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ON THE ORBITAL CIRCULARIZATION OF CLOSE BINARIES

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ABSTRACT

We suggest a modified approach to the tidal circularization of short-period binaries with convective envelopes. The proposed model is based on Zahn's old theory and is motivated by his recent comment about the reduction of stellar viscosity due to the fact that the tidal variation time scale is comparable to or shorter than the typical convection turnover time scale. In the modified approximation the circularization time scale for close binaries is proportional to the binary period to the power of 10/3. This exponent is smaller than in previously suggested theories. The presently available data, though admittedly sparse, are compared with the different approximations. The new exponent seems to fit the data better than those of the previous theories.

Subject headings: stars: binaries — turbulence

1. INTRODUCTION

In recent years there has been renewed interest in the orbital evolution of close binary systems. This is mainly due to the availability of large samples of spectroscopic binaries, made possible by the new generation of stellar speedometers in full operation at several observatories (see Latham 1985). In particular, the decay of the orbital eccentricity of binaries resulting from the tidal interaction between the two components has been studied. The circularization time scale strongly depends on the binary separation and stellar structure; however, its exact dependence on the binary parameters is still unclear.

Three different theoretical approaches to the mechanism of orbital circularization have been suggested. The first approach, proposed by Zahn (1966), assumes that the tidal interaction between the stars in a binary system induces a localized tidal bulge on each of two stars. Because of the viscosity, the stellar bulges lag (or precede) the line connecting the centers of the two stars, inducing torques which tend to circularize the binary orbit. The first approximate calculations indicated that the tidal interaction through the stellar main-sequence phase is strong enough to circularize the short-period binaries (Zahn 1966, 1977; Alexander 1973; Lecar, Wheeler, & McKee 1976; Hut 1981). Within this theory, the circularization time scale for late-type stars with convective envelopes is proportional to the orbital period of the binary to the power 16/3 (Zahn 1977; Mathieu & Mazeh 1988).

Tassoul (1988) has suggested a completely different mechanism for the circularization of short-period binaries. It involves large-scale, transient meridional currents induced by the tidal distortion of the stellar axial symmetry. The proposed mechanism (Tassoul 1987) was originally suggested to account for the synchronization or pseudo-synchronization observed in early-type stars with radiative envelopes. Tassoul (1988) further claimed that, when applied to late-type binaries, this novel mechanism is more efficient than the one suggested by Zahn (see also Tassoul & Tassoul 1990). Within this theory, the circularization time scale is proportional to the period of the binary system to the power 49/12.

A different approach to the orbital circularization was suggested recently by Zahn (1989). He noticed that the variation of the tidal stellar deformation occurs, in short-period binaries, on a time scale comparable to or shorter than a typical stellar convective turnover time scale. This effect tends to make the viscosity less effective and makes the circularization time scale much longer. Zahn & Bouchet (1989) followed this argument and claimed that for binary systems with masses ranging from 0.5 to 1.25 $M_{\odot}$, most of the orbital circularization occurs during the pre–main sequence phase. The subsequent decay of eccentricity on the main sequence is negligible.

One way to confront the different theories with the observations is to consider a coeval sample of binary systems with similar components. Tidal interaction will tend to circularize the orbits of the close binaries, while the wide binaries will retain their orbital eccentricities (see Koch & Hrivnak 1981). If the sample is large enough, the transition period between the circular and eccentric binaries can be easily observed. The circularization time scale of the transition period can then be compared with the evolutionary age of the sample. Mayor & Mermilliod (1984) indeed discovered this effect in the binaries of the open clusters of the Hyades and Praesepe, with a transition period of 5.7 days.

