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TWO DOUBLE-MODE RR LYRAE STARS IN THE FIELD

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ABSTRACT

Two new double-mode (RR*d*) field variables are identified in the Lick RR Lyrae survey field (RR VIII) in Draco. Only one field RR*d* star (AQ Leo) was previously known although some thirty are known in globular clusters. For variable RR VIII-58, $P_0 = 0^{d}.464718$ and $P_1 = 0^{d}.345389$; for variable RR VIII-10, $P_0 = 0^{d}.511844$ and $P_1 = 0^{d}.381275$. The mean B magnitudes of these stars are 14.9 and 16.5, respectively. The amplitudes of the two modes are comparable for RR VIII–58, but the amplitude of the primary (first-overtone) period is significantly greater in RR VIII-10. The mean $B-V$ colors (0.29 and 0.28) are intermediate between those found for RR Lyrae stars of type *c* and type *ab* in this field. The theoretical mass calibration of the P_1/P_0 versus P_0 (Petersen) diagram by Cox et al. was used to derive masses of 0.49, 0.58, and 0.66 $M_\odot$ for VIII-58, VIII-10 and AQ Leo, respectively. The star AQ Leo lies in an area of the Petersen diagram occupied by the RR*d* stars in Oosterhoff type II clusters, but the two new field RR*d* stars lie in areas that are not occupied by cluster RR*d* stars. [Fe/H] estimates of -1.90, -1.91, and -1.81, respectively were derived by the ΔS method using the calibration of Zinn & West. (The ΔS for AQ Leo was taken from Mendes de Oliveira & Smith). In all cases the abundance data were reduced with the method used for RR*c* type variables, and the implications of this are discussed. The difference between the Hβ and the mean of the Hγ and Hδ equivalent widths is also shown to be proportional to [Fe/H]; this relation is used to derive [Fe/H] values (of lower weight) of -1.4 and -1.7, respectively, for VIII-58 and VIII-10. These results give no evidence for a mass-metallicity relationship among these three RR*d* field stars, but, in the mean, they agree with the mass-metallicity relation given by the cluster RR*d* stars.

Subject headings: stars: pulsation — stars: RR Lyrae

1. INTRODUCTION

The study of double-mode RR Lyrae (RR*d*) stars is important because it provides information about the masses of horizontal branch stars. A number of RR*d* stars have been identified in globular clusters, and the masses calculated for them indicate a relationship between mass and metallicity. Sandage (1990) used the published data for M15, M3, and IC 4499, and the mass calibrations of Cox, Hodson, & Clancy (1983, hereafter CHC), to express this relationship in the form

$$\log M = -0.10[\mathrm{Fe/H}] - 0.41 . \qquad (1)$$

The masses determined for RR*d* stars in other clusters: M68 (Clement 1990), NGC 2419, and NGC 6426 (Clement & Nemec 1990) confirm this relationship. However, the evolutionary models of Sweigart, Renzini, & Tornambè (1987) and those of Lee, Demarque, & Zinn (1990) predict different relationships. It is necessary to know how the masses of RR Lyrae stars are related to metal abundance in order to determine the luminosity and to use these stars as reliable distance indicators. RR Lyrae stars in the field have a greater range of periods and metal abundances than those in globular clusters. Therefore, it is important to identify RR*d* stars in the field to test the mass-metal abundance relation. Up to the present, however, only one RRd star has been identified in the field. This is AQ Leo (Jerzykiewicz & Wenzel 1977). We present evidence here for the existence of two others.

2. THE PHOTOMETRIC OBSERVATIONS

The two variables were found in the course of the continuation of a survey for RR Lyrae stars with the Lick Astrograph (Paper VII by Kinman et al. 1990), using the multiple exposure technique described by Kinman (1965). They are located in Field RR VIII in Draco, and are designated VIII-10 and VIII-58, respectively. Their 1950.0 coordinates are $\alpha = 17^h05^m59^s.3$, $\delta = +55°31'25''$ (for VIII-10) and $\alpha = 17^h37^m36^s.1$, $\delta = +55°08'24''$ (for VIII-58). Finding charts are shown in Figure 1. These two variables were discovered on a series of plates, taken by the late C. A. Wirtanen between 1964 May and 1965 August. This plate series comprised 39 exposures of 30 minutes, on 103a-O emulsion, without filter. A second series of plates of this field was taken with the same telescope by E. A. Harlan between 1976 June and 1985 April; this consisted of 27 single exposures of 40 or 45 minutes on unfiltered IIa-O emulsion. The second series, which was generally of better photometric quality than the first, was taken so that reliable phases would be available for the interpretation of recent spectroscopic and photoelectric observations. The first series of plates was blinked by Wirtanen and Stryker, and amongst the 58

