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ON THE FORMATION OF GLOBULAR CLUSTERS. I. DYNAMICAL LIMITS ON GLOBULAR CLUSTER METALLICITIES

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ABSTRACT

We address the question of the efficiency with which supernova ejecta are dispersed into the ISM and its implications for the metal abundances of early stellar populations. Based upon the dynamical limits imposed by the dispersive physical processes, we derive a permitted range of metallicities for a closed system such as a globular cluster. The mean supernova progenitor mass, an adjustable parameter in these models, is taken to be $\sim 30\ M_{\odot}$. We discuss this choice in the context of observed abundance patterns and an assumed IMF. Our lower metallicity limit arises from the fact that at most $\sim 8\%$ of the input energy is available for gas motions (dispersion). The upper metallicity limit is rather a consequence of momentum conservation: the minimum energy that can be retained is $\sim 0.4\%$, implying very inefficient mixing and therefore a higher achieved metallicity. We find it to be both encouraging and suggestive that the permitted range, $\sim 10^{-2}$ to $10^{-1}\ Z_{\odot}$, is compatible with that observed for the Galactic halo globular clusters.

Subject headings: clusters: globular — stars: abundances — stars: evolution — stars: stellar statistics

1. INTRODUCTION

The origin and evolution of the Galactic globular cluster system, and its relation to the earliest stages of evolution of the Galaxy itself, remain subjects of considerable theoretical interest. At the present time, the critical question has yet to be resolved as to whether they were formed before, during, or after the collapse of the protogalaxy itself (Fall & Rees 1987). Current estimates of the ages of the globular clusters (Janes & Demarque 1983; Iben & Renzini 1984; VandenBerg 1988; Sandage & Cacciari 1990; VandenBerg, Bolte, & Stetson 1991) are comparable to those obtained for the Galaxy from nuclear chronometers (Fowler & Meisl 1986; Meyer & Schramm 1986; Mathews & Schramm 1988; Cowan, Thielemann, & Truran 1987, 1991), but somewhat greater than age estimates for the disk from white dwarf cooling (Winget et al. 1987; Iben & Laughlin 1989; Wood 1990). It is thus clear that the globular clusters represent some of the oldest objects known in our Galaxy.

A question of particular concern, the answer to which can provide an important boundary condition on the epoch of globular cluster formation, is whether the abundance patterns that are observed to characterize the current population of stars in globular clusters reflect (1) the composition of the gas from which the protoclusters formed or, rather, (2) the products of nucleosynthesis associated with an earlier generation of stars within the clusters themselves. Our investigation of this question has led us to the conclusion that the abundance patterns characterizing, specifically, the spheroidal component of the Galactic globular clusters are entirely attributable to self-enrichment. Observational support for this view is briefly reviewed below.

Important clues to the formation and early evolution of the globular clusters are provided by observations of their composition trends and specific abundance patterns. A variety of general trends and correlations in globular cluster abundance patterns have previously been noted: (1) The strikingly narrow abundance spreads within individual clusters, as inferred from the narrow widths of their giant branches in the color magnitude diagram (Kraft 1979; Freeman & Norris 1981), indicates that the currently observed stellar generation formed from homogenized gas. This forms the basis for a common argument against self-enrichment, although it provides equally stringent constraints on virtually all models for globular cluster formation. (2) The frequency distribution of globular cluster abundances (Fe/H) is bimodal (Zinn 1985), indicating the existence of distinct disk and halo globular cluster populations. (3) The globular clusters of the halo population exhibit no significant abundance gradient with galactocentric distance (Pilachowski 1984; Zinn 1985). (4) The metallicity distribution function of the globular cluster stars differs from that of the field halo stars (Laird et al. 1988), in the sense that the field stars exhibit a much broader range in metallicity than do the clusters. Specifically, a substantial fraction of the field stars are much more metal deficient than any of the globular clusters. (5) Observations of extragalactic globular cluster systems indicate that the globular clusters are systematically bluer at a given galactocentric distance than the mean colors of the underlying field star population (Forte, Strom, & Strom 1981; Harris 1986; Mould 1987).

The abundance trends noted above are largely based upon photometric studies of the light output of the cluster stars. Spectroscopic studies of individual stars in globular clusters now permit the determination of individual elemental abundances in these stars. The results of such studies to date reveal, specifically, that globular cluster stars exhibit abundance patterns which are both similar to those of the extremely metal-deficient field halo stars and quite distinct from solar system abundances (Anders & Grevesse 1989). Composition trends in field halo stars and globular cluster stars which reflect these similarities are briefly reviewed below.

Observations of the abundances in the most extremely metal-deficient field halo stars, recently reviewed by Spite & Spite (1985) and Wheeler, Sneden, & Truran (1989), reveal a number of interesting trends. High oxygen to iron ratios $[O/Fe] \approx +0.5$ are generally found to characterize the halo stars. Similar trends are evident for the intermediate mass elements Mg, Si, Ca, and Ti, which are again found to be enriched relative to iron by approximately 0.5 dex. In contrast, the abundances of both carbon and nitrogen, relative to iron, are compatible with solar abundances. The data also give evidence for the presence of a mild odd-even effect involving the products of explosive nucleosynthesis (Truran & Arnett 1971): specifically, elements containing odd numbers of protons (e.g., Na, Al, and Sc) seem perhaps to show somewhat greater relative deficiencies than neighboring even-Z nuclei in extremely metal-deficient stars.

