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THE COMPLETE SAMPLE OF 1 JANSKY BL LACERTAE OBJECTS. I. SUMMARY PROPERTIES

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ABSTRACT

This paper describes the first homogeneous, flux-limited sample of radio-selected BL Lac objects, taken from the 1 Jy survey of Kühr et al. These 34 BL Lac objects comprise the only well-defined sample of BL Lac objects outside the X-ray band. The selection criteria include flat radio spectra, optical counterparts visible on Sky Survey Plates, and optical emission lines weaker than 5 Å equivalent width in the rest frame. With two exceptions, all of the BL Lac objects also have high optical polarization, $P_{\rm opt} > 3\%$.

Redshifts from weak emission lines, usually narrow [O II] λ3727, [O III] λ5007, or broad Mg II λ2798, are available for $\sim\frac{3}{4}$ of the sample. The nearby ($z < 0.2$) BL Lac objects, at least, sit in elliptical galaxies and may occur in groups or small clusters of galaxies. Gravitational lensing appears to affect one of the high-redshift ($z \gtrsim 0.5$) BL Lac objects and could be important in others as well. The radio number counts derived from the 1 Jy BL Lac sample are roughly Euclidean over the range between 1 and ~5 Jy, and there is evidence for evolution at the 2 σ level. The local radio luminosity function of BL Lac objects is derived and the total number density for $P_{5\,\rm GHz} \gtrsim 6 \times 10^{31}$ ergs s^{-1} Hz^{-1} is ~40 Gpc^{-3}.

Subject headings: BL Lacertae objects — gravitational lenses — luminosity function — radio sources: galaxies

1. INTRODUCTION

The unusual properties of BL Lac objects (Stein, O'Dell, & Strittmatter 1976), namely the large and rapid radio and optical variability, the strong and highly variable radio and optical polarization, and the weak or absent emission lines in the optical spectrum, set them apart from most other classes of active galactic nuclei (AGN). Only the high polarized quasars (HPQs; Moore & Stockman 1981, 1984) exhibit a somewhat similar continuum behavior, but their emission line strengths are comparable to those of normal quasars.

BL Lac objects and HPQs together comprise the blazar class (Angel & Stockman 1980), and it is now widely argued that relativistic beaming, originally suggested by Blandford & Rees (1978) for BL Lac objects, is responsible for the common properties of blazars (e.g., Antonucci & Ulvestad 1985). According to this model, the emission lines of BL Lac objects are masked by the Doppler-boosted optical continuum. Because optical spectroscopy of an increasing number of BL Lac objects has resulted in the detection of weak narrow or even strong broad emission lines during low states of continuum emission, the explanation of BL Lac objects as a "central engine" without surrounding gas or without broad- and narrow-line regions (Guilbert, Fabian, & McCray 1983) can probably be ruled out.

Recently, amplification of the optical continuum by gravitational microlensing has been considered as an alternative explanation for the BL Lac phenomenon. Ostriker & Vietri (1985, 1990) proposed that BL Lac objects are actually distant ($z \gtrsim 1$), highly variable (OVV) quasars gravitationally microlensed by nearby ($z \lesssim 0.2$) elliptical galaxies which thereby appear as BL Lac host galaxies. In contrast, Nottale (1986, 1988) suggested that the strong variability and the weak emission lines of BL Lac objects during optical outbursts are caused by gravitational microlensing of distant ($z \gtrsim 1$) nonvariable quasars (see also Schneider & Weiss 1987); the foreground galaxies responsible for lensing, however, have generally not been seen in direct images but in a few objects may manifest themselves through intervening absorption-line redshift systems. Clearly, distinguishing among Doppler boosting and the two different gravitational lens scenarios for BL Lac objects is of great importance since physical models differ considerably in these three cases.

As with any astronomical objects, understanding the properties of the class requires well-defined samples, i.e., samples with known, quantifiable biases. The identification of X-ray sources from large surveys provided the first complete samples of X-ray–selected BL Lac objects (XBLs; Piccinotti et al. 1982; Giommi et al. 1989; Stocke et al. 1990). However, most of the known BL Lac objects (Ledden & O'Dell 1985; Burbidge & Hewitt 1987, 1989; Véron-Cetty & Véron 1989) have been discovered during the identification of radio sources with their optical counterparts. Because of the different telescopes and detectors used, there is a wide spread in the quality of the available optical spectra on which the classification of a particular radio source as BL Lac object is based. Radio-selected BL Lac objects (RBLs) known from earlier studies thus form a rather inhomogeneous set of objects, which does not allow a proper statistical study of the influence of relativistic beaming and/or gravitational lensing.

Furthermore, despite the fact that their X-ray luminosities are comparable, XBLs and RBLs have systematically different properties, with the RBLs having relatively more radio emission, stronger variability in the optical and radio, and higher optical polarization (Maraschi et al. 1986; Ledden & O'Dell 1985; Jannuzi, Smith, & Elston 1991). There may be a distribution of properties, with XBLs and RBLs representing the extremes. Both types possibly share the same underlying physical mechanism and differ only with respect to some secondary property such as the beaming angle, the Lorentz factor of the

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bulk relativistic motion, or the relative contribution of different wavelength bands. The statistical study of the XBL samples indeed shows that the redshift distribution and the number counts are consistent with the expectations from beamed Fanaroff-Riley type I (FR I) radio galaxies (Padovani & Urry 1990, 1991). Clearly, analysis of a corresponding well-defined radio-selected sample is essential for understanding the BL Lac phenomenon.

In § 2, we describe the selection of a radio flux–limited, homogeneous sample of BL Lac objects from the 1 Jy survey of Kühr et al. (1981b). A detailed description of the observational data base for this sample can be found in an accompanying paper (Stickel, Fried, & Kühr 1991, hereafter Paper II). A summary of the optical properties of the sample, from deep direct imaging and low-resolution spectroscopy, is given in § 3. We consider the redshift distribution and the number counts (log N–log S) in § 4; in § 5 we estimate the evolution and determine the first radio luminosity function of BL Lac objects. Finally, § 6 presents our discussion and conclusions. Throughout this paper, the values $H_0 = 50$ km s^{-1} Mpc^{-1} and $q_0 = 0$ have been used.

