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PECULIAR CLUSTER VELOCITIES FROM MEASUREMENTS OF THE KINEMATIC SUNYAEV-ZELDOVICH EFFECT

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ABSTRACT

A detailed discussion is given of the possibility of determining peculiar velocities of clusters of galaxies by measuring the kinematic Sunyaev-Zeldovich effect. From available optical and X-ray data we select a sample of clusters for which the effect might be measurable at the optimal frequency where the thermal Sunyaev-Zeldovich effect vanishes.

Subject headings: cosmic background radiation — galaxies: clustering — galaxies: redshifts

1. INTRODUCTION

Compton scattering of the cosmic microwave background (CMB) radiation by hot electrons in clusters of galaxies imprints on the incident photon spectrum a signature (hereafter the SZ effect) of considerable astrophysical significance, as originally proposed by Zeldovich & Sunyaev (1969) and Sunyaev & Zeldovich (1972), and reviewed by Sunyaev & Zeldovich (1980a, 1981) and, more recently, by Rephaeli (1990). Its detection has been the goal of many observational searches (see the reviews by Sarazin 1986 and Birkinshaw 1990). Observations were carried out mostly in the Rayleigh-Jeans (R-J) side of the spectrum, where the scattering leads to an intensity decrement. Interpretation of measurements in the R-J side is plagued by the possibility of high-frequency radio emission from sources within clusters, in addition to the basic limitation in that the intensity change is small. In the Wien side the intensity is enhanced relative to its Planckian value. Measurements of the effect near the Planckian peak and on the Wien side should in principle, be more attractive because the intensity enhancement can be larger than the magnitude of the R-J diminution (Gould & Rephaeli 1978).

The natural reference frame to describe the SZ effect due to the thermal motion of the scattering intracluster (IC) electrons is that of the CMB. If the cluster has zero velocity in this frame, then this thermal effect is the only signature imprinted on the CMB spectrum by the cluster. However, if the cluster has a peculiar velocity along the line of sight, then there is an additional Doppler shift. The significance of this kinematic effect has been mentioned (Sunyaev & Zeldovich 1972) and elaborated upon by Sunyaev & Zeldovich (1980b). This kinematic effect can serve as a most valuable probe of the motion of clusters of galaxies. These motions are important in theories for the evolution of the large-scale structure of the universe. Since peculiar velocities are determined by the entire mass distribution (assuming the gravitational instability picture), they probe the present cosmography of the local universe, as well as the primordial spectrum of density fluctuations.

The most accurate measurement of peculiar velocity of a galaxy is that of our own, deduced from the dipole anisotropy induced in the CMB radiation (for a review, see Lubin & Villela 1986). Observations of peculiar velocities of other galaxies became more accurate in recent years (see Gunn 1989 and Burstein 1989 for reviews). The measured line-of-sight peculiar velocity is $v_r = cz - rH_0$, where c is the speed of light, z is the redshift, r is the "true" distance and H_0^{-1} is the Hubble time.

While redshift measurements are very accurate, measuring an independent distance indicator is a difficult task. The best examples of "good" distance indicators are the infrared Tully-Fisher relation, (according to which, total luminosity of a spiral galaxy and its rotation velocity are related), and the Faber-Jackson relation (by which the luminosity or a photometric diameter of an elliptical galaxy are related to its stellar velocity dispersion). It is important to note that all these relations are empirical, and that it is assumed that they are independent of environment. The distance measurement is subject to various selection effects (e.g., the Malmquist bias). The error in the deduced distance is proportional to the distance; even the best distance indicators give a fractional error which is at least 20% per galaxy. This makes it very difficult to measure peculiar velocities beyond a redshift distance of (say) 15,000 km s^{-1}, even if distances to a number of galaxies are grouped together.

Rubin et al. (1976) were the first to measure peculiar velocities for a large sample of spirals and found that the sample of galaxies has a "bulk-flow" relative to the CMB. More recently Aaronson et al. (1986, 1989) have used the infrared Tully-Fisher relation to measure the peculiar velocity of the Local Group relative to clusters. The "Group of Seven", Lynden-Bell et al. (1988), have used a relation between photometric diameter and velocity dispersion to measure the peculiar velocities of about 400 ellipticals. They found a strong "bulk-flow" vector and also fitted the flow to a "Great Attractor" model.