In the heavy element region, the data now clearly establish the existence of depletions in the abundances of the designated *s*-process elements Sr and Ba, relative to iron, in stars of low Fe/H. The theoretical interpretation of these trends as due to the fact that the heavy element abundance patterns characteristic of extremely metal-deficient stars are dominated by *r*-process contributions (Truran 1981) has now been strongly confirmed by observations (Sneden & Pilachowski 1985; Gilroy et al. 1988).

Collectively, these abundance patterns in metal-deficient stars are therefore quite in agreement with those expected to be characteristic of the ejecta of massive stars (Truran 1983, 1989). It seems logical that this be so, since massive stars $M \gtrsim 10\ M_\odot$ of short lifetime $\tau \lesssim 10^8$ yr can be expected to be the major source of nucleosynthesis on a time scale compatible with a halo collapse time scale.

An intriguing result is the finding that similar elemental abundance trends are evident in the abundance patterns determined for globular cluster stars. We can note particularly the following features. Intermediate mass nuclei such as magnesium, silicon, calcium, and titanium (the "alpha" nuclei) are typically enriched, relative to iron, by up to approximately 0.5 dex, for clusters with values of [Fe/H] ranging from -2.52 to -0.81 (see Table 1). Such relative enrichments are not, however, characteristic of other iron-peak nuclei like chromium and nickel. High O/Fe ratios are also encountered in some globular clusters, although there generally remain serious questions concerning the oxygen abundances in these systems. Finally, the heavy element ($A > 60$) abundance patterns in globular cluster stars (see Table 2) reveal anomalies in the ratios Zr/Fe, Ba/Fe, and Eu/Fe similar to trends observed in extremely metal-deficient field halo stars, which have been determined to be *r*-process in nature. In particular, the systematically high values obtained for [La/Zr] for cluster stars, with a mean value $+0.32$, again seem suggestive of an *r*-process origin (Truran 1988). The fact that all of the heavy elements Zr, Ba, and Eu are systematically depleted relative to Fe for the studied cases is consistent, as well, with the suggestion of Mathews & Cowan (1990) that the *r*-process associated with massive stars $M > 10\ M_\odot$ which forms the heavy element pattern in the halo stars preferentially operates in stars in the mass range $10 \lesssim M \lesssim 20\ M_\odot$.

It is these patterns which we believe now begin to impose particularly interesting constraints on globular cluster chemical evolution. In particular, Truran (1983, 1988; see also Truran & Thielemann 1987) has called attention to the fact that the occurrence of heavy element abundance patterns in the most metal-rich clusters that reflect *only* the contributions from massive stars (stars of mass in excess of approximately 10 solar masses) and associated Type II supernovae is strongly suggestive of a self-enrichment process for the Galactic globular clusters. If self-enrichment were to occur, on a time scale short with respect to the time scale of evolution of low- and intermediate-mass stars, then one would expect to find that even the most metal-rich clusters would exhibit both the high O/Fe and Mg/Fe ratios and the *r*-process–dominated heavy element patterns which characterize massive star ejecta (and the abundances of the most metal-deficient field halo stars). The "metallicity" of a cluster, in this context, is simply a function of the total number of supernova which have contributed

TABLE 1

GLOBULAR CLUSTER ABUNDANCES: THE α-ELEMENTS

Cluster	References	[Fe/H]	[Mg/Fe]	[Si/Fe]	[Ca/Fe]	[Ti/Fe]
M71	2	−0.81	...	+0.38	+0.28	+0.24
M71	4	−0.6 → −1.0	+0.40	+0.15	+0.58	+0.30
NGC 362	1	−0.87	...	...	+0.65	+0.30
NGC 3201	1	−0.95	...	...	+0.15	+0.13
NGC 3201	2	−1.32	...	+0.23	+0.19	+0.18
47 Tuc	1	−1.09	...	...	+0.44	+0.53
47 Tuc	2	−0.83	...	+0.40	+0.13	+0.15
M4	3	−1.20	...	+0.65	+0.59	+0.20
M4	2	−1.32	...	+0.80	+0.46	+0.24
M5	1	−1.09	...	...	+0.37	+0.20
M5	2	−1.47	...	+0.59	+0.56	+0.39
M10	1	−1.28	...	...	+0.38	+0.31
M10	2	−1.42	...	+0.36	+0.45	+0.28
NGC 6752	1	−1.26	...	...	+0.36	+0.25
NGC 6752	2	−1.53	...	+0.48	+0.36	+0.17
NGC 4833	1	−1.34	...	...	+0.30	+0.36
NGC 4833	2	−1.74	...	+0.28	+0.48	+0.41
M79	2	−1.46	...	+0.17	+0.22	+0.35
M13	3	−1.55	...	+0.35	...	+0.60
M22	2	−1.56	...	+0.44	+0.36	+0.40
M22	3	−1.60	...	...	...	+0.10
M22	5	−1.70	...	...	+0.40	...
NGC 5897	2	−1.85	...	+0.38	+0.08	−0.04
NGC 6397	2	−1.88	...	+0.44	+0.27	+0.07
M68	2	−1.93	...	+0.44	+0.42	+0.02
M92	6	−2.52	+0.61	...	+0.31	+0.22

REFERENCES.—(1) Pilachowski et al. 1983; (2) Gratton & Sneden 1991; (3) Wallerstein et al. 1987; (4) Leep et al. 1987; (5) Lehnert et al. 1991; (6) Peterson et al. 1990.

