THE ASTROPHYSICAL JOURNAL, 376:1–7, 1991 July 20

THE ACCURACY OF GALAXY MASSES FROM THE TIMING ARGUMENT

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Received 1990 August 13; accepted 1991 January 15

ABSTRACT

The accuracy of the timing argument mass estimate is examined using binary galaxies found in an N-body simulation of an $\Omega = 1$ cold dark matter (CDM) cosmology. Masses are calculated assuming the binaries are bound, for both purely radial orbits and orbits with angular momentum. The timing masses, M_t, are comparable to the total mass within two spheres, centered on each galaxy, of radii one half the separation between the two galaxies. These "two-sphere" masses, M_{2s}, are estimated with little systematic bias for the angular momentum calculation. The corresponding radial orbit timing masses on average underestimate M_{2s} by a factor of 1.7. The standard deviation of $\log_{10}(M_t/M_{2s})$ ranges from a minimum of 15% for nonradial approaching orbits to a maximum of 34% for approaching binaries assuming radial orbits and the same number of complete orbits as the best angular momentum timing mass. Eighty percent of the binary galaxies present in the simulation have completed less than one and a half orbits. The lack of binaries in advanced stages of their orbital history likely indicates that isolated binary galaxies merge quickly. If the timing argument is indiscriminately applied to all of the binaries (bound and unbound) the timing masses do not correlate well with the measured masses.

This analysis suggests that the radial orbit timing mass of the M31 Galaxy system is quite likely 50% of the two sphere mass, M_{2s} with 1 σ range of 0.2 to $1.3M_{2s}$ if the system is bound. The mass within a radius of 50 kpc (assuming a Hubble constant of 50 km s^{-1} Mpc^{-1}) is of order 1/10 the timing mass for wide binaries that are similar to the M31 galaxy system. We conclude that the timing argument data for the M31 galaxy pair is consistent with an extensive massive halo in an $\Omega = 1$ universe.

Subject headings: cosmology — galaxies: clustering — galaxies: interactions

1. INTRODUCTION

The masses of galaxies at large radii are not well established. Various methods of estimating the masses have yielded a considerable range of values for the amount of dark matter present. Mass-to-light ratio determinations range from ~ 3 in the disk of our Galaxy to 200 in clusters of galaxies (see the review by Trimble 1987). The major obstacle in determining the mass of the Galaxy and the extent of the dark matter halo is the paucity of luminous tracer particles at large radii.

Mass determinations of the Galaxy within a radius of 50–100 kpc have yielded values in the wide range of 2–30 $\times 10^{11}$ $M_\odot$, (for a review, see Faber & Gallagher 1979; Trimble 1987). Hartwick & Sargent (1978) made an estimate of the mass of the Galaxy, $M(R < 60$ kpc) from the orbits of satellite galaxies. They found that the mass of the Galaxy within 60 kpc is $3.4 \pm 1.5 \times 10^{11}$ $M_\odot$ if the satellites are assumed to have radial orbits, while an isotropic velocity dispersion yields a value of $7.6 \pm 2.1 \times 10^{11}$ $M_\odot$. Little & Tremaine (1987) refined the satellite estimate by introducing a new statistical method and using more accurate radial velocities. The mass estimate they derived for an isotropic velocity distribution is $2.4^{+1.3}_{-0.7} \times 10^{11}$ $M_\odot$, much lower than Hartwick and Sargent's value. New data for the outer satellites (Zaritsky et al. 1989) raises the mass estimate to $12.5^{+8.4}_{-3.2} \times 10^{11}$ $M_\odot$. If the satellites are on mainly radial orbits the estimated mass is lowered.

The timing argument, a mass estimate at large radii, was first proposed by Kahn & Woltjer (1959) to determine the mass of the Local Group. The basic idea is that galaxies currently in a binary system were at approximately the same point in space shortly after the big bang. The Hubble expansion velocity between the two galaxies is slowed due to their mutual gravitational attraction, and their orbit is described by Newton's equations of motion. In principle, if a pair of galaxies is well represented by point masses and isolated, their total mass may be determined by measuring their separation, relative velocity, and the time since the expansion of the universe began.

The masses that are calculated using the timing argument ($\simeq 10^{12}$ $M_\odot$) are considerably larger than the mass derived from methods using visible tracers such as satellites, not completely unexpected since the timing masses sample to larger radii (100–500 kpc) than local methods. For the application of the timing argument to M31 and our Galaxy, Kahn & Woltjer (1959) argued that most of the observable mass is in the two main galaxies, and that the satellite galaxies are not dynamically important. They assumed that the system has a radial orbit and obtained 1.8×10^{12} $M_\odot$ as a minimum estimate for the reduced mass of the system. Other determinations of the mass of the Local Group using the timing argument with radial orbits have been made by Gunn (1975), Gott & Thuan (1978), Lynden-Bell (1982, 1983), Mishra (1985), and Sandage (1986). The range of values from these authors is 2–6 $\times 10^{12}$ $M_\odot$ for the total mass of the Local Group. Einasto & Lynden-Bell (1982) included angular momentum in their timing argument calculation and derived a total mass in the range of 3–6 $\times 10^{12}$ $M_\odot$. Raychaudhury & Lynden-Bell (1989) have estimated the effect of neighboring galaxies on the orbit of M31 and the Galaxy and found that the radial orbit timing mass of the Local Group is not greatly affected.

