

DOUBLE CORE EVOLUTION. IV. THE LATE STAGES OF EVOLUTION
OF A $2 M_{\odot}$ RED GIANT WITH A $1 M_{\odot}$ COMPANION¹

RONALD E. TAAM

Department of Physics and Astronomy, Northwestern University, Evanston, IL 60208

AND

PETER BODENHEIMER

University of California Observatories / Lick Observatory, Board of Studies in Astronomy and Astrophysics,
University of California, Santa Cruz, CA 95064*Received 1990 September 12; accepted 1990 October 31*

ABSTRACT

We report on the results of hydrodynamical simulations of the late phase of the common envelope stage of a binary consisting of a $2 M_{\odot}$ red giant and a $1 M_{\odot}$ main-sequence companion. The numerical results demonstrate that sufficient energy is released from the orbit to eject the mass within the common envelope without requiring the main-sequence companion to spiral into the white dwarf core of the red giant star. At the end of the simulation the orbital decay time scale increases rapidly to more than 160 yr. The long decay time scale reflects the removal of mass from the common envelope and its subsequent spin-up to near corotation. The ratio of the orbital decay time scale to the mass-loss time scale from the common envelope increases to more than 700 and the mass contained within the common envelope decreases to $\sim 0.01 M_{\odot}$ or less. It is argued that further orbital decay will be small and that the final binary period will be 1.2 days.

Subject headings: stars: binaries — stars: evolution — stars: interiors — stars: mass loss

1. INTRODUCTION

Among the various phases of binary evolution involving close interacting systems, the common envelope evolutionary stage is the least understood. It was first proposed by Ostriker (1975) and Paczyński (1976) to facilitate the formation of short-period binary systems containing a compact object. Examples of such systems include planetary nebulae with binary nuclei and their cataclysmic variable descendants (Bond 1985, 1989), low-mass X-ray binaries such as 4U 1820–30 (Bailyn & Grindlay 1987; Verbunt 1987), and binary radio pulsars (van den Heuvel & Taam 1984). A necessary requirement in their scenario was that upon mass transfer from a red giant star to its less massive companion a common envelope would be formed. In contrast to contact binaries (or W Ursae Majoris stars), the common envelope is not assumed to be in a state of corotation since the evolutionary time scale during this advanced stage is short compared to the time scale need for the establishment of a uniformly rotating configuration. Since the formation of a compact object (either a white dwarf or neutron star) necessarily requires the expansion of the progenitor star to a large radius as the core regions become centrally condensed, the transformation of the system from a long-period system to a short-period system must involve a severe loss of both mass and orbital angular momentum (see reviews by Bodenheimer & Taam 1986; Taam 1988, 1989; Livio 1989).

Numerous investigations of varying degrees of approximation have been performed (Taam, Bodenheimer, & Ostriker 1978; Meyer & Meyer-Hofmeister 1979; Delgado 1980; Bodenheimer & Taam 1984; Livio & Soker 1984, 1988; Taam & Bodenheimer 1989, hereafter Paper III) which have led to clarifications of some of the ideas set forth in the pioneering work of Ostriker (1975) and Paczyński (1976). Chief among them are the importance of energy transport processes (Taam,

Bodenheimer, & Ostriker 1978) in delaying the onset of mass lost from the common envelope and the role of multidimensional effects (Bodenheimer & Taam 1984; Livio & Soker 1988) in reducing the efficiency of the process. Fundamentally, the mass within the common envelope is driven to escape speed at the expense of the energy lost from the orbital motion of the binary components. Although the energy transport by radiation and convection is negligible at the advanced phases of common envelope evolution, the conversion of orbital energy to mass loss is not efficient since some mass is given more than the escape velocity. The efficiency of the mass loss process, defined as the ratio of the binding energy of the envelope to the energy loss from the orbital motion (see Iben & Tutukov 1984; Livio & Soker 1988), has been found to be as low as 30% (Paper III).