TABLE 2

Globular Cluster Abundances: The Heavy Elements

Cluster	References	[Fe/H]	[Zr/Fe]	[Ba/Fe]	[Eu/Fe]	[La/Zr]
M71	2	−0.81	...	−0.16	...	...
NGC 362	1	−0.87	...	−0.30	...	+0.60
NGC 3201	1	−0.95	−0.62	−0.55	−0.45	+0.38
47 Tuc	1	−1.09	−0.17	−0.34	−0.32	+0.26
47 Tuc	2	−0.82	...	−0.17	...	...
M4	3	−1.20	−0.20	...	...	...
M4	2	−1.32	...	+0.05	...	...
M5	1	−1.09	−0.36	−0.30	−0.25	+0.26
M5	2	−1.42	...	−0.32	...	...
NGC 6752	1	−1.26	−0.30	−0.21	−0.48	+0.31
NGC 6752	2	−1.53	...	−0.06	...	...
NGC 4833	1	−1.34	−0.06	−0.23	−0.19	+0.13

References.—(1) Pilachowski et al. 1983; (2) Gratton et al. 1987; (3) Wallerstein et al. 1987.

to the cluster enrichment. It is the presence of such abundance trends in globular clusters that has provided the motivation for this work.

Calculations of chemical evolution generally ignore the local effects of the dynamical processes which return enriched material to the interstellar medium. These processes play a negligible role in chemical evolution throughout most of the Galaxy's lifetime because the background metallicity is sufficiently high that the metals ejected by a few stars are insignificant compared to the metals in the ambient medium. However, in a primordial environment of low initial metallicity they may play a major role. In such an environment, stars formed before system-wide mixing can take place will have their metallicities determined by the efficiency with which metals are dispersed locally. The most obvious candidates for such local contamination are the Galactic globular clusters. These systems are small enough that their metal contents can have been generated by a few massive stars. Cayrel (1986) has shown that self-contamination could explain many of the observations concerning globular clusters. In addition, Fall & Rees (1985) have suggested that some degree of self-contamination may be unavoidable in globular clusters. In light of these various considerations, we therefore propose to examine the following model of globular cluster evolution.

1. Proto-globular-cluster clouds will condense in a collapsing protogalaxy. Those clouds above critical mass are gravitationally unstable and contract to form a first generation of primordial ($Z = 0$) stars. Fall & Rees (1985) have shown that the minimum mass for this gravitational instability is of order $10^6\ M_\odot$. We assume that the first generation contains sufficient numbers of massive ($M > 10\ M_\odot$) stars to contaminate a cloud to a metallicity $Z \sim 10^{-2}\ Z_\odot$, but that only a small fraction of the cloud's mass is converted into stars in this generation.

Since $Z = 0$ stars are rare at best (see, e.g., Bond 1981), and since there is some theoretical evidence that star formation in metal-poor environments can be biased toward high-mass stars (Shu, Adams, & Lizano 1987; Mouschovias 1987, 1989), it might be argued that this population contains primarily high-mass stars, and thus that there should be no residual component of zero metallicity stars to introduce a metallicity spread into the cluster. This may indeed be true. We believe, however, that the form of the initial IMF for this first stellar generation may not be a critical issue. Numerical calculations to be reported in a subsequent paper (Burkert, Brown, & Truran 1991) suggest that this stellar component will subsequently be left unbound, and be lost, when more than half of the initial gas mass is driven away.

2. As the massive stars evolve off the main sequence, stellar winds, ultraviolet photons, and supernova ejecta will sweep the surrounding gas into a "supershell" (McCray & Kafatos 1987; McCray & Mac Low 1988), which simultaneously sweeps up the surrounding H I cloud and is enriched by first generation supernovae. Turbulence behind the expanding shock wave is assumed to mix the enriched material with the ambient material. This phase continues until the entire cloud has been swept into the supershell.

3. Once the shell emerges from the cloud, it is rapidly decelerated by the pressure of the surrounding hot gas in which the cloud is embedded. Numerical simulations indicate that the shell slows to speeds of order 10^5 cm s^{-1} in a few million years. Once the shell has slowed to these speeds, it continues to expand quasi-statically as the first generation supernovae continue to heat the interior of the supershell. Typically, clouds reach this slow, quasi-static expansion approximately 5 to 10 Myr after the first supernova occurs.

4. In the slowly expanding supershell, a second generation of stars forms; this generation is the currently observed generation in the halo globular clusters. The stars do not, however, feel the pressure of the hot gas in the interior of the supershell, but rather are only affected by gravity. These stars can form a globular cluster only if the total mass of stars formed is sufficiently large that the gravitational binding energy of the stars (only) exceeds their kinetic energy:

$$\frac{1}{2}\frac{GM^2}{r} > \frac{1}{2}\,Mv^2 .$$

Supershells have sizes reaching typically 50 pc during this late stage of expansion, leading to a minimum globular cluster mass of about $10^4\ M_\odot$. Of course, the system must also be able to survive the Galactic tidal field, and this increases the minimum mass by about a factor of 10.

