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ARM CLASSIFICATION AND VELOCITY GRADIENTS IN SPIRAL GALAXIES

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ABSTRACT

On the basis of published rotation curves, we have compiled a list of velocity gradients for 94 galaxies. A significant correlation is found in this sample of galaxies between their gradients and arm classes (as given by Elmegreen and Elmegreen); galaxies with steeper curves tend to have a flocculent arm structure, and galaxies with flatter curves tend to have a grand design morphology. The correlation is true, since it is not induced by other correlations. Our result is in agreement with previous suggestions by Whitmore and with the recent result by Elmegreen and Elmegreen; it is also consistent with the predictions of density wave theory for the formation of the spiral structure.

Subject headings: galaxies: internal motions — galaxies: structure

1. INTRODUCTION

The classification system of spiral arms introduced by Elmegreen & Elmegreen (1982, hereafter EE82) has, in the last few years, stimulated many researchers to look for correlations with other galaxy properties, in order to understand the physical processes that may be responsible for the formation and/or the maintenance of these structures. The classification is based on the regularity of the spiral arm structure and stresses the properties of arm continuity, length, and symmetry. The galaxies were previously assigned to 12 arm classes (AC in the following), ranging from chaotic and fragmented arms (AC = 1), to two long, sharply defined, symmetric arms (AC = 12). In a later paper Elmegreen & Elmegreen (1987, hereafter EE87), reduced the number of classes by the elimination of AC = 10 and 11, because these were not directly related to the spiral structure. A major division (which we adopt in the following) was made by EE82 into *flocculent* (F) and *grand design* (G) galaxies, with AC ≤ 4 and AC ≥ 5, respectively.

Several properties of spirals have been examined for correlations with AC. G galaxies were suggested to be bluer (Romanishin 1985) and of lower surface brightness (Phillips & Disney 1985) than F galaxies, but these suggestions were not confirmed on a larger sample (Elmegreen & Elmegreen 1986). The average star formation rate does not differ between G and F galaxies (Elmegreen & Elmegreen 1986), nor does the neutral hydrogen content, the supernova rate, the CO surface brightness, the radio continuum, and X-ray emissions per unit light (Romanishin 1985; McCall & Schmidt 1986; Stark, Elmegreen, & Chance 1987; Giuricin, Mardirossian, & Mezzetti 1989). The environment may have an effect on the arm structure of a galaxy: the early suggestion that F galaxies do not tend to have companions or bars (EE82; Kormendy & Norman 1979, hereafter KN79) was not confirmed by a more recent analysis (Giuricin et al. 1989). The absolute magnitude (or physical size) is the only quantity which is known to significantly correlate with AC (EE82; EE87).

A correlation between the arm structure and the galaxy rotation curve has been found by Whitmore (1984, hereafter W84). The correlation was based on a sample of ~ 30 galaxies and showed a trend for galaxies with a flat rotation curve to have a stronger spiral structure than galaxies with a rising rotation curve. On the contrary, KN79 found that the differential rotation of the galaxy suppresses the global spiral pattern, while this is present in the region of rigid rotation, in a sample of 21 spirals without bars and companions. This discrepancy may be due to the different galaxy samples used, or to the different parameters investigated. In fact, KN79 did not use a velocity gradient in their analysis, but "the radius at which the rising part of the curve gives way to a nearly constant velocity." On the other hand, W84 used the outer gradient of the rotation curve (OG in the following), i.e., the percentage increase of the rotation curve from 0.4 to $0.8R_{25}$. Persic & Salucci (1986) reached similar conclusions as W84, using a different gradient parameter from a sample of 36 galaxies. KN79 divided their sample into galaxies with *m*-armed global patterns, NGC 2841–type disks (i.e., with short spiral filaments and no grand design structure), and transition cases. W84 used the DDO luminosity class as a parameter related to the arm structure (van den Bergh 1960a, b).

The correlation between the galaxy rotation curve and the arm structure is very important for understanding the spiral arm phenomenon. The *density wave theory* (Lin & Shu 1964) predicts that the well-developed two-arm structure should be found preferentially in galaxies with large disk–to–total mass ratio, *f*, while short and multiple arms should be most easily produced in galaxies with a very massive halo component (see, e.g., Wielen 1974; Toomre 1981; Carlberg & Freedman 1985; Carlberg 1986; Binney & Tremaine 1987; and references therein). Since the velocity gradient of the galaxy rotation curve is connected to the fractional mass in the disk (see, e.g., Whitmore 1989; Persic & Salucci 1988, hereafter PS88; and 1990a), the findings of W84 can be interpreted in the framework of the density wave theory (see also the numerical simulations by Sellwood & Carlberg 1984 and Carlberg & Freedman 1985).

When the present work was about to be completed, we received a recent preprint by Elmegreen & Elmegreen (1990, hereafter EE90), dealing independently from us with the same topic. The authors analyzed 41 rotation curves and found that galaxies with AC = 12 have a negative "extended gradient"

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(EG in what follows; see Whitmore, Forbes, & Rubin 1988), while the other galaxies have EG ≥ 0. These authors divided their sample into three groups: the flocculent galaxies, F, the multiple–long-arm galaxies, M ($5 \leq$ AC ≤ 9), and the galaxies with AC = 12. They did not find any significant decrease in EG from the F to the M sample, as would be expected in the context of the density wave theory, yet they suggested that this problem could be solved with the use of a larger sample.

In this paper we examine this topic with a sample of galaxies which is twice as large as the EE90 sample, and three times as large as the W84 one. We use the AC parameter that seems most suitable for studying the spiral structure (EE82). We choose two gradient parameters (see below), both more internal than EG, since we wish to examine the very region where the arm structure develops. A more extensive statistical analysis is performed, and we give quantitative estimates of the significance of the results. The conclusion we obtain is in line with W84 and the recent qualitative result by EE90, and hence with the prediction of the density wave theory. The present paper also solves the problem of the puzzling result found by EE90, i.e., that M galaxies seem to have their gradients similar to F galaxies, since we find a continuous, albeit much scattered, decrease of the gradient from AC = 1 to 12.

In § 2 we describe our galaxy sample; in § 3 we present both our analysis and results; in § 4, we give our conclusions.