2. THE 1 Jy BL LACERTAE SAMPLE

The 1 Jy catalog of radio sources (Kühr et al. 1981b) is complete over a large fraction of the sky (9.81 sr) and contains 518 radio sources with $S_\nu \geq 1$ Jy at 5 GHz, about half of which have flat or inverted radio spectra at centimeter wavelengths (Kühr & Schmidt 1990). Since the first publication of the 1 Jy catalog, the identifications and redshifts have continuously been updated from our own as well as other published observations and are now virtually complete for the optically brighter ($m < 20$ mag) flat spectrum sources. By virtue of its large area, its high identification fraction, and its short survey wavelength (which favors flat spectrum sources), the 1 Jy catalog is ideal for selecting BL Lac objects. The present BL Lac sample has been derived from this catalog using the following selection criteria:

1. The radio spectrum at centimeter wavelengths is flat or inverted with a radio spectral index between 11 and 6 cm of $\alpha \geq -0.5$ ($S_\nu \propto \nu^\alpha$).
2. The optical counterparts of the radio sources are brighter than 20 mag on Sky Survey Plates.
3. Emission lines in the optical spectrum are absent or weak with a rest-frame equivalent width of the strongest line of less than 5 Å.

The resulting sample consists of 34 BL Lac objects (Table 1). Because of the inclusion of the emission-line strength criterion, the sample of 1 Jy BL Lac objects given in Table 1 supersedes that described by Stickel, Fried, & Kühr (1989a) and Kühr & Schmidt (1990). We note that for all but two objects (0426−380 and 2005−489) the optical polarization exceeds 3%.

For each object in Table 1 we give the IAU name (the first column), other designations (the second column), and the radio position taken from the 1 Jy catalog (the third and fourth columns). The fifth column lists the brightness of the optical counterpart taken from the 1 Jy catalog (Kühr et al. 1981b), except for 0716+714 and 1803+784 (Biermann et al. 1981), 1144−379 (Nicolson et al. 1979), and 2005−489 (Wall et al. 1986). The sixth and seventh column give the radio flux density at 5 GHz and the two-point spectral index between 11 and 6 cm (2.7 and 5 GHz), respectively. The apparent steep radio spectral index of $\alpha = -0.73$ for 1749+701 is due to variability and the nonsimultaneous flux density measurements (Kühr 1990). It is included with the 1 Jy BL Lac objects as it is a typical flat spectrum radio source and fulfills all the other selection criteria, especially the emission line is very weak (Arp et al. 1976; Stickel, Fried, & Kühr 1989b). The redshifts, given in the last column, are from our own as well as other observations; a detailed description of each BL Lac object and extensive references can be found in Paper II.

The fraction of BL Lac objects among the 1 Jy flat spectrum radio sources brighter than 20 mag is about 15%. We note that this percentage is comparable to the highly efficient selection of XBLs from among the sources of the *Einstein* Extended Medium Sensitivity Survey (EMSS; Stocke et al. 1990), but twice as high as the fraction of RBLs among the complete sample of Parkes flat spectrum radio sources (Savage et al. 1990). At present nine additional radio sources from the 1 Jy catalog satisfy the first two selection criteria above, but spectroscopy is not yet available; they are thus potential BL Lac candidates. Assuming that BL Lac objects are not overrepresented in these unobserved sources, we expect that at most one or two BL Lac objects are missing from the 1 Jy BL Lac sample listed in Table 1.

The most problematic of the selection criteria listed above is the strength of the emission lines in the optical spectrum. There is no generally accepted spectroscopic criterion for the separation of BL Lac objects from HPQs. The choice of 5 Å equivalent width was deduced from an examination of the line strengths of a representative sample of flat spectrum radio sources from the Parkes catalog (Wilkes 1986), and it is the same limit used for the selection of the XBLs from the EMSS (Stocke et al. 1990). However, in contrast to the EMSS XBLs, and in order not to bias the 1 Jy BL Lac objects against high-redshift ($z > 0.5$) objects which are more common among RBLs, we used the rest frame value rather than the observed value [$\mathrm{EW_{rest}} = \mathrm{EW_{obs}}/(1 + z)$]. For most of the objects, this spectroscopic criterion is limited by our observations to the wavelength range 4000 Å $\lesssim \lambda \lesssim$ 7500 Å. Some of the 1 Jy radio sources that have been described by other observers as potential BL Lac candidates but revealed strong emission lines above the 5 Å limit in our first spectrum were not included in the 1 Jy BL Lac sample. The data for most of these sources can be found in Stickel et al. (1989b).

Although the optical brightness and thus the equivalent widths of emission lines are highly variable, only a few 1 Jy BL Lac objects have sometimes shown emission lines above the 5 Å limit. The probability of detecting strong emission lines in a particular object increases with the number of spectroscopic observations. Because the number of spectra per object is not uniform, we would certainly bias the sample if we removed all objects showing emission lines above the 5 Å limit after repeated measurements. Therefore these borderline objects were retained in the 1 Jy sample; they are briefly described in the following paragraph.

After extensive spectroscopic observations of 0235+164, Cohen et al. (1987) reported on one occasion a rest frame equivalent width of 8.1 Å for the Mg II λ2798 emission line which, however, was obtained only after an integration time of more than 8 hr. In all other cases, the line strengths were below 5 Å. For 0537−441, Wilkes (1986) measured a rest frame equivalent width for Mg II λ2798 of more than 5 Å, but our own observations showed on two occasions (one of which has been described by Stickel, Fried, & Kühr 1988c) a value below 5 Å. Moreover, the weakness of this emission line was also noted by Peterson et al. (1976), Cristiani (1985, 1986), and Falomo, Tanzi, & Treves (1989). Early observations of