5. As the supershell, which now contains both second generation stars and gas, continues to expand, the stellar component falls behind the gas and may become bound, while the residual gas is dispersed. For the stellar component to remain bound, we find that approximately 10%–20% of the gas must be converted into second generation stars. Even with this rather high efficiency, the system must typically lose 80%–90% of its initial mass. Any low-mass first generation stars will, therefore, be

lost from the system as well, since they formed in virial equilibrium with the initial potential.

6. We expect that systems with such high star formation efficiencies ($\gtrsim 10\%$) are rare, and that most of the systems are rather left unbound. In our model, this is presumed both to provide an explanation of the origin of the field halo stars and to explain the relative rarity of globular clusters, compared to field stars. If we assume a total initial gas mass of $10^{11}\ M_\odot$ (10^5 protoglobular clusters of $\sim 10^6\ M_\odot$), a star formation efficiency of 0.1, and a yield $y = 0.02$, we find that a total mass $\sim 2 \times 10^8\ M_\odot$ of metals would have been synthesized and reprocessed by halo stars. The protodisk gas, as it began to settle into the equatorial plane, would then have been enriched in metallicity to $Z \sim 0.1\ Z_\odot$, in agreement with observations. Since no more than 1% of the present (visible) halo mass is in globular clusters (Woltjer 1975), it is natural to expect that the process of globular cluster formation is very inefficient.

In this paper we will concentrate our attention on steps 2, 3, and 4, with our primary goal being to determine the metallicity of the second generation of stars. We will ignore, for the moment, the effect of gravity on the evolutions of the systems, because it can be demonstrated that the work done against gravity in systems of mass comparable to that of protoglobular clusters will be insignificant compared to the other energetic processes in the system. The question of the binding of such systems will be examined in a subsequent paper (Brown, Burkert, & Truran 1991). Section 2 identifies and reviews the critical features of the evolution of supernova remnants and derives simple analytic expressions for minimum and maximum expected metallicities of the globular clusters as a function of the mass ejected per supernova event. In § 3 these limits are examined in the context of current models of massive star evolution. Discussion and conclusions follow.

2. EVOLUTION OF THE SUPERNOVA REMNANTS

Broadly speaking, the evolution of supernova remnants is well understood (Spitzer 1978). Initially, the ejecta undergo a free expansion phase, until they have encountered and swept up a mass comparable to their own. At this point, a shock wave is formed and driven into the ejecta, thus heating it considerably. The ejecta and swept-up material become so hot that radiation is negligible, and the evolution is very well described by a Sedov (1959) blast wave. During this expansion, about 72% of the energy of the supernova remnant is in thermal energy, and the rest is kinetic energy. This phase lasts until radiative losses become important and the stored thermal energy goes into radiation, rather than into work on the interstellar medium. After this transition, the remnant radiates its thermal energy, and the shock becomes approximately isothermal. With the pressure of the postshock material no longer driving the remnant, it coasts into the ambient material at roughly constant momentum. This solution is more approximate than the Sedov solution, but it should nevertheless provide an adequate estimate of the mass of the interstellar medium polluted by the supernova, since the material accelerated during the adiabatic phase will continue to be diluted as long as the shock is present and continues to mix the ambient material into the shell. This phase comes to an end when the shell's velocity slows to ambient sound speed and the shock dissipates. Several authors have considered the detailed solutions for the expansion in relation to the problem of globular cluster formation (Dopita & Smith 1986; Morgan & Lake 1989). We consider this problem in the more general context of how the evolution of a number of high-mass stars might serve to enrich the local interstellar medium.

During the early, nonradiative phase of the expansion, the shell proceeds outward at constant kinetic energy. Therefore, as more mass is swept into the shell, its momentum increases. Once radiation becomes important, the pressure force is lost such that the shell must expand at constant momentum, and therefore the kinetic energy of the shell is radiated away. The mass of the shell, at the point when the shell velocity reaches local sound speed, is then a function of the amount of kinetic energy remaining in the system. In order to express this quantitatively, we will consider a system of N high-mass stars, each of which introduces an energy E and a mass of metals m_z into the surrounding medium. Defining M, Z, and V to be the final mass, metallicity, and velocity of the shell, and f_e to be the fraction of the initial energy which is not radiated away (i.e., the mechanical efficiency of the expansion in converting the input energy into gas motion), we can write down two relations among these quantities. The first of these relations provides the definition of f_e, while the second is the definition of Z:

$$\tfrac{1}{2}MV^2 = f_e NE\ , \tag{1a}$$

$$ZM = N\bar{m}_z\ . \tag{1b}$$

In equation (1b), the quantity $\bar{m}_z$ is to be understood to represent an average over an appropriate initial mass function $\phi(m)$,

$$\bar{m}_z = 1/N \int dm \quad \phi(m)m_z(m)\ , \tag{2}$$

where $m_z(m)$ can be approximated as shown in our equation (18). Taking the ratio of the two equations (1a) and (1b), we can eliminate the extrinsic parameters M and N. After rearranging terms, the final metallicity can then be expressed in terms of the remaining parameters as:

$$Z = f_e^{-1}(\tfrac{1}{2}\bar{m}_z V^2 E^{-1})\ . \tag{3}$$

For a real system, $\bar{m}_z$ and to a lesser extent E must be understood as average values. Nevertheless, most of the variation that occurs from system to system should be due to variations in f_e. While it may be very difficult to determine f_e for a particular system, even given many details of the system, it is fairly straightforward to determine upper and lower limits on f_e.