While the global flow patterns of ellipticals and spirals roughly agree, there are discrepancies between different observations (even for the same cluster). This is due to measurement errors, deviations from a universal distance indicator and the way in which galaxies are assigned to clusters and groups and their velocity is averaged. An interesting case is the cluster A2634, for which, using distances to elliptical galaxies, Lucey et al. (1990) deduced a peculiar velocity of -3000 ± 800 km s^{-1}, while the Tully-Fisher peculiar velocity of the cluster is only

300 ± 500 km s^{-1}. The method presented below will enable testing the reality of these different measurements.

Here we discuss the feasibility of determining peculiar cluster velocities from measurements of the kinematic SZ effect. The advantages of this method over the above distance-indicator method are clear: (1) there is no need to measure an inaccurate empirical distance indicator, (2) the method is independent of distance, and (3) the effect has a physical explanation. A basic difficulty is the expected relative smallness of the kinematic SZ effect with respect to the thermal effect. We will therefore expand on how the spectral characteristics of the two effects can be optimally exploited so as to enable determination of the kinetic intensity change. Using optical and X-ray data, we then compile a list of clusters whose estimated peculiar velocities render them good candidates for future observations.

2. THERMAL AND KINEMATIC SUNYAEV-ZELDOVICH EFFECTS

The interaction of a Planckian radiation field with a (nonrelativistic) Maxwellian electron gas can be well-approximated by means of a solution of the Kompaneets (1957) equation. The change in the CMB (spectral) intensity due to Compton scattering is (Zeldovich & Sunyaev 1969; Gould & Rephaeli 1978)

$$\Delta I_T = \frac{2(kT)^3}{(hc)^2}\, y g(x)\,, \tag{1}$$

where T is the CMB temperature, $x = h\nu/(kT)$, and

$$g(x) = \frac{x^4 e^x}{(e^x - 1)^2}\left[x \coth\left(\frac{x}{2}\right) - 4\right]. \tag{2}$$

The spectral form of the thermal effect is contained in the function $g(x)$, which is zero at the critical value $x_0 = 3.83$. This critical value corresponds to $\nu_0 = 218$ GHz. The (nonrelativistic) Comptonization parameter, y, is

$$y = \int (kT_e/mc^2) n\sigma_T\, dl\,, \tag{3}$$

where n and T_e are the electron density and temperature, σ_T is the Thomson cross section, and the integral is over a line of sight through the cluster. In the R-J region,

$$\Delta I_T/I = \Delta T_T/T \approx -2y\,. \tag{4}$$

Equations (1) and (2) describe the SZ effect due to the thermal motions of the electrons, an effect which has been searched for ever since the time it was suggested. Much less attention has been given to the *intensity* change due to the motion of the cluster with respect to the CMB frame (Sunyaev & Zeldovich 1972). This kinematic effect is proportional to the line-of-sight component of the peculiar velocity (in the CMB frame). As has been discussed by Sunyaev & Zeldovich (1980b), under typical conditions in clusters the kinematic intensity change, $\Delta I(x)_K$, is small in comparison with $\Delta I(x)_T$, unless the radial component of the peculiar velocity, v_r, is of order 10^3 km s^{-1}. As we have already discussed, interest in the possibility that some clusters might be moving with such velocities has increased recently, as a result of observational work stemming from the growing realization of the significance of the velocity field in studies of the large-scale structure.

While the Doppler change in the temperature of the radiation along a line of sight through the cluster is frequency-independent, $\Delta I(x)_K$ is not:

$$\Delta I_K = -\frac{2(kT)^3}{(hc)^2}\frac{v_r}{c}\,\tau h(x)\,, \tag{5}$$

where $\tau = \int \sigma_T n\, dl$, and

$$h(x) = \frac{x^4 e^x}{(e^x - 1)^2}\,. \tag{6}$$

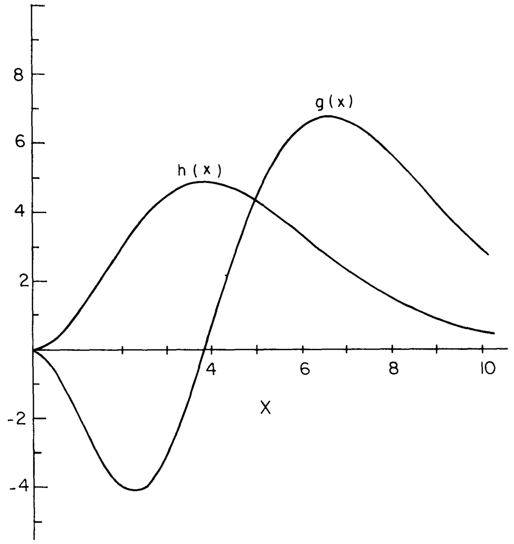

FIG. 1.—Spectral shapes of the kinematic and thermal SZ effects, the functions $h(x)$ and $g(x)$, respectively, are shown as functions of the (nondimensional) frequency x.