TABLE 1

THE 1 Jy SAMPLE OF BL LACERTAE OBJECTS

Object	Other Name(s)	RA (1950)	Dec (1950)	m	S_{5GHz}	α_{11-6}	z
0048 − 097	PKS,OB-080,MC3,PHL856	00 48 09.99	−09 45 24.6	17.0	1.98	+0.43	...
0118 − 272	PKS, OC -230.4	01 18 09.53	−27 17 06.9	16.5	1.14	+0.28	>0.557
0138 − 097	PKS, OC -065	01 38 56.80	−09 43 51.0	18.0	1.22	+0.88	>0.501
0235 + 164	PKS, AO, OD 160, MC3, GC	02 35 52.62	+16 24 04.0	19.0	2.85	+1.03	0.940
	MG 0238+1636						
0426 − 380	PKS	04 26 54.74	−38 02 52.4	19.0	1.17	+0.20	>1.030
0454 + 844	S5, 1H	04 54 57.02	+84 27 53.1	16.5	1.39	+0.37	...
0537 − 441	PKS, IRAS	05 37 21.07	−44 06 45.0	15.5	4.00	+0.06	0.896
0716 + 714	S5	07 16 12.98	+71 26 15.0	13.2	1.12	+0.22	...
0735 + 178	VRO 17.07.02, PKS, DA 237	07 35 14.13	+17 49 09.3	16.5	1.99	+0.05	>0.424
	OI 158, GC, MC, MG 0738+1742						
0814 + 425	OJ 425, S4	08 14 51.67	+42 32 07.7	18.5	1.69	−0.10	0.258?
0820 + 225	PKS, 4C 22.21, MG 0823+2223	08 20 28.57	+22 32 44.7	19.2	1.60	−0.20	0.951
0823 + 033	PKS, OJ 038, MG 0825+0309	08 23 13.56	+03 19 15.7	18.0	1.32	+0.77	0.506
0828 + 493	OJ 448, S4, BP 077	08 28 47.94	+49 23 33.4	18.5	1.03	−0.19	0.548
0851 + 202	PKS, OJ 287, B2, IRAS, GC	08 51 57.25	+20 17 58.4	14.5	2.62	+0.11	0.306
	VRO 20.08.01, MG 0854+2006						
0954 + 658	S4	09 54 57.86	+65 48 15.5	16.7	1.46	+0.35	0.367
1144 − 379	PKS, MC	11 44 30.87	−37 55 30.5	16.2	1.61	+0.66	1.048
1147 + 245	B2, OM 280, GC, MG 1150+2417	11 47 44.01	+24 34 34.6	16.5	1.01	+0.39	...
1308 + 326	AU Cvn, B2, OP 313, GC	13 08 07.59	+32 36 39.5	19.0	1.53	+0.20	0.997
	MG 1310+3220						
1418 + 546	OQ 530, B2, S4	14 18 06.20	+54 36 57.8	14.5	1.09	+0.38	0.152
1514 − 241	AP Lib, OK 225, B2, PKS, MC	15 14 45.24	−24 11 22.4	16.2	2.00	−0.16	0.049
1519 − 273	PKS	15 19 37.23	−27 19 29.6	18.5	2.35	+0.27	...
1538 + 149	4C 14.60, OR 165, PKS	15 38 30.18	+14 57 22.1	17.2	1.96	+0.34	0.605
	MG 1540+1447, GC, MC						
1652 + 398	Mkn 501, S4, OS 387, 1H	16 52 11.75	+39 50 24.6	14.0	1.42	+0.06	0.033
	4C 39.49, UGC 10599, 4U						
	VV 7-35-2, DA 426, B2						
1749 + 096	OT 081, 4C 09.57, PKS	17 49 10.39	+09 39 42.7	16.8	1.87	+1.01	0.320
	MG 1751+0938, GC, MC						
1749 + 701	S4, S5, 2H	17 49 03.38	+70 06 39.5	16.5	1.08	−0.73	0.770
1803 + 784	S5, IRAS	18 03 39.18	+78 27 54.2	16.4	2.62	+0.25	0.684
1807 + 698	3C 371.0, S4, 1H, IRAS	18 07 18:54	+69 48 57.0	14.2	2.26	+0.14	0.051
	VII Zw 768, NRAO 548						
	4C 69.24, UGC 11130						
1823 + 568	4C 56.27, S4, OU 539	18 23 14.99	+56 49 18.0	18.4	1.66	+0.22	0.664
2005 − 489	PKS	20 05 47.40	−48 58 45.0	15.3	1.23	+0.24	0.071
2007 + 777	S5	20 07 20.42	+77 43 58.0	16.5	1.26	+0.69	0.342
2131 − 021	4C -02.81, PKS, MC	21 31 35.34	−02 06 40.8	19.0	2.12	+0.13	0.557?
2200 + 420	BL Lac, PKS, B2, S4, MC	22 00 39.36	+42 02 08.6	14.0	4.77	−0.13	0.069
	OY 401, VRO 42.22.01						
2240 − 260	PKS, OY -268	22 40 41.82	−26 00 14.6	17.5	1.03	−0.08	0.774
2254 + 074	OY 091, CTD 135, PKS	22 54 45.97	+07 27 08.4	16.4	1.19	+1.19	0.190
	MG 2257+0743						

0851+202 (OJ 287) by Miller, French, & Hawley (1987) detected only weak emission lines, as was the case during our observation described in Stickel et al. (1989b); however, during a low state of continuum emission, Sitko & Junkkarinen (1985) detected relatively strong Hα λ6563. Spectroscopic observations of 1308+326 by Miller et al. (1978) and Wills & Wills (1979) failed to detect strong emission of Mg II λ2798, but one of our observations which apparently was made near minimum light (see Paper II) showed this emission line to be rather strong. Similarly, Morris & Ward (1988) have detected Hα λ6583/N II λ6583 in the spectrum of 1514−241 (AP Lib) only during a low state with a combined rest frame equivalent width of 11.6 Å. Finally, 1749+096, which was described as a weak-lined quasar by White et al. (1988), showed on a single occasion emission lines above the 5 Å limit (Stickel, Fried, & Kühr 1988a), while during another observation only weaker emission lines have been detected. Possibly these objects should not be called BL Lac objects; however, based on the optical spectroscopy for this sample, they are probably typical of the other BL Lac objects in the 1 Jy sample.

3. PROPERTIES OF THE SAMPLE

The properties of the 1 Jy BL objects sample will be discussed in greater detail in Paper II. Here we summarize some characteristics of the sample as a whole.