The efficiency f_e for the expansion of a single remnant, obtained by Spitzer (1978), is just the ratio of the final kinetic energy to the input energy. These factors, in turn, are simply related to the kinetic energy at the transition between the adiabatic and isothermal phases of the remnant's evolution. We write

$$f_e = \left(\frac{\frac{1}{2}MV^2}{\frac{1}{2}M_t V_t^2}\right)\left(\frac{\frac{1}{2}M_t V_t^2}{E}\right)\ , \tag{4}$$

where M_t and V_t are the mass and speed of the shell, respectively, at the point where radiation begins to dominate the shell's evolution. The second term in equation (4) is the ratio of kinetic energy to total energy during the adiabatic phase, numerically equal to 0.28 (Spitzer 1978). The momentum of the system is constant during the snowplow phase, so one power of mass-times-velocity cancels in the first term, yielding a simple expression for the efficiency factor,

$$f_e = 0.28 V V_t^{-1} \tag{5}$$

There are some uncertainties associated with this expression. First, the numerical value 0.28 is obtained for a blast wave associated with a single supernova. In the more general case we are considering, there may be many supernovae contributing to the shell. For such systems, the fraction of kinetic energy can be as high as 0.40 (Weaver et al. 1977). Second, the velocity in the shell will be a function of radius, and it will be close to but not identical to the shock speed. In spite of this uncertainty, the assumption that V_t is equal to the sound speed at 10^5 K yields the most efficient means of dispersing the ejecta, because radiation is thereby assumed to be negligible for as long as possible, prior to reaching the peak in a metal-free cooling function (Fall & Rees 1985; Shull & Silk 1979). Since this behavior does not depend on the total energy available, the result obtained is unaffected by the number of supernovae.

The lower limit on the efficiency is defined, alternatively, by allowing the maximum amount of energy loss by radiation. Intuitively, this is a more straightforward situation than the upper limit, since one need only skip the adiabatic expansion to ensure that no work is done on the surrounding medium. This then corresponds to a constant momentum expansion, and the efficiency is given by

$$f_e = \frac{\frac{1}{2}MV^2}{\frac{1}{2}M_e V_e^2}, \tag{6}$$

where M_e and V_e are the initial mass and velocity of the ejecta. As before, one factor of mass-times-velocity cancels, yielding

$$f_e = V V_e^{-1} . \tag{7}$$

The ejection velocity will depend on both the mass ejected and the energy released in each explosion. First, however, it is appropriate to consider whether this lower efficiency limit can be realized by a real supernova.

In the first stellar generation of a protoglobular cluster, we would expect to find a number of massive stars, rather like a modern OB association. The dynamics of the "supershells" of expanding materials swept up by early-type stars in such associations has been considered in detail by a number of authors (Weaver et al. 1977; McCray & Kafatos 1987; McCray & Mac Low 1988). In such systems, one usually supposes that the supernovae in the hot interior of a supershell evolve hydrodynamically, much like an isolated supernova. It is possible, however, that the evolution may not be treatable with the usual hydrodynamic equations. A shock forms around an isolated supernova after its ejecta has interacted with an ambient mass comparable to its own mass, but this shock forms only because there is a magnetic field of a few μG frozen into the ambient medium (Spitzer 1978). The ambient magnetic field will already have been largely removed, for supernovae occurring in the interior of the supershell, so the ejecta should interact with the residual interior gas over a thermal mean free path rather than a gyration radius.

The mean free path of an ejected particle in the low-density region interior to the shell can easily be found using standard results from plasma theory (e.g., see Nicholson 1983). Assuming a mean particle mass of half a proton mass, a velocity of 2×10^8 cm s^{-1} for the ejecta, and an interior temperature of 10^7 K, one obtains a minimum mean free path of 10 n_i^{-1} pc, for particles ejected in the supernova, where n_i is the interior particle density. For a typical particle density of 0.1 cm^{-3} in the hot interior of the supershell, the mean free path is comparable to or greater than the supershell's radius. For this condition, supernova remnants in the interior will undergo a free expansion until they reach the cold shell, and their energy will heat the shell rather than the interior. However, there is insufficient energy in a supernova to heat a supershell of $\gtrsim 10^5$ $M_\odot$ to the point at which its evolution becomes nonradiative. This means that virtually all of the energy is radiated away, and the supershell's momentum is increased only by an amount equal to the momentum of the ejecta itself.

A potentially interesting question here concerns the possible effects of stellar winds on our estimates of the efficiency of dispersing the supershells. In matter of solar composition, it is clear that winds should provide significant energy input to drive the shell. We note, however, that we are concerned with the evolution of the spheroidal component of our galaxy's globular clusters, for which the (primordial) composition of the first stellar generation is expected to be metal free. Kudritzki, Pauldrach, & Puls (1987) find that the effectiveness of such winds, as reflected in the energy output, is a strong function of the metallicity. For matter of composition $Z \approx 10^{-3}\, Z_\odot$, the energy in winds has fallen to less than 1% of its value for solar abundances, and clearly for $Z \equiv 0$ the effect should become more extreme. We therefore conclude that winds should not significantly influence the behavior of primordial globular clusters. We note, however, that winds may be relevant to the epoch of formation of disk globular clusters or of LMC or SMC globular clusters, which form from metal-enriched material.