In the last few years transition periods have been observed in a few samples of spectroscopic binaries: the pre–main-sequence stars (Mathieu, Walter, & Myers 1989), Hyades/Praesepe (Mayor & Mermilliod 1984; Burki & Mayor 1986), the open cluster M67 (Mathieu & Mazeh 1988; Mathieu, Latham, & Griffin 1990), and the Galactic halo stars (Latham et al. 1988a, b; Jasniewicz & Mayor 1988). The observed transition periods seem to increase with the sample age (Mazeh et al. 1990), indicating circularization processes which take place during the stellar main-sequence phase. However, the few observed transition periods do not seem to agree with either of the two exponents—16/3 and 49/12—suggested by the existing theories (see Fig. 2 of Jasneiwicz & Mayor 1988).

In this work we propose a modified model for the circularization time scale, based on Zahn's (1966) old approach and motivated by his recent comment (Zahn 1989). We take into account the reduction of the effective viscosity due to the fact that the convective time scale is comparable to or longer than the tidal time scale for short-period binaries. We show that the circularization time scale can still be approximated as a new power law of the binary period (§ 2). The new exponent, 10/3, is substantially smaller than that of the early theory of Zahn and is even smaller than the exponent derived by Tassoul (1988). We compare our results with the accumulated data (Mazeh et al. 1990). The data, though admittedly sparse, are consistent with this new exponent (§ 3). The limitations of the present data are discussed further in § 4.

2. THE MODIFIED MODEL

2.1. *Turbulent Viscosity*

The tidal interaction between two components of a stellar system induces a velocity gradient in the envelopes of the two stars. The viscosity in the envelopes causes a frictional force between adjacent fluid layers of different velocities, which leads to the synchronization of stellar rotation with the orbital period and to the circularization of the binary orbit. The time scales for these processes are *inversely proportional to the stellar viscosity*. Therefore, in any theory of circularization, the estimate of the viscosity is of prime importance. The basic source for viscosity, molecular viscosity, turns out to be too small to produce effective circularization and synchronization. However, the effective viscosity can be largely enhanced if there exists a turbulent velocity field. To discuss the induced turbulent viscosity in stellar envelopes, we turn first to consider very briefly some basic relevant aspects of turbulence.

In a fully developed turbulence (see, e.g., Batchelor 1973; Hinze 1975), the turbulent velocity is composed of components with different spatial scales, each characterized by a wavenumber k. It can be (heuristically) visualized as a superposition of eddies, where the eddy corresponding to the wavenumber k rolls over a size $\sim k^{-1}$ with a velocity $\sim v(k)$. The turbulent eddies interact among themselves. The interaction can be described in terms of the breakup of an eddy of wavenumber k into smaller eddies, resulting in a transfer of energy from the large-scale (small-k) eddies to the small-scale (large-k) ones. The time scale for this energy cascade is the eddy correlation time scale, $\tau(k)$, which is of the order of a rollover time of the eddy, $\sim k^{-1}v(k)^{-1}$ (Batchelor 1973; Hinze 1975). This time scale can be pictured as the lifetime of an eddy of wavenumber k before it breaks up into smaller eddies.

For the turbulence to be in a steady state, there must exist some stirring mechanism that generates the turbulence. Typically, the source feeds energy predominantly to the large-scale eddies with the largest scale k_0^{-1}, determined by the characteristics of the source. At each k, part of the energy is cascaded to smaller eddies and part is dissipated by the molecular viscosity into heat. At small k the transfer into larger k's dominates, but as k increases, the relative importance of the dissipation increases (see, e.g., Batchelor 1973). Consequently, the transfer of energy to smaller eddies terminates at some small enough scale.

Consider now a nonturbulent velocity gradient introduced into a turbulent medium. An eddy of wavenumber k generates a dissipative interaction between two adjacent fluid layers, at a distance $\sim k^{-1}$, that have different velocities owing to the velocity gradient. The combined effect of all modes in the turbulence spectrum is to produce the effective drag force that can be represented in terms of an effective viscosity. As shown in Appendix A, the total turbulent viscosity v_t is contributed mostly by the large-scale eddies and is given by

$$v_t = v_t(k_0) \sim v(k_0)k_0^{-1} , \tag{1}$$

where k_0 is the wavenumber of the largest eddy. The turbulent viscosity is thus a product of the velocity and size of the largest eddy.