2. THE DATA SAMPLE

In order to test the existence of a correlation between arm structure and fractional disk mass, it is necessary to have both parameters as observables. As far as the fractional mass is concerned, Persic & Salucci (1990a) have derived an expression for this parameter, relating it to the logarithmic gradient (LG in the following; see Persic & Salucci 1986) of the galaxy rotation curve, measured at the disk radius R_{25} (deduced from the $25B$ mag arcsec^{-2} isophotal contour), i.e.,

$$f = \frac{\mathscr{M}_{\rm disk}}{\mathscr{M}_{\rm total}} = \frac{0.8 - {\rm LG}}{0.1\,{\rm LG} + 1.1} \qquad (1)$$

with

$$ {\rm LG} = \left(\frac{\partial \log V}{\partial \log R}\right)_{R_{25}}, \qquad (2)$$

where V is the rotational velocity of the galaxy. It is then possible to use LG as an indicator of the fractional mass in the disk (f monotonically decreases with increasing LG). We adopt this parameter in our analysis, yet we also consider OG (see § 1), in order to be able to compare our results with those by W84.

The arm structure is best described by the AC parameter as defined by EE87 (see § 1); since classes 10 and 11 are no longer used, for the sake of continuity we have considered AC = 12 as a class that immediately follows AC = 9, in the statistical analysis below.

We have selected all the galaxies listed in EE87 for which the LG parameter was either known or obtainable from published rotation curves. We fitted each rotation curve with a straight line, via a standard least-squares method, with the velocity as a dependent variable. The fitting was made in the interval from $R_{\rm in}$ to $R_{\rm out}$, i.e., in the region where the galaxy rotation curve shows a constant gradient. $R_{\rm in}$ ranges typically from 0.1 to $0.3R_{25}$ (see PS88) in different galaxies, while $R_{\rm out}$ is the most external radius at which the rotation curve was sampled; when $R_{\rm out} \geq R_{25}$, only the first point beyond R_{25} was considered in our fit. R_{25} was taken from the Second Reference Catalogue of Bright Galaxies, by de Vaucouleurs, de Vaucouleurs, & Corwin (1976, hereafter RC2). When more than one rotation curve was available from the literature, we generally chose that with the largest $R_{\rm out}$. PS88 and Persic & Salucci (1990b) published the parameters of the fitting lines for several dozen galaxies; when available, we used their parameters to compute LG. Since the rotation curve is more easily observable for galaxies with higher inclination angles and the opposite is true for the determination of the arm morphology, we were able to collect only 84 galaxies with both LG and AC known. However, this sample is much larger than any other sample previously analyzed in the literature (KN79; W84; EE90).

Moreover we collected the following quantities: (1) the outer gradient, OG $= [V(0.8R_{25}) - V(0.4R_{25})]/V_{\rm max}$, taken from Whitmore et al. (1988) or W84, when available; otherwise computed assuming $V_{\rm max} = {\rm MAX}\ [V(0.4R_{25}),\ V(0.8R_{25})]$, (see PS88); (2) the DDO luminosity class, taken from A Revised Shapley-Ames Catalog of Bright Galaxies, by Sandage & Tammann (1981), or from RC2: we coded it with a number from 1 to 5, with fractional numbers corresponding to intermediate luminosity class (e.g., 2.5 corresponds to DDO class II/III); (3) the absolute blue total magnitudes, M_B, corrected for the absorption in our own Galaxy, and inclination, taken either from the Nearby Galaxies Catalog, by Tully (1988, hereafter NBG), or from PS88, when available, or computed from the apparent magnitudes (taken from RC2) using redshift estimates; (4) the 21 cm line width parameters W_R^i (corrected for non-rotational motions and inclination) either taken from NBG, or computed (according to Tully & Fouqué 1985) from the 21 cm line widths at the 20% level of the maximum velocity, V_{20}, as listed in Bottinelli et al. (1984), Bottinelli, Gouguenheim & Paturel (1982), or Huchtmeier et al. (1983); (5) the morphological Hubble type, T, coded as in NBG, taken either from NBG or from RC2; (6) the inclination angle of the galaxy, i, taken either from NBG or from Bottinelli et al. (1984), or computed after RC2; and (7) the distance moduli m-M, either taken from NBG or computed from the radial velocities listed in Persic & Salucci (1990a).

All the distance-dependent quantities have been computed or rescaled from published values, using $H_0 = 50$ km s^{-1} Mpc^{-1}. The relevant data for our sample are listed in Table 1.

3. ANALYSIS AND RESULTS

3.1. *Arm Class versus Velocity Gradient*

In order to evaluate whether AC depends on the velocity gradient, we estimate the linear regression correlation coefficient, r_P, and two rank correlation coefficients, Spearman's r_S and Kendall's r_K. We rely on these nonparametric coefficients to yield the statistical significance of the correlations (see, e.g., Kendall & Stuart 1977). Table 2 lists the main correlations performed; only the percent significance level associated with r_K is listed, since it always turns out to be nearly identical to the significance level of r_S.

We deal with LG as our most trusted velocity gradient parameter, yet, for the sake of comparison with W84, we also use OG to check the basic results. However, these two parameters are well correlated (see Table 2).

Our analysis shows the existence of a correlation between AC and LG at a 99.79% significance level, which appears even stronger when OG is used instead of LG. In Figure 1 we plot