Incorporating the maximum and minimum efficiencies obtained in equations (5) and (7) into equation (3), and using the fact that $V_e = (2E/m)^{0.5}$, one obtains, respectively, the minimum and maximum possible metallicities that can be expected to be achieved:

$$z_{\min} = 1.79 \bar{m}_z V V_t E^{-1} , \tag{8}$$

$$z_{\max} = 0.71 \bar{m}_z V m^{-0.5} E^{-0.5} . \tag{9}$$

We then have only to determine V to have our desired limits. Previous authors have taken V to be the local sound speed (Spitzer 1978). For velocities below sound speed, no additional ambient material is swept into the shell, since a pressure wave (rather than a shock) now propagates ahead of the shell, and V is the speed below which the ambient material is no longer mixed into the shell. V is equal to the local sound speed only if the shell remains well mixed down to that speed. If this is the case, then *the shell and the second generation of stars will have a homogeneous composition.* This is a critical issue, since the usual objection to models of self-enrichment is that the resulting systems must be inhomogeneous, while the globular clusters (with a few exceptions) are observed to be extremely homogeneous. It is natural to expect that, in any system experiencing *continuous star formation*, there will be a spread in metallicity. However, in a system with *two distinct generations*, separated in time, such as we describe, the second generation will form at a single metallicity, provided the material from which the second generation forms is itself well mixed. This is the critical question. (It is essential to note that, in the context of our present discussion, we are concerned entirely with the implied metallicity spread of the *second stellar generation.* In proceeding in this manner, we are tacitly assuming that, for whatever reason, there exists no residual stellar component of zero metal stars. The justification for this assumption is presented in a later section.)

Mixing in the shell should occur, powered by the energy released as gas in the ambient medium is swept into the shell. The shell should remain well mixed as long as the mixing time is less than the mass accretion time. The shell accretes mass at approximately constant momentum, so the rate at which energy is dissipated per unit mass is

$$\dot{e} = \dot{E}/m = 0.5\dot{m}/mv^2 = 1.5v^3r^{-1} , \tag{10}$$

where v is the speed of the supershell and r is its radius. From the theory of turbulence (Batchelor 1953), the specific rate of energy input needed to maintain a turbulent velocity V_T at a length scale L is

$$\dot{e}_T = V_T^3 L^{-1} . \tag{11}$$

This equation is valid for any velocity. Use of this expression will now allow us to show that there is sufficient energy released by the accretion of the surrounding gas to drive mixing on a time scale that is short compared to the dynamical time of the shell.

Equating the two energy dissipation rates then yields

$$V/V_T = 0.87(r/L)^{0.33} . \tag{12}$$

The mixing time and the mass accretion time are

$$t_{\rm mix} = LV_T^{-1} , \tag{13}$$

$$t_{\rm acc} = m/\dot{m} = 0.33rv^{-1} . \tag{14}$$

Taking the ratio of equations (13) and (14) and eliminating the velocities with equation (12), the ratio of the time scales becomes

$$\frac{t_{\rm mix}}{t_{\rm acc}} = 2.6\left(\frac{r}{L}\right)^{0.67} . \tag{15}$$

In the present context, L corresponds to the thickness of the shell. Using the detailed similarity solution for the snowplow phase, one finds that L/r is approximately 0.01. Therefore, the shell should remain well mixed down to local sound speed, and V is equal to the local sound speed.

3. RESULTS AND DISCUSSION

Our aim in this paper is to establish numerical limits on the achievable metallicity of a globular cluster-like stellar population. To accomplish this, we also require a knowledge of the metal production of the early population of massive stars. It is simplest, and sufficiently accurate for present purposes, to assume that the nucleosynthesis products of these stars are the same as have been calculated in detail for Population I objects. Weaver & Woosley (1986) give the following simple approximation for the mass (m_z) of metals formed as a function of the mass (m) of the precursor star:

$$m_z = 0.4m - 4.2 . \tag{16}$$

Once again, m and m_z are to be understood as averages ($\bar{m}$ and $\bar{m}_z$) over an appropriate IMF. The upper and lower limits are obtained by incorporating equation (18) into equations (9) and (10). In order to obtain numerical estimates, we adopt the following values for the quantities of interest: $E = 10^{51}$ ergs; $V = 10^6$ cm s^{-1}, corresponding to an ambient temperature of 10^4 K; and $V_t = 3 \times 10^6$ cm s^{-1}, corresponding to a transition temperature of 10^5 K. For these choices,