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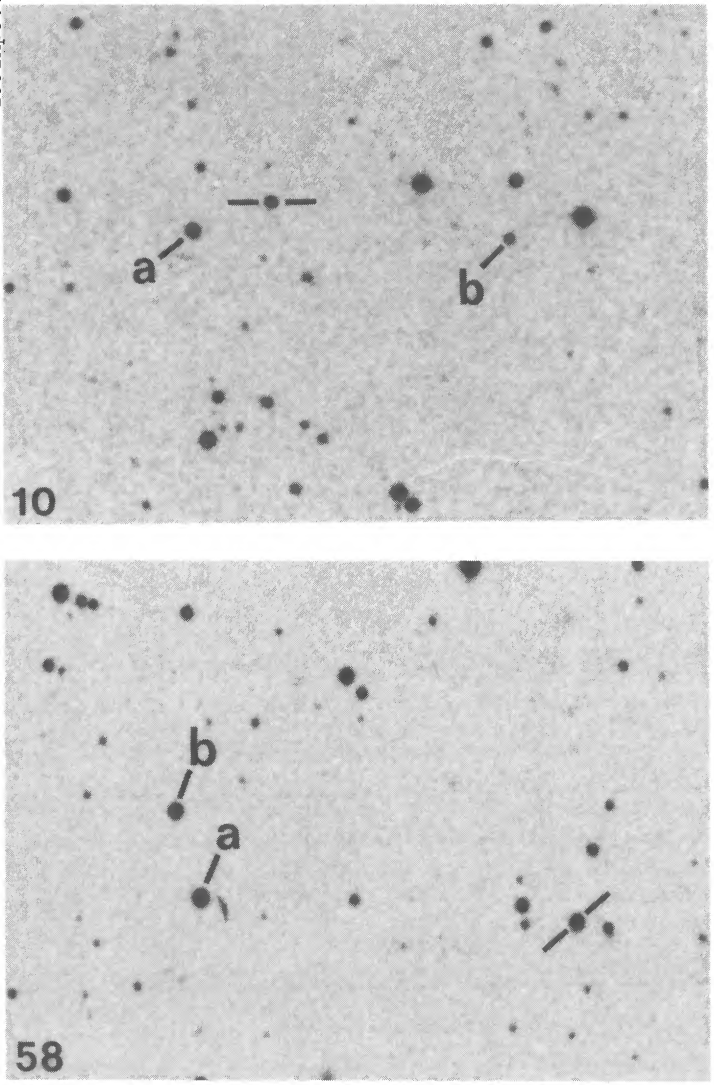

FIG. 1.—Finding charts for the two variables. The charts are 6′ × 8′, with North to the top and East to the left. They are taken from the Palomar Sky Survey (blue). The variables are marked between two lines and the comparison stars are marked a and b in each case. The finding charts are one above the other with 10 and 58 in the lower left corner to indicate VIII-10 and VIII-58. The adopted B magnitudes for the standard stars are: a (14.95), b (16.94) for VIII-10 and a (14.22), b (15.17) for VIII-58. Photoelectric observations suggest that these adopted magnitudes may be ~0.06 too faint.

variables which they found, periods have been determined for 19 RR Lyrae stars of type *ab* and eight of type *c*. These results, as well as all the magnitudes and the details of the data reduction, will be published in Paper VII (Kinman et al. 1990). The two stars, VIII-10 and VIII-58, appeared to be of short period, but satisfactory light curves were not found for them. The mean $B-V$ colors of 0.28 and 0.29 for VIII-10 and VIII-58 respectively, based on seven photoelectric observations of each star, are intermediate between the mean color of the type *c* variables (0.23 from 10 observations of six stars) and that of the type *ab* variables (0.34 from 85 observations of 19 stars) in this field. Since RR*d* stars occur at the transition between the first overtone and fundamental modes, these two stars were considered to be good candidates for double-mode pulsation.

3. THE PERIODS AND LIGHT CURVES

The observations from both sets of plates were combined for the analysis. A computer program based on Stellingwerf's (1978) phase-dispersion minimization (PDM) technique was used to determine the best periods. In each case, a (5, 2) bin structure was used, and mean light curves were derived using cubic-spline fitting functions. The procedures described by Nemec (1985a, b) and Stobie (1970) were followed for the prewhitening of the data. In the search for secondary periods, the interval over which the period searches were carried out was limited to values of P_0 for which the ratio P_1/P_0 was between 0.73 and 0.76, the range that is considered to be physically realistic (Fitch 1970, CHC, Nemec 1985b). For both stars, the primary mode of pulsation turned out to be the first overtone, but significant secondary oscillations were detected, with periods in the range expected for the fundamental mode. The θ-transforms (plots of Stellingwerf's 1978 θ-statistic versus period, in the vicinity of our adopted periods) are shown in Figure 2.

Figure 2*a* shows the θ-transforms for RR VIII-10. The top panel shows the result of the search of the original magnitudes to find the primary (first-overtone) period. The second panel shows the search of the residuals to the mean light curve to find the secondary (fundamental) period. The third panel shows the search of the original magnitudes, corrected using the mean secondary light curve, to find an improved value of the primary period, and the fourth panel shows the search of the residuals obtained from the original photometry and the improved mean primary light curve, to find an improved value of the secondary period. It is evident from Figure 2*a* that there is no significant change in either the primary or secondary period after the prewhitening, and that the minimum θ-values for both periods are lower after the other pulsation mode has been removed. This indicates that the periods we have found are accurate to within 10^{-5} days. In Figure 2*b*, the θ-transforms for RR VIII-58 are shown.