The energy drained from the velocity gradient into the turbulence is transferred to smaller scales by the turbulence cascade and is dissipated at the smallest scales by the molecular viscosity. In Appendix B we show that, for the case under consideration here, the rate of energy drained into the turbulent convection from the tidal velocity gradient is much smaller than the original rate of energy cascade in this turbulence; thus the turbulence is essentially unchanged by the interaction with the tidal velocity gradient.

2.2. *Reduction of the Turbulent Viscosity for Short Orbital Periods*

Let us consider a close binary with a small eccentricity in synchronous rotation. In this case the tidal velocity gradient varies on a time scale of the orbital period P. The above description of the interaction between the turbulence and the tidal velocity gradient tacitly assumed that the time scale for variation of the tidal velocity gradient is much larger than the convective time scale. Consider now a situation where this assumption does *not* apply, because the orbital period P is comparable to or shorter than $\tau(k_0)$, the time scale for the interaction of the largest eddies with the tidal velocity gradient. Thus, over a time shorter than the period P, there will not be enough time for the interaction to take place. On the other hand, over a time comparable to $\tau(k_0)$, the large eddies will effectively interact with some time average of the tidal velocity gradient. Thus, either way, one expects that the effective turbulent viscosity exerted on the velocity gradient will be drastically reduced. Zahn (1989) was the first to realize that such a reduction is relevant for the problem of circularization of close binaries, since the orbital periods are indeed comparable to or shorter than the time scale of the largest eddies of the convection in stellar envelopes.

To account for the reduction factor in binaries with $P \leq \tau(k_0)$, Zahn (1989) proposed (following Zahn 1966) that the effective free path of the largest eddy is now $v(k_0)P/2$ instead of k_0^{-1}. This follows from the assumption that the largest eddies interact with the tidal velocity gradient on a time scale $P/2$. Within this time span, the viscous interaction can take place between stellar layers separated by a distance $v(k_0)P/2$. The resulting reduction in the turbulent viscosity is proportional to $P/\tau(k_0)$. Goldreich & Keeley (1977) faced a similar problem in the context of the damping of solar pulsations by the turbulent convention. Their approach to the problem was quite different: they suggested that when the pulsation period P_p is smaller than $\tau(k_0)$, no eddies with $\tau(k)$ larger than P_p could contribute to the effective turbulent viscosity. Taking the turbulent spectrum to be of the Kolmogorov form (see Appendix A), they found a reduction proportional to $[P_p/\tau(k_0)]^2$. Note that the two different suggestions imply *different functional dependences* of the circularization time scale on the binary period P.

We find the suggestion of Zahn (1966, 1989) unconvincing, since the largest eddy does not break before rolling over a distance comparable to its size, in a time $\sim \tau(k_0)$. Therefore, we agree with Goldreich & Keeley (1977) that the largest eddies will not contribute to the turbulent viscosity. The presence of the tidal velocity gradient does not change this fact, since, as noted above, the turbulence in the present case is effectively unchanged by the energy it drains from the tidal velocity gradient.

To clarify this point further, let us use an analogy with molecular viscosity, which is due to transfer of momentum by molecules over a distance comparable to their mean free path (see, e.g., Reif 1965). Consider a velocity gradient that changes on a time scale P shorter than the mean free time between collisions τ. Molecules with an intercollision time comparable to or larger than P will interact with the average velocity gradient and will not contribute to the viscosity. Therefore, the molecular viscosity will be contributed only by those molecules that spend time between collisions small compared with P. This corresponds to a reduced mean free path of $\lambda P v$, with v being the mean molecular thermal velocity and λ an arbitrary constant smaller than unity. This mean free path is reduced by $\lambda P/\tau$ compared with the mean free path $v\tau$. This is analogous to Zahn's (1966, 1989) suggestion (choosing $\lambda = \frac{1}{2}$). However, it is important to remember that an average molecule *will not suffer a collision* after a time shorter than τ. Only a fraction