3.1. *Optical Spectra*

The optical spectra of the BL Lac sample are, by definition, devoid of strong features. However, usually with large effort, weak features have been seen in most of the 1 Jy BL Lac objects. On occasion, narrow [O II] λ3727 is present, though it is always weak. For the low-redshift BL Lac objects, a narrow [O III] λ5007 line is sometimes present, but given the limited spectral range, [O III] would not be seen at redshifts $z \gtrsim 0.5$. A histogram comparing the luminosity of the [O III] emission line in the 1 Jy BL Lac objects and in a sample of quasars (Yee

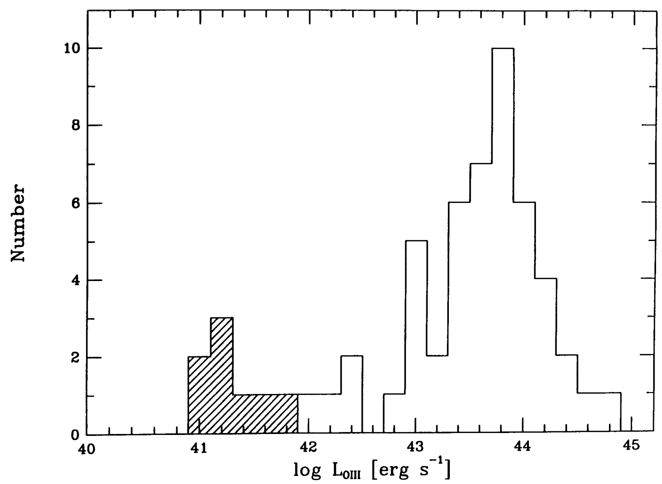

FIG. 1*a*

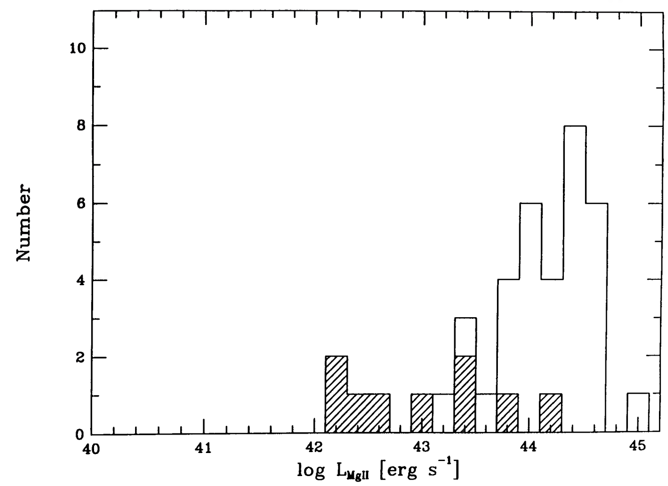

FIG. 1*b*

FIG. 1.—(*a*) The distribution of [O III] λ5007 line luminosities for the 1 Jy BL Lac objects (*hatched area*) and a sample of quasars (Yee 1980; Oke et al. 1984). The BL Lac line luminosities are much lower than the quasars, but are comparable to radio galaxy line luminosities. (*b*) The distribution of Mg II λ2798 line luminosities for the 1 Jy BL Lac objects (*hatched area*) and a sample of quasars (Oke et al. 1984). The mean line luminosity is lower for BL Lac objects than for quasars, but there is some overlap.

1980; Oke, Shields, & Korycanski 1984) is shown in Figure 1*a*. All the BL Lac line luminosities are relatively low; in fact they are comparable to those of radio galaxies (see, e.g., Miller et al. 1978; Sitko & Junkkarinen 1985).

In the high-redshift objects, the strongest feature usually seen is broad Mg II λ2798, shown in the histogram in Figure 1*b*, again contrasted to the same line in quasars. This feature enters the observed optical spectra for redshifts $z \gtrsim 0.4$ and is thus detected only in the high-redshift BL Lac objects. Whether a similar feature is present at a comparable strength in the low-redshift 1 Jy BL Lac objects is not yet clear because individual *IUE* spectra are usually less sensitive to weak lines than the optical spectra discussed here and in Paper II. While the Mg II luminosity is generally lower in BL Lac objects than in quasars, a few BL Lac objects have line luminosities as high as some quasars (in contrast to the [O III] case). The distinction between types of spectral features, both the presence and luminosity of narrow [O III] and broad Mg II with respect to quasars, may indicate a difference between high- and low-redshift BL Lac objects or it may be caused by selection effects.

3.2. *Optical Morphology*

Optical CCD images were obtained for all objects in the BL Lac sample with typical seeing of 1"–1".5. For the seven nearby objects with $z < 0.2$, the galaxy surrounding the BL Lac point source is easily visible. In each case, the observed image has been fitted with a two-dimensional brightness profile of the form $I(r) = I_0 \exp(-r/r_c)^{\beta}$, where $\beta = 0.25$ corresponds to a de Vaucouleurs profile (de Vaucouleurs 1948), appropriate for an elliptical galaxy, and $\beta = 1$ corresponds to a disk galaxy (Freeman 1970). The inner parts of the images ($r < 5''$) were excluded from the fits to minimize the influence of the central point source.

The fitted values of β, listed in Table 2, are generally much less than one, indicating the host galaxies to be most likely ellipticals. Only for 2200+420 (BL Lacertae) is β relatively high, which may be due either to the presence of two nearby bright stars affecting the fit or to an intermediate-type host galaxy, as suggested previously by Hutchings (1987). Also listed in Table 2 are the effective radii and total apparent *R* magnitudes derived from fitting the observed image with a de Vaucouleurs law. The absolute magnitudes of the host galaxies take into account *K*-corrections (Coleman, Wu, & Weedman 1980), a constant color index $(V-R) = 0.9$, and Galactic absorption (Burstein & Heiles 1982). The mean absolute magnitude is $\langle M_V \rangle = -22.9 \pm 0.3$, comparable to the brightest cluster galaxies (Kristian, Sandage, & Westphal 1978; Wilkinson & Oke 1978) and to the value found in previous studies (e.g., Miller 1981; Burbidge & Hewitt 1987, 1989; Ulrich 1989), although with a smaller dispersion, possibly because we are dealing with more homogeneous data.

Among the nine BL Lac objects with reliable redshifts $0.2 \lesssim z \lesssim 0.7$, four (0823+033, 0851+202, 1749+096, and 1823+568) are spatially resolved. Subtracting a point source from the images did not yield convincing results, so the magnitude of the underlying galaxy could not be determined. It is remarkable that 2007+777, although similar to 1749+096 in brightness and redshift, is unresolved, while 1823+568, which has a redshift nearly twice as high, is resolved. The latter is one of the highest redshift objects that appears extended on deep images.