Light curves for the two stars are shown in Figure 3, and the characteristics of these curves are given in Table 1. The periods and epochs of maximum light for each mode of pulsation were determined from an analysis of the combined data, but in the light curves of Figure 3, the early and late data were plotted separately. The amplitudes listed in Table 1 were determined from mean light curves (derived using cubic-spline fitting functions) for the 1976–1985 data alone, and the σ-values are the standard deviations of the points about these mean light curves. In each panel of Figure 3, there are three curves. The upper curve is plotted with the measured magnitudes and the primary period (P), the middle curve is plotted with the residuals and the secondary period (P_0), and the bottom curve is a plot of the magnitudes with the revised primary period (P_1), after prewhitening with the secondary period. The 1976 observations were all made in an interval of one week at the end of June, and these are plotted as crosses on the curves. If the stars are true double-mode pulsators, these observations alone should give satisfactory light curves for both the fundamental and first overtone periods. The light curves for VIII-58 indicate that it is undoubtedly an RR*d* star. However, the curves for the fainter star, VIII-10, show more scatter. (Note the larger values of σ for A_0 in Table 1.) Also, the gap in phase, at about 0.5, with two unusually bright points on the secondary light curve plotted for the 1964–1965 observations is a matter of some concern. The secondary period for this star is

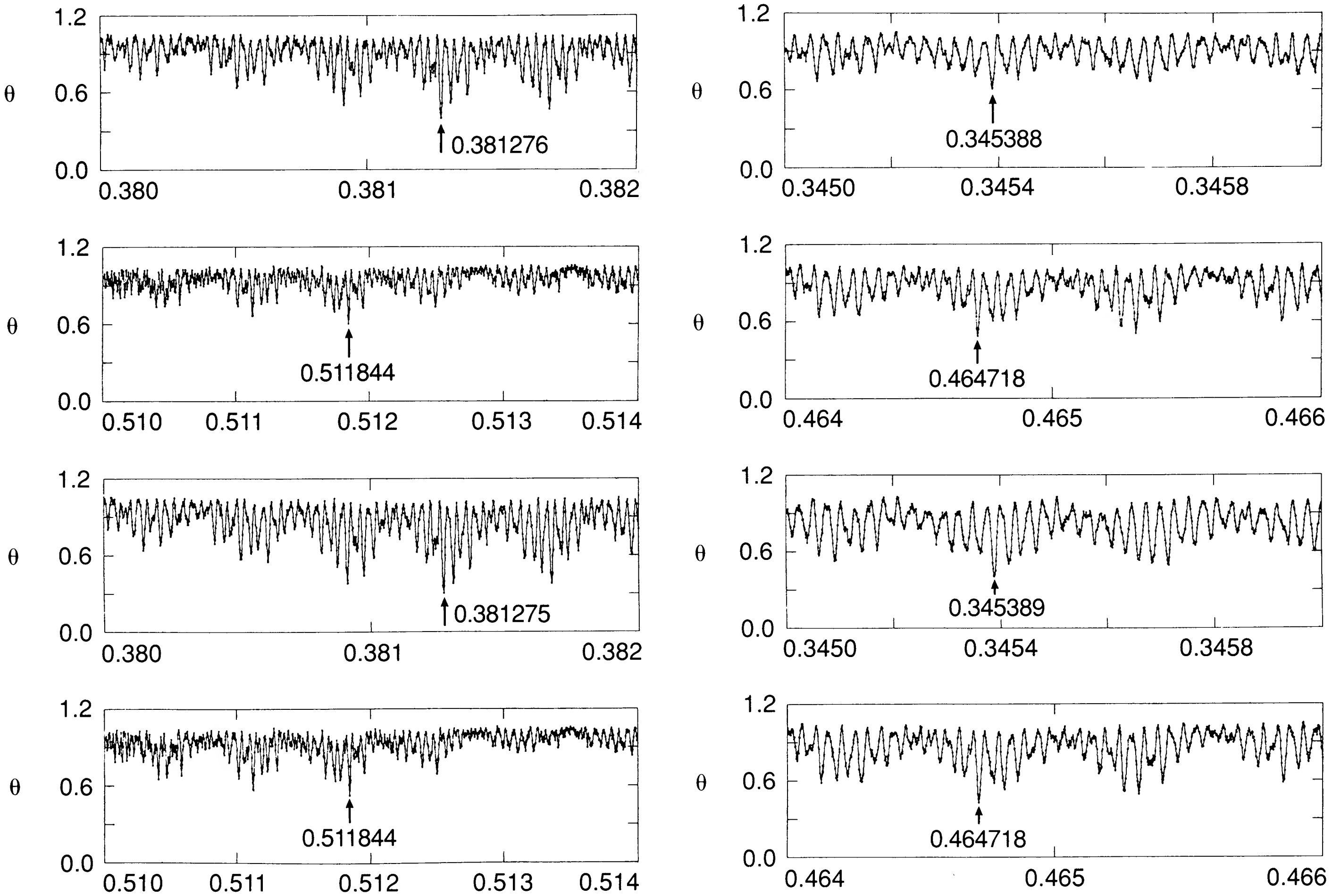

FIG. 2*a*

FIG. 2*b*

FIG. 2.—(*a*) θ transforms for VIII-10. (*top panel*) Period search of the original magnitudes. (*second panel*) Period search of the residuals derived from the mean primary light curve. (*third panel*) Period search of the original magnitudes after correcting them with the mean secondary light curve. (*bottom panel*) Period search of the residuals obtained from the original magnitudes and the improved mean primary light curve. (*b*) θ transforms for VIII-58. The sequence of curves is the same as in Fig. 2*a*.

close to half a day and could be spurious. Because of this, it would be worthwhile to make CCD observations of VIII-10 on a few consecutive nights to establish that it really is an RR*d* star.