$$Z_{\rm min} = 4.3 \times 10^{-6}(\bar{m} - 10.5) ; \tag{17}$$

$$Z_{\rm max} = 4.0 \times 10^{-4}(\bar{m} - 10.5)\bar{m}^{-0.5} . \tag{18}$$

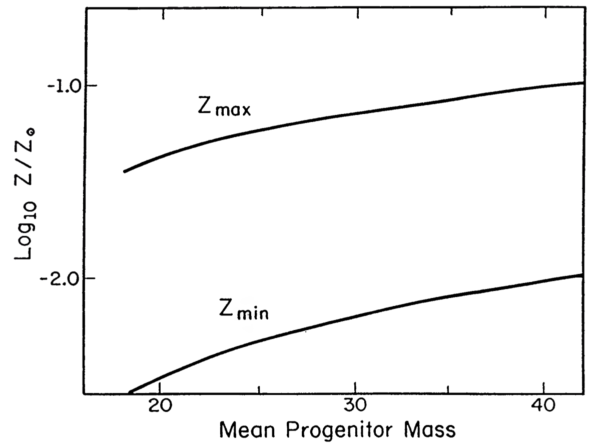

FIG. 1.—Derived limits on self-enrichment as a function of mean progenitor mass

These limits are plotted in Figure 1, as a function of the mean mass $\bar{m}$. A comparison of these results with a histogram of [Fe/H] for the Galactic globular clusters (Figure 2) reveals an excellent agreement between the theoretical range of allowed metallicities and the range observed in the metal-poor globular clusters, for the choice of a mean mass of approximately 30 $M_{\odot}$. This metal-poor component is essentially Zinn's (1985) halo population, which has the kinematic characteristics of a spheroidal population, as opposed to the disklike kinematics of the metal-rich clusters. The agreement between our derived limits and the observed range leads us to conclude that the heavy element abundances in the metal-poor clusters are very likely attributable to self-contamination.

There are several additional constraints which are satisfied by this model. First, the lower limit is a quite general result, since the effects of multiple supernovae tend to increase abundances. Also, the inclusion of the effects of gravity will tend only to increase metallicities. This implies that stars forming in any system will rarely form near or below the lower limit derived here. Bond (1981) suggests that there is a cutoff in iron abundance in the halo at -2.6, which is reasonably consistent with this result. While a deeper, more recent survey has revealed more stars below this limit (Beers, Preston, & Shectman 1985), the relative scarcity of such stars is well established.

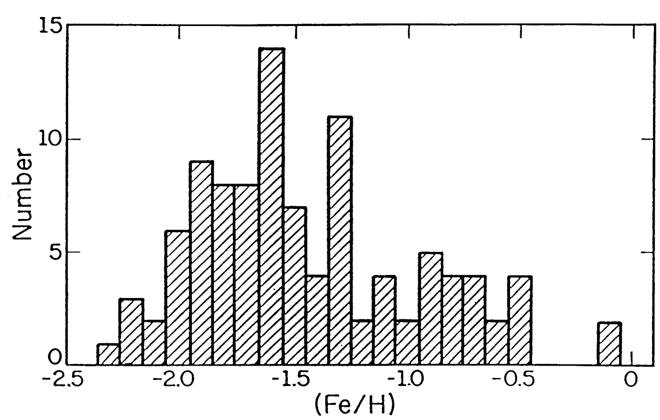

FIG. 2.—Histogram of [Fe/H] for Galactic globular clusters. Data is from Webbink (1985); only those clusters with well-measured subgiant branch colors and reddenings are included.

Furthermore, the range of allowed metallicities is determined almost entirely by the choice of E, and it is comparable to the width of the metal-poor peak, 1.0–1.3 dex, for any credible choice of E. Since the metal-poor stars show evidence of having been contaminated only by supernovae (Truran 1983, 1987; Pilachowski 1984), it is natural to seek to explain the observed lower limit in terms of the efficiency with which supernovae disperse their ejecta. If one establishes the lower limit on this basis, then the allowed range of metallicities in second generation stars must fit the range observed in the halo clusters.

Second, nucleosynthesis provides additional external constraints which should be satisfied. One should find that the relative abundance patterns in the halo globular clusters reflect the nucleosynthesis expected from a typical progenitor. Truran (1983) noted that the metallicity pattern was consistent with that of the ejecta of a 35–50 star; Cayrel (1986) obtained a typical precursor mass of 25 $M_{\odot}$. In general, reasonable agreement with observed abundance patterns follows from the assumption that the massive star ($M \gtrsim 10\ M_{\odot}$) progenitors of Type II (and perhaps Type Ib) supernovae are the dominant contributors. While the uncertainties in this approach are considerable, it is clear that a mean progenitor mass of $\sim 30\ M_{\odot}$ is consistent with the abundance patterns observed in the halo globular clusters, so nucleosynthesis constraints are satisfied as well.

Finally, we wish to ensure that the model is self-consistent. The mean progenitor mass of 30 $M_{\odot}$ was selected initially because it was found to be consistent with both nucleosynthesis and time scale constraints. We now explore this latter question more carefully. Clearly, only those stars which can evolve while the shell is well mixed (i.e., prior to the onset of star formation) will have their ejecta mixed into the shell. The lower mass limit compatible with this time scale can readily be estimated. Turbulence (and associated mixing) will continue only as long as there is energy available to drive it. The assumed rate of energy loss requires that the kinetic energy lost in the shell be radiated, but this assumption remains valid only as long as the shell moves supersonically. Once the shell becomes subsonic, a pressure wave moves ahead of the shell, and the energy is dissipated in PdV work against the external pressure. This provides a constraint on the masses of those stars which can contribute to the enrichment of the shell.