The functions $g(x)$ and $h(x)$ are drawn in Figure 1. The fact that $h(x)$ attains its maximum value exactly at the point where $g(x)$ vanishes is an interesting one, and can potentially be very useful.

Relativistic corrections cannot be ignored (Wright 1979; Fabbri 1981; Loeb, McKee, & Lahav 1990) when an exact determination of the intensity change is needed over a wide range of frequencies. Here, however, we are mainly interested in the value of the intensity changes at or near x_0, which is very well-approximated by the nonrelativistic treatment.

The above expressions give the thermal and kinematic intensity changes when it is assumed that before traversing the cluster the photons have an exact Planckian distribution. We have considered elsewhere (Rephaeli 1980) the more general case of an additional distortion due to hot intergalactic (IG) gas. The recent *COBE* (Mather et al. 1990) measurements set an upper limit of 0.001 on the y Comptonization parameter characterizing such a distortion. (This is in sharp conflict with the earlier Matsumoto et al. 1988 rocket measurements of submillimetric excess CMB emission with a corresponding value of y higher by about one order of magnitude than the *COBE* limit.) The possibility that the CMB spectrum is a Comptonized Planckian could have been, in principle, quite relevant to our discussion here. Since the critical frequency at which $g(x)$ vanishes will be used below, one would have needed to evaluate this frequency for a Comptonized Planckian. However, the *COBE* upper limit on the IG value of y is sufficiently low that the change in the value of x_0 (as compared with its value for a pure Planckian) is negligible.

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The kinematic change in the CMB intensity depends linearly on v_r and τ. The motion of a cluster transverse to the line of sight, with a tangential velocity v_t, induces linear polarization of the CMB along lines of sight through the cluster (Sunyaev & Zeldovich 1980b). However, the degree of polarization is extremely small, as it is determined by terms proportional to $(v_t)^2\tau$ or $v_t\tau^2$. Thus, the determination of the tangential motion of a cluster through CMB polarization measurements is an unrealistic goal in the near future (Birkinshaw & Gull 1983 proposed another way of measuring the tangential velocity from the change in the CMB temperature induced by the interaction of the CMB photons with the cluster as a moving gravitational lens.)

All dependence on the gas properties is contained in y, which is essentially a line integral of the electron pressure. The gas density, temperature, and their radial profiles are deduced from X-ray spectral and spatial measurements. The gas is usually modeled as a polytrope $T_e \propto n^{(\gamma-1)}$, with the density profile commonly taken to be of the form $n = n_0(1 + r^2/a^2)^{-\alpha}$, where γ, α, and the core radius a are best fits to the X-ray data (Gull & Northover 1975; Cavaliere & Fusco-Femiano 1976). In rich clusters the values of these three parameters usually fall in the (rough) ranges $a = 0.2$–0.5 Mpc, $\gamma = 1$–$5/3$, and $\alpha = 1$–1.5 (see, e.g., Sarazin 1986). Since $n(r)$ and $T_e(r)$ are monotonically decreasing functions of (the radial coordinate) r, y is maximal for a line of sight through the center of the cluster. In a moderately X-ray luminous cluster, $n_0 \approx 3 \times 10^{-3}$ cm^{-3}, and $T_0 \approx 5$ keV, so $y \approx 1 \times 10^{-4}$, and $\tau \approx 0.01$, if we take mean values for a and α. Under these conditions, $\Delta T_T/T \approx -2y \approx -2 \times 10^{-4}$, in the R-J region. (Note, however, that the relative intensity change on the Wien side is significantly higher.)