$$(1 - e^{-\lambda P/\tau}) \sim \lambda P/\tau$$

of the molecules will do so. As a result, the molecular viscosity will be reduced by a factor of $(\lambda P/\tau)^2$. Thus, if one is willing, like Zahn (1966, 1989), to consider a breakup of the largest eddies in time scales shorter than $\tau(k_0)$, the probability for this to happen cannot be taken equal to unity. A more detailed calculation that considers the contribution from molecules with all possible values of the time between collisions (taking into account the probability distribution of this free time) yields the same result.

In the alternative approach (Goldreich & Keeley 1977) an eddy of wavenumber k interacts with the velocity gradient on a time scale comparable to its correlation time scale $\tau(k)$. Therefore, only the part of the turbulence spectrum with $\tau(k)$ small enough compared with P will contribute to the effective turbulent viscosity. Since the correlation time scale is typically a decreasing function of k (Batchelor 1973), only eddies smaller than some size k_*^{-1} will contribute to the turbulent viscosity. Consequently, the effective turbulent viscosity in short-period binaries, $\nu_{t,\text{short}}$, will be given not by equation (1) but instead by an analogous expression with k_* replacing k_0:

$$\nu_{t,\text{short}} = \nu_t(k_*) \sim v(k_*)k_*^{-1} . \tag{2}$$

We will denote by $\nu_t(k_0)$ the nonreduced turbulent viscosity given in equation (1).

For short-period binaries with $P \leq \tau(k_0)$ the value of k_* is determined by setting

$$\tau(k_*) = \lambda P , \tag{3}$$

with $\lambda < 1$ a dimensionless coefficient; $1/(2\pi)$ was adopted by Goldreich & Keeley (1977). The specific functional dependence of the reduced turbulent viscosity on P is determined by the form of the turbulence spectrum. In Appendix A we argue that for turbulent convection in solar-type stars, and for the values of the orbital periods relevant here, the spectrum is of the Kolmogorov form yielding (see eq. [A10])

$$\nu_t = \nu_{t,\text{short}} = A\lambda^2\nu_t(k_0)[P/\tau(k_0)]^2 , \quad \lambda P < \tau(k_0) , \tag{4a}$$

while for long orbital periods

$$\nu_t = \nu_t(k_0) , \qquad P \gg \tau(k_0) . \tag{4b}$$

Here $A \sim 3$, because at k_0 the slope of the turbulence spectral function is smaller than that of the Kolmogorov spectrum. Since the circularization time scale T_circ is proportional to $\nu_t^{-1}P^{16/3}$, we get

$$T_\text{circ} \propto P^{10/3} , \qquad \lambda P < \tau(k_0) , \tag{5a}$$

and

$$T_\text{circ} \propto P^{16/3} , \qquad P \gg \tau(k_0) . \tag{5b}$$

Equation (4a) was used to express the reduced $\nu_{t,\text{short}}$ in terms of the nonreduced $\nu_t(k_0)$.

The periods of the binaries with circularized orbits considered here are distributed between 2 and 20 days (see Zahn 1989; Mazeh et al. 1990). In order to find out which of the above equations apply to these binaries, we have to estimate the convective time scale $\tau(k_0)$. The estimate is complicated by the fact that this time scale is a function of depth in the convection layer. However, the weighted average of the convection time scale will be close to its value at the lower part of the convection zone. This is so, since the tidal energy dissipated per unit volume is proportional to the gas density times the turbulent viscosity, both of which increase with depth. For solar-type stars the convective time scale at the deepest part of the convective zone is estimated to be $\simeq 20$ days in the model of Spruit (1974), while the corresponding value in the model of Goldreich & Keeley (1977) is $\simeq 12$ days. Thus, it seems plausible that $\tau(k_0)$ is in the range 10–20 days. For $\lambda \leq 0.5$, equation (5a) applies to all binaries with periods shorter than 20–40 days, and therefore 10/3 is the relevant power law for the discussion of the circularization time scale.