It has been demonstrated by these previous studies that a large fraction of the initial mass and orbital angular momentum can be lost, but the final stages of this phase have not been followed to the point where the common envelope phase has terminated. This last phase is especially important because it has yet to be demonstrated that a short-period binary is formed. In fact, it is possible that the two stars may still merge with each other if the time scale for orbital decay (τ_s) is comparable to or less than the time scale of the ejection of mass (τ_M) from the common envelope during the late stages. In general, one expects that both outcomes are possible and that the properties of the individual components of the binary determine whether one outcome is favored over the other. At the termination (primarily because of numerical difficulties) of the previous two-dimensional hydrodynamical calculation in Paper III for a $5 M_{\odot}$ primary and a main-sequence companion of $1 M_{\odot}$, the ratio τ_s/τ_M increased to a value of ~ 10 . However, the result did not definitively determine the final state of the common envelope evolution because τ_s was still short (~ 1 yr) and the orbital separation had decreased to $\sim 3 R_{\odot}$, possibly too close to avoid further orbital decay.

¹ Lick Observatory Bulletin, No. 1186.

In this paper we report on new results of two-dimensional hydrodynamical calculations in which common envelope evolution is followed to a still later stage through use of different parameters (a $2 M_{\odot}$ primary) and improved numerical techniques. In particular, we find that the rate of orbital decay slows appreciably due to the continuous loss of mass from the common envelope and the spin-up of the envelope to near corotation. In the next section we present the numerical results of the detailed calculations, and in the final section we discuss the implication and the generality of our results.

2. RESULTS

As an initial model, a $2 M_{\odot}$ asymptotic red giant star with a $0.67 M_{\odot}$ carbon-oxygen (C/O) core was constructed. In this case the mass lying between the white dwarf core and a typical orbital separation of $7 R_{\odot}$ amounted to only $0.01 M_{\odot}$. In contrast, the case studied earlier (Paper III) of a $5 M_{\odot}$ asymptotic red giant star with a $0.62 M_{\odot}$ C/O core had $0.5 M_{\odot}$ contained within the same region. It was the eventual fate of this large reservoir of mass interior to the binary orbit and exterior to the white dwarf core of the red giant which could not be addressed quantitatively in Paper III. As we shall see below, the much smaller mass contained in this region in the initial lower-mass asymptotic giant branch star is a property that can be exploited to allow one to examine the final phases of the common envelope evolution in some detail. To graphically illustrate the differences between these two red giant models, the density distribution corresponding to each is shown in Figure 1. It can be seen that the density declines very rapidly, by nearly five orders of magnitude, over a radial extent of 10^9 cm for the $2 M_{\odot}$ red giant whereas a similar density contrast encompasses a much larger region ($\sim 3 \times 10^9$ cm) for the $5 M_{\odot}$ model in its particular evolutionary stage.

We describe the common envelope in cylindrical coordinates (R, Z) with a main sequence-like star (hereafter referred to as the secondary star) of mass $1 M_{\odot}$ initially located at 5×10^{11} cm from the center of the red giant with a radius of 2×10^{13} cm. The choice of initial starting location of the main-

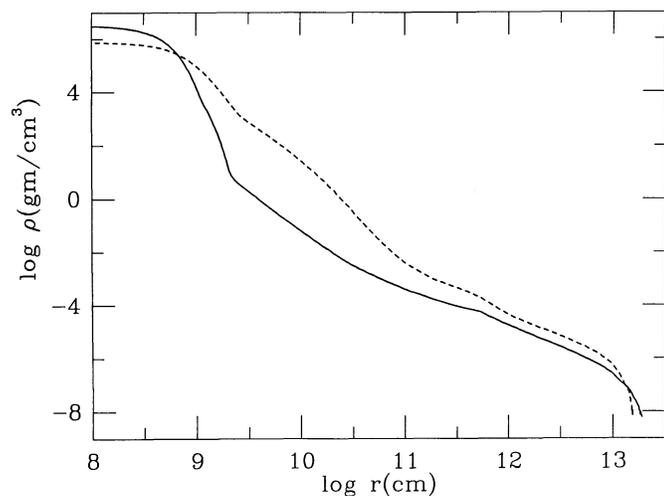


FIG. 1.—The initial density distribution of the red giant as a function of radius for the $2 M_{\odot}$ model (solid line) and a $5 M_{\odot}$ model (dashed line). The white dwarf core of the $2 M_{\odot}$ and $5 M_{\odot}$ models correspond to $0.67 M_{\odot}$ and $0.62 M_{\odot}$, respectively. Note the sharp decline in the density distribution in the $2 M_{\odot}$ model in comparison with the $5 M_{\odot}$ model. This feature in the initial red giant model facilitates the termination of the spiral phase (see text).