Peebles et al. (1989) have modeled the formation of the Local Group by gravitational accretion onto two seed masses. The velocities of the outer members of the Local Group are best described by a model with a Hubble constant of 80 km s^{-1} and $\Omega = 1$. The mass estimate they derive for the entire Local Group is $5 \times 10^{12} M_{\odot}$.

The accuracy of the timing argument may be questioned due to the neglect of the effects of neighbors, the absence of measured tangential velocities, and the assumption that two distinct galaxies formed at $t = 0$ and have moved as single particles since then. The purpose of this paper is to compare the masses determined by the timing argument with the measured masses in an $\Omega = 1$ CDM N-body simulation. In the next section the theory of the timing argument is summarized for radial and elliptical orbits. The data are discussed in the third section. The results are presented in § 4. The last section contains a discussion of the results.

2. THEORY: THE TIMING ARGUMENT

If we approximate a curent epoch wide binary as two point masses, unaffected by surrounding material, that began expanding with the big bang then their orbits and the system's mass are readily determined. This case is relevant to the determination of the mass of the Local Group since we cannot presently measure the transverse velocity of M31. The mass calculated assuming a radial orbit, i.e., zero angular momentum, will be a lower bound if the orbit is actually elliptical. For completeness, the following section gives the previously derived equations for estimating timing masses.

2.1. *Radial Orbit Masses*

At the time of the big bang two isolated galaxies (or their corresponding protogalaxies) are at approximately the same place and begin to travel away from one another. Their Hubble expansion velocities are slowed by their gravitational attraction and if there is enough mass present within the two galaxies they eventually stop and fall back along a bound orbit. In an assumed Keplerian potential the separation of the binary and time since pericenter are described by $r = a(1 - e \cos \chi)$ and $t = (a^3/GM)^{1/2} (\chi - e \sin \chi)$, where r is the separation, t is the time, a is the semimajor axis, e is the eccentricity, χ is the eccentric anomaly, and M is the total mass of the binary system. Initially, at $t = 0$, $r = 0$, and for a radial orbit $e = 1$. Differentiating the above two equations with respect to χ and dividing the results yields

$$\frac{dr}{dt} = \frac{dr/d\chi}{dt/d\chi} = \sqrt{\frac{GM}{a}} \frac{\sin \chi}{1 - \cos \chi} = \frac{r \sin \chi(\chi - \sin \chi)}{t(1 - \cos \chi)^2} . \quad (1)$$

Thus, given the separation, the radial velocity and the time since expansion began, we can find the eccentric anomaly, semimajor axis, and the total mass of the binary system.

2.2. *Orbits with Angular Momentum*

The orbit of a binary system is inevitably subjected to torques from other galaxies and surrounding matter. The timing masses for elliptical orbits were derived by Einasto & Lynden-Bell (1982), and the following summarizes their results.

For the orbit of a binary we have the following equation $h^2/GMr = 1 + e \cos \phi$, where ϕ is the true anomaly (see Green 1985), $h = rV_t$, is the angular momentum per unit mass, and e is the eccentricity.

The ratio of radial and transverse velocity components is $V_r/V_t = e \sin \phi/(1 + e \cos \phi)$. The eccentric anomaly and the true anomaly are related by $\cos \phi = (\cos \chi - e)/(1 - e \cos \chi)$. Substituting for M, e, and a into the equation of the time since pericenter gives

$$t = \frac{r \sin \chi[\chi(V_t^2 \sin^2 \chi + V_r^2)^{1/2} \mp V_r \sin \chi]}{[\mp(V_t^2 \sin^2 \chi + V_r^2)^{1/2} + V_r \cos \chi]^2} , \quad (2)$$

where the negative (positive) sign corresponds to receding (approaching) binaries.

Therefore, given the separation, relative transverse and radial velocities, and the time since the expansion began, the eccentric anomaly may be found and hence the total mass, period and eccentricity of the binary.

3. DATA

The data are from Carlberg & Couchman's (1989) dissipative N-body simulation. They examined the evolution of a CDM spectrum with a total density parameter, $\Omega = 1$, density parameter for baryonic matter, $\Omega_b = 1/11$ and a Hubble constant of 50 km s^{-1} Mpc^{-1}. Three cube sizes were used, 40, 80, and 200 Mpc. The 40 Mpc cube data are used in this analysis. This cube contains 524,288 particles. Half of the particles were baryonic, initially all labeled gas, and using a gasdynamical scheme they were allowed to cool and form stars. The remaining particles are dark matter.

Scaling gives each gas and dark matter particle a mass of $1.54 \times 10^9 M_{\odot}$ and $1.69 \times 10^{10} M_{\odot}$, respectively. The experiment was evolved to 42.99 time units (one time unit is 3.03×10^8 yr), a time identified as the current epoch. In the simulation one grid unit is equal to 0.625 Mpc and the velocity scale is 2015 km s^{-1} at the current epoch. The unit of mass that is used throughout this analysis is the mass of a gas particle.