TABLE 1

DATA SAMPLES

Name[a] (1)	AC[b] (2)	DDO[c] (3)	T[d] (4)	LG[e] (5)	OG[f] (6)	R_{out}/R_{25}[g] (7)	M_B[h] (8)	W_R^i[i] (9)	$m-M$[j] (10)	References[k] (11)
IC 342	9	1.5	6X	0.18	9.	≥1.00	−22.44	...	28.84	1
IC 4182	2	...	9A	1.22	65.	≥1.00	−1650	69	29.10	2
NGC 45	1	3.0	8A	1.02	45.	≥1.00	−18.70	204	29.73	3
NGC 300	5	2.0	7A	0.56	*22.	≥1.00	−17.76	200	26.30	4
NGC 450	2	2.0	6X	0.43	20.	0.87	−20.34	265	32.63	5
NGC 488	3	1.0	3A	*0.21	9.	0.57	−22.24	574	33.21	6
NGC 598	5	2.5	6A	0.38	14.	0.83	−19.19	202	25.11	7
NGC 681	2	...	2X	0.28	...	0.59	−20.05	436	32.55	8
NGC 753	9	1.0	4X	*0.11	*0.	0.55	−22.46	418	34.93	9
NGC 925	1	2.5	7X	0.75	34.	≥1.00	−20.54	231	30.74	10
NGC 1084	5	2.0	5A	−1.14	−37.	0.86	−21.14	321	32.05	11
NGC 1087	2	3.0	5X	*0.41	*34.	0.75	−21.09	272	32.27	12
NGC 1300	12	1.0	4B	*0.00	0.	0.52	−21.30	373	32.25	13
NGC 1357	12	...	1A	0.30	*0.	0.83	−20.50	423	32.84	14
NGC 1417	12	1.0	3X	*0.26	*21.	0.79	−22.28	577	34.55	9
NGC 1569	1	4.0	10B	0.91	42.	≥1.00	−17.25	108	26.94	15
NGC 1792	3	2.0	4A	0.64	19.	0.71	−21.01	306	31.54	16
NGC 1832	5	1.0	4B	0.39	10.	0.62	−21.05	292	32.73	17
NGC 1961	4	2.0	5X	0.59	26.	0.67	−23.40	−834	34.55	18
NGC 2146	1	2.0	2PB	−0.81	...	0.45	−21.12	776	32.06	19
NGC 2336	9	1.0	4X	*0.02	5.	≥1.00	−22.88	493	33.53	20
NGC 2403	4	3.0	6X	0.25	*10.	≥1.00	−20.56	248	29.02	4
NGC 2608	12	2.0	3B	*0.52	*0.	0.81	−20.83	271	33.34	12
NGC 2742	9	2.0	5A	*0.40	*14.	0.88	−20.71	329	32.16	9
NGC 2775	3	...	2A	−0.19	*−18.	0.73	−21.01	447	32.03	14
NGC 2776	9	1.0	5X	−0.01	−9.	0.63	−21.65	...	33.82	21
NGC 2805	5	...	7X	0.63	33.	0.97	−21.60	121	33.12	22
NGC 2841	3	1.0	3A	0.02	*3.	≥1.00	−21.46	636	31.27	23
NGC 2903	7	1.5	4X	−0.02	*−2.	≥1.00	−20.73	391	29.86	4
NGC 2997	9	1.0	5X	*0.00	0.	0.57	−21.62	302	31.59	24
NGC 2998	9	1.0	5X	*0.19	*2.	0.98	−22.79	...	34.91	9
NGC 3031	12	1.5	2A	−0.30	*−18.	≥1.00	−19.17	473	26.55	4
NGC 3054	9	1.0	3X	*0.29	*6.	0.86	−21.43	493	33.23	9
NGC 3145	2	1.0	4B	*0.08	*2.	0.85	−22.56	508	34.44	9
NGC 3310	1	2.0	4PX	0.31	...	0.55	−20.97	...	32.24	25
NGC 3351C	6	2.0	3B	−0.32	−31.	0.95	−20.14	302	30.42	26
NGC 3504	8	1.5	2X	*0.38	28.	0.69	−21.26	341	33.00	27
NGC 3521	3	2.5	4X	0.28	−4.	0.72	−20.76	489	30.16	28
NGC 3672	2	1.5	5A	*0.06	*5.	≥1.00	−22.05	406	33.15	9
NGC 3684	5	2.0	4A	1.06	...	0.44	−20.57	262	32.72	29
NGC 3686	5	1.5	4B	0.80	...	0.45	−20.82	262	32.74	29
NGC 3786	8	...	1PX	0.24	...	0.51	−21.28	200	33.98	30
NGC 3898	3	1.0	2A	*0.29	*0.	≥1.00	−20.98	633	32.59	9
NGC 3963	12	1.5	4X	*0.03	−2.	≥1.00	−21.81	...	34.03	31
NGC 3992C	9	1.0	4B	*0.09	9.	≥1.00	−21.63	517	32.03	32
NGC 4062C	3	2.5	5A	*0.43	*14.	0.55	−19.15	291	30.81	9
NGC 4088C	12	2.5	5PX	0.23	18.	≥1.00	−21.35	357	32.03	33
NGC 4136C	9	1.5	5X	0.93	52.	≥1.00	−19.11	...	30.81	2
NGC 4151	5	...	2X	0.05	−5.	≥1.00	−21.34	202	32.42	7
NGC 4237C	2	2.0	5X	0.71	24.	≥1.00	−19.75	434	32.01	34
NGC 4254C	9	1.0	5A	*0.29	*15.	0.78	−21.72	426	32.01	35
NGC 4303	9	1.0	4X	−1.53	*3.	0.56	−21.59	...	31.79	36
NGC 4321C	12	1.0	4X	*0.20	*14.	0.68	−22.01	444	32.01	9
NGC 4378	6	...	1A	−0.19	*−4.	0.96	−21.41	...	33.61	9
NGC 4433	1	3.0	4X	0.82	39.	0.50	−21.69	420	33.99	37
NGC 4450C	12	...	2A	0.25	*12.	0.96	−21.24	331	32.01	34
NGC 4490	1	3.0	7PB	0.68	37.	≥1.00	−20.43	230	30.34	38
NGC 4501C	9	2.0	3A	0.23	*7.	≥1.00	−22.11	548	32.01	34
NGC 4535C	9	1.0	5X	0.45	*11.	≥1.00	−21.48	332	32.01	34
NGC 4548C	5	1.5	3B	0.62	*29.	≥1.00	−21.16	352	32.01	34
NGC 4568C	2	2.5	4A	0.38	17.	≥1.00	−20.69	335	32.01	34
NGC 4579C	9	2.0	3X	0.08	*−6.	0.78	−21.55	605	32.01	34
NGC 4618	4	2.0	9B	0.47	20.	≥1.00	−19.00	192	30.19	20
NGC 4625	4	...	9X	0.46	20.	≥1.00	−17.57	...	30.45	20
NGC 4639C	2	2.0	3X	−0.37	−11.	0.64	−19.98	353	32.01	36
NGC 4647C	3	3.0	5X	0.70	32.	≥1.00	−20.22	446	32.01	34
NGC 4651C	9	1.5	5A	0.13	8.	0.90	−20.95	437	32.01	36
NGC 4654C	4	2.0	6X	0.13	17.	≥1.00	−21.19	316	32.01	34
NGC 4682	2	2.0	5X	*0.24	*6.	0.89	−20.85	...	33.17	9
NGC 4689C	3	2.0	5A	0.32	19.	0.86	−20.50	290	32.01	34