For 19 BL Lac objects in the sample, we have spectra of galaxies in the same field (see Paper II for details). Although there have been previous reports of BL Lac objects being members of groups of galaxies (e.g., Arp 1970; Ulrich 1978; Balick & Heckman 1982), our study represents the largest single collection of data on the environments of BL Lac

TABLE 2

PROPERTIES OF BL LACERTAE HOST GALAXIES

Object	z	β_5	r_e (kpc)	m_R (mag)	$M_{V,c}$ (mag)
1418+546	0.152	0.28	6.8	16.2	−23.0
1514−241	0.049	0.22	11.5	14.4	−22.6
1652+398	0.033	0.18	18.3	12.8	−22.9
1807+698	0.051	0.17	20.5	14.4	−22.4
2005−489	0.071	0.38	5.2	14.2	−23.3
2200+420	0.069	0.66	10.5	15.4	−23.0
2254+074	0.190	0.43	14.5	16.6	−23.2

objects. Three of the four low-redshift ($z < 0.2$) BL Lac objects in this subsample have one to three neighboring galaxies at the same redshift, possibly indicating the presence of a group or small cluster of galaxies. There may indeed be additional cluster galaxies in the field, but because the field objects are faint relative to the BL Lac object, spectra were usually obtained only for one or two galaxies.

For the higher redshift ($z > 0.2$) BL Lac objects, the character of the environment is less clear. In contrast to studies of quasars where companion galaxies out to $z \sim 0.5$ were found (Stockton 1978; Hintzen 1984), the present, comparable effort revealed no companion galaxies near the high-redshift BL Lac objects. Thus, if these objects do have companions, they must be several magnitudes fainter than the BL Lac object. On the other hand, some of the high-redshift objects are surrounded by asymmetric fuzz that may represent an unresolved, fainter galaxy near the host galaxy of the BL Lac object.

3.3. *Gravitational Lensing*

An important result of the investigation of the 1 Jy BL Lac sample is the identification of several candidates for gravitational lensing. The most prominent objects, 0235+164 and 0537−441, have been discussed previously by Stickel, Fried, & Kühr (1988b, c). Although both have redshifts near $z \sim 1$ (measured from Mg II λ2798 emission lines), they appear fuzzy on deep direct images, which can be explained by a foreground galaxy on the line of sight. In the case of 0235+164, independent evidence for a foreground galaxy comes from an intervening redshift system (Cohen et al. 1987). More recently, our optical image of the radio source 1308+326 ($z = 0.996$) revealed that it appears to be spatially resolved, too (see Paper II), again suggesting a foreground galaxy along the line of sight. Also the strong variability and the high bolometric luminosity of 1308+326 are similar that of 0235+164 and 0537−441.

In these three cases a foreground galaxy may be macrolensing the background BL Lac objects, which might explain the extremely high bolometric luminosity of these objects (Impey & Neugebauer 1988). Given the central star densities in galaxies, microlensing is also likely to be important. Possibly the importance of microlensing can be evaluated from the variability characteristics (Schneider & Weiss 1987; Kayser et al. 1989).

Four BL Lac objects (0118−272, 0138−097, 0426−380, and 0735+178) in the 1 Jy sample have relatively high-redshift absorption lines (used here as lower limits to the source redshift). By analogy to the early observations of 0235+164, where only intervening absorption lines had been seen (Rieke et al. 1976; Burbidge et al. 1976), these objects may be gravitationally lensed as well, but no foreground galaxies have been detected in the images. These four BL Lac objects are good candidates for gravitational lens studies with higher spatial resolution. A similar suggestion was recently made by Falomo (1990) for the case of 0823−223, which also shows only an absorption-line redshift system in its spectrum.

4. REDSHIFT DISTRIBUTION AND NUMBER COUNTS

Spectroscopic redshifts from emission lines are available for 25 of the 34 1 Jy BL Lac objects (see Table 1), only two (0814+425 and 2131−021) of which must be considered highly uncertain. In four (0118−272, 0138−097, 0426−380, and 0735+178) of the remaining nine objects, absorption lines of cosmological origin were detected and have been used to derive lower limits to the emission-line redshifts, assuming the absorption line is Mg II λ2798 in intervening material along the line of sight. In only five BL Lac objects (0048−096, 0454+844, 0716+714, 1147+245, and 1519−273) neither emission nor absorption lines have been detected. In each case, a lower limit of $z > 0.2$ can be derived from the stellar appearance on the direct images (see Paper II).

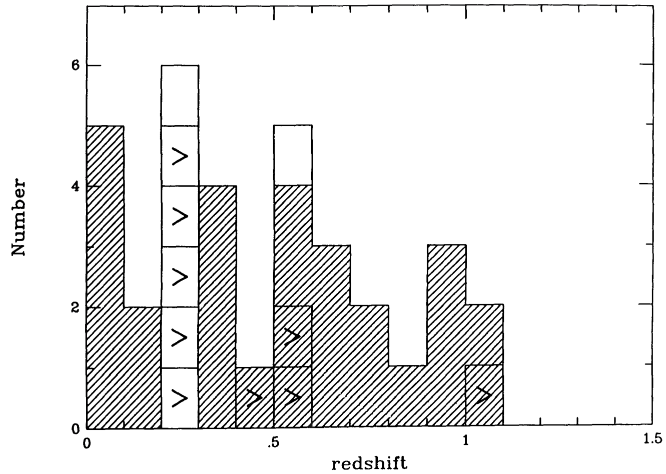

FIG. 2.—The redshift distribution of the 1 Jy BL Lac objects. The hatched area represents the redshifts from emission lines and/or galaxian absorption lines, the open area represents the uncertain emission line redshifts, and the lower limits are the limits derived from intervening absorption lines (*hatched*) or from direct images (*open*).

The resulting redshift distribution of the 1 Jy BL Lac sample (Fig. 2), is roughly uniform up to redshifts of $z \sim 1$ and does not show a strong increase toward low redshifts, as was previously found from incomplete groups of BL Lac objects (Woltjer & Setti 1982; Browne 1989).

