THE ASTROPHYSICAL JOURNAL, 372:L33–L36, 1991 May 1


# A SIMPLE ACCRETION/DIFFUSION MODEL FOR $\lambda$ BOOTIS STARS

PAUL CHARBONNEAU[1]
High Altitude Observatory, National Center for Atmospheric Research,[2] P.O. Box 3000, Boulder, CO 80307-3000
*Received 1991 January 28; accepted 1991 February 13*

## ABSTRACT

It has recently been suggested that the peculiar abundance patterns observed in the $\lambda$ Bootis stars could be understood in terms of accretion of gas previously depleted in metals by means of grain formation in the interstellar medium. A simple analytical model is presented, describing the evolution of elemental abundances in these stars, under the combined influence of accretion and chemical separation. The only arbitrary parameter involved is the accretion rate. A rate of order $10^{-13}$ $M_\odot$ yr$^{-1}$ is found to naturally reproduce many peculiar characteristics of $\lambda$ Bootis stars, in particular their restriction to the spectral type range A0–F0. This lends quantitative support to the accretion hypothesis as a key component toward an understanding of the $\lambda$ Bootis phenomenon.

*Subject headings:* diffusion — stars: abundances — stars: accretion — stars: peculiar A

## 1. INTRODUCTION

The $\lambda$ Bootis stars (Morgan, Keenan, & Kellman 1943) are a class of chemically peculiar A stars characterized by moderate underabundances of many metals. They are Population I objects (Baschek & Searle 1969, and references therein) and show no observable magnetic fields (Bohlender & Landstreet 1990). Possibly because of their relative paucity—to this date, hardly more than a dozen stars are classified as $\lambda$ Bootis stars—these have often been overlooked by observers and theoreticians studying chemically peculiar stars. However, recent years have witnessed a renewed interest in these unusual stars, yielding a better defined picture of their basic characteristics (Hauck & Slettebak 1983; Abt 1984; Baschek et al. 1984; Baschek & Slettebak 1988; Gray 1988; Venn & Lambert 1990). In particular, the careful spectroscopic work of Gray (1988) has shown that (1) $\lambda$ Bootis stars are unevolved objects, often lying very close to the zero-age main sequence (ZAMS); (2) When hydrogen line profiles are used to estimate effective temperatures, all $\lambda$ Bootis stars are found to lie in a restricted spectral type interval, namely between A0 and F0; (3) Their observed $v \sin i$ values rarely exceed 100 km s$^{-1}$, although this may be because of observational selection effects. Yet there does not seem to be any slowly rotating ($v \sin i \lesssim 50$ km s$^{-1}$) $\lambda$ Bootis star.

It is also becoming apparent that a significant fraction of $\lambda$ Bootis stars show evidence for the presence of circumstellar dust and/or gas, as inferred from infrared excesses (see Sadakane & Nishida 1986; also the discussion in § 4 of Venn & Lambert 1990) or absorption in the core of the Ca II K line (observed in one star, HD 110411/HR 4881; see Table 1 of Gray 1988).

From the standpoint of chemical abundances, $\lambda$ Bootis stars have been traditionally characterized by normal or slightly overabundant light metals (C, N, and O), underabundances by factors of order 3 in iron-group elements, and somewhat larger underabundances in Mg and Ca (Baschek & Slettebak 1988, and references therein). This view has recently been challenged by Venn & Lambert (1990), who found much larger underabundances (by factors of up to 50–100) for many metals in the three $\lambda$ Bootis stars that they observed. Whether or not these larger underabundances are typical of all $\lambda$ Bootis stars, such abundance patterns are strikingly different from those found in the more common FmAm stars, another class of nonmagnetic chemically peculiar stars populating the same region of the H-R diagram (see, e.g., Preston 1974).