TABLE 1—*Continued*

Name[a] (1)	AC[b] (2)	DDO[c] (3)	T[d] (4)	LG[e] (5)	OG[f] (6)	R_{out}/R_{25}[g] (7)	M_B[h] (8)	W_R^i[i] (9)	$m-M$[j] (10)	References[k] (11)
NGC 4698C......	3	...	2A	0.57	*32.	0.75	−19.20	443	32.01	9
NGC 4731C......	5	3.0	6B	0.59	36.	≥1.00	−22.05	227	32.95	32
NGC 4736........	3	2.0	2A	−0.25	*2.	≥1.00	−20.25	373	29.05	4
NGC 4800........	2	2.5	3A	*0.20	*9.	0.82	−19.60	468	31.79	9
NGC 5033........	9	1.5	5A	*0.09	*3.	≥1.00	−21.91	454	32.24	39
NGC 5055........	3	2.5	4A	*0.00	*1.	≥1.00	−21.02	447	30.15	7
NGC 5371C......	9	1.0	4X	*0.04	8.	≥1.00	−22.45	585	33.77	39
NGC 5383C......	12	1.5	3PB	*−0.07	−1.	≥1.00	−21.84	608	33.77	40
NGC 5426........	2	1.0	5PA	*0.00	1.	0.71	−21.21	437	33.65	41
NGC 5427........	9	1.0	5PA	−0.59	−13.	0.72	−21.82	...	33.79	41
NGC 5457........	9	1.0	6X	0.10	8.	≥1.00	−21.33	...	29.53	42
NGC 5595........	4	2.0	5X	0.35	17.	≥1.00	−21.91	164	33.81	36
NGC 5728........	2	2.0	1X	0.79	36.	0.74	−22.55	591	34.01	43
NGC 5905........	12	1.0	3B	*0.11	8.	≥1.00	−22.32	620	34.03	44
NGC 5963........	2	...	12P	0.19	3.	0.86	−19.63	330	31.73	45
NGC 6643........	5	2.0	5A	0.04	−2.	0.75	−21.67	357	32.92	46
NGC 6764........	5	...	4B	1.36	73.	0.71	−21.42	369	33.72	47
NGC 6946........	9	2.0	6X	0.07	8.	≥1.00	−21.66	306	28.39	1
NGC 7171........	5	1.0	3B	*0.13	*−10.	0.61	−20.91	495	33.58	9
NGC 7217........	3	2.5	2A	−0.11	*−4.	0.74	−21.26	651	31.90	48
NGC 7331........	3	1.5	4A	*0.06	*12.	≥1.00	−21.98	522	31.65	39
NGC 7723........	5	1.5	3B	*0.00	10.	0.66	−21.17	873	32.76	49
NGC 7793........	2	4.0	8A	0.61	28.	0.95	−18.57	208	28.09	50
UGC 2259........	12	...	8B	*0.52	*11.	0.98	−17.77	167	30.87	4

[a] Galaxy name; a letter C following the name means the galaxy belongs to one of the clusters identified by Tully 1987.
[b] Arm class.
[c] DDO luminosity class.
[d] Hubble morphological type, coded as in NBG.
[e] Logarithmic gradient of the rotation curve; a starred superscript on the value means that is has not been computed from the rotation curve but from Persic & Salucci 1990b or PS88.
[f] Outer gradient of the rotation curve; a starred superscipt on the value means that it has not been computed from the rotation curve but taken from Whitmore, Forbes, & Rubin 1988 or W84.
[g] Furthermost sampling radius of the rotation curve, in units of the optical radius of the galaxy, used in the determination of the logarithmic gradient.
[h] Absolute blue total corrected magnitude.
[i] 21 cm line width parameter W_R^i, km s^{-1}.
[j] Distance modulus $m-M$.
[k] Reference to the rotation curve used in the determination of the gradients (the reference concerns LG only, when the value OG is preceded by a starred superscript; in this case the reference to OG can be found in Whitmore, Forbes, & Rubin 1988 or W84). (1) Young and Scoville 1982; (2) Allsopp 1979; (3) Lewis 1972; (4) Kent 1987; (5) Rubin & Ford 1983; (6) Peterson 1980; (7) Bosma 1981a; (8) Kyazumov & Barabanov 1980; (9) Rubin et al. 1985; (10) Marcelin, Boulesteix, & Courtès 1982; (11) Kyazumov 1981b; (12) Rubin, Ford, & Thonnard 1980; (13) Peterson & Huntley 1980; (14) Kent 1988; (15) Reakes 1980; (16) Rubin et al. 1964; (17) Burbidge & burbidge 1968; (18) Rubin, Ford, & Roberts 1979; (19) Benvenuti, Capaccioli, & D'Odorico 1975; (20) van Moorsel 1983c; (21) Carozzi-Meyssonier 1979; (22) Reakes 1979; (23) Bosma 1981b; (24) Peterson 1978; (25) van der Kruit 1976; (26) Peterson et al. 1976; (27) Peterson 1982; (28) Burbidge et al. 1964; (29) Blackman 1980; (30) Afanas'ev 1982; (31) van Moorsel 1983a; (32) Gottesman et al. 1984; (33) van Moorsel 1983b; (34) Guthathakurta et al. 1988; (35) Chincarini & de Souza 1985; (36) Distefano et al. 1990; (37) Chromey 1974; (38) Viallefond, Allen, & de Boer 1980; (39) Begeman 1988; (40) Sancisi, Allen, & Sullivan 1979; (41) Blackman 1982; (42) Huchtmeier 1975; (43) Rubin 1980; (44) van Moorsel 1982; (45) Romanishin, Strom, & Strom 1982; (46) Kyazumov 1981a; (47) Rubin, Thonnard, & Ford 1975; (48) Kent 1986; (49) Chevalier & Furenlid 1978; (50) Carignan & Freeman 1985.

the mean LG (*a*) and OG (*b*) per arm class, versus AC. Despite the scatter in the mean values of the gradients, we notice a trend for the gradients to decrease toward higher values of AC. This result is confirmed by the Rank sum test (see, e.g., Kendall & Stuart 1977; Hoel 1971), applied to the cumulative distributions of the LG (or OG) parameter for the two subsamples of G and F galaxies: there is a very low probability (1.93% and 0.32%, respectively, for LG and OG) that the two subsamples have the same parent distribution. In Figure 2 we plot the two cumulative distributions of LGs (*a*) and OGs (*b*) for G (*solid line*) and F galaxies (*broken line*); the difference in the two distributions is evident.