The galaxies were found using a link method, with a link length of 0.1 grid units. The algorithm first starts a tree with any star particle (the method is independent of which particle is chosen initially), then finds the nearest particle that is unattached and less than one link length from nearest particle in the tree and joins it to the tree. This procedure is repeated until no more particles can be added to the tree. This process is continued for the remaining unattached paticles. The algorithm produces a minimal spanning tree which is unique. Trees containing five or more star particles form a galaxy. Eleven percent of the total of 89,224 star particles make up the 985 galaxies found in the 40 Mpc box. The average galaxy contains 10.4 star particles, has a mean half-mass radius of 0.069 grid units (43 kpc), and has on the average a virialized halo with ~ 5 to 10 times as many dark particles. The minimum galaxy size corresponds to a minimum luminosity of $M_B = -18.5$, assuming $M_{\rm star}/L = 2$.

The algorithm for finding the binary galaxies is, for each galaxy:

1. Find the closest neighbor galaxy and its distance, r_1.
2. Find the next closest galaxy to either of the two galaxies in the pair and its distance from the closer member of the pair, r_2.
3. Calculate the ratio r_2/r_1 to determine how isolated the pair of galaxies is and whether or not it should be considered a binary as opposed to part of a group.

A limit of $r_2/r_1 > 2$ is imposed to eliminate galaxies that are members of a group of three or more galaxies. A higher limit of $r_2/r_1 > 3$ finds nearly the same results for bound systems. The binaries found are independent of the order in which the gaslaxies are examined, so that the list of binaries is unique. With this method 100 binary galaxies are found, of which 54

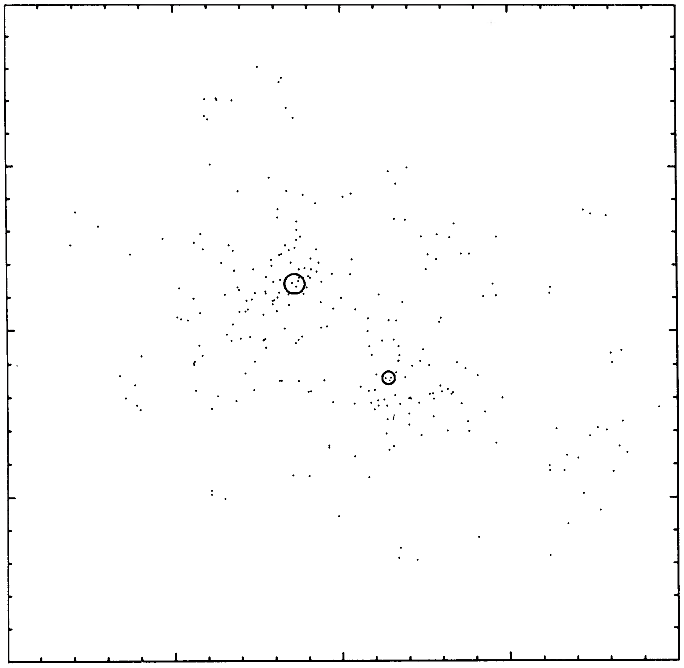

FIG. 1.—Example of a wide binary. The galaxies are indicated by a circle of half mass radius. The dark matter particles are also shown in this x-y projection. The length of the side of the box is 2 grid units.

are approaching and 46 are receding from one another. One of the wide binaries is shown in Figure 1.

The isolation of a binary may also be tested by determining the tidal influence of a third galaxy. Assuming circular orbits and a constant mass-to-light ratio the fractional perturbation in deduced mass due to a third galaxy, f, may be expressed by the following inequality (Schweizer 1987)

$$f \geq \frac{3M_3 r_1^3}{(M_1 + M_2)D^3}, \tag{3}$$

where M_1, M_2, and M_3 are the luminous masses of the galaxies and D is the distance from the center of mass of the binary to the perturber. For all of the binaries with $r_2/r_1 > 1$ the largest value of

$$f_t = \frac{3M_3 r_1^3}{(M_1 + M_2)r_2^3} \tag{4}$$

is found, where M_3 is the galaxy with the largest tidal influence on the binary, not necessarily the nearest galaxy. Note that f_t is approximately equal to or larger than the right-hand side of equation (3) since r_2 is not the center of mass distance to the perturber but the distance from the perturber to the closest galaxy of the binary. All but four of the binaries with $r_2/r_1 > 2$ have $f_t \lesssim 0.6$.

The measured mass of a binary is defined in four different ways. The stellar mass of the galaxies is added to the dark matter mass (the amount of gas in the galactic halos is negligible) within: (1) two spheres centered on each galaxy with radii equal to 50 kpc, M_{50}, (2) two spheres of radii 100 kpc, M_{100}, (3) two spheres of radii $r_1/2$, one half the separation between the two galaxies, M_{2s}; (4) one sphere of radius r_1, the separation, centered midway between the two galaxies, M_{1S}.