The absolute value of the circularization time scale depends, of course, on the turbulent viscosity $\nu_t(k_0)$, the value of which depends on the specific model used for the convection layers. In particular, it depends on the ratio between the mixing length and the pressure scale height, and also on whether the model includes the possibility of overshooting of the convective eddies into the radiative stellar interior (VandenBerg & Poll 1989). (Andersen, Nordström, & Clausen 1990 recently found the overshooting effect to be very important also in more massive stars with convective cores.) Different models yield different values for the depth of the convection zone and for the convective velocities, and therefore different values for the turbulent viscosity.

To estimate the absolute value of the circularization time scale, we consider a binary consisting of two 1 $M_\odot$ stars with a period of 5.7 days—the transition period of the Hyades and Praesepe. For the convection model used by Zahn (1989) and the standard approach of equation (4b), we find (see his eq. [15]) that the circularization time scale for such a binary is 1.5×10^9 yr. A similar value of 1.6×10^9 yr was obtained for these parameters by Mathieu & Mazeh (1988; see their Table 1). With regard to the crudeness of our estimate of the turbulent viscosity, these values are surprisingly close to the age of the Hyades, which is estimated to be 0.8×10^9 yr. Other convection models would yield different values. In particular, incorporating overshooting into the model will increase the effective width of the convective zone and thus increase (see eq. [1]) the turbulent viscosity and shorten the circularization time scale. Moreover, even a small amount of overshooting can be important, since the rate of energy dissipation per unit volume is proportional to the density, which is higher in the radiative interior.

The reduction of the turbulent viscosity in equation (4a), compared with the standard value of equation (4b), depends quadratically on the (unknown) values of λ and $\tau(k_0)$. Here we present this factor normalized to $\lambda = 0.5$, $P = 6$ days, and $\tau(k_0) = 10$ days, as

$$A^{-1}\left[\frac{\lambda P}{\tau(k_0)}\right]^{-2} \sim 4\left(\frac{\lambda}{0.5}\right)^{-2}\left(\frac{P}{6\ \text{days}}\right)^{-2}\left[\frac{\tau(k_0}{10\ \text{days}}\right]^2 . \tag{6}$$

We see that the reduction factor is not necessarily large for the binaries discussed here; for the adopted parameters of equation (6) it is of the order of 4. Therefore, if we adopt the crude estimates of the nonreduced viscosity discussed above, the discrepancy between the circularization time scale of the modified approach and the evolutionary age of the Hyades is somewhat less than a factor of 10. Given the uncertainties of the theory, we find this difference acceptable.

3. COMPARISON WITH THE OBSERVATIONS

The proposed modified approach for circularization of short-period binaries with convective envelopes predicts that $T_{\rm circ} \propto P^{10/3}$. This model as well as the previous ones, should be tested against the presently available data. The rapidly accumulating data about spectroscopic binaries have revealed so far distinct (albeit different) transitions between circular and eccentric orbits at *three* coeval samples of late-type main-sequence close binaries. In the three samples, all binaries with substantial eccentricities have periods longer than the transition period. The results for the three samples taken from the compilation of Mazeh et al. (1990) are summarized in Table 1; see also Jasniewicz & Mayor (1988).