Motivated by the success of the diffusion model for FmAm stars (Michaud et al. 1983; see also Charbonneau & Michaud 1991, hereafter CM91), Michaud & Charland (1986) proposed a diffusion model for $\lambda$ Bootis stars, with higher mass loss being the factor leading to abundance patterns differing from FmAm stars. By assuming a mass-loss rate of order $10^{-13}$ $M_\odot$ yr$^{-1}$, Michaud & Charland showed that underabundances by factors of order 2–5 for most metals do materialize after $\simeq 10^9$ yr. This model has come under criticism on the grounds that rotationally induced meridional circulation should prevent chemical separation in the more rapidly rotating $\lambda$ Bootis stars (Baschek & Slettebak 1988), and that for A stars, the $\lambda$ Bootis "signature" would become observable only at the end of the main-sequence lifetime (Gray 1988). Although CM91 have shown that chemical separation in general remains possible for many elements, even for equatorial rotational velocities ($v_e$) as high as 200 km s$^{-1}$ (see their Figs. 6 and 7), further calculations of chemical separation in the presence of mass loss *and* meridional circulation suggest that there exists no value of a weak mass-loss rate that leads to the development of generalized *under*abundances at any stage of main-sequence evolution, even for $v_e$ as low as 75 km s$^{-1}$ (Charbonneau 1991). This raises doubts as to the validity of the mass-loss/diffusion model for $\lambda$ Bootis stars.

Noting the high degree of similarity between abundance patterns in interstellar matter and in $\lambda$ Bootis stars, Venn & Lambert (1990) have suggested that the latter may simply be main-sequence stars having accreted gas previously depleted in metals following grain formation in the interstellar medium. Although their model accounts naturally for the peculiar abundance patterns of $\lambda$ Bootis stars, it leaves some important questions unanswered: why is the $\lambda$ Bootis phenomenon restricted

[1] Postdoctoral Fellow of the National Science and Engineering Research Council of Canada.

[2] The National Center for Atmospheric Research is sponsored by the National Science Foundation.

to a few spectral types between A0 and F0? Why is it apparently restricted to a relatively narrow $v \sin i$ interval? Are the observed star-to-star variations in abundances compatible with the model? What is the range of acceptable accretion rates? Although Venn & Lambert tentatively address some of these points in a qualitative fashion, a quantitative investigation is relevant at this juncture.

The aim of this Letter is to provide a quantitative basis to the Venn & Lambert model, suitably modified to include the effects of chemical separation. The required accretion rate is estimated, along with the effective temperature and $v \sin i$ intervals in which the $\lambda$ Bootis phenomenon is expected to occur.

## 2. THE DIFFUSION/ACCRETION MODEL

The three basic working hypotheses of the present accretion/diffusion model are the following: (1) Convective mixing and overshooting into the atmosphere ensure that the portion of the envelope extending from the surface down to the bottom of the superficial convection zone (SCZ) is thoroughly mixed, implying that observed abundances are representative of abundances in the SCZ. Mixing is considered instantaneous, as the associated time scales are much smaller than those relevant to all other transport processes considered (see Schatzman 1969). (2) There exists an accretion mechanism such that (depleted) interstellar gas, but not grains, is accreted onto the stellar surface. (3) The radiative envelope is stable enough to allow chemical separation to occur *below* the SCZ.

The time evolution of observed abundances is then driven by accretion at the surface and concomitant mixing within the SCZ, *and* by particle transport at its base. The latter is also influenced by accretion; in the limit of small accretion rates, the perturbation of local hydrostatic equilibrium generates an inward-directed global drift velocity ($v_A$), whose magnitude is obtained by the requirement of mass conservation. Assuming spherical symmetry, one readily finds

$$v_A(r) = \frac{\dot{M}}{4\pi r^2 \rho}\,, \tag{1}$$

where $\dot{M}$ is the accretion rate, and other symbols have their usual meaning. This is a global drift velocity, affecting all chemical species in an identical fashion. When investigating the abundance evolution of a given element, one must add to it a second component ($v_D$), including contributions from gravitational settling, differential thermal diffusion, diffusion driven by differential radiative acceleration, and so on.

The resulting transport problem for a trace element usually translates into a two-dimensional initial-boundary value problem described by a linear second-order partial differential equation, whose solution often poses serious numerical difficulties (see, e.g., CM91). Consider, however, a situation where meridional circulation is neglected, no large concentration gradient exists at the base of the SCZ, and the accretion rate is large enough so that $v_A \geq v_D$ at its base. The SCZ then behaves as a homogeneous reservoir being drained from below and replenished from above at a constant rate (which is a function of both the accretion rate and elemental abundances of the accreted material). One can then write a single ordinary differential equation governing the time evolution within the SCZ of a trace element's concentration ($c$) as