To determine if the binary is bound the sign of the energy is calculated using the formula for the total energy of a two body system,

$$\frac{E}{M_1 M_2} = \frac{v^2}{2M} - \frac{G}{r_1}, \tag{5}$$

where E is the total energy of the binary, M_1 and M_2 are the masses of the individual galaxies in the binary, and $M = M_1 + M_2$ is the total measured mass of the binary as measured by one of M_{50}, M_{100}, M_{2s}, or M_{1S}. Of course this total mass is easily measured in the simulation but is inaccessible to observation. The individual galaxy masses, M_1 and M_2, are not defined for measured mass M_{1S}, therefore the right-hand side of equation (5) will be evaluated to ascertain whether or not the binary is bound for all of the measured masses. In this analysis the timing masses are calculated assuming that the binaries are always bound, which may not be the case. If the energy of the binary is known to be positive the correct radial orbit calculation uses a somewhat different set of equations, containing hyperbolic functions instead of the trigonometric functions. The binding energy is calculated as a theoretical tool for identifying binary systems that satisfy the binary selection criteria but which are expected to give grossly inaccurate mass estimates. We emphasize that this indicative binding energy is calculated assuming point mass potentials and is dependent on the total measured mass. Approaching binaries that are marginally unbound by this criterion are likely to be bound when mass beyond the imposed measurement spheres is included.

The bound approaching binaries on average are more isolated than the unbound approaching binaries. The mean r_2/r_1 is 4.33 (2.24) for the bound (unbound) approaching binaries with a minimum separation of 1 grid unit (625 kpc). If the minimum separation is decreased to 0.5 grid units the mean r_2/r_1 is 6.50 for the bound approaching binaries and 2.96 for the unbound approaching binaries.

The binary galaxies may be at various phases in their orbits. The radial orbit timing mass, M_{tr} and the angular momentum timing mass, M_{th} for each binary are found assuming 0, 1, and 2 complete orbits. The eccentric anomaly, χ, for an approaching binary that has not completed one orbit has a value between π and 2π. If the binary has orbited once (twice) the eccentric anomaly will be between 3π and 4π (5π and 6π). The eccentric anomalies for receding binaries are between 0 and π for zero complete orbits, 2π and 3π for one complete orbit and 4π and 5π for two complete orbits.

4. RESULTS

Approaching binary galaxies that have not completed one orbit in a Hubble time are of interest since (1) no halo stripping will have taken place so the masses may provide an estimate of the total halo mass; (2) M31 and our Galaxy are thought to be on their first approach. The criteria for a "wide" binary are a crossing time greater than the time since the expansion began and a free-fall crossing time greater than half that time. Therefore the following equations are used to determine whether or not an approaching binary is "wide":

$$t_c = \frac{r}{|dr/dt|} > t\,, \quad t_{ffc} = \frac{\pi}{2}\sqrt{\frac{r^3}{2GM}} > \frac{t}{2}, \tag{6}$$

where t_c is the crossing time, M is the total measured mass of the galaxies and t_{ffc} is the free-fall crossing time. The free-fall crossing time is equal to $t_{dyn}/2^{1/2}$, where t_{dyn} is the dynamical time. The approaching binaries are placed into five categories. The widest binaries are those that satisfy both equations (6).

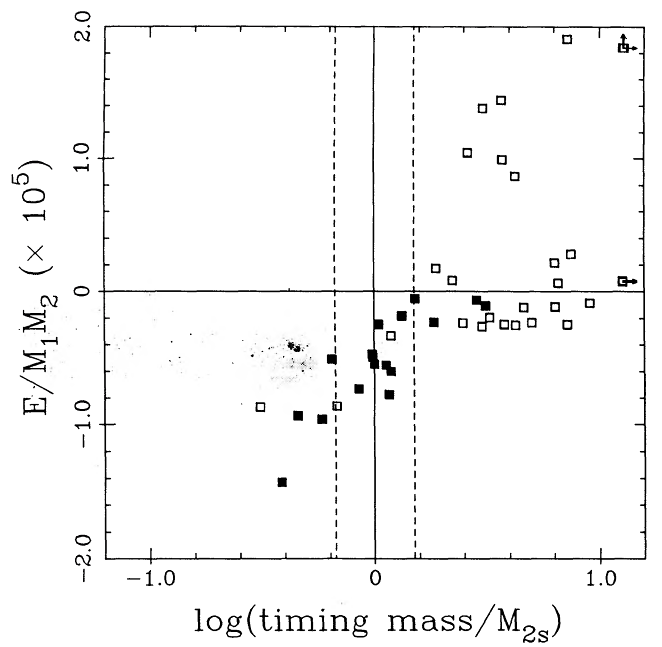

FIG. 2.—$E/M_1 M_2$ vs. the ratio of M_{th} and measured mass M_{2s} assuming one complete orbit for the receding binaries. The filled squares represent the bound binaries and the open squares the unbound binaries. The dashed line to the left (right) of the solid line indicates where the timing mass is 2/3 (3/2) of the measured mass.

The next three categories satisfy the following successively relaxed conditions but do not contain binary galaxies in any of the previous categories,

$$t_c > \frac{t}{n+1}\ ; \quad t_{ffc} > \frac{t}{2(n+1)}\ , \quad \text{for} \quad n = 1, 2, 3\ . \tag{7}$$

The fifth category contains the binary galaxies that do not fit into any of the above.