The transition periods given in Table 1 are preliminary, since the distinction between eccentric and circular binaries as a function of their period is not sharp. The samples include two binary systems—G65-22 in the Galactic halo ($P = 18.74$ days) and J331 in the Hyades ($P = 8.5$ days)—with circular or nearly circular orbits. The interpretation of these outliers is not clear (Zahn & Bouchet 1989; Mazeh et al. 1990). In addition, a few binaries with small, yet nonzero, ecentricities are found with periods shorter than the transition period (Mazeh et al. 1990). Mazeh (1990) noted, however, that these eccentricities could be due to a third star in the system (Mazeh & Shaham 1979). We therefore need large samples of binaries to detect the correct transition periods, and the present table should be regarded as a *very* preliminary result.

The fourth coeval sample discussed by Mazeh et al. (1990) is the pre–main-sequence sample of known spectroscopic binaries of Mathieu, Walter, & Myers (1989), with an indication for a transition period of about 4 days. This sample is very different from the other three and therefore can*not* be used to find the correct functional dependence of the circularization time scale on the binary period. Despite the fact that we are left with only three late-type main-sequence samples, it would be of interest to compare the theoretical approach introduced here with the present observational results.

Following Jasniewicz & Mayor (1988), we plot in Figure 1 the three transition periods on the (log $P_{\rm trans}$, log Age)-plane. The transition periods of the M67 and halo samples were taken at the center of the period range of Table 1. No error bars were attributed to the age of the samples or to the transition periods. Assuming that the age of each sample is equal to the circularization time scale of a binary with the observed $P_{\rm trans}$, we plot on the same figure our proposed power law, together with the other two previous functions. All functions were calibrated to

TABLE 1

MAIN-SEQUENCE SAMPLES WITH TRANSITION PERIODS

Sample	Evolution Age (10^9 yr)	Transition Period (days)
Hyades/Praesepe	0.8	5.7
M67	5	10.3–11.0
Halo	15	12.4–13.7

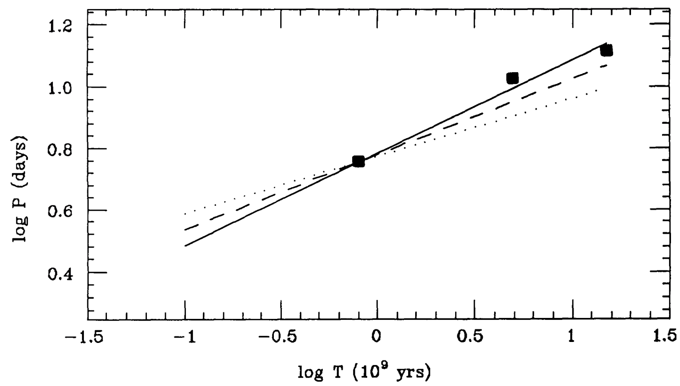

FIG. 1.—Transition period as function of sample age T. *Filled squares:* Observed transition periods of Table 1. *Solid line:* Present theory; exponent of 10/3. *Dashed line:* Tassoul (1988); exponent of 49/12. *Dotted line:* Zahn (1977), Mathieu & Mazeh (1988); exponent of 16/3.

fit the 5.7 day transition period of the Hyades/Praesepe. We could obtain this somewhat arbitrary calibration only by assuming for our proposed model a nonreduced viscosity larger than the one assumed for the Zahn original theory, as discussed in § 2. The figure suggests that the 10/3 exponent fits the data better than the other exponents.

4. DISCUSSION

Several words of caution should be emphasized here. We note first that in order to make the comparison between the theoretical graphs and the observations meaningful, the circularization processes have to take place during the main-sequence phase. This point was discussed in length by Mazeh et al. (1990). They argued that overall evidence suggests that the tidal circularization is effective throughout the main-sequence phase of stellar evolution. On the other hand, Zahn & Bouchet (1989) claimed that all circular binaries have been circularized during the pre–main-sequence phase of those systems. Their argument is based on Zahn's (1989) revised estimate of the *absolute* time scale of tidal circularization, for which he used the mixing-length theory and the prescription for the reduced turbulent viscosity mentioned in § 2. However, we have pointed out that the crudeness of our present understanding of stellar convective layers, that somewhat arbitrariness of the scale-height parameter, and the lack of confrontation with the observations, except in the case of the Sun, make the absolute calibration of the circularization uncertain. In any case, the observations do not seem to support the Zahn and Bouchet argument (Mazeh et al. 1990). We therefore prefer to assume that the circular binaries of the samples considered here were circularized during their main-sequence lifetime. In our proposed theoretical approach we chose to concentrate on the functional dependence of the circularization time scale on the binary period, which can be tested observationally, and leave the absolute calibration for later stages of the study.