The integral number counts of the radio-selected 1 Jy BL Lac sample are shown in Figure 3 (*filled circles*), where a point is plotted for each object. (Note that in an integral representa-

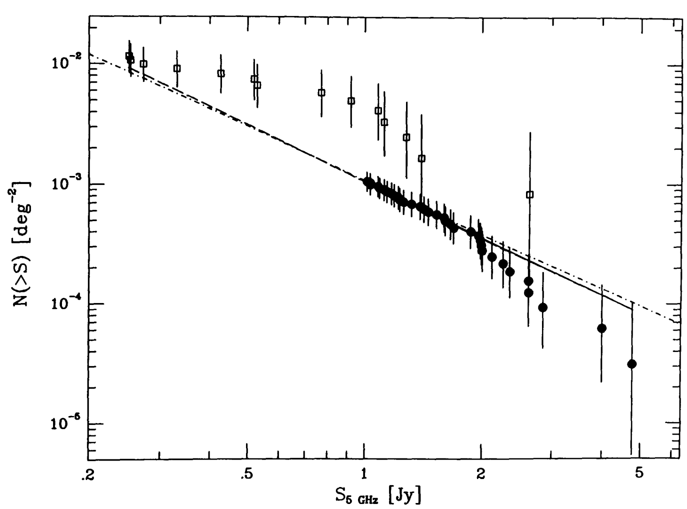

FIG. 3.—The integral number counts for the 1 Jy BL Lac objects (*filled circles*) with the error bars representing the 1 σ Poisson errors (Gehrels 1986). The solid line represents the best-fit slope derived from a maximum likelihood method, and the dashed line represents its extrapolation to lower fluxes. The dotted-dashed line is the Euclidean relationship, normalized to 1 Jy. The integral number counts of the S5 survey BL Lac objects are also shown (*open squares*; Kühr et al. 1987; Kühr & Schmidt 1990).

tion the data do not have to be binned and the data points are not strictly independent.) The dotted-dashed line represents the Euclidean relationship $[N(>S) \propto S^{-1.5}]$, normalized at 1 Jy. The best way to estimate the slope of the number counts using all the available information is through the maximum likelihood method (Crawford, Jauncey, & Murdoch 1970, hereafter CJM). Assuming that $N(>S) \propto S^{-a}$, the best-fit value of a is the one that minimizes the likelihood function (eq. [6] of CJM)

$$L = N \ln a - a \sum \ln\left(\frac{S_i}{S_0}\right) - N \ln\left[1 - \left(\frac{S_m}{S_0}\right)^{-a}\right], \quad (1)$$

where N is the total number of sources and S_i are the individual fluxes, extending over the range S_0–S_m.

The resulting best fit value with its 1 σ confidence interval (derived from eq. [10] of CJM) is $a = 1.58 \pm 0.44$. Although Figure 3 shows some steepening for $S_\nu \gtrsim 2$ Jy, a single power law of index $a = 1.58$ is a good description of the integral counts (at the 99.9% confidence level), as found by applying the Kolmogorov-Smirnov (KS) test (see eq. [14] of CJM).

Figure 3 also shows (*open squares*) the integral counts for the 14 BL Lac objects in the S5 survey (Kühr et al. 1981a, 1987; Kühr & Schmidt 1990), which extends to lower fluxes ($S_{5\,\mathrm{GHz}} \geq 0.25$ Jy) and covers a smaller solid angle (0.368 sr) than the 1 Jy catalog. (Five objects are common to both surveys.) The surface density of BL Lac objects from the S5 survey having $S_\nu > 1$ Jy is $4.1^{+2.8}_{-1.8} \times 10^{-3}$ per square degree, higher than the surface density inferred from the 1 Jy survey, $N(>1\ \mathrm{Jy}) = 10^{-3}$ per square degree, but only by 2 σ. It is interesting to note that the S5 survey at low fluxes falls on the extrapolation of the single power-law distribution $[N(>S) \propto S^{-1.58}]$ derived from the 1 Jy sample (Fig. 3, *dashed line*).

5. EVOLUTION AND THE LUMINOSITY FUNCTION

The evolution of X-ray–selected BL Lac objects appears to be substantially less than for quasars (see, e.g., Stocke et al. 1988). Padovani & Urry (1990) put a lower limit to their evolutionary time scale in the beaming scenario of approximately one-third the Hubble time. With the present sample, we can now address the question of evolution at radio wavelengths.

To do this we applied the V/V_m test (Schmidt 1968), where V_m is the maximum accessible volume within which an object could be detected above the flux limit of the sample, taking into account the amount of sky surveyed (in this case 9.81 sr). In the absence of evolution the quantity V/V_m has the property of being uniformly distributed between 0 and 1, with a mean value of 0.5 (Schmidt 1968).

The V/V_m value depends mostly on the flux and increases weakly with redshift; it is always larger than the value corresponding to the extreme case of the Euclidean approximation [i.e., $V/V_m = (S/S_{\mathrm{lim}})^{-1.5}$]. The inclusion of the nine objects with lower limits on the redshift therefore does not compromise the application of the V/V_m test—if anything, the mean value $\langle V/V_m \rangle$ will be underestimated. The five objects with $z > 0.2$ were set to $z = 0.56$, the mean value of the whole sample taking into account the presence of lower limits using the Survival Analysis package (Feigelson & Isobe 1988) of the Space Telescope Science Institute Science Data Analysis System (STSDAS). The four objects with absorption lines only were set to their lower limits on the redshift. The value of V_m was computed using the expression for the comoving volume appropriate for a Friedmann-Robertson-Walker cosmology, taking into account the K-correction by using the individual spectral slopes. In this case, where there are both radio ($S \geq 1$ Jy) and optical ($m \leq 20$) flux limits, V_m corresponds to the more restrictive one, which turned out to be the radio for all but two objects.

The mean value of the whole sample of 34 objects is $\langle V/V_m \rangle = 0.60 \pm 0.05$, probably indicative of evolution. This can be quantified for particular evolutionary laws; for the sake of brevity we assume here a luminosity evolution of the kind used by Danese et al. (1987) to explain the evolutionary behavior of flat-spectrum radio sources: $P(z) = P(0) \exp[T(z)/\tau]$, where $T(z)$ is the look-back time and τ is the time scale of the evolution in units of the Hubble time ($\tau \sim 0.2$ for flat spectrum radio sources; Danese et al. 1987). We did investigate whether two distinct subgroups are present in our sample, namely the low-redshift ($z < 0.5$) and the high-redshift ($z > 0.5$) objects; however, the values for $\langle V/V_m \rangle$ differ only at the $< 2\,\sigma$ level, so that separate analyses of their evolutionary properties are not warranted. A much larger sample of radio-selected BL Lac objects is needed to explore this possibility further.