3.2. *Partial Correlation Analysis*

Other correlations could lead to an apparent dependence of AC from LG. There is a strong dependence of AC on the absolute magnitude of the galaxy (see § 1). The late Hubble-type spirals tend to avoid grand design configurations (EE82). The rotation curves of spiral galaxies depend both on the luminosity and the maximum rotational velocity (Rubin et al. 1985; Persic & Salucci 1991). The results of the correlations of AC and LG with these quantities are shown in Table 2, for our sample; M_B is strongly related both to AC and LG, W_R^i correlates with LG, but not with AC; the Hubble type T has a low correlation with AC and a strong correlation with LG.

In order to estimate how much of the correlation between LG and AC is not spurious, we estimate the Kendall's *partial* correlation coefficient, $r_{xy.w}$ (Kendall 1948), which is a measure of the correlation between two data sets, x and y, independently of their correlation with a third data set, w. Since the sampling distribution of $r_{xy.w}$ is unknown, we here adopted the bootstrap method of resamplings (see, e.g., Efron 1979; Efron

TABLE 2

CORRELATION ANALYSIS

Correlation Number[a] (1)	x[b] (2)	y[b] (3)	Number of Galaxies[c] (4)	r_P[d] (5)	r_S[e] (6)	r_K[f] (7)	Significance[g] (8)	Notes (9)
1..............	LG	OG	88	0.83	0.86	0.70	>99.99	
2..............	LG	AC	94	−0.25	−0.29	−0.20	99.79	
3..............	OG	AC	88	−0.31	−0.34	−0.25	99.97	
4..............	AC	M_B	94	−0.33	−0.38	−0.29	>99.99	
5..............	AC	W_R^i	82	0.11	0.12	0.10	91.74	
6..............	AC	T	85	−0.17	−0.15	−0.12	94.36	Peculiars excluded
7..............	M_B	LG	94	0.33	0.35	0.24	99.96	
8..............	W_R^i	LG	82	−0.40	−0.48	−0.33	>99.99	
9..............	T	LG	85	0.35	0.37	0.28	99.99	Peculiars excluded
10..............	M_B	OG	88	0.29	0.20	0.14	97.07	
11..............	W_R^i	OG	77	−0.30	−0.38	−0.26	99.95	
12..............	T	OG	80	0.45	0.49	0.38	>99.99	Peculiars excluded
13..............	OG	DDO	63	0.43	0.39	0.29	99.96	Multiple repression
14..............	M_B	RES	75	−0.23	−0.25	−0.17	98.59	Residuals AC vs. LG
15..............	W_R^i	RES	75	0.06	0.02	0.02	<90.00	Residuals AC vs. LG
16..............	T	RES	75	−0.11	−0.05	−0.05	<90.00	Residuals AC vs. LG
17..............	M_B	RES	72	−0.23	−0.27	−0.17	98.46	Residuals AC vs. OG
18..............	W_R^i	RES	72	0.06	0.00	0.00	<90.00	Residuals AC vs. OG
19..............	T	RES	72	−0.09	−0.02	−0.02	<90.00	Residuals AC vs. OG
20..............	AC	M_B	22	−0.75	−0.71	−0.52	99.96	Cluster galaxies
21..............	LG	AC	34	−0.25	−0.23	−0.17	92.07	Unbarred (A)
22..............	LG	AC	59	−0.29	−0.39	−0.28	99.91	Barred (B+X)
23..............	OG	AC	33	−0.33	−0.37	−0.28	98.90	Unbarred (A)
24..............	OG	AC	54	−0.38	−0.41	−0.30	99.94	Barred (B+X)
25..............	LG	AC	46	−0.36	−0.38	−0.27	99.65	Low distance
26..............	LG	AC	48	−0.12	−0.21	−0.15	92.76	High distance
27..............	LG	AC	47	−0.22	−0.24	−0.16	94.90	Low i
28..............	LG	AC	47	−0.28	−0.33	−0.23	98.91	High i
29..............	LG	AC	47	−0.17	−0.16	−0.11	<90.00	Low R_{out}/R_{25}
30..............	LG	AC	47	−0.45	−0.41	−0.29	99.80	High R_{out}/R_{25}
31..............	LG	AC	39	−0.49	−0.47	−0.35	99.93	$R_{out}/R_{25} \geq 1$
32..............	LG	AC	82	−0.26	−0.29	−0.21	99.74	AC = 12 excluded
33..............	OG	AC	76	−0.29	−0.34	−0.25	99.94	AC = 12 excluded

[a] Progressive correlation number.
[b] Parameters considered in the correlation, x and y, respectively.
[c] Number of galaxies in the sample.
[d] Linear regression correlation coefficient, r_P.
[e] Spearman's rank correlation coefficient, r_S.
[f] Kendall's rank correlation coefficient, r_K.
[g] Percent significance level assigned to r_K.

& Tibshirani 1985) in order to compute its statistical significance (we performed 5000 bootstrap resamplings for each correlation). The results are listed in Table 3; they clearly indicate that the correlation between LG (or OG) and AC is not induced by other correlations, since the significance of the gradient–arm class correlation, cleaned from spurious secondary dependences, still remains at a level >97%.

3.3. *Multiple Regression Analysis*

In order to evaluate the weight of each single parameter in the determination of AC, we have to perform a multiple linear regression analysis (see, e.g., Flury & Riedwyl 1988); yet if we do this, we have to give up the nonparametric methods employed so far. We feel confident in doing this since the parametric coefficient, r_P, has yielded results that are not very different from nonparametric ones, r_S and r_K (compare cols. [5] and [6] in Table 2). We consider the restricted sample of 65 galaxies for which we know all of the following quantities: LG, M_B, W_R^i, T, taken as independent variables (regressors), AC, taken as the dependent variable, and also (see below) the DDO class (when OG is considered in place of LG, the sample is restricted to 63 galaxies).

We adopt the method of *backward elimination* (Flury & Riedwyl 1988) in order to identify the fundamental regressors for the dependent variable AC. If AC is correlated with a certain number p of regressors, we first compute the coefficient of determination, R_p^2, with all p regressors, and then with $p-1$ regressors, eliminating each independent variable one at a time. We eliminate the variable which gives the smallest contribution to R_p^2, i.e., the variable whose elimination leads to the smallest diminution in R_p^2. The method proceeds by eliminating all variables except one. This analysis gives qualitative results: the less important regressors are eliminated first, and an abrupt change in R_p^2 suggests that the variable eliminated is important in the determination of AC.