Another crucial point for the comparison of the theory with the observed transition periods is the similarity of the stars of the three different samples. The relevance of this point was addressed by Mathieu & Mazeh (1988), who considered the effect of varying the spectral type and metallicity on the circularization time scale. They have found that as long as the spectral type of the components of the binary system is later than F, the spread in the circularization time scale is less than a factor of 2. In this preliminary stage of the study, even such a factor can drastically change the conclusions. Hence, we have to *assume* for now that the three samples consist of similar stars. Figure 1 should therefore be regarded only as a *very* preliminary step toward a real confrontation of the different models with observations.

The different models for circularization should be compared in the future with more extensive data. More main-sequence binaries are one source of observational importance. Another source is the chromospheric active binaries with short periods. As Hall & Henry (1990) have convincingly shown, these systems can also test the circularization and synchronization theories, despite the fact that the different stars in the sample have different ages. After finding the correct theory of circularization, we can turn the reasoning backwards and use the observed transition period as an independent "clock" to estimate the sample age, as suggested by Mathieu & Mazeh (1988). This method can, in principle, be used to check stellar evolution models.

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APPENDIX A

In what follows we derive equation (4a). Let us consider a fully developed, homogeneous, and isotropic steady state turbulent velocity field characterized by the spectral function $F(k)$, which equals twice the turbulent kinetic energy per unit mass per unit wavenumber, at the wavenumber k. The averaged squared velocity of an eddy of size k^{-1} is (Hinze 1975)

$$v^2(k) = \int_k^\infty F(k')dk' \, . \tag{A1}$$

The turbulent viscosity due to all wavenumbers larger than k is given by

$$\nu_t(k) = \int_k^\infty \tau(k')F(k')dk' \, , \tag{A2}$$

and results in energy transfer (cascade) to wavenumbers larger than k. Here, $\tau(k)$ is the eddy correlation time scale given by (Heisenberg 1948)

$$\tau(k) \sim F(k)^{-1/2}k^{-3/2} \, . \tag{A3}$$

From equation (A2) it follows that the turbulent viscosity is a decreasing function of k. Moreover, from equations (A1), (A2), and (A3) it can be shown that most of the contribution to $\nu_t(k)$ is from wavenumbers close to k and that $\nu_t \sim v(k)k^{-1}$. In this paper we are interested in the interaction of the turbulence with a velocity gradient (more precisely a velocity shear) coexisting in the same volume. For a shear with a length scale larger than k_0^{-1}, $\nu_t(k_0)$ is the effective viscosity acting on the shear (Hinze 1975). The energy drained from the shear is transferred into the turbulence cascade and is dissipated at the small scales by the molecular viscosity. In our specific problem, the scale length of the shear is $\sim R$, the stellar radius (Lecar et al. 1976). The latter is indeed larger than the turbulence length scale, which is of the order of the pressure scale height, and even at the bottom of the convective zone is less than $0.3R$.