The evolution parameter τ can be determined by finding the value that makes $\langle V/V_m \rangle$ equal to 0.5. (The same method has been applied, for example, by Maccacaro et al. 1983 for X-ray–selected AGNs.) The goodness of fit is given by the uniformity of the distribution of the V/V_m once the effect of evolution has been taken into account, and the 1 σ interval of τ corresponds to the values for which $\langle V/V_m \rangle = 0.5 \pm \sigma$ with $\sigma = 1/(12N)^{1/2} = 0.05$ (Longair & Scheuer 1970). The best fit for the evolution parameter is $\tau = 0.32$ with an associated 1 σ interval of [0.24, 0.59]. For this value the distribution of the individual V/V_m is consistent with being uniform between 0 and 1 according to a K-S test. The results for the restricted subsample of 23 objects with firm redshift determinations are similar. We note that the mean value of V/V_m for our sample, although larger than 0.5, is much smaller than the value found by Impey & Tapia (1990) for a sample of only six RBLs (selected with somewhat different criteria).

Using the derived value for the evolution parameter τ in our sample, we can estimate the first local luminosity function of RBLs by computing the zero redshift luminosity, $P(0) = P(z) \exp[-T(z)/0.32]$, for each object in the subsample with complete redshift determinations. Our derivation of the luminosity function is based on Schmidt's (1968) estimator $\sum 1/V_m$, where V_m takes into account the effects of evolution.

The resulting 5 GHz local luminosity function is shown in Figure 4, with the error bars representing the 1 σ Poisson errors (Gehrels 1986). The luminosity function can be well represented by a single power law of the form $\phi(P) \propto P^{-B_r}$. Varying the binning, the fitted differential slope was in the range $2.4 \lesssim B_r \lesssim 2.7$, while the total number density, derived by integrating the power-law fit over the observed luminosity range, varied by about a factor of 2. For a bin size $\Delta \log P = 0.3$, a weighted least-squares fit yielded a luminosity function of the form $\phi(P_r) \simeq 2.0 \times 10^{50} P_r^{-2.53 \pm 0.15}$ ($\chi^2_\nu \simeq 0.67$ for 6 degrees of freedom). It follows that the local number density of BL Lac objects in the range (6.0×10^{31})–(3.4×10^{34}) ergs s^{-1} Hz^{-1} (at 5 GHz) is $N \simeq 40$ Gpc^{-3}.

The inclusion of the sample objects with bounded or uncertain redshifts will increase slightly the total number density and flatten the fit to the luminosity function. This is because these objects have quite high luminosities and therefore do not contribute much to the total number density. Specifically, the

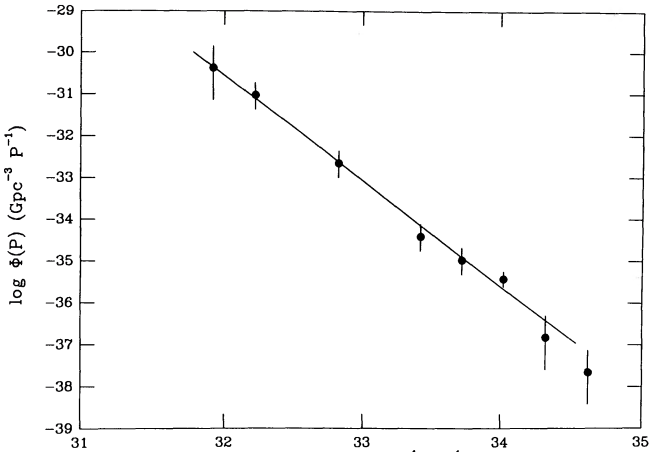

FIG. 4.—The local radio luminosity function for the 1 Jy BL Lac objects with firm redshifts. Luminosities have been computed at $z = 0$ according to the evolutionary law derived in § 5. The solid line is a weighted least-squares fit to the data.

five BL Lac objects with redshift limits $z > 0.2$ have $P_r > 10^{33}$ ergs s^{-1} Hz^{-1}, and the remaining objects without firm redshifts have even higher luminosities (unless the uncertain redshifts have been seriously overestimated). For example, including the uncertain redshifts, setting the limits $z > 0.2$ equal to $\langle z \rangle = 0.56$, and considering the other lower limits as actual estimates, the fit to the luminosity function is slightly flatter (typically by $\sim 1\ \sigma$ in the slope) while the number density increases up to 10% (depending on the bin size) but on average does not change.

The derived luminosity function depends on τ, in the sense that higher values of τ (smaller evolution), for example, will flatten the luminosity function. If we consider the limiting case of no evolution, which cannot be exluded by the data, we obtain a luminosity function of the form $\phi(P_r) \simeq 8.9 \times 10^{38} P_r^{-2.18 \pm 0.13}$ ($\chi^2_\nu \simeq 1.0$ for 5 degrees of freedom). In this case, the local number density of BL Lac objects in the range (6.6×10^{31})–(1.5×10^{35}) ergs s^{-1} Hz^{-1} is $N \simeq 31$ Gpc^{-3}.

As a final check of our results, we compared the number counts and redshift distribution predicted by the derived luminosity function (pertaining to the subsample with well-determined redshifts) and evolution ($\tau = 0.32$) with the observations. We used a spectral slope equal to the mean value of our sample, $\alpha = 0.27$, and carried out the computation up to a redshift $z_{max} = 1.5$. The agreement is good. The best fit to the observed counts would require a renormalization to a total number density ~ 46 Gpc^{-3} (i.e., an increase in number $\sim 15\%$), but this is not surprising, since the adopted luminosity function does not refer to the whole sample, while the counts do.

6. DISCUSSION AND CONCLUSIONS

Great insight as to the origins of the BL Lac phenomenon have been afforded by extensive studies of the known BL Lac objects. However, a fundamental limiting factor in discriminating among the possible explanations has been the absence of well-defined samples. Simple quantities like the luminosity function of BL Lac objects (or even just the integrated space density), essential for testing any model, are only rough guesses without such samples. Even the implications of observed properties of known BL Lac objects are complicated by inhomogeneous samples.