Receding binaries with a negative energy according to equation (5) may be unbound if there is insufficient mass to reverse the expansion. In Figure 2 the quantity $E/M_1 M_2$ is plotted versus the ratio of M_{th} (assuming one complete orbit) and measured mass M_{2s} for the receding binaries. There is a group of binaries for which the timing mass is too large below the zero binding energy line. The corresponding plot with measured mass M_{1S} also shows a similar group. The calculation of the energy assumes a $1/r$ external potential which is incorrect for a more extended mass distribution. Therefore, a receding binary with a crossing time that exceeds the age of the universe or a free-fall time larger than half that age, i.e., a receding binary that satisfies either of equations (6) will be considered to be unbound. Imposing this restriction eliminates almost all of the binaries that have both a large timing mass and a negative energy. The bound binaries are indicated by filled squares and the unbound binaries by open squares in Figure 2.

A plot of the relative radial velocity versus the separation is shown in Figure 3. The filled symbols represent the binaries that are bound and the open symbols the unbound binaries. The binaries that are wide or have a high relative radial velocity tend to be unbound.

In order to determine which measured mass is closest to the timing mass the timing masses for the approaching bound binaries, M_{tr} and M_{th} for 0, 1, and 2 complete orbits are compared with the four measured masses. The corresponding comparisons are made for the receding bound binaries, but only for 1 and 2 complete orbits. It should be noted that very few receding binaries have solutions for χ in the range 0 to π. Most receding binaries are traveling apart too fast to be bound for their separation and the age of the universe.

An unsuccessful attempt was made to discover the orbital history of the wide binaries by looking at the simulation data at earlier times. The major problem is that galaxies are continually being created from pre-existing smaller galaxies and subgalactic fragments. It is therefore difficult to trace the orbits of two distinct galaxies in a binary system. This emphasizes one of the limitations of this analysis, the assumption of distinct galaxies from the beginning. Nevertheless the purpose of this analysis is not to trace the orbits of the binary systems but to determine how accurately the current epoch data estimates the mass of the current epoch galaxy systems.

For each binary the timing mass closest to each of the measured masses is found and is called the best timing mass. The best timing mass is the one which produces the ratio of the timing mass to the measured mass closest to unity. The mean and standard deviation of the logarithm of the ratio of the angular momentum timing mass to the measured mass are displayed in Table 1 for the approaching, receding, and all of the bound binaries. In the three samples the standard deviation is smaller for measured mass M_{2s} than for M_{1S}. The F-test (see for instance Pollard 1977) is applied to the M_{2s} and M_{1S} data to determine if the difference in the variances is significant. It is found that for the approaching binaries the probability of obtaining the variances given if the samples actually have the same variance is 14%, for receding binaries 98%, and for all of the bound binaries 26%. Application of the F-test indicates no significant preference for one or the other of these two masses.

The mean and standard deviation of the logarithm of the

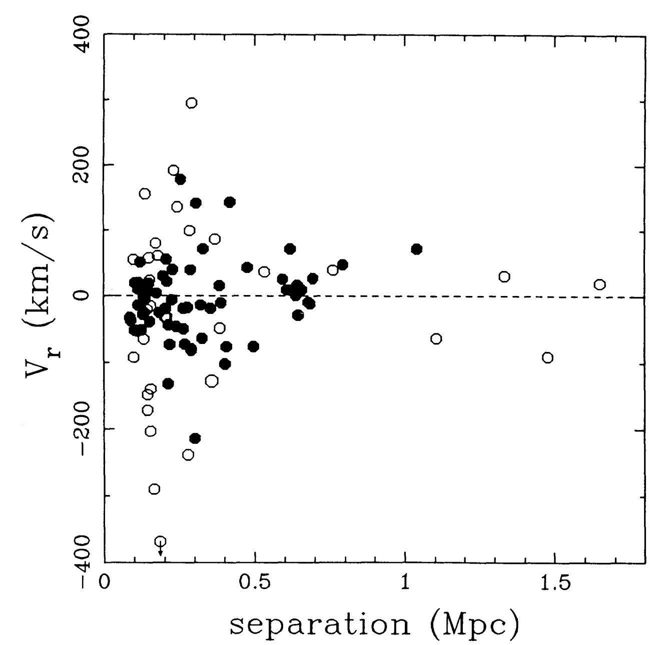

FIG. 3.—Relative radial velocity vs. the separation, for all binaries. The filled markers represent the binaries that are bound for measurement mass M_{2s}, the open markers the unbound binaries.

TABLE 1

TIMING MASS ACCURACY[a]

Group	Mass	Number	Radial Orbits Mean[c]	Radial Orbits σ	Radial Orbits[b] Mean	Radial Orbits[b] σ	Angular Momentum Mean	Angular Momentum σ
Approaching	M_{2s}	34	−0.059	0.183	−0.262	0.337	0.025	0.151
Approaching	M_{1S}	43	−0.170	0.244	−0.302	0.327	−0.031	0.193
Approaching	M_{100}	31	−0.161	0.341	−0.307	0.439	0.013	0.299
Approaching	M_{50}	14	0.156	0.370	−0.055	0.573	0.243	0.355
Receding	M_{2s}	17	0.019	0.216	−0.164	0.310	0.078	0.183
Receding	M_{1S}	22	−0.207	0.248	−0.288	0.253	−0.061	0.184
Receding	M_{100}	16	−0.212	0.270	−0.318	0.252	−0.050	0.211
Receding	M_{50}	9	0.007	0.149	−0.165	0.298	0.166	0.220
All	M_{2s}	51	−0.033	0.196	−0.230	0.329	0.043	0.162
All	M_{1S}	65	−0.182	0.244	−0.298	0.302	−0.041	0.189
All	M_{100}	47	−0.178	0.317	−0.311	0.382	−0.008	0.272
All	M_{50}	23	0.098	0.307	−0.098	0.479	0.213	0.306

[a] For bound binaries only.