The time scale of the turbulent convection, $\tau(k_0) \sim$ 10–20 days (depending on the specific modeling of the solar convection zone) is comparable to or larger than the orbital periods under consideration ($\sim$2–20 days). As argued in § 2, only eddies smaller than some size, k_*^{-1}, such that

$$\tau(k_*) = \lambda P \, , \tag{A4}$$

with $\lambda < 1$ $[\sim\frac{1}{2}\text{–}1/(2\pi)]$, will contribute to the turbulent viscosity. Their contribution is obtained by taking the lower limit in equation (A2) to be k_*. To derive the effective viscosity, we need to know $F(k)$. For a power-law spectral function

$$F(k) \propto k^{-m} , \tag{A5}$$

one gets, by using equations (A2) and (A3),

$$\tau(k_*) \propto k_*^{(m-3)/2} \tag{A6}$$

and

$$\nu_t(k_*) \propto k_*^{-(m+1)/2} \, . \tag{A7}$$

Therefore,

$$\nu_t(k_*) \propto \tau(k_*)^{(1+m)/(3-m)} \, . \tag{A8}$$

In a typical case the turbulent spectral function does not have a power-law shape through the whole range of wavenumbers (Hinze 1975). However, in the case of turbulent convection in a solar-type star, $F(k)$ takes the Kolmogorov form

$$F(k) \propto k^{-5/3} \tag{A9}$$

for $k \geq (2\text{–}3)k_0$ (see, e.g., Canuto, Goldman, & Chasnov 1987). This power-law range corresponds to $\tau(k) \leq \frac{1}{2}\tau(k_0)$, and therefore includes $\tau(k_*)$, which appears in equation (A4). We can, therefore, use equation (A8) with $m = 5/3$ to get

$$v_t(k_*) = Av_t(k_0)[\lambda P/\tau(k_0)]^2 , \quad (A10)$$

with A a dimensionless constant. Since the actual $F(k_0)$ is smaller than the value obtained by extrapolating the Kolmogorov inertial spectrum back to k_0, $A > 1$. Using spectra for convective turbulence from Canuto et al. (1987), we estimate $A \sim 3$.

APPENDIX B

We show here that the turbulence is unchanged by the interaction with the tidal shear, since $\epsilon_{\rm shear} \ll \epsilon$. (Here ϵ is the total energy per unit time and per unit mass fed into the turbulence by the source generating it, and $\epsilon_{\rm shear}$ is the energy drained from the shear per unit time and per unit mass.) In order to do so, note that (Canuto, Goldman, & Chasnow 1988 and references therein)

$$\epsilon \sim v_t(k_0)\tau(k_0)^{-2} . \quad (B1)$$

From the Navier-Stokes equations it follows (Monin & Yaglom 1971; Hinze 1975) that the energy per unit mass per unit time drained out from the shear into the turbulence, $\epsilon_{\rm shear}$, is given by

$$\epsilon_{\rm shear} = \tfrac{1}{2}\tau_{ij}S_{ij} , \quad (B2)$$

where τ_{ij} and S_{ij} are the turbulence stress tensor and the velocity shear, respectively. It is usually assumed following Boussinesq (1877, 1879; cited in Monin & Yaglom 1971), Taylor (1915), and Prandtl (1925) that

$$\tau_{ij} = v_t S_{ij} , \quad (B3)$$

resulting in

$$\epsilon_{\rm shear} = \tfrac{1}{2}v_t S^2 , \quad (B4)$$

where

$$S = (S_{ij}S_{ij})^{1/2} . \quad (B5)$$

is the absolute value of the velocity shear tensor. Using equations (A10), (B1), and (B4), we get

$$\epsilon_{\rm shear}/\epsilon \sim A[\lambda P/\tau(k_0)]^2[S\tau(k_0)]^2 . \quad (B6)$$

The amplitude of S for synchronous rotation is estimated from Lecar et al. (1976) to be

$$S \sim e\,\frac{2\pi}{P}\left(\frac{R}{a}\right)^3 , \quad (B7)$$

so that

$$\epsilon_{\rm shear}/\epsilon \sim (2\pi\lambda)^2 e^2 (R/a)^6 \ll 1 , \quad (B8)$$

where R is the stellar radius, a is the semimajor axis, and e is the orbital eccentricity.

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