In this paper we have presented the first well-defined, homogeneous radio sample of 34 BL Lac objects selected from a large-area survey of sources brighter than 1 Jy at 5 GHz (Kühr et al. 1981b). In addition to the radio flux limit, the selection criteria include flat radio spectra between 11 and 6 cm ($\alpha \geq -0.5$), optical counterparts brighter than 20th mag on Sky Survey Plates and optical emission lines weaker than 5 Å equivalent width in the rest frame. Since almost every BL Lac object also has high optical polarization ($P_{opt} > 3\%$), this could effectively have been a fourth selection criterion.

Extensive optical spectroscopy has revealed weak emission lines in roughly $\frac{3}{4}$ of the 34 BL Lac objects in the sample. Most are weaker than the 5 Å selection limit (and all are weaker, by definition, in the first spectrum taken), but repeated spectroscopy sometimes yields a spectrum with stronger features, possibly when the source gets faint. The emission lines most commonly observed are narrow [O III] $\lambda 5007$ for the low-redshift objects and broad Mg II $\lambda 2798$ for the high-redshift objects. The mean line luminosities for the BL Lac objects are one or more orders of magnitude lower than the same lines in quasars, although for Mg II there is some overlap. The difference between the two groups of high- and low-redshift objects might be due to selection effects, which infrared and ultraviolet spectra could clarify.

Optical imaging reveals that the nearby BL Lac objects ($z < 0.2$) are not stellar, and that the surface brightness distribution of the surrounding fuzz is consistent with the host galaxies being bright ellipticals. For obvious reasons there are no equivalent data for the high-redshift objects, but this will be interesting to pursue with the high angular resolution instruments of ground-based new technology telescopes.

Our optical images also reveal, for many of the low-redshift ($z \lesssim 0.2$) objects, other resolved (fuzzy) sources near the BL Lac object. We have obtained spectra of several of these, and they often turn out to be galaxies at the redshift of the BL Lac object. Thus BL Lac objects may occur in groups or small clusters of galaxies. Again, for the high-redshift ($z \gtrsim 0.2$) objects, the evidence is less clear because of the available angular resolution and sensitivity. (It would have taken far more observing time to determine the redshifts of the faint candidate companion galaxies than it did to obtain the BL Lac redshift, itself a difficult proposition.) This too will be an important observation to follow up given the predictions of, for example, a beaming scenario with FR I galaxies as the parent population (Padovani & Urry 1990, 1991; Urry, Padovani, & Stickel 1991).

There is a suggestion that gravitational lensing may affect three of the 17 high-redshift ($z \gtrsim 0.5$) BL Lac objects in our sample (0235+164, 0537−441, and 1308+326). The optical images are fuzzy, which is not expected from the host galaxy but instead could be indicating an intervening lensing galaxy. Supporting evidence in the case of 0235+164 comes from the well-known intervening redshift system at $z = 0.524$, while in the other two cases an intervening redshift system attributable to the foreground galaxy has not yet been seen. Four other BL Lac objects (0118−272, 0138−097, 0426−380, and 0735+178) in the 1 Jy sample show only absorption line redshift systems in their spectra, most likely due to intervening matter along the line of sight, so they, too, are lensing candidates. These observations suggest that gravitational lensing may be responsible for some of the observed properties of at least part of the sample.

Our observations do not support a microlensing scenario for the low-redshift objects, as was originally proposed by

Ostriker & Vietri (1985, 1990); instead they are in accord with the idea of Nottale (1986, 1988) that the high-redshift BL Lac objects are microlensed by a foreground galaxy that is difficult to detect in direct images. It remains to be shown whether the strong variability and lineless spectra are due to gravitational microlensing, but in any case gravitational macrolensing must be amplifying the observed flux density at all wavelengths. Therefore, some or all of these seven lensing candidates are probably included in the 1 Jy sample only because of amplification by lensing. Since this represents up to half of the high-redshift ($z > 0.5$) subgroup, the properties of the high-redshift BL Lac objects as a whole (e.g., properties like the mean value of V/V_m) may be affected by lensing.

We have used the 1 Jy BL Lac sample to derive the radio number counts of BL Lac objects. The slope is roughly Euclidean, although it is uncertain because the range in flux is small (roughly 1 to 5 Jy). The surface density of radio selected BL Lac objects above 1 Jy is $\sim 10^{-3}$ deg^{-2}, much smaller than the corresponding surface density of XBLs (see Urry et al. 1991).

The sample shows evidence for evolution at the 2 σ level ($\langle V/V_m \rangle = 0.60 \pm 0.05$). We tested an exponential model for the evolution, $P(z) \propto \exp[T(z)/\tau]$, and the best-fit evolutionary time scale in units of the Hubble time is $\tau = 0.32^{+0.27}_{-0.08}$, which excludes neither the case of no evolution ($\tau > 1$) nor an evolution as strong as flat spectrum radio quasars ($\tau \sim 0.2$).

Since a high fraction of the redshifts are known, the luminosity function could be determined. The local radio luminosity function of BL Lac objects was derived by deevolving the sample: it is well represented by a power law with differential slope $B_r = 2.5$, and the corresponding number density in the luminosity range (at 5 GHz) of (6.0×10^{31})–(3.4×10^{34}) ergs s^{-1} Hz^{-1} is $N \simeq 40$ Gpc^{-3}. In the case of no evolution, the derived slope and the number density decrease by about 15% and 20%, respectively.

The number density of BL Lac objects is much less than the number density of FR I galaxies, $\sim 3.5 \times 10^5$ Gpc^{-3} for $2.4 \times 10^{27} \lesssim P_{5\,\mathrm{GHz}} \lesssim 3.2 \times 10^{33}$ ergs s^{-1} Hz^{-1} (Urry et al. 1991), which have been discussed as the likely parent population in the context of the beaming scenario (e.g., Padovani & Urry 1990). The constraints on this hypothesis available from the 1 Jy sample of BL Lac objects are discussed in a subsequent paper (Urry et al. 1991).

The 1 Jy BL Lac sample gives for the first time a reliable picture of the line properties, host galaxies, environments, surface density, and luminosity function of radio-selected BL Lac objects. As a result, it will be a valuable tool in diagnosing the BL Lac phenomenon. It is sobering to contemplate that even with the enormous effort represented by the selection of these 34 objects, the sample is undoubtedly inhomogeneous (e.g., some BL Lac objects are affected by lensing). A means of enlarging this sample would clearly be valuable. We note that since the radio counts are still steep, unlike the X-ray counts, surveys at fainter radio flux levels will still yield a large increase in the number of BL Lac objects.

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