[b] Same number of complete orbits as best M_{th}.

[c] Mean of logarithm of ratio timing mass and measured mass.

ratio of the timing mass and measured mass are also shown in Table 1 for the best radial orbit timing masses and for the radial orbit timing masses with the same number of orbits as the best angular momentum timing mass. The best timing masses that are obtained with only the radial velocity information for the bound binaries are on average 0.93 of the measured mass M_{2s} with a 1 σ range of 0.59–1.46. The radial orbit timing masses that correspond to the best angular momentum timing masses are on average 0.59 of the measured mass M_{2s} with a 1 σ range of 0.28–1.25. Assuming that all the approaching (receding) bound binaries are on their first (second) orbit gives a mean of 0.39 (0.58) and a 1 σ range of 0.13–1.16 (0.25 to 1.31). The measured mass inside of 50 or 100 kpc does not correlate well with the timing argument mass, having nearly twice the dispersion in mass ratio of either M_{2s} or M_{1S}.

The timing mass calculation with angular momentum will be considered in the following discussion. If M_{2s} is the best measured mass 79% of the approaching binaries are on their first approach and 82% of the receding binaries have completed one orbit. If the best measured mass is M_{1S} only 44% of the approaching binaries are approaching for the first time and 23% are approaching for the second time. Most of the receding binaries are on their third orbit (64%).

A dynamical argument can be used to suggest which of the two measured masses is the best by calculating the time scale for frictional orbit decay of a satellite galaxy. The halo of the parent galaxy may be approximated by an isothermal sphere. The orbital radius of the satellite galaxy decreases as (Tremaine 1981),

$$r^2(t) = r^2(t_0) - 0.605\,\frac{Gm_s t}{\sigma}\ln\Lambda\,, \tag{8}$$

where r is the orbital radius, m_s is the mass of the satellite, t is the time, σ is the one-dimensional velocity dispersion, $\Lambda \approx r/r_s$, and r_s is the size of the satellite. If we assume the orbit is circular and the rotation curve of the parent galaxy is flat with a velocity $v_c = 2\frac{1}{2}\sigma$, the above equation is rewritten as

$$t = \frac{[r^2(t_0) - r^2(t)]v_c}{0.428Gm_s \ln\Lambda}\,. \tag{9}$$

To estimate the merger time of a binary we take $\ln\Lambda \approx 1$ and define χ by $t \approx \chi r(t_0)/v_c$:

$$\chi \approx \frac{r(t_0)v_c^2}{0.43Gm_s}\,. \tag{10}$$

If it is also assumed that the satellite and parent galaxies have approximately the same mass, then

$$v_c^2 = \frac{Gm_s}{r(t_0)}\,. \tag{11}$$

Substituting equation (11) into equation (10) at the time of the merger yields $\chi \sim 2.3$. From this rough calculation it can be seen that most of the binary galaxies will merge quickly and that one would not expect to see a large number of binaries that have completed 1 or more orbits. The study of mergers using N-body simulations has also shown that isolated binaries merge quickly (White 1978; Carlberg 1982). From the results of this approximation and the lower variances in M_{2s} it will be assumed M_{2s} is the measured mass that best represents the timing mass.

In Figure 4 the best M_{th} and the M_{tr} with the same number of complete orbits as the best M_{th} (not the best M_{tr}) are plotted versus the measured mass M_{2s} for the bound galaxies. The corresponding plot is shown for all the binaries in Figure 5. The skeletal markers represent the radial orbit calculation (M_{tr}) and the filled markers the angular momentum calculation (M_{th}). The number of vertices on a marker represents the number of complete orbits offset by three. Figure 5 reinforces the need for the restriction that the binaries are bound.

The distribution of eccentricities for isotropic orbits assuming a Keplerian potential is $n(e, e + de) = 2Ne\,de$, where N is the total number of binary systems (Binney & Tremaine 1987). The χ^2 test shows that there are no significant differences between the eccentricities for the best angular momentum timing masses and an isotropic distribution. It should be noted that in the most radial bin ($e = 0.9$ to 1.0) there is a weak tendency for more binaries to be present than for an isotropic distribution.