$$\frac{dc(t)}{dt} = A_2 c_{\rm ISM} - A_1 c(t)\,, \tag{2}$$

with

$$A_1 = \frac{4\pi r^2 \rho}{M_{\rm CZ}}(v_A - v_D)\,, \quad A_2 = \frac{\dot{M}}{M_{\rm CZ}} \tag{3}$$

where $M_{\rm CZ}$ is the mass in the convection zone, $c_{\rm ISM}$ the element's concentration in the accreted material, and $v_A$, $v_D$, and $\dot{M}$ are all positive quantities. Such an expression cannot be written if $v_D > v_A$, since it is then impossible to specify a priori at all times the contaminant flux entering the SCZ from below, although approximate treatments remain possible under specific circumstances (see Michaud & Charland 1986, § 3). Integration of equation (2) readily yields the following solution:

$$c(t) = \left[\left(c_0 - \frac{A_2 c_{\rm ISM}}{A_1}\right)\exp(-A_1 t)\right] + \frac{A_2 c_{\rm ISM}}{A_1}\,, \tag{4}$$

where $c_0$ is the initial concentration in the SCZ. It is immediately apparent from equation (4) that $c$ tends exponentially to a steady state value $c_{\rm (ss)} = A_2 c_{\rm ISM}/A_1$, on a time scale

$$\tau = A_1^{-1}\,, \tag{5}$$

and that in the limit $v_A \gg v_D$, one has $c_{\rm (ss)} \rightarrow c_{\rm ISM}$ and $\tau \rightarrow M_{\rm CZ}/\dot{M}$. At low effective temperatures ($T_{\rm eff} \leq 7500$ K, say), $M_{\rm CZ}$ increases rapidly with decreasing effective temperature, so that $\tau$ generally increases with decreasing $T_{\rm eff}$. At higher values of $T_{\rm eff}$, $M_{\rm CZ}$ exhibits a much weaker dependence on surface temperature. On the other hand, because of the global increase of radiative acceleration with effective temperature (see, e.g., Michaud et al. 1978), for a given accretion rate the ratio $v_D/v_A$ will generally tend to increase with increasing $T_{\rm eff}$, leading to a concomitant increase in $\tau$ in the high $T_{\rm eff}$ limit. It then follows that, for a given $\dot{M}$, there exists a $T_{\rm eff}$ at which $\tau$ reaches a minimum. This defines an effective temperature domain in which abundance anomalies triggered by accretion are most likely to appear.

This is illustrated on Figure 1, where variations of $\tau$ with effective temperature are shown for a few chemical elements. Curves are coded according to accretion rates, in $M_\odot$ yr$^{-1}$ (see caption for further details). For titanium in a 8000 K model, the condition $v_A > v_D$ translates into the requirement that $\dot{M} \gtrsim 2 \times 10^{-14}$ $M_\odot$ yr$^{-1}$; smaller accretion rates obviously cannot be ruled out, but the simple model developed here is no longer valid in these cases. Note the broad, shallow minima in $\tau$ for most elements, as well as the rapid increase in $\tau$ on either side of these minima. The fact that time scales for calcium do not increase in the high $T_{\rm eff}$ range can be traced to variations in radiative acceleration (calcium, being in a rare gas configuration under the SCZ for the hotter models, is not supported). For all elements and accretion rates considered here, curves are nearly identical at low values of $T_{\rm eff}$, but they can vary greatly from element to element at high values of $T_{\rm eff}$. Note also the sensitive dependence of the minima's widths on accretion rate; for $\dot{M} \lesssim 10^{-14}$ $M_\odot$ yr$^{-1}$, underabundances simply cannot materialize for most elements, while for accretion rates larger than about $5 \times 10^{-13}$ $M_\odot$ yr$^{-1}$, time scales remain extremely short up to $T_{\rm eff} \simeq 20{,}000$ K for all but a few elements. An increase in $\alpha$ (ratio of mixing length to pressure scale height) would shift all curves toward higher effective temperature, the displacement being more pronounced at low values of $T_{\rm eff}$: for titanium, the shift is by 400 K at 7000 K, as compared to $\simeq 100$ K at 10,000 K, when using $\alpha = 2.0$ instead of $\alpha = 1.4$. Uncertainties on $g_{\rm R}$ affect mostly the point at which $\tau$ starts increasing again at high $T_{\rm eff}$; arbitrarily increasing $g_{\rm R}$ would shift to