sequence-like star was dictated by the requirement that the rate of energy deposition by the interaction within the common envelope become comparable to the maximum rate of energy transport by convection. For the purpose of estimating the efficiency of energy transport we assume an equatorial outflow over an angular extent of 0.4 radians (see Taam & Bodenheimer 1989). In this case, the convective velocity is ~ 0.2 times the local sound speed. For greater initial starting radii the orbital energy is efficiently transported away from the interaction region by convection, whereas for smaller starting radii hydrodynamic effects are indicated. The evolution is followed with a modified version of a code developed by Różyczka (1985). An implicit assumption underlying our study is that the description of the hydrodynamical flow is symmetric with respect to the rotation axis, Z . During the relevant phases of the evolution, where τ_c is much longer than the binary orbital period, this approximation is well justified. The computational grid (composed of 75×75 zones) extends out to a radius of 5×10^{12} cm, which includes more than 80% of the envelope mass. The inner boundary of the calculated grid lies at 2×10^{10} cm; the white-dwarf core is treated as a point mass. Typical grid resolution is 4×10^9 cm near the inner boundary and 3×10^{10} cm near the orbital position of the secondary.

The input physics is similar to that described in Paper III with several improvements. First, the gravitational potential of the secondary is calculated by means of a Legendre-polynomial expansion with up to 30 terms. Second, center-of-mass corrections were included in the determination of the orbit. Third, the energy dissipation rate and the torque exerted on the secondary, caused by the frictional interaction between the secondary and the common envelope, are calculated locally over its surface. Specifically, the energy dissipation rate per unit volume, ϵ , generated by the double core is expressed as

$$\epsilon = \frac{f\rho V_{\text{rel}}^3}{2\pi r} \quad (1)$$

where ρ , V_{rel} , and r are the local density, relative velocity between the orbital motion of the secondary and the common envelope, and the radius respectively. The factor f is a number of order unity and is obtained from interpolation of the results of Shima et al. (1985). The total dissipation rate is given by the volume integral of ϵ over an annulus with cross-sectional area determined by the accretion radius, R_a . Here, the accretion radius is given by (see Paper III)

$$R_a = H(H/R_0 + R_0/4H)^{-1} \quad (2)$$

where H is the local density scale height at the position of the secondary and R_0 is the generalized Bondi (1952) radius given as

$$R_0 = \frac{2GM}{V_{\text{rel}}^2 + C^2}, \quad (3)$$

with C denoting the local speed of sound in the common envelope. The torque per unit volume is similarly given by an expression of the form

$$j = \frac{f\rho V_{\text{rel}}^2}{2\pi}, \quad (4)$$

which when integrated over the volume of the annulus yields the angular momentum loss rate from the secondary's orbit.

At the initial time the secondary was already engulfed within the common envelope at a binary orbital period of 41 hr. After

some readjustment, the energy dissipation rate amounted to $\sim 10^7 L_\odot$ during which time the orbital decay time scale was typically 1–2 yr. As a consequence of the rapid deposition of this energy in the orbital plane, matter was driven outward in a fan-shaped pattern at supersonic velocities ($\sim 100 \text{ km s}^{-1}$) comparable to the escape speed at $5 \times 10^{12} \text{ cm}$. As found in the earlier investigation, complicated circulation patterns developed, however, the net transport of angular momentum away from the orbit by mass motion led to only a slight spin-up of the interior regions (with the envelope velocity ~ 0.3 times the orbital velocity). As the orbit decayed, mass was ejected from the common envelope at a rate of $\sim 1 M_\odot \text{ yr}^{-1}$. As a consequence, the density within the common envelope at the position of the secondary decreased despite the shrinkage of the orbit. Thus, the energy dissipation rate decreased and the orbital decay time scale increased.

Toward the later phases of the common envelope stage, the frictional luminosity generated at the expense of the orbital motion declined by a factor of 1000 to $10^4 L_\odot$. This resulted not only from decline in the density at the position of the secondary, but also from the reduction in the relative velocity between the secondary and the common envelope. The velocity field and density distribution at an evolution time of 0.7 yr is illustrated in Figure 2. The circulation pattern is smaller in size than in the earlier phases because of the low density of matter flowing from the polar directions. This follows from the fact that the reduction in density (due to the continuous loss of matter) leads to a lower energy dissipation rate and, hence, to a lower outflow velocity. Since the velocities are $\sim 20 \text{ km s}^{-1}$, the remaining mass is not lost from the common envelope.