4. THE SPECTROSCOPIC DATA

Spectroscopic observations of the two suspected RR*d* variables were made by Suntzeff, Kinman, & Kraft (1991, hereafter SKK), in their study of metal abundances of RR Lyrae variables at intermediate galactic latitudes. These observations were made in 1986 and 1987 with the KPNO 2.1 m telescope, and the basic spectral data are presented in Table 2. Column (1) lists the star, (2) lists the heliocentric Julian date at the time of the observation, (3) lists the integration time in minutes, (4) lists the phase on the primary light curve, (5)–(7) list the "raw" spectral types based on the strength of the Hβ, Hγ, and Hδ lines, and (8) lists the spectral type based on the strength of the Ca II K line at 3934 Å. These "raw" spectral types were

TABLE 1

CHARACTERISTICS OF THE LIGHT CURVES

Star	Period (days)	Epoch of maximum (JD − 2400000)	Amplitude (mag)	σ (mag)	P_1/P_0	A_1/A_0
VIII-10..........	$P = 0.381276$	42956.775	$A = 0.80$	0.13	0.7449	2.03
	$P_0 = 0.511844$	42957.934	$A_0 = 0.34$	0.09		
	$P_1 = 0.381275$	42956.778	$A_1 = 0.69$	0.08		
VIII-58..........	$P = 0.345388$	42956.775	$A = 0.82$	0.15	0.7432	1.18
	$P_0 = 0.464718$	42957.767	$A_0 = 0.49$	0.06		
	$P_1 = 0.345389$	42956.777	$A_1 = 0.58$	0.06		

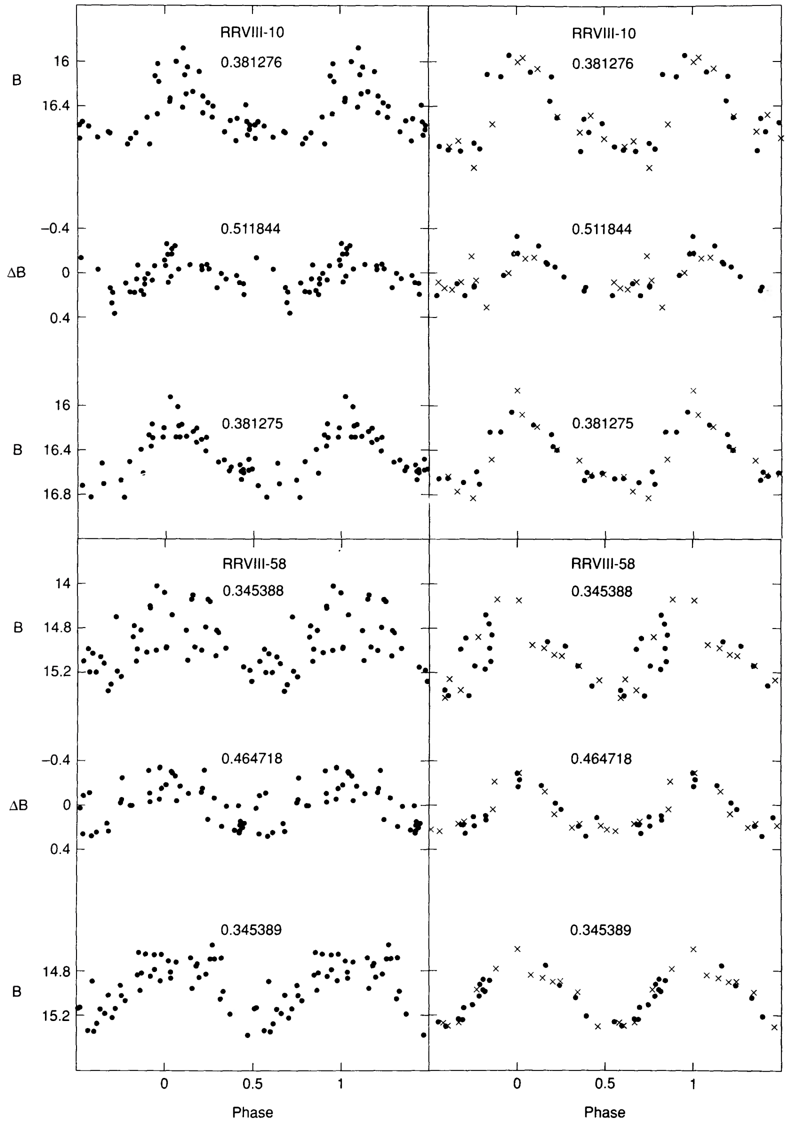

FIG. 3.—Light curves for the two RR*d* candidates. The two data sets are plotted separately, with the earlier data (1964–1965) on the left, and the later data (1976–1985) on the right. The crosses represent the 1976 observations which were all made in an interval of one week. In each panel, the upper curve is plotted with the original data, the middle curve is with the residuals from the mean primary light curve, and the bottom curve is after prewhitening with the secondary period.

derived from a linear transformation which relates pseudoequivalent width of the feature to spectral type, based on the spectra of dwarf stars in the Coma star cluster. (SKK give a more complete discussion of the calibrations.)

These observations were used to determine the metal abundance according to the ΔS method. Preston (1959) defined ΔS as the difference at minimum light:

$$\Delta S = 10 \times [\mathrm{SpT(H)} - \mathrm{SpT(K)}] \;, \tag{2}$$

where SpT refers to the spectral type based on either the calcium or hydrogen lines. Before deriving ΔS for an RR Lyrae variable, it is necessary to establish whether the star is a type *ab*

TABLE 2

SPECTRAL DATA FOR THE RR*d* STARS IN FIELD RR VIII IN DRACO

STAR (1)	DATE (JD – 2400000) (2)	INTEGRATION TIME (minutes) (3)	PHASE (4)	RAW SpT(H) Hβ (5)	 Hγ (6)	 Hδ (7)	SpT(K) (8)
VIII-58	46555.923	16	0.59	F5.2	F3.6	F3.3	A3.8
VIII-58[a]	46559.849	16	0.96	A8.5	A7.4	A7.0	A0.8
VIII-10	46965.738	32	0.60	F5.1	F2.6	F2.4	A2.8

[a] The observation was made near maximum light and has not been used for determining [Fe/H].

