster, $M^{(C)} \sim 1\text{--}3M_{15}$ (The & White 1986; Merritt 1986; Hughes 1989), which can be taken as a lower bound to the mass of the cluster + halo. Moreover, the fraction of mass in clusters + halos corresponding to this choice of parameters is consistent with the fraction of galaxies associated with clusters (15%–25%) derived by Bahcall (1986).

The other method is to consider the clusters as (more or less) biased tracers of the whole mass distribution. With this assumption one has that $v_p = H_0 d_{\text{conv}} \Omega_0^{0.6} b_{cp}^{-1} \mathcal{Q}/\mathcal{M}_c$, where one assumes a linear biasing, and $v_p = 600 \text{ km s}^{-1}$ to be the whole LG motion. The experimental value at the convergence depth are $\mathcal{Q}_c/\mathcal{M}_c \approx 0.26\text{--}0.29$ (with and without mass weighting). In linear theory, one then has $b_{cp} \approx b_{cg} b_{gp}$, with b_{cp} bias of clusters to matter, b_{cg} bias of clusters to galaxies, and b_{gp} bias of galaxies to matter. One expects $b_{cg} \approx (r_{0,cc}/r_{0,gg})^{\gamma/2} \approx 3$, where $\gamma \approx 1.8$ is the slope of the correlation function and $r_{0,cc} \approx 20h^{-1} \text{ Mpc}$ and $r_{0,gg} \approx 6h^{-1} \text{ Mpc}$ are the correlation lengths for clusters (Bahcall & Soneira 1983; Huchra et al. 1990) and galaxies (Davis & Peebles 1983; de Lapparent, Geller, & Huchra 1988), respectively. We then obtain $\mathcal{Q}_g/\mathcal{M}_g = b_{cg}^{-1} \mathcal{Q}_c/\mathcal{M}_c \approx 9 \times 10^{-2}$, and $\Omega_0^{0.6} \approx b_{gp}/2.7$. Therefore we have $\Omega_0 \approx 0.2, 0.4, 1$ when $b_{gp} \approx 1, 1.5, 2.7$, that is when galaxies are unbiased, or biased, or strongly biased. These results are only slightly changed by extrapolation in the missing strip (the $\mathcal{Q}/\mathcal{M}$ ratio typically increases by $\sim 10\%$, so we then have $\Omega_0^{0.6} \approx b_{gp}/3$).

Given the various uncertainties, we then have indications of an open universe ($\Omega_0 \approx 0.2\text{--}0.4$), in agreement with most experimental results (see the review of Peebles 1987), when a very likely range of the bias parameter is used ($b_{gp} \approx 1\text{--}1.5$).

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