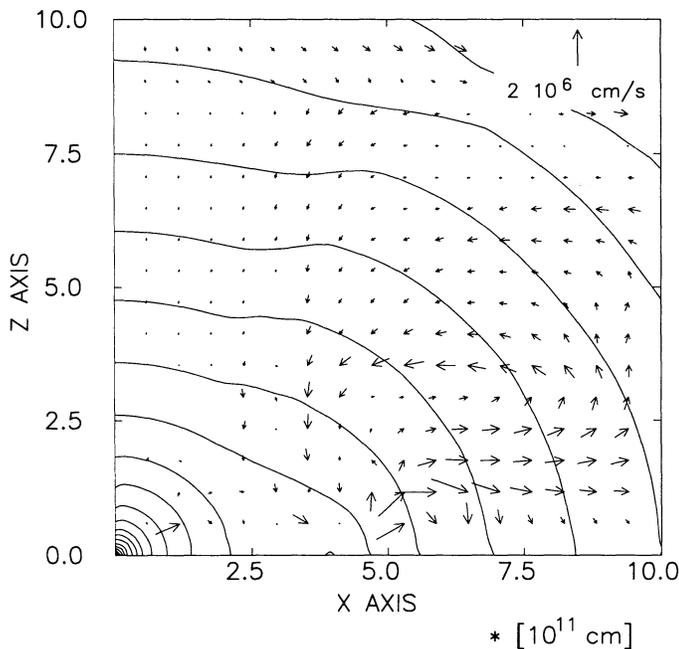


FIG. 2.—The velocity field and density distribution at time 0.7 yr within the inner 10^{12} cm of the common envelope. The maximum velocity in the grid is $2 \times 10^6 \text{ cm s}^{-1}$, and the density contours are logarithmically spaced from $\log \rho = -6.3$ to $\log \rho = 0.35$. Note the circulating flow pattern in the vicinity of the companion star, here located at $4 \times 10^{11} \text{ cm}$. This pattern results from the low dissipation rates generated in the common envelope. As a consequence of this flow angular momentum is not removed from the orbit location very efficiently. Hence, the relative velocity between the companion star and the common envelope decreases accentuating the decline in the rate of energy dissipation.

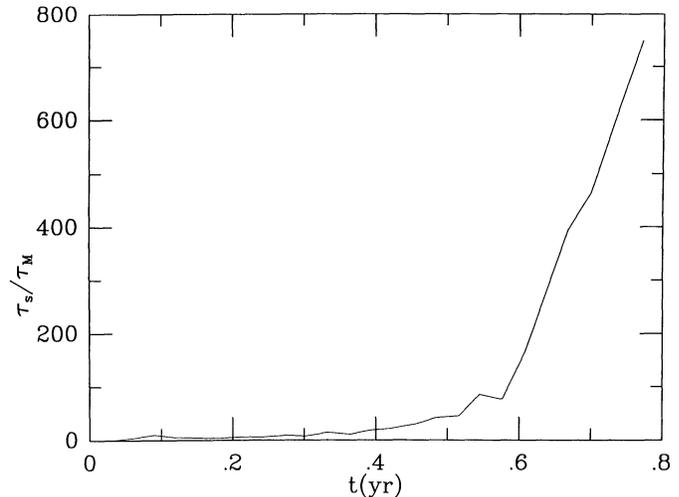


FIG. 3.—The ratio of the spiral time scale, τ_s , to the mass loss time scale, τ_M , for matter external to the white dwarf core of the red giant is illustrated as a function of evolution time for the common envelope configuration consisting of a $2 M_\odot$ red giant and a $1 M_\odot$ main-sequence star. Note that after a time of $\sim 0.6 \text{ yr}$ this ratio dramatically increases primarily reflecting the reduction in the frictional luminosity generated in the double core.

Instead, on account of the lower pressures along the polar axis as compared to the pressures along the equatorial axis the matter just circulates back to the core regions. Because of this circulation the angular momentum is not advected outward as efficiently as in the earlier phases of the evolution. Consequently, the specific angular momentum of the matter is higher in the vicinity of the secondary than in earlier phases resulting in a spin up of the common envelope (to within 16% of corotation) and to a concomitant reduction in the relative velocity between the secondary and the common envelope. Because the energy dissipation rate depends on the third power of the relative velocity (see eq. [1]), the energy dissipation rate is significantly reduced. This nonlinear feedback leads to a much smaller energy dissipation rate in comparison to a frictional luminosity which only depends on the local density within the common envelope.