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or type *c* variable because the reduction procedures for the two types are different. The main reason for this is that the final ΔS for an RR*ab* is referred to minimum light, which occurs at a later spectral type (and lower $T_{\rm eff}$) than in an RR*c* variable. Another reason is that an RR*ab* star has a larger variation in $T_{\rm eff}$ through its light cycle, so that the correction to $T_{\rm eff}$ at minimum light is different from that for an RR*c* variable. In the present investigation, we wish to determine ΔS for two RR*d* variables, and therefore have to decide which procedure is appropriate for RR*d* variables.

SKK assumed that, for the purposes of measuring ΔS, these two RR*d* variables should be treated in the same manner as RR*c* variables. They made this assumption because of the early SpT(H) for the two stars at minimum light. Their Figure 1 (SKK) is a plot of hydrogen spectral type as a function of the phase during the light cycle for a sample of field RR Lyrae variables. At minimum light (phase between 0.5 and 0.75), the hydrogen spectral type of an RR*ab* variable is in the range F5 to F7, but an RR*c* variable has an earlier spectral type. According to Table 2, the observation of VIII-58 on JD 2446555, and also that of VIII-10 on JD 2446965, was made near minimum light. In both cases, the hydrogen spectral type was earlier than that expected for an RR*ab* variable. (The other observation of VIII-58 was made near maximum light and was not used for determining ΔS.) Another argument for treating VIII-10 as an RR*c* variable is that its dominant mode of pulsation is the first overtone ($A_1/A_0 = 2.03$). For VIII-58, the amplitudes of the two modes are approximately equal, and so this argument does not apply. However, the epochs of maximum light listed in Table 1 for this star indicate that, on JD 2446555.923, it was at minimum in both the first overtone and fundamental mode cycles. Thus its early spectral type suggests that, for calculating ΔS, it should also be treated as an RR*c* variable. We explore in more detail the nature of this calculation.

Normally, ΔS for an RR*ab* variable, is calculated by the following procedure. The equivalent widths of Hδ, Hγ, Hβ, and the Ca II K line are calibrated as a function of dwarf spectral types (A0 to G0) by reference to the Coma star cluster, the Pleiades star cluster, or field MK standards. The hydrogen spectral type [SpT(H)] is defined as the average of the spectral types based on the Hδ and Hγ lines. These dwarf spectral types (the "raw" spectral types) are then transformed to giant spectral types via a non-linear transformation described, but not given, by Butler (1975). This is done so that the resulting spectral types (and ΔS) can be compared to the original Preston (1959) work, where giant spectral types were used. One of the effects of this transformation is to change the hydrogen spectral type at minimum light from the "raw" value of F6 to F4. ΔS is then calculated according to equation (2) where SpT refers to the "transformed" (equivalent) giant spectral type at the time of the observation. The observed ΔS and SpT(H) are then plotted in Figure 2 of Butler (1975) or Figure 3 of Smith (1986). These figures contain ridge lines for the changing values of ΔS as an RR*ab* star goes through its light cycle. The final value of ΔS is simply the interpolated ridge line that the point [SpT(H), ΔS] falls on, and is equivalent to the value of ΔS that the star would have had if it had been observed at minimum light. Butler (1975) showed that ΔS defined in this fashion was equivalent to the ΔS published by Preston (1959).

While Butler (1975) included a few RR*c* variables in his 1975 work, his ΔS values for these stars were determined by the method appropriate for the RR*ab* variables. Kemper (1982) showed that the total variation in ΔS during the light cycle of an RR*c* variable was ± 1 unit in ΔS, and since this is the same as the error of measurement in ΔS (Butler & Deming 1979), he concluded that the normal correction of ΔS to minimum light, as defined by Butler (1975), was not needed. Kemper suggested the following procedure instead. The observed ΔS is calculated from the "raw" spectral types of hydrogen and the Ca II K line. This ΔS is then corrected by the transformation $(0.98 \times \Delta S + 0.41)$, which is given in Figure 2 of Kemper (1982). Butler et al. (1982) suggested a different calibration of ΔS for the RR*c* variables. They calculated the ΔS for an RR*c* variable as if it were an RR*ab* variable, and then transformed this ΔS by the formula $0.7 \times \Delta S + 3.0$. This transformation is based on 16 RR*c* variables in common between Kemper's (1982) and Butler's (1975) investigations.

SKK applied the Kemper (1982) calibration of ΔS for RR*c* variables to the two RR*d* variables and then made one further correction. In an analysis of 16 observations of 11 bright field RR*c* variables from the Lick archive of RR Lyrae spectra observed between 1972 and 1978, they found that the difference between the published ΔS given by Kemper (1982), and their measured ΔS using the Kemper receipe, was 0.48 ± 0.42 (with a dispersion of 1.38). They therefore subtracted 0.5 units from the observed value to arrive at the values tabulated in SKK. These values are given in column (3) of Table 3. In the present investigation, we have also calculated ΔS from the Butler et al. (1982) recipe for both RR*ab* and RR*c* variables. In Table 3, column (4), we list the ΔS calculated as if the star were an RR*ab*, and in column (5), we apply Butler's (1975) linear transformation (given above) to correct this ΔS value to the RR*c* value. The agreement between all the techniques is very good.