The isolation of the binary and the crossing time do not

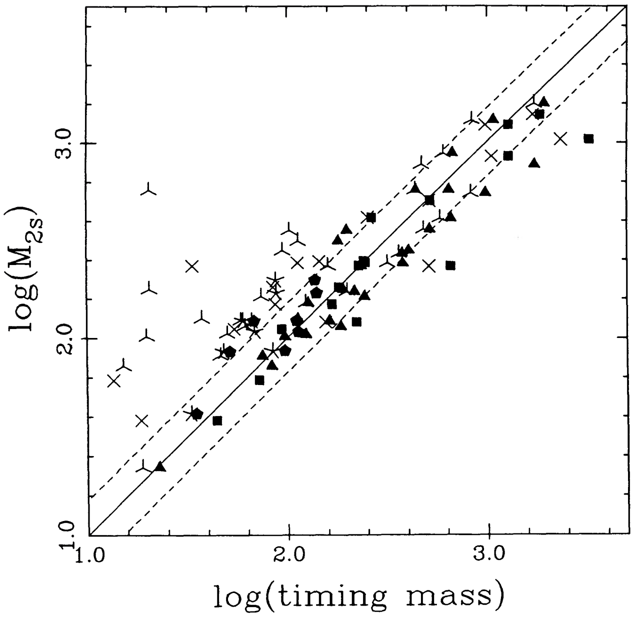

FIG. 4.—Best timing mass vs. the measured mass M_{2s} for the bound galaxies. Filled markers represent the best M_{th} and skeletal markers represent the M_{tr} that has the same number of complete orbits as M_{th}. The standard deviation of $\log_{10}(M_{th}/M_{2s})$ is 0.162.

seem to be important in determining whether or not the timing masses are accurate for the sample with $r_2/r_1 > 2$. All but two of the bound binaries have a tidal parameter $f_t < 0.095$ ($r_2/r_1 > 2$). The angular momentum timing mass is a good estimate of M_{2s} if $f_t \lesssim 0.13$ for the sample with $r_2/r_1 > 1$. There is a strong dependence of increasing timing argument mass accuracy with separation, free-fall crossing time, and relative radial velocity.

There is a correspondence between how wide an approaching bound binary is (eqs. [6] and [7]) and how many orbits have taken place. The timing masses that are most accurate for the widest binaries are those found assuming the galaxies are approaching each other for the first time. The binaries for which the first orbit calculation underestimates their mass are not present until the fourth widest category ($n = 3$) is included. This may indicate that those binaries have orbited at least once and/or are in the process of merging.

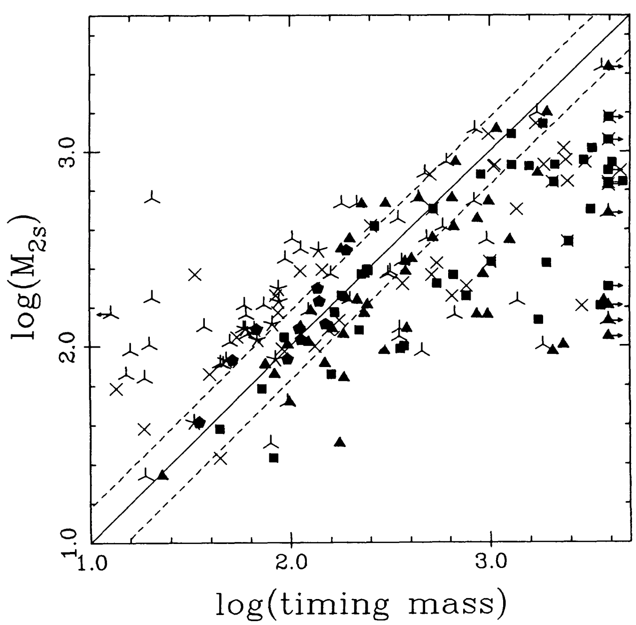

FIG. 5.—Same as Fig. 4 except all binaries are plotted. Note that the restriction that the energy of the binary be negative is important to reduce the scatter.

There are five wide approaching binaries, 14, 30, 50, 53, and 63 which are approximate analogs of the M31 Galaxy system. Binaries 14 and 30 are definitely unbound. The timing masses that are found for 14 are too large by a factor of at least 20. Binary 14 has the largest separation, 2.36 grid units or 1.48 Mpc. Binary 30 has the second largest separation and the timing masses found are again quite high. Both of these wide binaries are unbound according to equation (5). In a straight line orbit the closest approach of binaries 14 and 30 are 0.94 and 1.07 Mpc, respectively. The large separations and high relative velocities of these two binaries precludes them as good analogs to the M31 galaxy system. A third binary, 63, is probably also unbound since $E/M_1 M_2$ is very close to zero. For the first approach M_{tr} is $0.730 \times 10^{12}\ M_\odot$ and M_{th} is $2.67 \times 10^{12}\ M_\odot$ (the eccentricity is 0.290). The angular momentum calculation gives a value which is significantly higher than the measured masses $M_{2s} = 1.19 \times 10^{12}\ M_\odot$ and $M_{1s} = 1.70 \times 10^{12}\ M_\odot$.

The timing argument works well for binaries 50 and 53. The measured masses M_{2s} and M_{1s} of binary 50 are $1.36 \times 10^{12}\ M_\odot$ and $2.38 \times 10^{12}\ M_\odot$, respectively. The timing masses M_{tr} and M_{th} are $0.935 \times 10^{12}\ M_\odot$ and $1.04 \times 10^{12}\ M_\odot$ ($e = 0.921$) for zero complete orbits. The argument in favor of mass M_{2s} being closer than M_{1s} to the timing mass is strengthened by the fact that the mass M_{1s} is too large by more than a factor of 2. If one complete orbit has taken place the timing masses are $\sim 4.6 \times 10^{12}\ M_\odot$. It is obvious that for this wide binary the galaxies are on their first approach. Binary 53 has a mass $M_{2s} = 0.882 \times 10^{12}\ M_\odot$ and mass $M_{1s} = 1.07 \times 10^{12}\ M_\odot$. The timing masses are approximately the same ($0.676 \times 10^{12}\ M_\odot$ and $0.681 \times 10^{12}\ M_\odot$) for both calculations since the orbit is nearly radial ($e = 0.995$). The timing masses for the second approach again are near $4.6 \times 10^{12}\ M_\odot$. The two widest bound binaries, 50 and 53, are well isolated, approaching each other for the first time, and have timing masses that are within 30% of the measured mass M_{2s}.