The results are summarized in Table 4, where we list the values of R_p^2 with p with p decreasing from 4 (all regressors) to 1. The regressors used in the correlation with AC are listed in column (1), in the order of their subsequent elimination. The behavior of R_p^2 versus p is also shown in Figure 3 (the gradient parameter used is LG in panel [*a*] and OG in panel [*b*]; panel

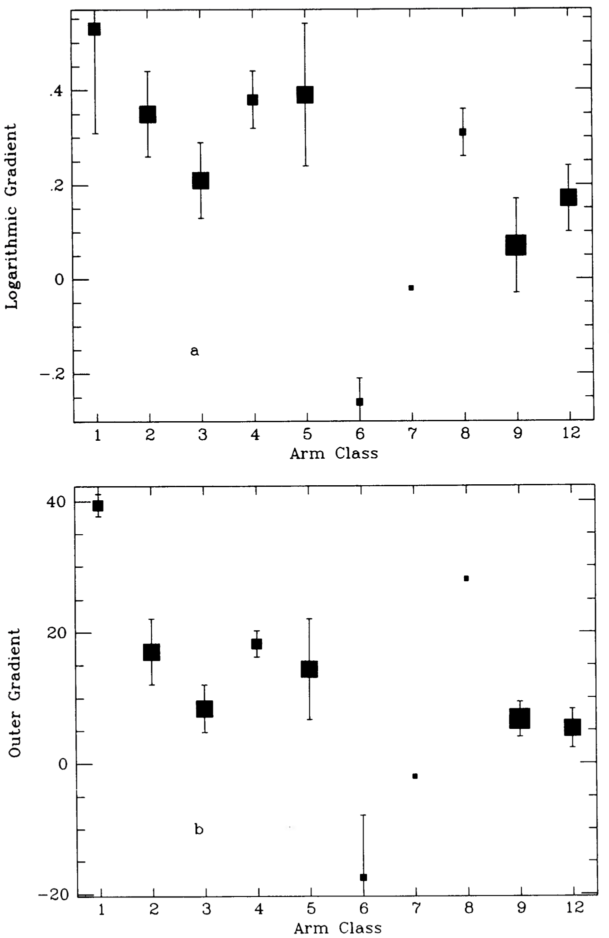

FIG. 1.—(*a*) Mean logarithmic gradient LG = $[\partial \log V/\partial \log R]_{R_{25}}$, per arm class, vs. arm class; the area of the squares is proportional to the number of points used to compute the mean; the error bars are ± one standard deviation from the mean. (*b*) Same as (*a*), with the outer gradient OG = $[V(0.8R_{25}) - V(0.4R_{25})]/V_{\max}$, in lieu of LG. A square without error bars represents a single galaxy.

[*c*] will be described below), where the regressors used are explicitly listed on the *x*-axis for each value of R_p^2. The gradient parameter (LG as well as OG) is a fundamental regressor for AC; in fact, it is the last regressor eliminated, and its elimination causes an abrupt change in R^2. Among the other parameters considered here, only M_B shows a similar behavior, and AC is determined by LG (or OG) and M_B almost as well as by the whole set of parameters.

Following W84, we have considered the correlation between DDO and OG; it is highly significant and gives $r_P = 0.43$ on 63 data points, in agreement with the result of W84. This value is very close to the value, $R = 0.48$, of the multiple regression coefficient (i.e., the square root of R^2) of AC with the two regressors OG and M_B. This fact is possibly related to the definition of the DDO parameter, which is based on both the arm structure and the brightness of the galaxy, and somehow incorporates the two quantities AC and M_B.

We further investigate the importance of different parameters in the determination of AC, by taking the residual (labeled RES) of the least-squares lines fitted on the samples of data-points (LG, AC), and (OG, AC). Only M_B significantly correlates with these residuals (see Table 2). The importance of M_B in the AC determination was stressed by EE82 and EE87. We reexamine the AC-M_B correlation for a sample of cluster galaxies (labeled with a C in Table 1), in order to avoid large errors in the distance determinations (in fact, cluster distances are usually better defined than the distances to single galaxies). The sample is reduced to 22 galaxies only; however, the M_B versus AC correlation is still significant (see Table 2).

3.4. *Bars*

The importance of bars in determining the arm phenomenology is not fully understood (see, e.g., Elmegreen & Elmegreen 1989; EE82; Carlberg 1986; Toomre 1981; Binney & Tremaine 1987; KN79). In early-type spirals, bars seem to be connected to the grand design two-armed spiral structure, yet the opposite seems to hold for late-type galaxies (see, e.g., Elmegreen & Elmegreen 1985, 1989). Sellwood (1981) has found that a necessary condition for a bar formation is the presence of a rising rotation curve in the inner part of a galaxy, but we are dealing with outer, not inner, gradients. We perform the correlation analysis on our subsamples of barred (B) and unbarred