We have one more method for measuring ΔS of these stars. SKK noted that the SpT(H) based on Hβ was not the same as that based on the other two hydrogen lines and was, in fact, a strong function of metallicity. In Figure 4, we plot the difference between the hydrogen spectral type determined from Hβ and the average of the hydrogen spectral type determined from Hδ and Hγ against ΔS for the bright field RR Lyrae stars in the SKK sample. We have only considered stars which have SpT(H) later than F0 for this discussion. The best-fit linear

TABLE 3

ΔS VALUES FOR THE RR*d* STARS IN FIELD RR VIII IN DRACO

		ΔS				
STAR (1)	DATE (JD − 2400000) (2)	Kemper *c* (3)	Butler *ab* (4)	Butler *c* (5)	SpT(H) (6)	Adopted (7)
VIII-58..........	46555.923	9.4	9.3	9.5	6.1	8.6
VIII-10..........	46965.738	9.4	9.6	9.7	8.3	9.2

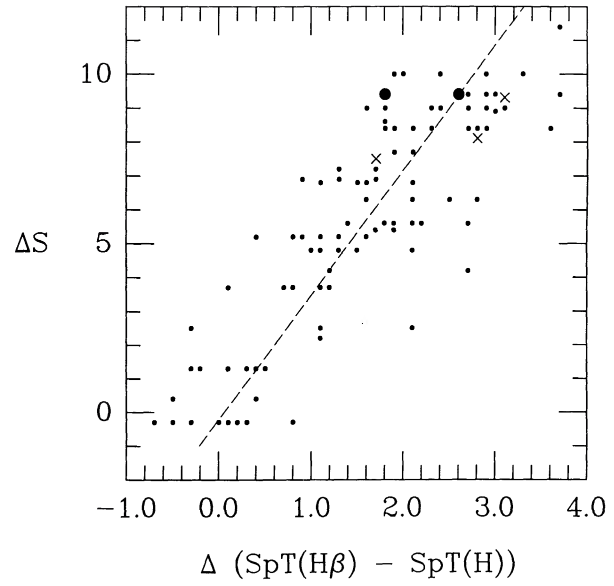

FIG. 4.—The ΔS for the bright field RR Lyrae variables studied by Suntzeff, Kinman, & Kraft (1991) as a function of the difference in hydrogen spectral type based on the Hβ line and the average of the Hδ and Hγ lines. The sense of the trend is that the more metal rich RR Lyraes have stronger Hβ lines than the strength of Hδ and Hγ would indicate. The RR*c* variables are plotted as crosses and the two RR*d* stars in field RR VIII are plotted as large filled circles. The best-fit regression is plotted as a dashed line.

regression

$$\Delta[\mathrm{SpT(H}\beta) - \mathrm{SpT(H)}] = 0.061 + 0.271 \times \Delta S \qquad (3)$$

is also plotted in Figure 4. The trend alluded to in SKK is clearly present, in the sense that the more metal rich RR Lyrae stars have *earlier* Hβ spectral types. That is, the more metal-rich the RR Lyrae, the stronger Hβ is relative to Hδ and Hγ. The RR*c* variables in the SKK study are plotted as crosses in Figure 4. There is no large offset between the RR*ab* and RR*c* stars in this diagram, but since there are only a few RR*c* stars, we hesitate to draw any firm conclusions from this. The dispersion in the difference in ΔS determined from the Hβ spectral type and the published ΔS given in SKK for 101 observations of field stars is 1.5 units. Since most of the ΔS-values for the field stars given in SKK were from multiple observations, the dispersion of 1.5 must represent the error in the ΔS based on SpT(Hβ) for the bright field stars. (For the program stars in SKK, which are fainter, the dispersion about the regression line is 2.2 units in ΔS.) Using the relationship shown in Figure 4, we have estimated the ΔS from SpT(Hβ) for the two RR*d* variables, and the results are listed in column (6) of Table 3. These stars are plotted as large solid circles in Figure 4. For VIII-10, the two values of ΔS agree to within the errors, but for VIII-58, the ΔS from SpT(Hβ) is more metal rich at somewhat more than the 1 σ level. We conclude that ΔS based on Hβ supports the conclusion that the two RR*d* stars are metal-poor, but the larger error precludes a more detailed comparison.

Our final adopted values of ΔS for the two stars were determined from the data in Table 3 in the following manner. The mean of columns (3) and (5) was given a weight of 3 and averaged with column (6), which was given a weight of 1. These adopted values of ΔS are listed in column (7). It is difficult to place a real error on the value of ΔS for the RR*d* stars because we do not know whether the various recipes for calculating ΔS can be applied to the RR*d* variables. We do know the typical errors in ΔS for stars known to be *ab* or *c* types. SKK showed that the typical error for the RR*ab* stars in their study was 1.0 unit. The S/N ratio of the spectra of the RR*d* variables was probably better than the typical S/N ratio for the stars of the SKK study. The agreement of the hydrogen spectral types based on the Hδ and the Hγ lines as given in Table 2 also indicates that the spectra were of good quality. The typical error in ΔS for an RR*c* variable was given by Kemper (1982) as ±1 unit. To check this, we have restudied the RR*c* variables from the Lick archive of RR Lyrae spectra. From eight observations of three bright field RR*c* variables, the estimated error in a single observation is 1.2 units in ΔS. The difference in ΔS for 16 RR*c* variables between the Kemper (1982) published values and the measured values has a dispersion of 1.38 units in ΔS. If the error is divided equally between the two values, an error of 1.0 unit in ΔS for a single observation of a bright field RR*c* variable is implied. We do not have multiple observations for fainter RR*c* variables to directly calculate the error of a program RR*c* variable in the SKK study. However, it was shown by SKK that the ΔS-values for both the bright and faint RR*ab* variables were similar, probably due to the fact that the errors in ΔS for all but the very faintest stars in the program were dominated by the inaccuracies in the ΔS transformation and irregularities in the light cycles of the variables, and not by the S/N ratio of the spectra. We would therefore expect that the error in a single observation of a program RR*c* variable is between 1.0 and 1.5 units, with 1.2 units being our best estimate.