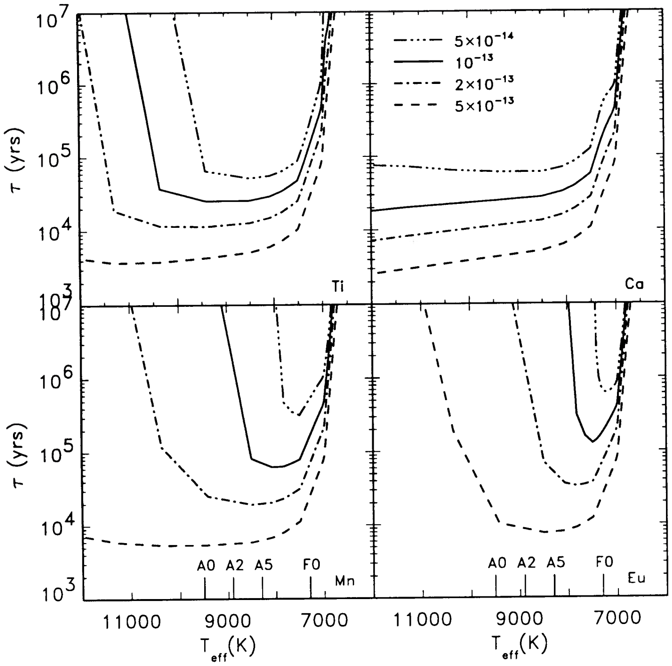


FIG. 1.—Abundance variation time scales within the accretion/diffusion model vs. effective temperature for a sample of elements. Calculations were carried out in a sequence of main-sequence envelopes having log $g = 4.3$, solar abundances, and $\alpha = 1.4$ (ratio of mixing length to pressure scale height). Curves are coded according to accretion rates, in $M_\odot$ yr$^{-1}$. Radiative accelerations are taken from CM91. Note the broad but flat minima in $\tau$ and the overall sensitive dependence on accretion rate.

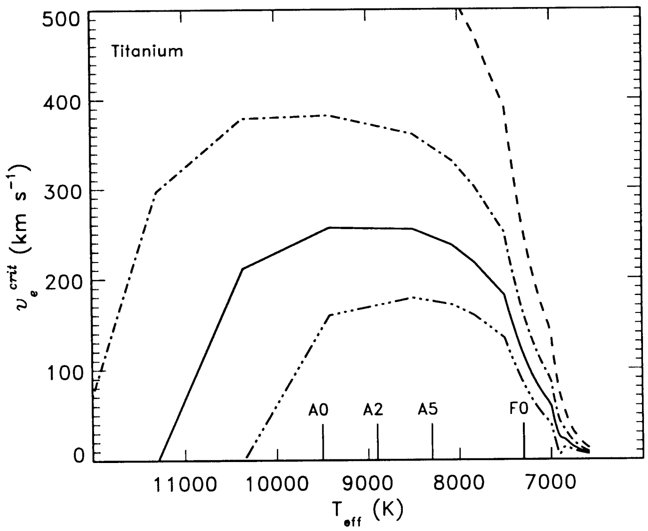


FIG. 2.—Critical equatorial rotational velocity vs. effective temperature. For equatorial rotational velocities larger than this critical velocity, meridional circulation is expected to " win " over accretion at the base of the SCZ and thus reduce the predicted abundance anomalies. This can thus be (loosely) interpreted as an absolute $v$ sin $i$ upper limit to the $\lambda$ Bootis phenomenon. Curves are again coded according to accretion rate (see Fig. 1).

lower effective temperature the hot end of the $\tau$ versus $T_{\rm eff}$ curves, with the opposite behavior if $g_{\rm R}$ is reduced instead.

While uncertainties on $\alpha$ and $g_{\rm R}$ are likely to influence, for a given chemical element, the boundaries of the $T_{\rm eff}$ range in which $\tau$ is sufficiently small to allow the appearance of accretion-triggered abundance anomalies, it remains clear that the observational restriction of the $\lambda$ Bootis phenomenon to the spectral type range A0–F0 translates into an extremely tight constraint on allowed accretion rates within the simple model presented here.