3. DETERMINATION OF PECULIAR VELOCITIES OF CLUSTERS

We discuss now the possibility of determining radial components of peculiar velocities of clusters from measurements of the kinematic SZ effect. Measuring $\Delta I(x)_K$ yields v_r directly if τ is known, as is clear from equation (5). The basic difficulty is that $\Delta I(x)_K$ cannot be negligible as compared with $\Delta I(x)_T$, or else the total measured intensity change will be dominated by the thermal effect. From equations (1) and (5), it follows that $\Delta I(x)_K = \Delta I(x)_T$ for $vh(x)/c = kT_e g(x)/mc^2$. Now, IC gas temperatures are typically 5 keV and higher, meaning that if x is such that $h(x)$ is comparable to $g(x)$, then the two intensity changes are comparable if $v_r = O(10^3)$ km s^{-1}. Even if v_r were so large, it still would have been very difficult to separate out the thermal and kinematic changes.

Clearly, therefore, it is essential to carry out measurements at an optimally selected frequency which maximizes the ratio $h(x)/g(x)$. As was pointed out in the previous section, $h(x)$ attains its maximum, at x_0, exactly where $g(x)$ is zero. Thus, x_0 is the optimal frequency for measurements of the kinematic effect; since $x_0 = 3.83$, and $T_0 = 2.735$ K, then $\nu_0 = 218$ GHz. While $h(x)$ is essentially constant near x_0, $g(x)$ varies very steeply there. But even a moderate spectral resolution will suffice to almost eliminate the contribution of $\Delta I(x)_T$ to the measured signal.

The optical depth of the IC gas to Compton scattering, τ, can be obtained from spectral and spatial X-ray measurements. IC gas temperatures can be determined relatively accurately. The gas density and its spatial profile are currently known for only a few clusters, but future X-ray satellites are expected to yield these quantities in many clusters. For rough estimates, $T_0 = 5$ keV, $\tau = 0.01$, so that $y \approx 1 \times 10^{-4}$, may be regarded as typical values in well-relaxed, rich clusters. Scaling to these values, we calculate the integrated intensity changes over a 4 GHz band centered on $\nu_0 = 212$ GHz to be

$$(\Delta I/I)_T \approx 2 \times 10^{-8}(y/10^{-4}) , \tag{7}$$

while

$$(\Delta I/I)_K \approx -5 \times 10^{-4}(v_r/10^3)(\tau/0.01) . \tag{8}$$

These estimates clearly indicate the basic advantage in measurements very close to the optimal frequency ν_0. The value of ΔI_K in equation (7) corresponds to a temperature change $\Delta T = -v_r \tau T/c = -0.9(v_r/10^3)(\tau/0.01)$ mK, using $T_0 = 2.74$ K. The estimated value of ΔT is close to the current detectability threshold (which should improve appreciably in the near future), so that the kinematic SZ effect can be measured in clusters moving with $v_r > 10^3$ km s^{-1}.

We have compiled a list of high-velocity clusters which can be good candidates for the measurement of $\Delta I(x)_K$ and the deduction of v_r. The list (Table 1) is based on Tully-Fisher (for spirals) and diameter-velocity dispersion (for ellipticals) relations for rich clusters with estimated values of $v_r > 10^3$ km s^{-1}.

TABLE 1

CANDIDATES FOR THE MEASUREMENT OF THE KINEMATIC SZ EFFECT

Cluster Name	R.A.	Decl.	z	Richness	log L_x	V_{pec}	Source
A426 (Perseus)	$03^h15^m3^s$	+41°20″	0.0183	2	44.96	−880	G7
A569	07 05.4	+48 42	0.0196	0	<41.74	−1123	G7
Cen 30	12 46.0	−41 02	0.0101	...	43.76	+1110	G7
						+240	LC
A2199	16 26.9	+39 38	0.0309	2	44.47	−2919	G7
A2634	23 35.8	+26 45	0.0312	1	<42.76	−3000	LGCT
						+300	A
A2218*	16 35.7	+66 19	0.171	4	44.11		
A665*	08 26.2	+66 03	0.1816	5	44.69		
0016 + 16*	00 16	+16	0.541	...	...		
A576*	07 17.3	+55 50	0.0381	1	44.04		
A401*	02 56.2	+13 23	0.0748	2	45.45		
A2319*	19 19.2	+43 52	0.0564	1	45.21		

NOTES.—An asterisk (*) indicates that the cluster is included based on a claimed detection of the thermal SZ effect. Peculiar velocity sources: G7 (Faber et al. 1989), LC (Lucey & Carter 1988), A (Aaronson et al. 1989), LGCT (Lucey et al. 1990).