SKK established a relationship

$$[\mathrm{Fe/H}] = -0.158\Delta S - 0.408 \qquad (4)$$

to convert the ΔS-values to [Fe/H] on the system used by Zinn & West (1984) for the globular clusters. We have used equation (4) to calculate [Fe/H] for the two stars: −1.86 and −1.77 for VIII-10 and VIII-58, respectively. The error in measurement of [Fe/H], corresponding to an error of 1.2 units in ΔS, is estimated to be 0.19 dex. However, we note that the question of how to determine ΔS for RR*d* variables cannot be resolved until ΔS is studied in an RR*d* star in a globular cluster of known metallicity.

5. THE MASS-METALLICITY RELATION

The masses of RR*d* stars may be determined according to a method devised by Jorgensen & Petersen (1967). The position of a star in the Petersen diagram, which is a plot of P_1/P_0 versus P_0, depends on its mass. Mass calibrations for stars with Population II compositions have been made by Cox, King, & Hodson (1980), CHC, and Kovács (1985), but the results do not all agree. Consequently, Cox (1987) has stated that there is still some uncertainty about the true values of the masses. A Petersen diagram is shown in Figure 5. The solid lines indicate the theoretical mass calibrations of CHC and the circles (open and closed) indicate the observed periods and period ratios for the RR*d* stars in globular clusters. Also plotted (as crosses) are the positions of the three field RR*d* stars: VIII-10, VIII-58, and AQ Leo. It can be readily seen that the existing models do not cover a sufficiently large range of values for the periods and period ratios. There are points above the 0.65 $M_\odot$ line and below the 0.55 $M_\odot$ line so that the masses for these stars can only be obtained by extrapolation. Clearly, it would be desirable to have more model calculations so that the masses can be determined more accurately. In Table 4, we list the periods and masses (interpolated or extrapolated from Figure 5) for the

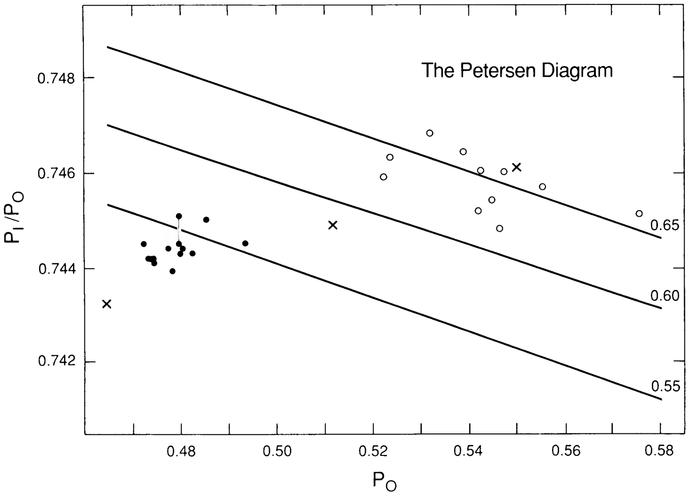

FIG. 5.—Petersen (P_1/P_0 vs. P_0) diagram. Solid lines represent the mass calibrations of Cox, Hodson, & Clancy (1983) for models with the King Ia ($Y = 0.299$, $Z = 0.001$) composition and masses 0.55, 0.60, and 0.65 $M_\odot$. The solid circles denote the RR*d* stars in the Oosterhoff type I globular clusters, the open circles the RR*d* stars in the Oosterhoff type II globular clusters, and the crosses denote the RR*d* stars in the field.

three field RR*d* stars. The periods for AQ Leo are taken from Jerzykiewicz & Wenzel (1977). From Figure 5, one can see that AQ Leo lies in the region of the diagram occupied by the RR*d* stars in the Oosterhoff type II clusters, but our two new RR*d* candidates lie in areas not occupied by cluster RR*d* stars. This is particularly interesting because systematic analyses of observations of the RR*c* stars in 25 globular clusters have been made recently (Clement & Nemec 1990; Clement & Walker 1990) in an attempt to find more cluster RR*d* stars, and none were found in these regions of the diagram.