5. CONCLUSIONS

The accuracy of the timing argument for binary galaxies is examined using data from an N-body simulation of an $\Omega = 1$ CDM cosmology. Modeling the binary galaxies as two point masses proves to be a remarkably good approximation even in the presence of tidal fields and continuing infall of surrounding material. The timing masses of the bound galaxies are most comparable to M_{2s}, the mass within two spheres centered upon each galaxy with radius equal to half the separation of the two galaxies. The masses inside of 50 and 100 kpc underestimate the timing masses and show a poor correlation with the timing argument masses.

The best angular momentum timing masses are on average 1.10 of the measured mass M_{2s} with a 1 σ range of 0.76–1.60. The radial orbit timing masses that correspond to the best angular momentum timing masses are on average 0.59 of the measured mass M_{2s} with a 1 σ range of 0.28–1.25. The orbits of the binaries are consistent with an isotropic distribution.

Wide approaching binary galaxies are examined in detail. Two of the five binaries are unbound and consequently have erroneously excessively large timing masses. A third binary is near the zero binding energy limit and is probably unbound, since the angular momentum timing mass is too high by a factor of more than 2. The timing argument works well for the two remaining binaries; the timing masses are within 30% of the mass M_{2s}.

The timing argument, assuming the binaries are on their first approach, does not work well if the separation is too small or too large. If the binary has a separation less than ~ 200 kpc there is some ambiguity in how many orbits have taken place. On the other hand if the separation is greater than ~ 1 Mpc the binary may have been influenced by other galaxies earlier in its history. The average r_2/r_1 is lower for unbound binaries with a large separation than for the bound galaxies. The relatively close neighbors can add angular momentum to the system, and there will be a large tangential velocity that will cause the binary to be unbound. Therefore the timing argument can be applied to isolated approaching binaries that have moderate (~ 200 kpc to 1 Mpc) with some confidence.

From this analysis we expect that the radial orbit timing argument applied to the M31 galaxy pair gives a slight underestimate of the M_{2s} mass of the system, that is, the timing mass is likely to be ~ 0.53 of two sphere mass, with a 1 σ confidence interval ranging from 0.2 to 1.3 about that mean value. Because the approximately isothermal galaxy halos in this model universe typically extend out several megaparsecs, it is not surprising that masses measured on the scales of 50 kpc are up to an order of magnitude smaller than those indicated in binary systems with separations of 500 kpc.

We would like to thank Scott Tremaine for his comments. Financial support was provided by NSERC.

REFERENCES

Binney, J. J., & Tremaine, S., 1987, Galactic Dynamics (Princeton: Princeton University Press)
Carlberg, R. G. 1982, MNRAS, 199, 1159
Carlberg, R. G., & Couchman, H. M. P. 1989, ApJ, 340, 47
Einasto, J., & Lynden-Bell, D. 1982, MNRAS, 199, 67
Faber, S. M., & Gallagher, J. S. 1979, ARAA, 17, 135
Gott, J. R., III, & Thuan, T. X. 1978, ApJ, 223, 426
Green, R. M., 1985, Spherical Astronomy (Cambridge: Cambridge University Press)
Gunn, J. E. 1974, Comm. Ap.Space, Phys., 6, 7
Hartwick, F. D. A., & Sargent, W. L. W. 1978, ApJ, 221, 512
Kahn, F. D., & Woltjer, L. 1959, ApJ, 130, 705
Little, B., & Tremaine, S. 1987, ApJ, 320, 493
Lynden-Bell, D. 1982, in Proc. Study Week on Cosmology and Fundamental Physics, Astrophysical Cosmology, Vol. 48, ed. H. A. Bruck, G. V. Coyne, & M. S. Longair (Rome: Specola Vaticana), 85
Lynden-Bell, D. 1983, in Kinematics, Dynamics and Structure of the Milky Way, ed. W. H. L. Shuter (Dordrecht: Reidel), 349
Mishra, R. 1985, MNRAS, 212, 163
Peebles, P. J. E., et al. 1989, ApJ, 345, 108
Pollard, J. H. 1977, A Handbook of Numerical and Statistical Techniques (Cambridge: Cambridge University Press)
Raychaudhury, S., & Lynden-Bell, D. 1989, MNRAS, 240, 195
Sandage, A. 1986, ApJ, 307, 1
Schweizer, L. Y. 1987, ApJS, 64, 427
Tremaine, S. 1981, in The Structure and Evolution of Normal Galaxies, ed. S. M. Fall & D. Lynden-Bell (Cambridge: Cambridge University Press), 67
Trimble, V. 1987, ARAA, 25, 425
White, S. D. M. 1978, MNRAS, 184, 185
Zaritsky, D., et al. 1989, ApJ, 345, 759