The shell receives the bulk of its energy after the most massive stars leave the main sequence, on time scales 3–5 Myr for stellar masses of 30–80 $M_{\odot}$. Subsequently, the supershell evolves on a dynamical time of $\sim 0.6r/c$, yielding 1–6 Myr for a radius of 20–100 pc. Taking the midpoints of these ranges, we find that all stars with lifetimes less than ~ 7.5 Myr will contribute. However, the stars do not necessarily all form at the same time, so we expect that there may be a spread in ages comparable to the star formation time scale, of order 10 Myr. This time includes the stellar evolution time, so stars with lifetimes as long as 13 Myr can evolve. Converting these lifetimes into masses, we conclude that only the ejecta of stars with masses greater than ~ 17–24 $M_{\odot}$ will be mixed into the shell. For such a lower mass limit, the mean progenitor mass (assuming a Salpeter IMF) is approximately 30 $M_{\odot}$, consistent with our earlier assumption. It is important to emphasize that *this mean value is obtained from hydrodynamic considerations*, and *not from the observed metallicities*. The self-enrichment scenario described herein imposes this constraint on the mean stellar mass. In contrast, a value for the mean mass less than 20 $M_{\odot}$ would be appropriate if all "massive" stars ($\gtrsim 10\ M_{\odot}$) contributed to the enrichment of the second generation. (Unfortunately, our knowledge of the nucleosynthesis contributions as a function of mass for stars in the range $M > 10\ M_{\odot}$ is not sufficiently precise to allow us to discriminate observationally between these two cases).

Proceeding on this basis, we can estimate the internal spread in metallicity of the second generation within an individual cluster. The conservative estimate is obtained by assuming that all metals produced after the shell becomes subsonic are only partially mixed into the shell. The ejecta from all stars above 20 $M_{\odot}$ is expected to be well mixed. If the second generation also takes 10 Myr to form, the range in metallicities should be comparable to the fraction of metals which is not well mixed—that component resulting from the evolution of stars of mass below 20 $M_{\odot}$. A 12 $M_{\odot}$ star evolves in about 20 Myr, so this fraction is given by the ratio of all stars between 12 $M_{\odot}$ and 20 $M_{\odot}$. For a Salpeter IMF, we obtain a value of 0.84, implying a variation in [Fe/H] of less than 0.1 dex. This is not the most favorable case, since in fact there is no apparent mechanism available to enable even partial mixing to occur. Turbulent mixing should end when the shell reaches local sound speed, while diffusion takes place on too long a time scale. It is also more consistent to have mixing end at V_f, which is taken to be the local sound speed. If this is the case, then the metals produced by the stars below about 20 $M_{\odot}$ will not be incorporated into the second generation at any level. This further insures that the second generation will be highly homogeneous, since all of its metal content will have been well mixed.

4. CONCLUSIONS

Our model for the formation and early evolution of *the spheroidal component of the Milky Way globular clusters* is based on the assumption that a first generation of stars formed in the central region of the proto-globular-cluster cloud on a time scale of the order of 10×10^6 yr. Stellar winds and supernova explosions of high-mass stars then formed a metal-enriched supershell, which swept up the surrounding gas in its outward progress. We assume that the globular clusters observed today consist mainly of (a second generation of) stars that formed in this shell. We can draw the following conclusions concerning the relation between our model and observed globular clusters.

1. A small, entirely self-enriched system of stars will typically achieve a metallicity in the range $\sim 10^{-2}$ to $10^{-1}\ Z_{\odot}$ observed for the metal-poor (or halo) globular clusters. We have shown that the metallicities are constrained to this range by dynamical limits imposed by the dispersive physical processes associated with supernova ejection. The metallicity depends mainly on f_e, the fraction of the initial energy which is not radiated away during the shell expansion. The upper and lower limits on f_e are given by the conditions that at most 8%, and at least 0.4%, of the input energy is available. This implies a metallicity of globular clusters in the range of $\sim 10^{-2}$ to $10^{-1}\ Z_{\odot}$, if the mean progenitor mass is $\sim 30\ M_{\odot}$. This is approximately the mean mass of those stars that can evolve on the dynamical time scale of the supershell.

2. The second stellar generation, and therefore the globular cluster as viewed today, should be expected to be extremely homogeneous in composition. Such a small internal metallicity spread, which is in agreement with observations, results from efficient mixing in the shell prior to the onset of star formation in the shell.

3. One critical question concerns the possible effects of the presence of a residual component of zero-metallicity low-mass stars from the (first) stellar generation that produced the observed abundance levels. It is possible that such a population never existed, because of the fact that only more massive stars $M \gtrsim 10\ M_\odot$ were formed in the first generation of zero-metallicity stars (Mouschovias 1989). An alternative explanation of the absence of such stars is provided by the possibility that, during the subsequent evolution of the system, the residual stars of the first stellar population became gravitationally unbound and were lost from the cluster (contributing to the lower metallicity component of halo field stars). In a subsequent paper (Burkert et al. 1991), we will present the results of numerical studies which suggest that this may indeed occur.

4. Our studies to date suggest that the chemical evolution of the spheroidal component of the galaxy was dominated by local mixing rather than global motions.

5. In conclusion, we wish to emphasize that our finding that the self-enrichment of globular clusters leads to metallicities in the range $\sim 10^{-2}$ to $10^{-1}\ Z_\odot$ is in no way inconsistent with the fact that there exist, for example, both Galactic and LMC globular clusters of higher metallicities. We do not argue that there cannot exist metal-rich clusters that were presumably born with significant primordial concentrations of heavy elements. Rather, our models indicate only that *the natural level of self-contamination lies in the range* $\sim 10^{-2}$ to $10^{-1}\ Z_\odot$, such that clusters containing no initial concentrations of heavy elements will necessarily achieve a metallicity in this range.

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