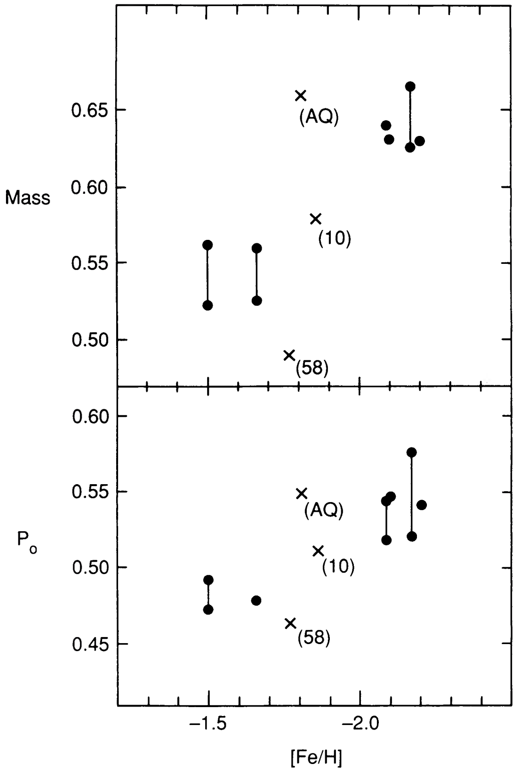

FIG. 6.—(*Upper panel*) Mass vs. [Fe/H] for RR*d* stars in the Galaxy. The RR*d* stars in clusters are plotted as solid circles and the field RR*d* stars are plotted as crosses. (AQ, 10, and 58 designate AQ Leo, VIII-10, and VIII-58, respectively.) In the clusters with more than one RR*d* star, there is usually a range of masses. In these cases, the extreme values are plotted and connected by a vertical line. The [Fe/H] values for the clusters are the values published by Suntzeff et al. (1991). (*Lower panel*) P_0 vs. [Fe/H] for RR*d* stars. The symbols are the same as in the upper panel.

Since the three field RR Lyrae stars have different periods and masses, they should form an appropriate dataset for testing the mass-metallicity relationship. Mendes de Oliveira & Smith (1990) recently determined $\Delta S = 8.9$ for AQ Leo, and we have used equation (4) to calculate $[\mathrm{Fe/H}] = -1.81$ on the system on Zinn & West (1984) for this star. Our adopted values of ΔS, [Fe/H], and the galactocentric distances for the three stars are listed in Table 4. Their [Fe/H] values all agree to within the estimated errors, but their masses differ substantially. In the upper panel of Figure 6, the relationship between mass and metallicity is plotted. The RR*d* stars in clusters are plotted as solid circles. In clusters where the RR*d* stars have a range in masses, the extreme values are plotted and connected by a vertical line. In the lower panel of Figure 6, the fundamental periods (P_0) for the RR*d* stars are plotted against [Fe/H]. The clusters with $[\mathrm{Fe/H}] > -1.7$ are the Oosterhoff type I clusters (IC 4499 and M3) and the clusters with $[\mathrm{Fe/H}] < -2.0$ are the Oosterhoff type II clusters (M68, NGC 2419, M15, and NGC 6426). The field RR*d* stars are plotted as crosses. One can readily see that the globular cluster RR*d* stars fall into two distinct groups, and that these field stars all have metallicities intermediate between the two groups. However, the masses determined for the field stars from the CHC calibration have a much larger range at a given metallicity than those in the clusters. We therefore conclude that, in the mean, the mass-metallicity relationship for the globular cluster RR*d* stars holds for the field stars, but the field stars show a large scatter. One sees that the field RR*d* stars also have a greater range of P_0-values than the cluster RR*d* stars at a given metallicity, but not so great as for the mass.

TABLE 4

PROPERTIES OF FIELD RR*d* STARS

Star	P_0 (days)	P_1/P_0	Mass ($M_\odot$)	ΔS	[Fe/H]	R_G (kpc)
VIII-10	0.511844	0.7449	0.58	9.2	−1.86	15
VIII-58	0.464718	0.7432	0.49	8.6	−1.77	11
AQ Leo	0.5497527	0.7461	0.66	8.9	−1.81	9

A possible explanation of the lack of correlation between the mass and [Fe/H] in our small sample of field RR*d* stars is that there is an age spread among them. Significant age differences have been found among globular clusters with $[\mathrm{Fe/H}] \gtrsim -1.6$, but not among those with $[\mathrm{Fe/H}] \sim -2.0$ (VandenBerg, Bolte, & Stetson 1990). An age spread at the intermediate [Fe/H] of our three field RR*d* stars has yet to be demonstrated. Zinn (1988), Suntzeff (1990), and Preston et al. (1990) have discussed the possibility of an age gradient in the Galactic halo. According to Webbink (1985), the galactocentric distances for the five "RR*d*" clusters M3, M15, M68, IC 4499, and NGC 6426 are all in the range between 10 and 15 kpc, and the data in Table 4 show that the field RR*d* stars have a similar distribution. The cluster NGC 2419 has a much greater distance (99 kpc from the Galactic center), but the mass of its RR*d* star is comparable to that of the RR*d* stars in other clusters of similar metal abundance.

There is also evidence that the mass-metallicity relation does not hold for all cluster RR Lyrae variables. Simon (1990) has used a Fourier decomposition technique to determine relative masses for the RR*c* variables in ω Centauri and found that the mass does not depend on the metal abundance. This suggests that there must be another important parameter which affects the relationship between mass and metal abundance for RR Lyrae variables.

These findings have implications for the reliability of the Petersen diagram for the mass determination of RR Lyrae stars and they may possibly suggest that the RR Lyraes in clusters and in the field have different characteristics. Our present sample of field RR*d* stars is small and should be enlarged. It is still unclear why RR*d* stars are common in some clusters but absent in others. At first sight, it is surprising that two RR*d* should be found in a single survey field. However, it is to be remembered that these are low-amplitude variables. Not only is the probability of detection not high by traditional blinking techniques, but the errors in the photographic magnitudes derived from survey plates (particularly from 103a-O emulsion) are large enough to make the required analyses of the light curves quite difficult. It is to be hoped that the use of modern techniques (e.g., the use of CCDs) will greatly facilitate the discovery of the RR*d* variables in the field.

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