The above discussion has altogether neglected the effects of rotation. In particular, the contribution of rotationally induced meridional circulation to particle transport at the base of the SCZ has not been taken into account. This will become important when the outward flux of (undepleted) matter in the polar regions becomes comparable to the accretion-induced inward flux at the base of the SCZ. As an order-of-magnitude estimate, one may compute a critical equatorial rotational velocity ($v_e^{\rm crit}$) for which the total velocity (diffusion + accretion + meridional circulation, the latter being taken from Tassoul & Tassoul 1982) in polar regions vanishes at the base of the SCZ. This is shown on Figure 2 for titanium. The coding of curves in terms of accretion rates is identical to that used on Figure 1. The decrease in $v_e^{\rm crit}$ at high $T_{\rm eff}$ can be traced to the increasing contribution of radiative acceleration, which can oppose the accretion-induced flow in a more and more efficient manner as $T_{\rm eff}$ increases (see Fig. 1). For accretion rates of order 1–2 $\times$ $10^{-13}$ $M_\odot$ yr$^{-1}$, the computed critical velocities are significantly larger (by factors of 2–3) than typical $v$ sin $i$ values for $\lambda$ Bootis stars. Nevertheless, the order-of-magnitude agreement certainly justifies performing detailed calculations of chemical separation in the presence of accretion and meridional circulation (currently in progress). Accretion itself may also be hampered by fast rotation, although it is not possible at this point to make even an order-of-magnitude estimate of this effect.

### 3. DISCUSSION

The greatest uncertainty associated with the accretion hypothesis of Venn & Lambert (1900) lies with the accretion mechanism itself. As these authors already pointed out, it is critical that (depleted) interstellar gas be accreted without concomitant accretion of grains; such a behavior is not *a priori* compatible with current models of grain formation and accretion in protostellar nebulae. Further work along these lines is clearly required. A truly convincing accretion-based explanation of the $\lambda$ Bootis phenomenon should include a detailed accretion model, allowing the evaluation of the gas-to-grain accretion efficiency ratio, as well as its dependence on luminosity and rotation. Note, however, that the accretion rates required by the present model are extremely small. A star accreting at a rate of $10^{-13}$ $M_\odot$ yr$^{-1}$ over a time interval of $10^8$ yr would accrete a total mass corresponding to 1% of Jupiter's mass.

Nevertheless, the analytical accretion-diffusion model described above allows one to quantify some important aspects of the hypothesis put forth by Venn & Lambert (1990) regarding the nature of $\lambda$ Bootis stars and to address some of the questions raised by these authors. Within the context of the simple model developed here, the required accretion rate is of order $10^{-13}$ $M_\odot$ yr$^{-1}$. Time scales for the development of underabundances become too high and restricted to too narrow a $T_{\rm eff}$ range if $\dot{M} \lesssim 5 \times 10^{-14}$. On the other hand, if $\dot{M}$ exceeds $5 \times 10^{-13}$ $M_\odot$ yr$^{-1}$, accretion would lead to the appearance of the $\lambda$ Bootis phenomenon in stars of too early spectral types. This sensitive dependence of the predicted $T_{\rm eff}$ range for the $\lambda$ Bootis phenomenon—in particular its upper

$T_{\rm eff}$ limit—appears problematic, although difficulties in accreting gas on a highly luminous star and/or decrease of the grain-to-gas accretion efficiency ratio with increasing luminosity may also contribute to preventing the appearance of the $\lambda$ Bootis phenomenon in stars earlier than A0. It is not currently possible to quantify this effect in the absence of a suitable model for the accretion process per se.

It is not necessary to arbitrarily constrain the accretion phase to stars of spectral types A0–F0. Although the postulated accretion phase may well occur in main-sequence stars of all effective temperatures, the restriction of the $\lambda$ Bootis phenomenon to a few spectral types centered around A2 arises as a consequence of the increase in convection zones masses at later spectral types, and of the general increases in $g_{\rm R}$ at the base of superficial convection zones at earlier spectral types. Even if abundances in accreted material are nearly identical in all cases, significant star-to-star variations in abundances are expected (and observed; see, e.g., Baschek & Slettebak 1988; Venn & Lambert 1990), because of the sensitive dependence of $g_{\rm R}$ at the base of SCZ on $T_{\rm eff}$.