The 2–6 keV X-ray luminosity (with $H_0 = 50$ km s^{-1} Mpc^{-1}) for A569, A665, A2218, and A2634, is from Kowalski et al. (1984), and the 2–10 keV luminosities for the other clusters are from Lahav et al. (1989).

In some cases (e.g., Cen 30 and A2634) there is a strong disagreement between different observations of the same cluster. The method presented could in fact discriminate between these possibilities. We also included six clusters for which a detection of the thermal SZ effect has been claimed (at the level of about $\Delta T = -10^{-3}$ K at the R-J region of the spectrum). The (possible) detection of the thermal effect indicates that their optical depth τ is sufficiently large to produce a detectable effect (see the review by Birkinshaw 1990, and the recent work on the SZ effect in A2218 by Klein et al. 1990). The table gives the cluster name, its equatorial coordinates (1950), redshift in the Local Group frame (mainly from Struble & Rood 1987), richness (Abell 1958), X-ray luminosity (in units of h_{50}^{-2} ergs s^{-1}) from the compilation of A. Edge (Lahav et al. 1989) or from Kowalski et al. (1984), and the peculiar velocities from Faber et al. (1989), Lucey & Carter (1988), and Lucey et al. (1990).

There are only a few clusters for which we know all the quantities needed for the calculation of y and τ. While we do not have all the relevant data to calculate y and τ exactly for the clusters in Table 1, many of the listed clusters are richer and more luminous than the Coma cluster. Therefore, the estimates in equations (7) and (8) (which are based on known values in Coma) are quite realistic. For an exact determination of v_r of a given cluster, the X-ray spectrum and brightness profile have to be measured. It is important to note, however, that if only X-ray spectral data are available, it is possible to substitute a measurement of ΔI_T for the X-ray brightness profile: The value of y can be determined from a measurement of the thermal intensity change at a frequency different than x_0 (preferably on the Wien side; Gould & Rephaeli 1978). The mean gas temperature, $\langle T_e \rangle$, is determined from the X-ray spectrum, and since the temperature is usually very nearly constant in the central regions of clusters, $\tau \approx y/\langle T_e \rangle$.

4. DISCUSSION

Determination of the (radial component) of peculiar cluster velocity by measurements of the kinematic SZ effect is the only method which is based on a truly cluster entity—the gaseous core. This method is much more suitable than any method which relies heavily on averaging over individual motions of member galaxies to obtain the mean cluster velocity. With current sensitivities, detection of the kinematic effect is feasible towards rich clusters moving at about 1000 km s^{-1}. This is useful in particular for distant clusters. Note that a measurement of the kinematic effect may, in general, be necessary for a careful detection of the thermal effect, in order to separate out the relative contribution of the kinematic effect to the total measured signal. The measurements of the SZ effects will become soon easier with the developments of new instruments, such as the Cambridge Very Small Array, and planned balloon-borne measurements.

The peculiar velocities of clusters are important probes of the matter distribution. The streaming motions can be compared with predictions of theoretical models (e.g., cold dark matter, hot dark matter and isocurvature baryon models) using various statistics, such as bulk flows, and velocity-velocity correlation functions (e.g., Bertschinger & Juskiewicz 1988; Kaiser 1988; Kaiser & Lahav 1989; Gorski et al. 1989; Groth, Juszkiewicz, & Ostriker 1989). The line-of-sight velocities could also be used to reconstruct the potential and the matter density distribution in the local universe (Dekel, Bertschinger, & Faber 1990). The comparison with the galaxy distribution could be used to determine the density parameter Ω_0 (if light traces mass). Moreover the ability to accurately measure peculiar cluster velocities (in the CMB frame) will help to detect our Local Group motion relative to a distant shell of clusters, and to compare it with the CMB dipole.

In principle, peculiar velocities of gas-rich galaxies can also be determined by measurements of the kinematic SZ effect. A typical velocity of a galaxy in a rich cluster can be as high as 2000 km s^{-1}. Thus, a large, gas-rich spiral galaxy in a rich cluster may cause a temperature change $O(10^{-5})$ K. Such galaxies may also contribute appreciably to the total intensity change across a cluster, and therefore constitute a source of contamination in measurements of the thermal and kinematic effects of the cluster as a whole. This has to be taken into account along with other considerations (e.g., see Gould & Rephaeli 1978; Rephaeli 1987; Lake & Partridge 1980) in observational strategies for successful detections of the SZ effects.

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