As a result of the lower energy dissipation rate the orbital evolution slows appreciably with the decay time scale increasing to greater than 160 yr. The mass loss from the common envelope also decreases as the rate of mass loss declines to $\sim 0.1 M_\odot \text{ yr}^{-1}$ or less. However, because of the nearly complete ejection of mass from the common envelope the timescale for removal of the remaining mass has not changed significantly during the latter 0.25 yr of the evolution, increasing from 0.1 to 0.2 yr. These results are summarized graphically in Figure 3 where the ratio of the spiral decay time scale, τ_s , to the mass-loss time scale of matter above the white dwarf core, τ_M , is plotted as a function of evolution time. It can be seen that τ_s/τ_M increases with time and that the increase is dramatic for times greater than 0.6 yr. By the end of the calculation τ_s/τ_M exceeds 700 and all the mass exterior to the orbit has been ejected in an equatorial outflow, leaving a remaining $0.01 M_\odot$ in the common envelope.

The final orbital period of the binary system is 29.2 hr, and it is unlikely that the orbital parameters will change during its emergence from the common envelope stage due to the small mass remaining in the envelope. Because the time scale for orbital decay has increased by more than two orders of magnitude the inner regions interior to the orbit are able to respond

thermally since the thermal time scales are now comparable to the orbital decay time scale. In fact, it is expected that the matter interior to the orbit will contract to the white dwarf core region on a time scale of ~ 100 – 1000 yr. Consequently, the local density in the neighborhood of the secondary is expected to decrease even faster than calculated above. This contraction results from the fact that the inner layers are unable to continue to produce the nuclear energy at the rate required to maintain the configuration in its distended state since there is so little mass above the nuclear burning shells. As a consequence, matter in the common envelope will recede from the secondary star much in the same manner as the surface of a red giant recedes from its Roche lobe when its envelope mass is exhausted (see Kippenhahn, Kohl, & Weigert 1967; Taam 1983). Hence, the common envelope phase will terminate rather mildly.

3. DISCUSSION

We have demonstrated by detailed numerical calculation that a common envelope configuration consisting of a $2 M_{\odot}$ asymptotic red giant and a $1 M_{\odot}$ main-sequence-like star can evolve into a short-period binary system. Sufficient energy can be extracted from the orbit to eject all the matter in a common envelope without requiring the secondary to spiral to the white dwarf core of the red giant. The final phase of evolution is a continuation of the previous phase in which the rate of orbital decay increases. A significant finding in the present study is that the orbital decay is decelerated rapidly by the reduction in density accompanying mass loss and by the spin-up of the common envelope. Rotation of the envelope, although unimportant during the early phases, is very important during the late phases. The envelope does not come into a state of corotation, but the energy dissipation rate is reduced sufficiently to significantly slow the rate of orbital decay to prevent a merger. The termination of the common envelope stage results from the continuous loss of mass from above the white dwarf core.

These results differ, in detail from earlier investigations in which it was found that spin-up could be important in the evolution of the common envelope. Specifically, the one-dimensional study by Taam, Bodenheimer, & Ostriker (1978) found that rotational effects could be important when the specific angular momentum was assumed to be uniform throughout the common envelope. However, the study by Taam (1979) showed that the rotational effects were unimportant if the common envelope rotated rigidly. Yet, the fact that hydrodynamic expansion and ejection occurred for very evolved configurations in both cases indicated that the outcome, in this approximation, depends more sensitively on the energy dissipation rather than on the angular momentum distribution in the common envelope. Similarly, in the more recent study by

Livio & Soker (1988) and Bond & Livio (1990) following the earlier work by de Kool (1987), the spin-up response of the common envelope was estimated by means of general arguments based upon simplifying assumptions regarding the redistribution of angular momentum in the common envelope with no regard of the detailed angular momentum transport by advection associated with fluid flow. Implicit in their analysis is that the angular momentum is not transported outward toward the surface of the common envelope, but only inward of the binary orbit. These very rough estimates based upon rather restrictive approximations provided an indication of a possible tendency of the effect of rotation in the common envelope, but did not convincingly demonstrate that a nearly corotating configuration could be achieved. Consequently, the final phase, in which the envelope and the core region interior to the orbit evolve independently, could not be estimated.