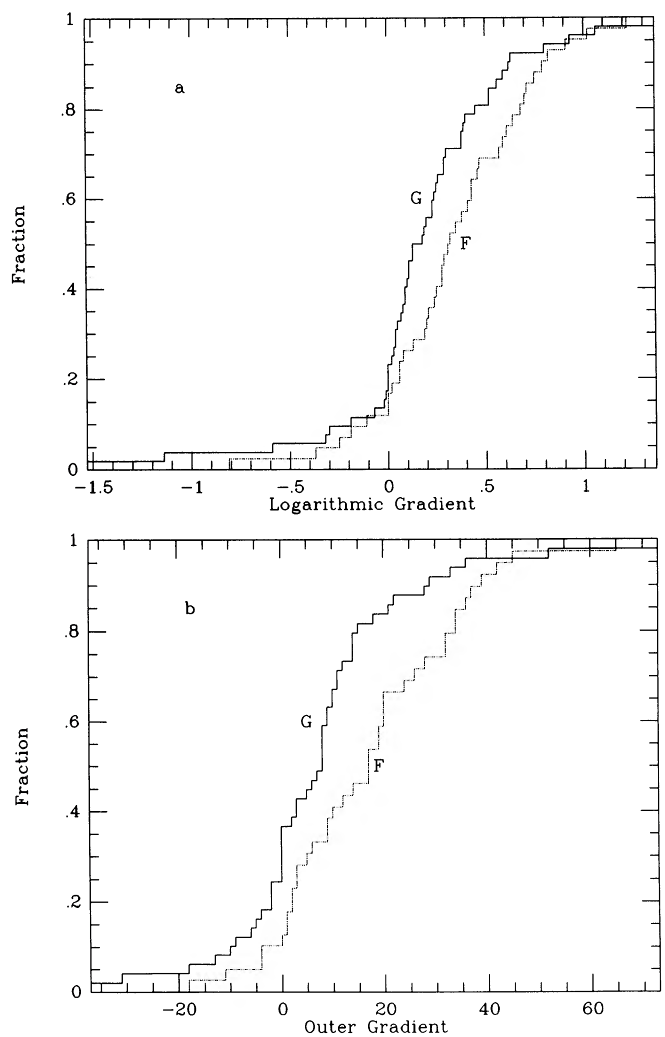

FIG. 2.—(*a*) Cumulative distribution functions of the logarithmic gradients LG = $[\partial \log V/\partial \log R]_{R_{25}}$, for grand design (*solid line*) and flocculent (*broken line*) galaxies. (*b*) Same as (*a*), with the outer gradients OG = $[V(0.8R_{25}) - V(0.4R_{25})]/V_{\max}$, in lieu of LGs.

TABLE 3

PARTIAL CORRELATION ANALISIS

Correlation Number[a] (1)	x^b (2)	y^b (3)	w^b (4)	Number of Galaxies[c] (5)	$r_{xy.w}$[d] (6)	Significance[e] (7)	Notes (8)
1	LG	AC	M_B	94	−0.14	97.34	
2	LG	AC	W_R^i	82	−0.16	97.44	
3	LG	AC	T	85	−0.19	99.62	
4	OG	AC	M_B	88	−0.22	99.86	
5	OG	AC	W_R^i	77	−0.22	99.72	
6	OG	AC	T	80	−0.22	99.66	
7	LG	AC	M_B	39	−0.24	97.42	$R_{out}/R_{25} \geq 1$

[a] Progressive correlation number.
[b] Parameters considered in the correlation, x, y, and w, respectively.
[c] Number of galaxies in the sample.
[d] Kendall's partial correlation coefficient, $r_{xy.w}$.
[e] Percent significance level assigned to $r_{xy.w}$, based on 5000 bootstrap resamplings.

(A) galaxies; intermediate cases (X) are considered together with barred galaxies. The correlation between AC and LG (or OG) seems to be higher in the B+X sample than in the unbarred A galaxy sample, but the difference is not significant (Kendall & Stuart 1977, vol. 2, p. 315).

3.5. *Selection Effects*

Some selection effects could in principle affect the correlation found between AC and the gradient. When distance increases, the arm classification can suffer from decreased spatial resolution, and the rotation curve may approach a delta function, difficult to resolve. Also the inclination angle of the galaxy, i, can affect the observations, higher i galaxies having better determined gradients, and lower i galaxies, better determined arm classes. Moreover, the nonextrapolated gradients ($R_{out}/R_{25} \geq 1$) are probably better determined than others.

We try to estimate the importance of these biases by taking the residuals from the least-squares line fitted on the data point (LG, AC), and correlating these with the distance moduli, the inclinations, and the sampling parameters R_{out}/R_{25}. The correlations are all significant at a $\sim$95% level. This means that the dispersion in the AC versus LG correlation can be at least partically accounted for by these biases. We divide our data set in two subsamples, three times: (1) galaxies with distance modulus lower and higher than the median distance-modulus; (2) galaxies with inclination lower and higher than the median inclination; (3) galaxies with sampling parameter R_{out}/R_{25}, lower and higher than the median value. We investigate the existence of the AC-LG correlation on these subsamples. The correlation coefficient is larger when acting in the subsamples of nearby galaxies, high-inclination galaxies, and galaxies with well-sampled rotation curve (see Table 2, results 25–30).

These results suggest that the LG-AC correlation is improved when we select data (gradients, in particular) of good quality. Therefore, we select galaxies according to the accuracy of their gradients, i.e., those with $R_{out}/R_{25} \geq 1$; a subsample of 39 galaxies is thus obtained. The LG-AC correlation is confirmed and strengthened in this subsample (see Table 2). The AC-LG diagram is plotted in Figure 4 (the meaning of the symbols is the same as in Fig. 1), and Figure 5 shows the cumulative distribution of the LG parameter for the G and F galaxies of this subsample. The Rank Sum Test yields a probability of 2.38% that the two samples are taken from the same distribution, in agreement with the above result for the total sample. The multiple regression analysis confirms the comparable importance of LG and M_B in determining AC (see Table 4 and Fig. 3*c*). Moreover, the coefficient of determination rises to 0.329, i.e., almost one-third of the variation in AC is determined by LG and M_B (see Flury & Riedwyl 1988, p. 63). We estimate the coefficient of partial correlation between AC and LG at fixed M_B: although the significance is reduced, the correlation is still present (see Table 3).

TABLE 4

MULTIPLE REGRESSION ANALYSIS

Regressors[a] (1)	Number of Galaxies[b] (2)	Dependent Variable: AC R_4^2[c] (3)	R_3^2[c] (4)	R_2^2[c] (5)	R_1^2[c] (6)	Notes[d] (7)
T, W_R^i, LG, M_B	65	0.190	0.186	0.181	0.131	P.E., DDO
T, W_R^i, OG, M_B	63	0.232	0.231	0.224	0.131	P.E., DDO
T, W_R^i, M_B, LG	31	0.329	0.322	0.315	0.278	P.E., $R_{out}/R_{in} \geq 1$

[a] Parameters used in the multiple regression analysis as regressors for the dependent variable AC; they are listed in order of subsequent elimination.
[b] Number of galaxies in the sample.
[c] Coefficient of determination R_p^2 of AC with p regressors considered; the regressors are eliminated one at a time, in the order in which they are listed in col. (1).
[d] Notes on the sample considered. P.E.: the galaxies classified peculiar are excluded from the sample; DDO: only galaxies with DDO class known have been considered.