The observed paucity of $\lambda$ Bootis stars follows naturally if one assumes that the accretion phase is either rare, short-lived, and/or limited to a specific phase of main-sequence evolution. The observations and analysis of Gray (1988) would lend support to the (speculative) idea that the $\lambda$ Bootis phenomenon be identified with a late accretion phase occurring in a fraction of stars at some point following their arrival on the ZAMS.

The reason behind the apparent restriction of the $\lambda$ Bootis phenomenon to a relatively narrow $v \sin i$ interval is more uncertain. It must first be emphasized that the lack of rapidly rotating $\lambda$ Bootis stars may well be an observational selection effect due to the inherent difficulty of measuring weak lines in a rapidly rotating star (R. O. Gray, private communication). While meridional circulation can be expected to homogenize SCZ abundances from below in the more rapidly rotating stars, the influence of stellar rotation on accretion efficiency may also be important. The lack of slowly rotating $\lambda$ Bootis stars can be explained by the fact that slowly rotating ZAMS A stars normally become FmAm stars after a few $10^6$ yr (see Michaud et al. 1983; CM91, § 3.1). In the absence of accretion, this occurs for stars with $v_e \lesssim 90$ km s$^{-1}$. However, the requirement that the He SCZ should be able to disappear through settling—the sine qua non condition for the appearance of the FmAm phenomenon—is likely to yield an upper limit to allowed accretion rates, as well as a lower limit on allowed rotational velocities for $\lambda$ Bootis stars. Detailed calculations of He settling in the presence of accretion and meridional circulation are needed to clarify this point.

Critical tests involving abundance determinations of specific elements have already been discussed by Venn & Lambert (1990, § 5). Additional tests can be inferred, based on the above model. Accretion-triggered underabundances should not materialize in stars later than F0. Star-to-star and element-to-element variations in abundances should be more pronounced among the hotter $\lambda$ Bootis stars than among the cooler ones. More detailed calculations of particle transport in the presence of accretion are required to identify more constraining observational tests.

To sum up, the simple accretion/diffusion model presented here lends support to the accretion hypothesis proposed by Venn & Lambert (1990) as an explanation of the $\lambda$ Bootis phenomenon. Once the existence of an accretion mechanism allowing accretion of gas but not grains is accepted, the only arbitrary parameter involved is the accretion rate; a value of $10^{-13}$ $M_\odot$ yr$^{-1}$ appears to account for most observational characteristics of $\lambda$ Bootis stars. A detailed modeling of the postulated accretion mechanism appears nevertheless essential, prior to reaching firmer conclusions as to the nature of the $\lambda$ Bootis phenomenon.

Stimulating discussions with T. J. Bogdan, K. B. MacGregor, and G. Michaud are gratefully acknowledged. Thanks are also due the referee, R. O. Gray, for many constructive comments and suggestions.

## REFERENCES

Abt, H. A. 1984, in The MK Process and Stellar Classification, ed. R. F. Garrison (Toronto: David Dunlap Observatory, University of Toronto), 340
Baschek, B., Heck, A., Jäschek, C., Jaschek, M., Köppen, J., Scholz, M., & Wehrse, R. 1984, A&A, 131, 378
Baschek, B., & Searle, L. 1969, ApJ, 155, 537
Baschek, B., & Slettebak, A. 1988, A&A, 207, 112
Bohlender, D. A., & Landstreet, J. D. 1990, MNRAS, 247, 606
Charbonneau, P. 1991, in preparation
Charbonneau, P., & Michaud, G. 1991, ApJ, 370, 693 (CM91)
Gray, R. O. 1988, AJ, 95, 220
Hauck, B., & Slettebak, A. 1983, A&A, 127, 231
Michaud, G., & Charland, Y. 1986, ApJ, 311, 326
Michaud, G., Charland, Y., Vauclair, S., & Vauclair, G. 1978, ApJ, 210, 447
Michaud, G., Tarasick, D., Charland, Y., & Pelletier, C. 1983, ApJ, 269, 239
Morgan, W. W., Keenan, P. C., & Kellman, E. 1943, An Atlas of Stellar Spectra (Chicago: University of Chicago Press)
Preston, G. W. 1974, ARA&A, 12, 257
Sadakane, K., & Nishida, M. 1986, PASP, 98, 685
Schatzman, E. 1969, A&A, 3, 331
Tassoul, J.-L., & Tassoul, M. 1982, ApJS, 49, 317
Venn, K. A., & Lambert, D. L. 1990, ApJ, 363, 234