A key requirement which facilitates the termination of the common envelope phase for red giant primaries is the presence of a very steep density gradient immediately exterior to the nuclear burning shells of a red giant star. We note that this tendency for more evolved configurations reiterates the conclusions of Taam, Bodenheimer, & Ostriker (1978), Taam (1979), Livio & Soker (1988), and Bond & Livio (1990) from a different viewpoint. If one makes use of this property then the results obtained in the present study can be generalized to include stars of other masses. Specifically, for more massive asymptotic red giant stars, the occurrence of steep density gradients occurs at more advanced evolutionary stages where the C/O white dwarf is more massive. For example, for a $5 M_{\odot}$ asymptotic red giant, such a density structure occurs for white dwarf cores greater than $\sim 1 M_{\odot}$. Similarly, for stars ascending the red giant branch for the first time such density contrasts occur for helium white dwarf cores more massive than $\sim 0.35 M_{\odot}$. Such advanced evolutionary stages are also favored for the successful ejection of the entire common envelope by lower mass secondary components since the binding energy of the common envelope is correspondingly smaller.

Future work should be directed in determining the generality of our results and to determine whether other physical processes operate to facilitate mass ejection for other binary systems where the structure of the more massive star is not highly centrally condensed.

This work was supported in part by the National Science Foundation under grant AST-8608291. We thank M. Różyczka for the use of his two-dimensional hydrocode, J. P. Ostriker for his stimulating suggestions, and M. Hjellming for discussions and for construction of the initial red giant model. R. E. T. thanks the director of Lick Observatory for the hospitality that was extended to him during his sabbatical visit.

REFERENCES

- Bailyn, C. D., & Grindlay, J. E. 1987, *ApJ*, 316, L25
 Bodenheimer, P., & Taam, R. E. 1984, *ApJ*, 280, 771
 ———. 1986, in *The Evolution of Galactic X-Ray Binaries*, ed. J. Truemper, W. H. G. Lewin, & W. Brinkmann (Dordrecht: Reidel), p. 207
 Bond, H. E. 1985, in *Cataclysmic Variables and Low Mass X-Ray Binaries*, ed. D. Q. Lamb & J. Patterson (Dordrecht: Reidel), p. 15
 ———. 1989, in *IAU Symp. 131, Planetary Nebulae*, ed. S. Tores-Peimbert (Dordrecht: Reidel), p. 251
 Bond, H. E., & Livio, M. 1990, *ApJ*, 355, 568
 Bondi, H. 1952, *MNRAS*, 112, 195
 de Kool, M. 1987, Ph.D. thesis, University of Amsterdam.
 Delgado, A. J. 1980, *A&A*, 87, 343
 Iben, I. Jr., & Tutukov, A. V. 1984, *ApJS*, 54, 335
 Kippenhahn, R., Kohl, K., & Weigert, A. 1967, *Zs. Ap.*, 66, 58
 Livio, M. 1989, *Space Sci. Rev.*, 50, 299
 Livio, M., & Soker, N. 1984, *MNRAS*, 208, 763
 ———. 1988, *ApJ*, 329, 764
 Meyer, F., & Meyer-Hofmeister, E. 1979, *A&A*, 78, 167
 Ostriker, J. P. 1975, talk presented at IAU Symposium 73, *The Structure and Evolution of Close Binary Systems*
 Paczyński, B. 1976, *IAU Symp. 73, The Structure and Evolution of Close Binary Systems*, ed. P. Eggleton, S. Mitton, & J. Whelan (Dordrecht: Reidel), p. 75
 Różyczka, M. 1985, *A&A*, 143, 59
 Shima, E., Matsuda, T., Takeda, H., & Sawada, K. 1985, *MNRAS*, 217, 367
 Taam, R. E. 1979, *Ap. Letters*, 20, 29
 ———. 1983, *ApJ*, 270, 694
 ———. 1988, in *Critical Observations vs. Physical Models For Close Binary Systems*, ed. K. C. Leung, (New York: Gordon & Breach), p. 365
 ———. 1989, *Highlights Astr.*, 8, 155
 Taam, R. E., & Bodenheimer, P. 1989, *ApJ*, 337, 849 (Paper III)
 Taam, R. E., Bodenheimer, P., & Ostriker, J. P. 1978, *ApJ*, 222, 269
 van den Heuvel, E. P. J., & Taam, R. E. 1984, *Nature*, 309, 235
 Verbunt, F. 1987, *ApJ*, 312, L23