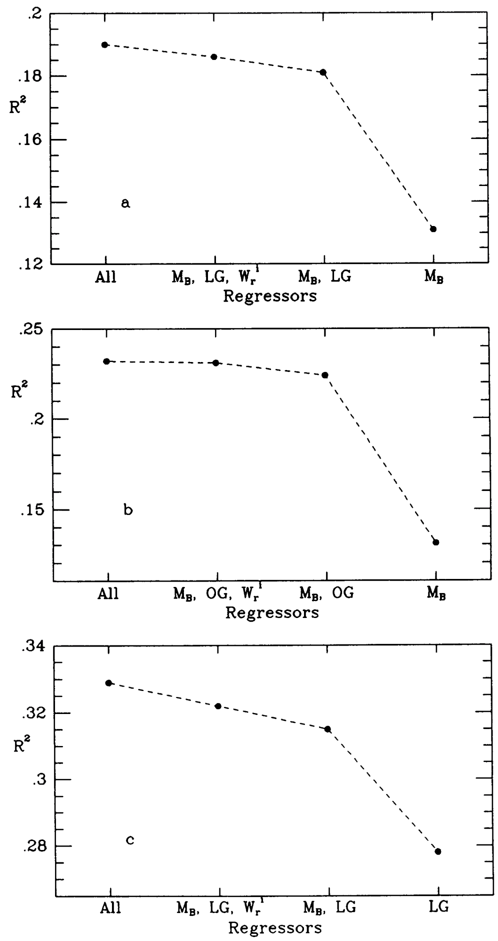

FIG. 3.—Multiple correlation analysis between the dependent variable AC and the regressors LG (or OG), M_B, W_R^i, T; plot of the coefficient of determination R_p^2 vs. the number p of regressors, listed on the x-axis; the label "All" refers to all the regressors used in the computation, i.e., (a) and (c): LG, M_B, W_R^i, and T; (b): OG, M_B, W_R^i, and T. Only the galaxies for which all parameters used in the regression are known, have been considered in the samples. The samples of panels (a) and (b) are further restricted to the galaxies with known DDO class. The sample of panel (c) is restricted to galaxies with sampling parameter $R_{out}/R_{25} \geq 1$. Peculiar galaxies are always excluded.

4. DISCUSSION

We have found that the arm class depends on the gradient of the rotation curve, in the sense of F galaxies having higher gradients than G galaxies. Our result is consistent with those obtained by W84 and EE90. The inconsistency between such more recent works and KN79 can be ascribed to the fact that KN79 lack good rotation curves, and good arm class and gradient parameters.

The similarity between the gradients of F and M galaxies, found by EE90, is no longer present in our sample: a continuous decrease of the gradient with increasing AC can be seen in our figures, and a significant correlation between AC and LG (or OG) still exists, even when AC = 12 galaxies are excluded from our sample (see Table 2, results 32 and 33).

The works by W84, EE87, EE90, and the present paper provide a strong constraint on the theories dealing with the generation and/or preservation of the arm structure: two parameters combined, the optical luminosity and the gradient

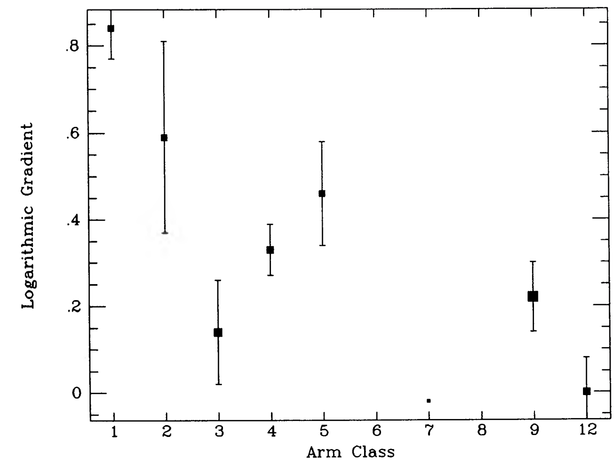

FIG. 4.—Mean logarithmic gradient LG = $(\partial \log V/\partial \log R)_{R_{25}}$, per arm class, vs. arm class, for the restricted sample of galaxies with sampling parameter $R_{out}/R_{25} \geq 1$. The meaning of the symbols is the same as for Fig. 1.

of the rotation curve of galaxies, can account for about one-third of the variation in AC.

The meaning of the correlation between the arm structure and the optical luminosity is not yet understood: either the prominent wave modes are more easily generated in larger galaxies, or the presence of a well-developed density wave enhances the star formation (see Elmegreen & Elmegreen 1986 for the lack of observational evidence on this latter point). On the contrary, the correlation between the arm structure and the gradient of the rotation curve, can be understood in the framework of the density wave theory (see § 1), if the gradient is indeed a fair indicator of the fractional mass in the disk (see, e.g., Whitmore 1989; PS88; Persic & Salucci 1990a; and references therein). This result suggests that the spiral structure is not a transient feature, although it can be regenerated by several recurrent episodes of triggering and amplification of wave modes, as long as the disk retains its mass.

The density wave theory has been previously supported mostly by simulations (see, e.g., Toomre 1981; Carlberg &

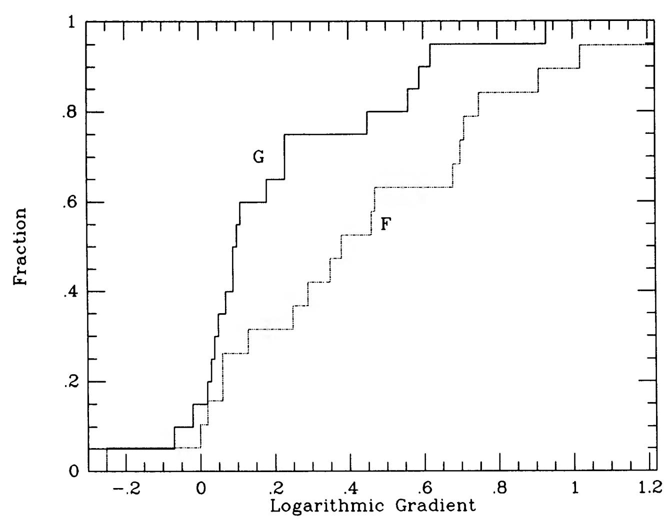

FIG. 5.—Cumulative distribution functions of the logarithmic gradients LG = $(\partial \log V/\partial \log R)_{R_{25}}$, for grand design (*solid line*) and flocculent (*broken line*) galaxies, for the restricted sample of galaxies with sampling parameter $R_{out}/R_{25} \geq 1$.

Freedman 1985; Sundelius et al. 1987). Now, the agreement among W84, EE90, and the present work adds an observational support to this theory. Spiral arms no longer appear fancy perturbations in galaxy disks but useful indicators of galaxy dynamics.

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